Switching Circuits and Logic Design
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Lecture - 18
Minimization Using Karnaugh Maps (Part -III)

So, we continue with our decision on minimizing switching expressions using Karnaugh map. If, you recall we talked about how to minimize 3 variable and four variable functions. So, let us continue from that point onward this is the third part of the lecture.

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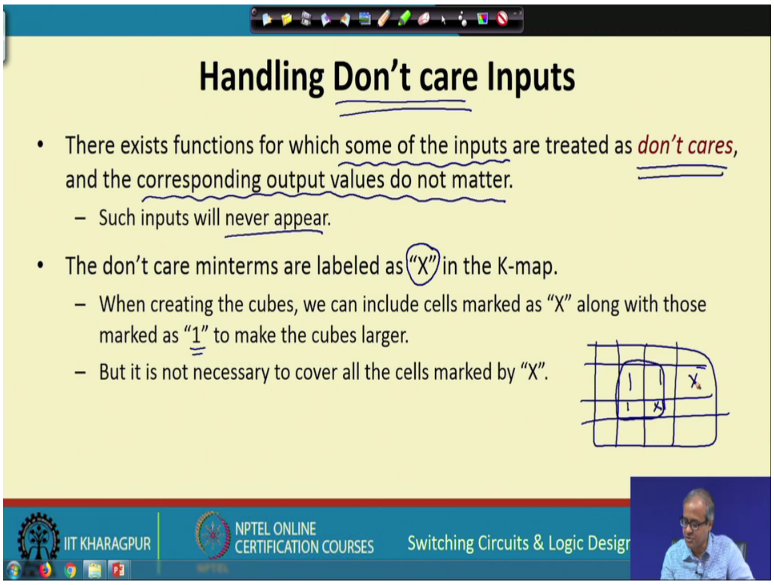

So, the first thing that we talk about today, now in the examples we have taken we did not considered don't care inputs so far. So, what is don't care input you recall we mention this earlier also, don't care is a kind of input combination which normally will never come, or appear like you think of an example were the input numbers are BCD. There is a four bit input which is coming which is a BCD number, BCD digit. Now, we know in BCD the 4 bit can be from 0 0 0 0 up to 9 1 0 0 1, the remaining 6 combinations 10 11 12 13 14 15, they are considered to be invalid they will never come as input.

So, what the output will be under this 6 invalid input combinations is immaterial we mark them as don't care, the output can be 0 it can be 1 also I don't care, because the corresponding input value will never come in practice, these are the don't cares ok. So, what I mean to say is that their exists functions were for some of the inputs, which we call as don't cares they will never appear, they will never appear and so, the corresponding output values do not matter.

We refer to them as don't cares and they are denoted by this X letter usually in the Karnaugh map. Now, the ideas follow you see when you make the Karnaugh map of course, we mark the 2 minterms right by using once, then additions there will be some excess in the Karnaugh map.

Let us take an example of let us say in a typical Karnaugh map, let us as I have a 1 here, I have a 1 here, I have a 1 here there is a X here. Normally if the X was not there I will make 1 cube like this and another cube like this, but because X is a don't care which means I can assume it to be 0 or 1 as per my convenience. In this case if I considered this X to be a 1, then I can make a bigger cube like this.

So, I can make a bigger cube including X, but suppose there is another X here. So, I need not have to cover this X. So, if I cover the once the 2 minterm that is sufficient, the only thing that I can use it X is that, I can use it to make a bigger cube whenever required. So, when I am saying the same thing I am mentioning here is that when creating the cubes, we can include cell marked with X to make the cubes larger. And, it is to be noted as I said that it is not necessarily to cover all the X marks sets, they are meant only for your purpose of making the cube larger and nothing else ok, they are really don't cares.

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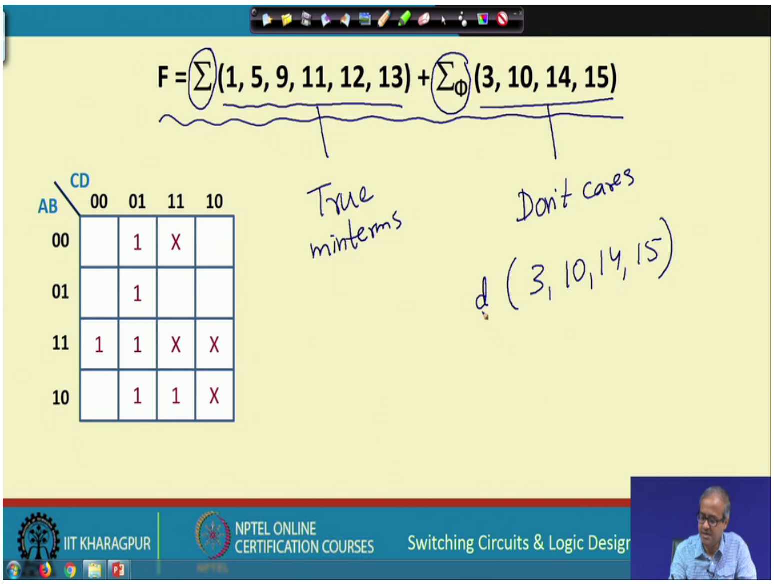

Let us take an example, now the first thing let me tell you is that how do you represent a function with don't cares, this is one way in which we can refer this summation, or sigma notation indicates that these are the true minterms, these are the true minterms. And sigma with the 5, 5 denotes don't care indicates that these 4 are the don't cares, in some books.

We will find instead of sigma 5, they have also denoted like this D 3, 10, 14, 15, means the same thing, they are don't cares D means don't cares. Now, let us look at this example this 1, 5, 9, 11, 12, 13 this 6 are the true minterms which are noted down here this are the 6 true minterms. And there are four don't cares 3, 10, 14, 15 this is 3, this is 10, this is 14, and this is 15 ok.

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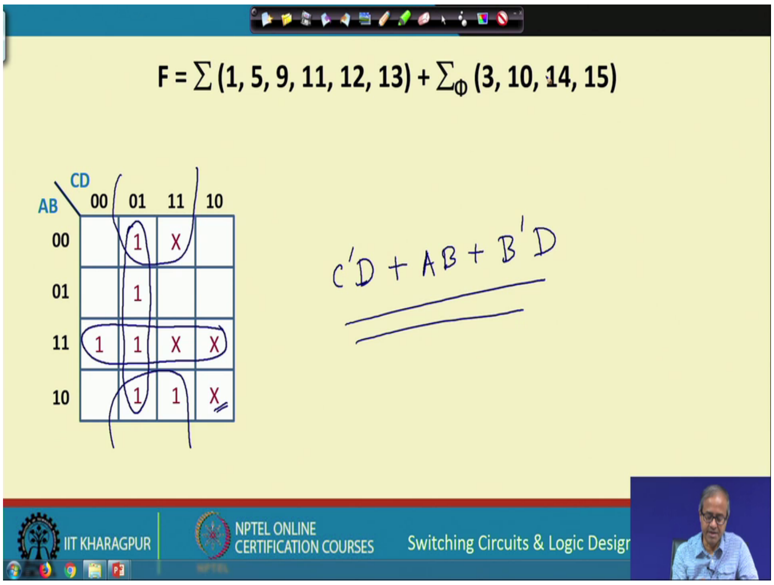

Now, with this one's and X's let us form the cubes well 1 large cube I can see this, you can make a cube like this. Now, with this X's I can make larger cube like for example, I can make a cube like this to cover this 1 and this 1 single 1 is still remaining, I can make another big cube including 1 of the don't cares here like this. But you see I do not need to cover all the don't cares this X remains let it remain, but I have covered all the ones that is that is what I want. So, this long 1 will C bar D plus this one will be A B and this will 0 0 0 B bar and D, this is the minimized form in the presence of don't cares right.

So, when there are don't cares you can use this X's to your advantage like this, right ok.

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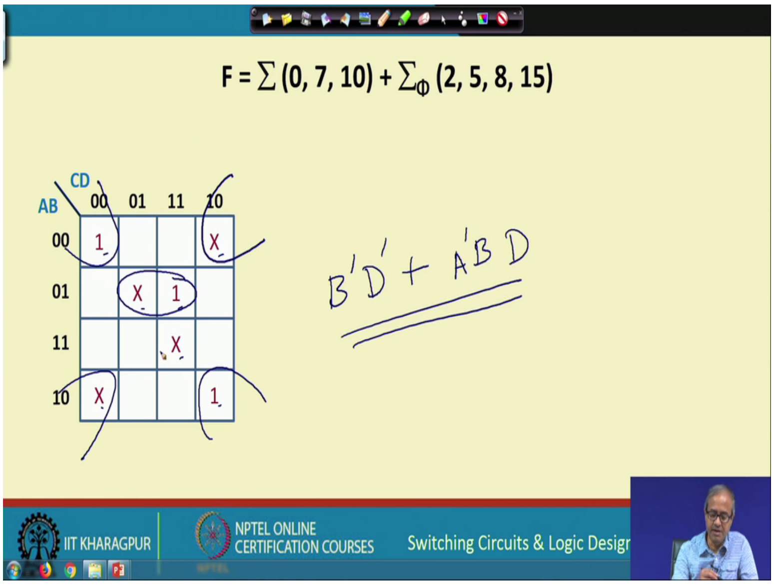

Let us take another example, these are the examples where there are 3 2 minterms 0 7 and 10, this is 0 this is 7 and, this is 10 2 5 8 15, this is 2 5 8 and 15 well here. you see there are 2 ones in the corner and 2 X's in the corner. So, you can make 1 cube out of this 4 and, there is 1 single 1 left out you can either make this or this.

Let us make this is fine this is done. So, the 4 corner will be B bar and D bar and this cube will be 0 1, which is which is A bar B and 0 1 1 is D and D, this is the minimized form. So, these examples actually tell you or show you that how to minimize using K map, when some of the minterms are marked as don't cares.

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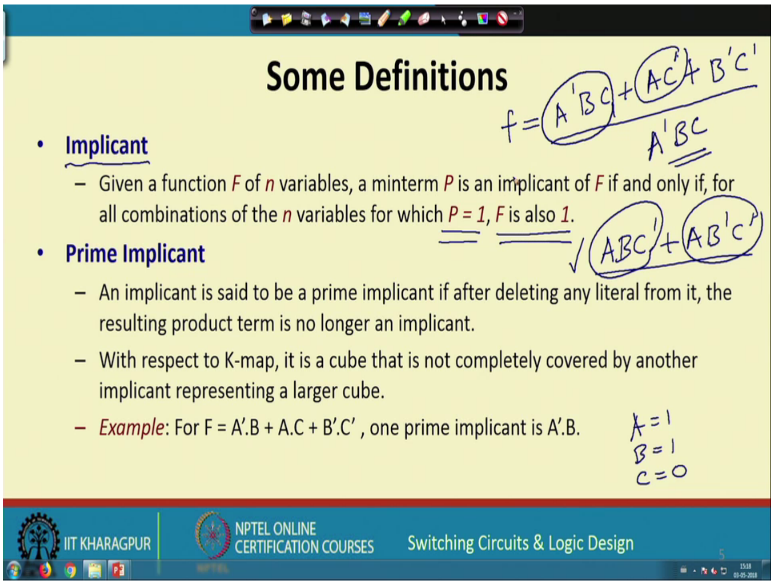

Now, let us come to some important definitions with respect to Karnaugh maps, which will be required in our next method of minimization that will be conceding after this. Well, we know what is meant by minterms, true minterms, false minterms. Now, we introduce some new concepts called implicants, prime implicants, essential prime implicants let us see what these are we start with implicant. Well, implicant means as the definition says, suppose I have a function of a n variables. Let us take an example suppose I have a function f, let us see of 3 variables suppose a function is A bar B C or A C A C bar, or let us say B bar C bar let us say this is my function.

Now, a minterm see a minterms will be an implicant, if an only if for all combinations of the variables, whenever the minterms is 1 F is also a 1 here, let us see what I am saying is that let us see this A bar B C this A bar B C, this can be an implicant, this implicant means that whenever for some input combination this I means this implicant is 1. If function will also be 1, but you think of this A C bar, this A C bar you can right as A B C bar or A B bar C bar, this ac bar you can expand by be like this, these A B C bar is a minterm A B bar C bar is also a minterm.

So, you call this as implicants, this implicant means you see, whenever A equal to 1, B equal to 1 and C equal to 0, let us say this first minterm or implicant will be 1 A B C bar. Now, because in f this is one of the terms F will also be 1 that is the definition, a minterm is an implicant if an only for all combination of the variables, whenever this implicant is 1, F is also 1 right. This is a necessary condition.

Now, prime implicant, is an special kind of an implicant prime implicant says that it is it is an implicant, where if I delete any literal for it from it do not remain an implicant any more. Let us take an example here, it is given here F equal to A bar B A C B bar C bar, here what I say is that this A bar B here we are saying, A bar B is a prime implicant why this A bar B means what. So, for what input combinations A bar B is 1.

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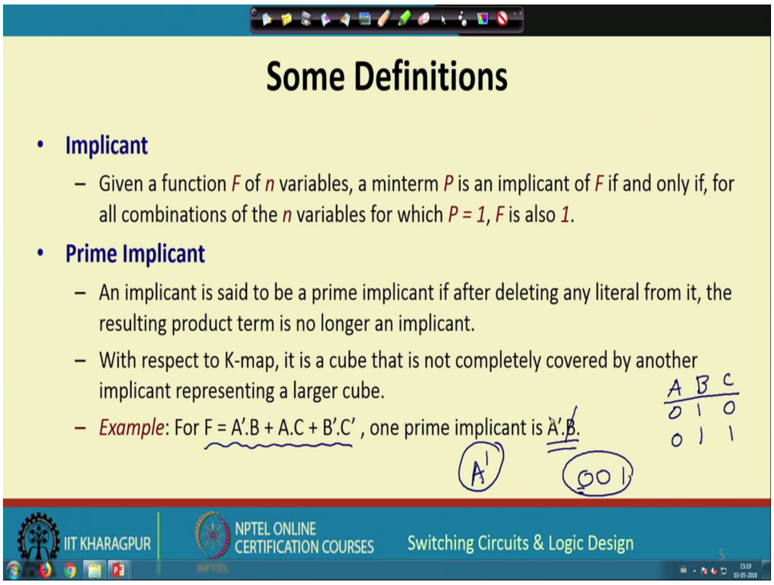

Let us say for A B C value, if I want to list A bar B means what is 0 B 1 so, C can be either 0, C can be either 1, these are the two combinations, this is an implicant, because whenever A bar B is 1 the function F is also 1. But what it says is that if you delete one literal from here, like if I remove B bar I make it only A bar this A bar is not an implicant any more.

Because for A bar I can haven an input combination 0 0 0 also, or 0 0 1 also A is 1 B C is something. So, A bar will be true, but this is not an implicant. Therefore, I say that this A bar is not a prime implicant, a prime implicant is something which is in some sort in some way it is miniminal, like I cannot remove or delete any literal from it. And even after removal the property of implicants still holds right.

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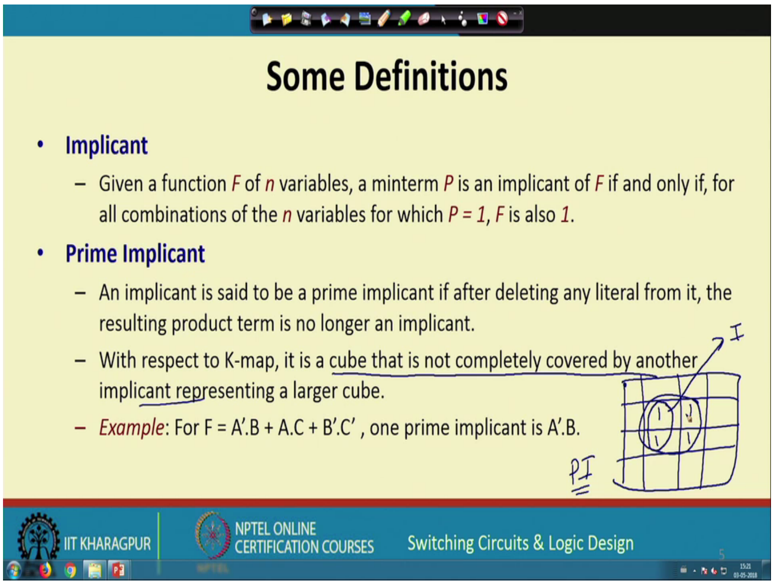

Now, with respect to the Karnaugh map what does that mean, it means that it is a cube that is not completely covered by another implicant, you see with respect to the Karnaugh maps, what is the meaning of this prime implicant and implicant. Suppose I have a four variable Karnaugh map.

Let us say there are four ones here and, I have a cube like this, this is a prime implicant. Now, if I consider a smaller cube like this, which is a proper subset of the larger cube, this will be an implicant, but not a prime implicant. Because from this one I can delete 1 of the variable still it will not change I will have a example later I should show you, this is the basis idea let us proceed.

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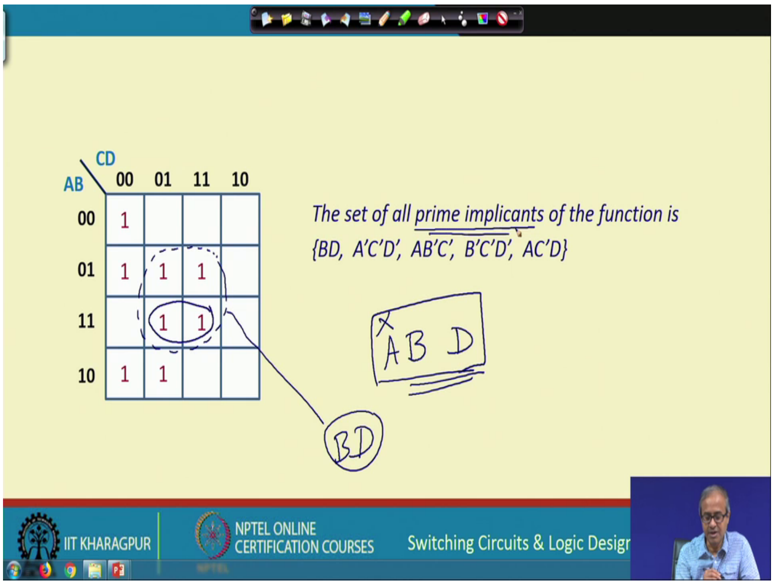

Let us take an example here; here we have a 4 variable map. Now, in this map if you look at the cubes this is 1 cube, this is 1 cube, this is 1 all possible cubes I am showing not the minimum 1. This is 1 cube this is also a cube so, how many 1 2 3 4 5.

So, corresponding to this 5 cubes the product terms are like this you can check, this one means A bar C bar D bar this one, this one means A bar B C bar, A bar B C bar, A bar B C bar it should be this 1, I think A B bar C bar any way this 1, let us see this 1 is A C bar D A C bar is this the big 1 is B D B D and B bar C bar D is B bar this 1 this 1 this and this, this and this, this 1 is this 1 ok.

So, here you can add all the implicants like this these are all set of prime implicants, but what I am saying is that: if we have let us say I make a cube like this, what does this cube mean this cube means A B and D A B D right. But I could have also had a bigger cube like this, the bigger cube what does that bigger cube mean B D, bigger cube is also a prime implicant.

It says that A B D is not a prime implicant because, if I delete one of the literal let us say A, whatever remains B D that is also a prime implicant, So, from the karnuagh map it means any cube which is not the largest 1 will not be the prime implicant, if you considered the largest possible cubes only they will all be the prime implicants ok. From the karnuagh map this is the meaning ok.

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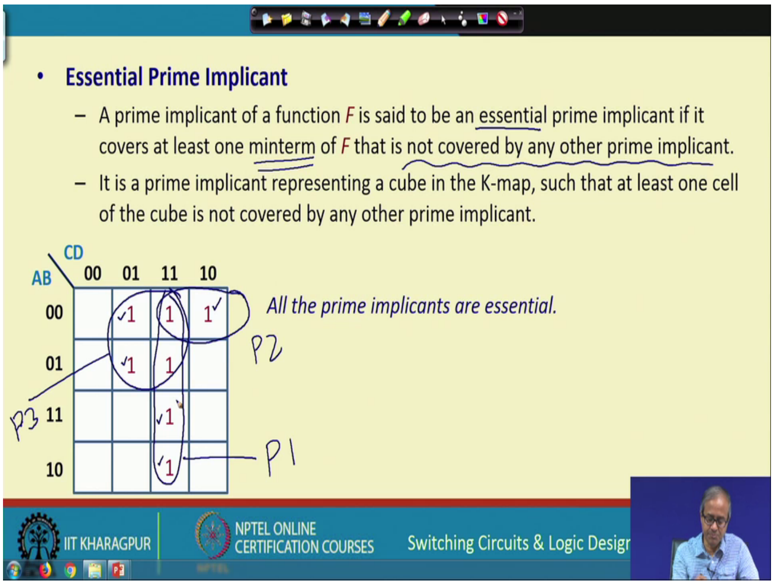

And there is a notion of essential prime implicant, some of the prime implicant may not be essentials some of the prime implicant can be essential. The notion is like this, some of the prime implicant is called essential, if that prime implicant covers at least one minterm of the function, which is not covered by any other prime implicant. like with respect to the Karnaugh map. Let us say here, this is 1 implicant, this is 1 prime implicant, this is 1 prime implicant, let us call this 3 prime implicant, let us call this as P 1, let us call this as P 2 and this 1 as P 3.

So, if you look at P 1, P 1 covers these two cells, which are not covered by P 2 or P 3, P 2 covers this cell, which is not covered by P 1 or P 3 and P 3 covers, these two cells which are not covered by P 1 or P 2 therefore, all the three prime implicants P 1, P 2, P 3 are essential. So, with the respect to the cubes in the Karnaugh map, the condition is that at least one cell of the cube is not covered by any other prime implicant, this is what I just now explained ok. Such prime implicants are called essential prime implicants.

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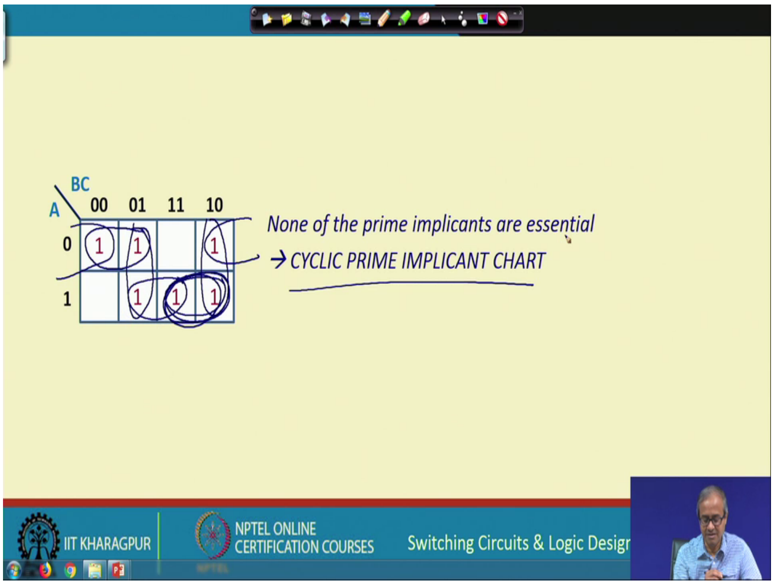

Now, take an example here where the prime implicants are not essential, why it is so, you look at the cubes here this is the 3 variable function, 1 cube is this, 1 cube is this, 1 cube is this 1 is this, 1 is this and 1 is this. So, this like a cyclic one this is called as cyclic prime implicant chart you see this they are all connected in a chain and, you cannot identity any 1 prime implicant here, which is covering one literal, or cell which is not covered by any other prime implicant.

Like for example if you consider this one, both the ones that covered by some other cube also this 1 is covered by this cube, this 1 is covered by this cube. So, none of the prime implicants here are essential here, there are 1 2 3 4 5 6 prime implicants, but none of them are essential ok.

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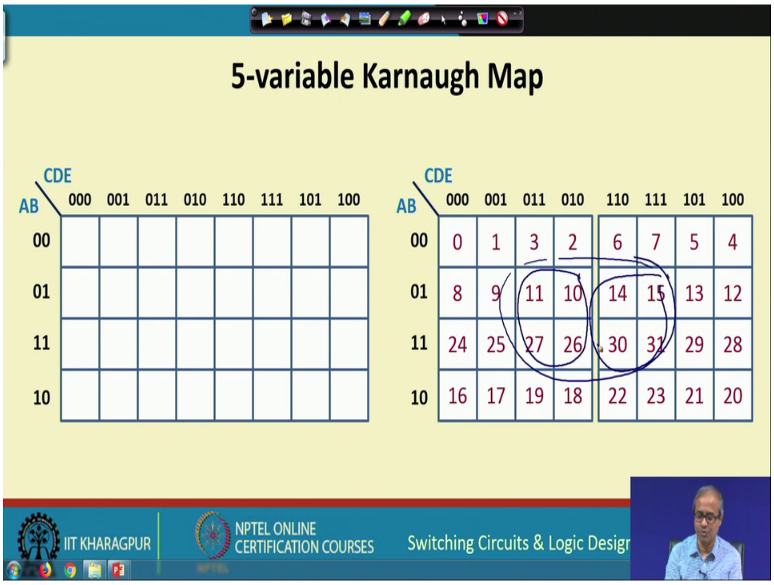

Now, I am not giving example just showing you how a 5 variable Karnaugh map looks like, just to tell you that it is it is a little more complex, you see 5 variable Karnaugh map will be having 5 variables here. Let us say 2 variables on this direction, the other 3 variable on this direction. You see here is here as I said the numbering would be something similar to the grey code numbering, they will differing in one position 0 0 0 1 1 1 1 0 here, you see similarly 0 0 0 0 0 1 0 1 1 0 1 0 1 1 0 1 1 1 1 0 and again back to 0 0 0, you see between adjacent cells it is differing by exactly 1 bit position.

And A B C D if you represent and write down the decimal numbers it will be something like this you can check, 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15, because grey code is a reflected code you can see there is a reflection kind of a thing across this, it is a the mirror image 0 1 2 3 from other side you count 4 5 6 7 8 9 10 11 12 13 14 15 like that ok; 6 variable again there will be 1 here and 1 here it will become much bigger it will become very complicated forming the cubes like when you forming the cubes you cannot form a cube like this for example, 3 from this side and 1 from this side, this is not a valid cube.

The rule is that you should have power of 2 from this side and the power of 2 from this side, they will be formed into a single cube. So, the rules are much more complicated here. So, I am showing you an example in this case 5 5.

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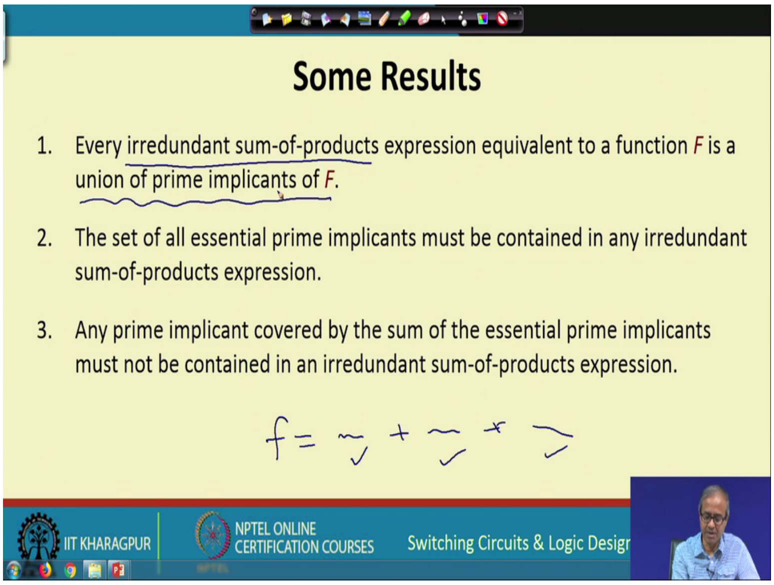

Now, there some results which you can just summarized with respect, what we have talked about the prime implicants and implicants essential prime implicants and so, on. These results are the important in understanding what we shall be discussing next. Now, I am just showing of few of the results here, first result says that any irredundant some of product, irredundant means it is a minimized form, irredundant some of product means all the product terms are essential. If you remove one of the product terms the function will become different.

But you may have some function where even if you remove product term, the function does not change which means, that product term was irredundant it was not required ok, when I see irredundant it mean it is a minimized form, that can be expression for a function F is union of prime implicants of F. This is important. It says that any minimized expression sum of product form.

Whatever you write something plus something plus something, these somethings most all be prime implicants, this is the first result, that in any minimized expressions all the product terms must be corresponding to prime implicants right. And second point is the essential prime implicants are those you recall, which will be covering some minterm that are not covered by any other prime implicant. So, in any minimized expression those essential prime implicants must always be present, otherwise it cannot represent the function some of the minterm it cannot cover right.

So, the second point says that the set of all essential prime implicants must be present in any irredundant some of product expression and, the third point is just a corollary of it, it says any prime implicant covered by the some of the essential prime implicants must not be contained in any irredundant it says that.

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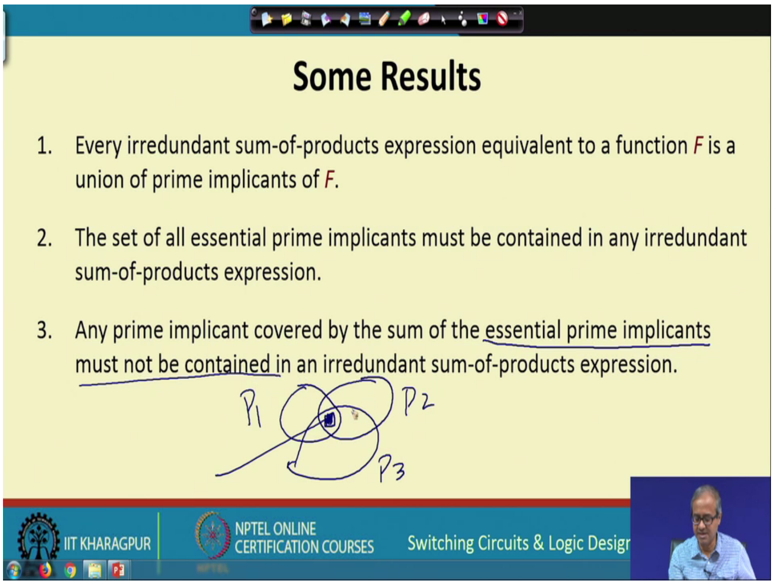

Suppose I have some essential prime implicant let us say I have this one essential prime implicant, another essential another essential let us say P 1, P 2, P 3. Let us say there is some prime not there is some prime implicant may not be essential here in the common area in this P 1, P 2, P 3.

Now, if this is here then, this fellow must not be contained in an irredundant expression, because P 1, P 2, P 3 because they are essential they will always be there, but because they are always there anything which is already covered must not be there. So, any prime implicant covered by the union, or the sum of the essential prime implicants must not be present in any minimized form, that is the idea ok. These are some of the results.

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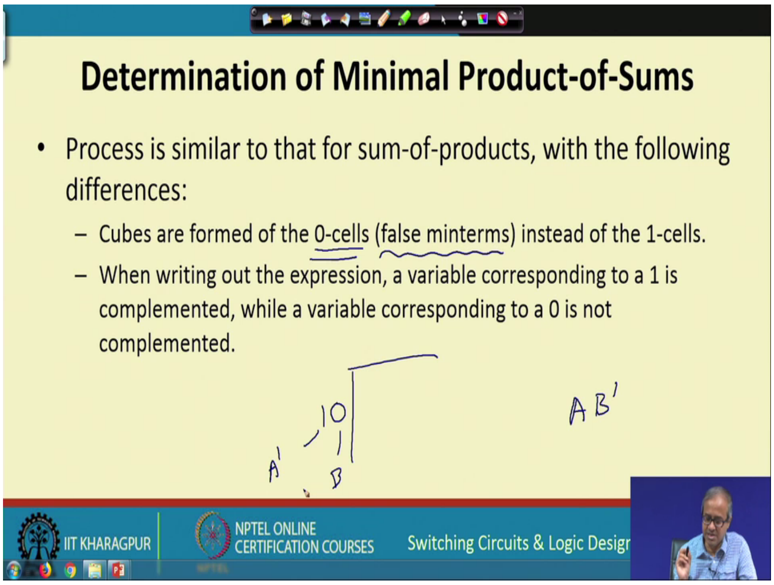

Now, just 1 thing let me tell you briefly we talked about how to generate minimum sum of products expression using Karnaugh map, but whatever product of sums because, of the principle of duality, if you can do sum of products you should be also able to do product of sums. So, using Karnaugh map in fact you can also do that I shall just take one example to show you, how to do it without doing into too much detail ok.

So, the point to notice that the process is somewhat similar, there are couple of differences the first difference is that, when you form the cubes you form them using the 0 cells or the false minterms not the 1 cells, which you do for sum of product. And lastly when you write down the expression for example, when you write down the expression for sum of product, if there as 1 0 here, you are writing A B bar 1 means A, 0 means B bar.

But here the convention will be different the variable corresponding to be 1 will be complimented, while a variable corresponding 0 will not complimented. So, for 1 you will be writing A bar for 0 you will writing let us say B these are the two changes.

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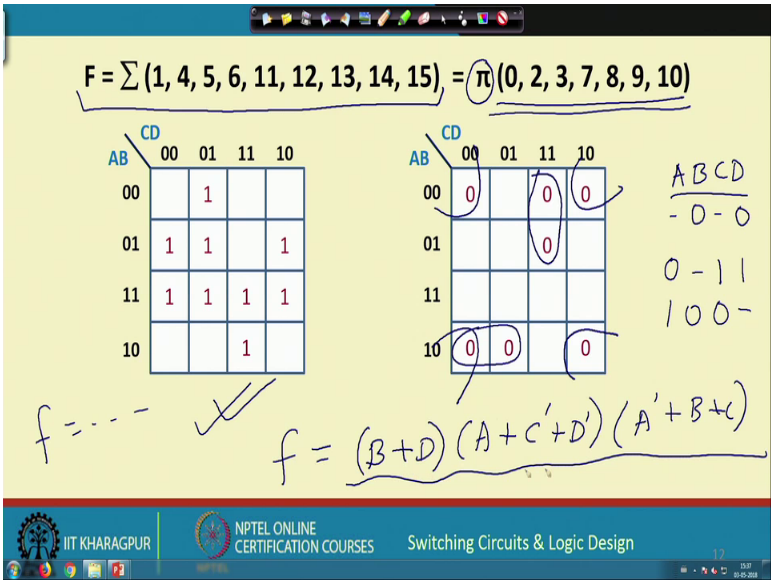

Let us take an example to show you how it works let us see what this slide means, consider we have a function like this, it is in the sum of products form, these are the true minterms 1 four 5 6 11 12 13 14 15, which is shown by this Karnaugh map these are the ones. Now, the remaining cells will be 0's, there 11 so the remaining there how many 5 9.

So, the remaining 7 will be 0's so, sometimes this remaining cells which are 0's are written in this product or pi notation, pi of this means these are the false minterms of the function, sigma means these are the true minterms, pi means these are the false minterms. So, when I talk about product of sum we need to talk about the false minterms and, you make a cube just like you did earlier. There are 4 corner cells, make a cube out of them, there will be one cube like this and let us say I make one cube like this that is all I have covered everything.

Now, out of the four corner cells let us see what are the thing, just I am writing in terms of A B C D four corner cells here and here 0 0 and 1 0 so, B is 0 B is 0 and here 0 0 and 1 0 D is 0. So, I said the convention is just reverse B is 0 D is 0 means, they will be in the uncomplemented form, I write B or D this is one of my sum term take this. In this 0 0 0 1 means A is 0 so, A is 0 and here 1 1. So, C is 1 D is 1 this is these are not here, so, A is 0 C is 1 D is 1. So, term will be A bar or sorry this should be A, this will be A plus C bar or D bar and the last one these 2 these A 1, B 0 so, A 1 B 0 and C is 0 D is not there.

So, it will be A bar or B or C this will be the product of sum expression, this is the rule. You consider the 0's try to cover the 0's and, when you write down the expression same way for 0 0 you use uncomplemented form and, for ones you use complicated forms, but instead of sum of product write them in product of sum form, so, this will my function in this case. So, if you it in sum of product form you will be getting some expression, if you write in product or sum form you will be getting some other expression ok.

So, with this we come to the end of this lecture. The next lecture we shall see how we can follow some more systematic approach, there is something called a tabular approach which can be done more systematically, which you can also use for larger functions, functions with more than 4 5 or 6 variables. We shall be discussing about that in our next lecture.

Thank you.