Switching Circuits and Logic Design
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Lecture - 16
Minimization Using Karnaugh Maps (Part - I)

So, if you recall we have been talking about various algebraic methods of manipulating switching expression switching functions and also we saw that using some basic rules or theorems we can apply them in a suitable way to reduce the number of product terms, reduce the number of literals which means some sort of minimization.

So, today we start our discussion on some more systematic ways of minimizing switching functions which in the algebraic for is a little complicated or complex. So, the method that we shall be starting with is using something called Karnaugh maps minimization using Karnaugh maps this is what we shall be starting our discussion on.

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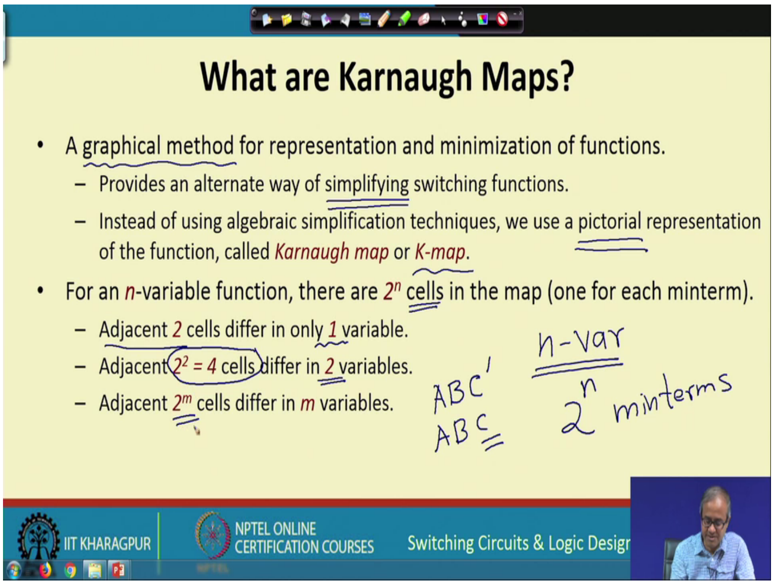

So, first let us try to understand what is a Karnaugh map? First thing let me tell you Karnaugh map is some kind of visual or graphical representation of the function. See any function any switching function that you want to minimize or manipulate we should be able to express it in some way.

We have already seen some of the methods like the truth table, like the like the some of products or the product of some expression forms. Now Karnaugh map is another way of representing a function which is visual or pictorial in a sense that all the minterms of the function can be represented in a matrix form and all the true minterms are marked in that matrix. Depending on the positions of the true minterms and their relationships, we can apply some simple rules to create some kind of minimization procedure so that what will get finally, will be minimized form of expression either sum of products or product of sums.

So basically speaking what I am talking is that Karnaugh map is nothing, but a graphical method for representing and minimization of functions. Well we use Karnaugh map for simplifying or minimizing switching functions, we earlier already have seen how we can use algebraic technique to do the same, but instead of algebraic methods we would be using this Karnaugh map which I shall said is some kind of pictorial representation and sometimes in short we refer to it as K-map ok. So, there was some characteristics of a Karnaugh map that I would like to talk about.

Well let us consider we have a n variable function I have a n variable function. So; obviously, there will be 2 to the power n minterms in this function. So, in the Karnaugh map there is a concept of cells and the cell has a one to one correspondence with these minterms; there are 2 to the power n minterms the K-map will also be having 2 to the power n cells. And this minterms are mapped to the cells in a very specific way such that the adjacent cells. Let us say adjacent 2 cells pair of cells will differ in only 1 variable; like let me take an example suppose in a 3 variable function one cell corresponds to ABC bar and its neighboring cell may corresponds to ABC where C is the only 1 variable which is differing right.

Similarly the Karnaugh map we can also talk about 2 square of 4 cells; now if we have set of adjacent 4 cells they will be differ in 2 variables. In general, we can have any part of 2 two to the power m cells which will be differing in m number of variables. So, this we shall be see through examples.

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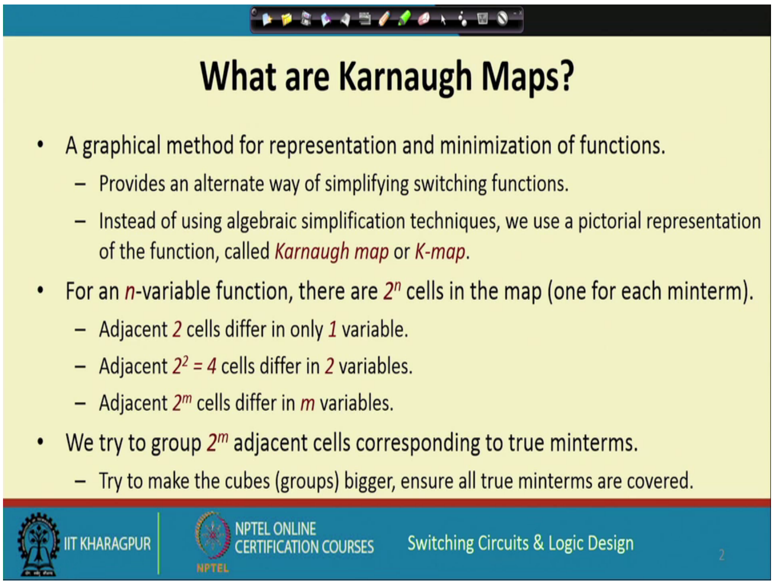

So, what we try to do is we try to group the cells as I said we have the Karnaugh map; we have the cells on the map where each cell corresponding corresponds to a minterm; we try to group the cells together and each group will be having 2 to the m adjacent cells some power of 2 1, 2, 4, 8, 16 like that.

So, we try to group some adjacent cells which are some power of 2 corresponding to the true minterms and you always try to make the size of these cubes bigger, because bigger the number of cells in a group more the minimization that we have been able to achieve. So, our target will be to make this cubes or this groups as large as possible this is our basic objective.

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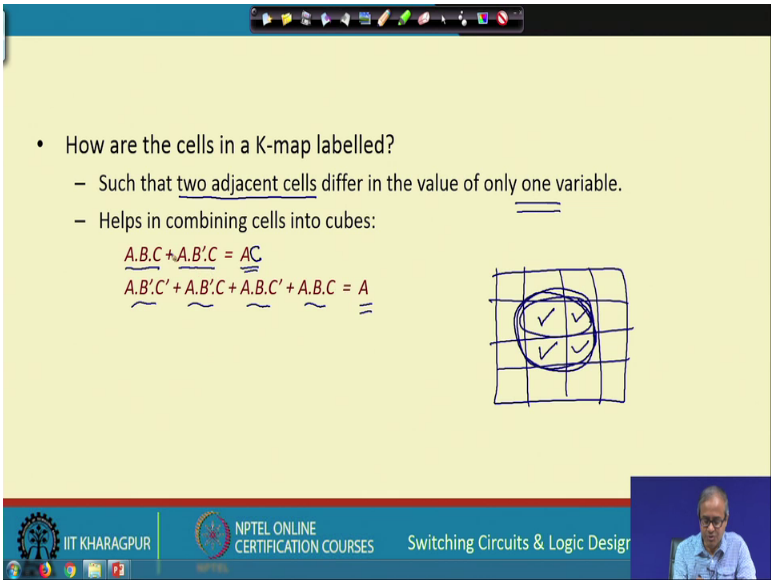

So, now the question is how do we label the cells in a Karnaugh map such that these property is ensured. Now this is something which we have already mentioned that we have mapped the minterms to the cells in such a way that a pair of adjacent cells will differ only 1 variable right. Now if we if we can have that for example, I will show you have a cube looks like. Suppose I have a cube like this let us say the 4 by 4; let us say I have one cell out here one cell out here; these 2 are adjacent cells. Let us suppose this one of the cell corresponds to the minterm ABC the other one corresponds to be minterm A B bar C which differs in only 1 variable B.

So, I am saying that if I have a pair of such neighboring true minterms; I can combine them into one. So, the idea of combining is this if I combine them then you see the rule of simplification if you take A C common is B and B bar will cancel out this will of course, C C is missing this will be AC ok. So, if you take AC common B plus B bar this will be AC. So, there is a minimization; similarly if you have 4 such cubes together let us say these 2 in addition this and this.

So, there is a bigger cube I can form of size 4; then this 4 cells may corresponds to these 4 minterms and using rules of switching algebra you can easily see by combining them in pairs that this is ultimately equal to A. So, you see the bigger the size of the cube; more is the minimization if you can combine 2 cubes I have something like AC if I can combine

4 cubes I have only a single literal. So, the number of literals go on decreasing as we make the cubes larger and larger.

Suppose, originally I have an invariable function if I make a 2 cube it becomes n minus one if I make a 4 cube it becomes n minus 2, if I make 8 cubes it becomes n minus 3 and so, on. So, the cubes will become smaller and smaller ok.

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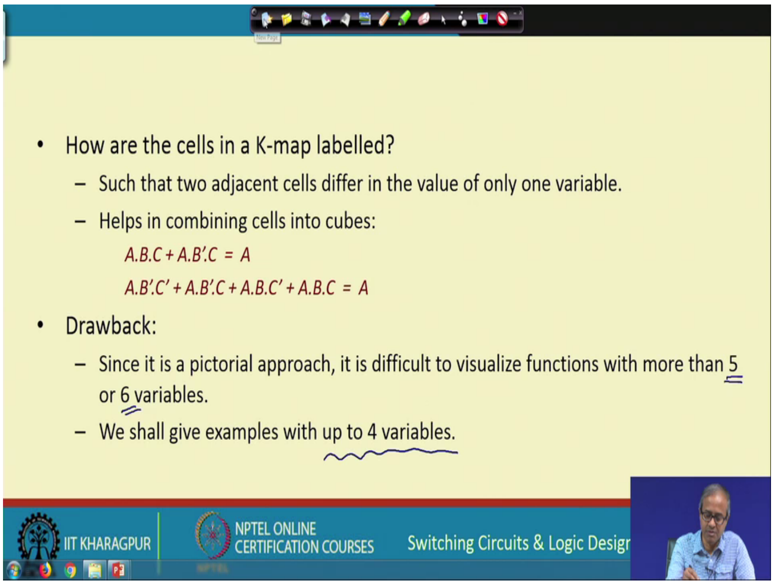

So, this Karnaugh map is fine, but of course, there is some drawback also the main drawback is that because it is a pictorial approach; well we cannot draw a figure like that with more than 5 at most 6 variables. While even a 5 variables it becomes complicated; so, here all the examples that we shall be showing will be up to 4 variables only 5 and 6 variables becomes complicated, beyond 6 it becomes impossible this is the main drawback of Karnaugh map.

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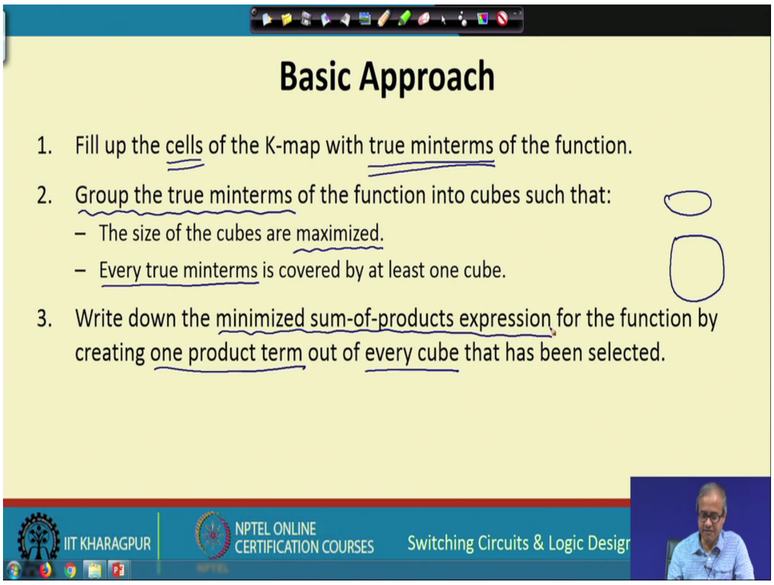

Now let us see what is the basic approach in this method the Karnaugh map based minimization method?

So, what we do as I had said Karnaugh map is a 2 dimensional matrix of cells, we try to fill up the cells with the true minterms of the function, I shall be illustrating this with an example. So, each cell corresponds to minterms; so, we consider only the true minterms which are the minterms? That are one for the function. So, we mark those true minterms and then we try to group these true minterms based on their neighborhood.

So, as I said I can group 2 of them together, I can group 4 of them together, I can group 8 of them together and so, on. So, this grouping will be carried out in such a way that we shall be trying to make the cubes as large as possible. We shall be trying to maximize the size of the cube and also should ensure that every true minterm in the function should be covered by at least one cube ok.

So, once we have done that; every cube will be corresponding to a product term to create one product term out of every cube. So, once you have done this grouping the true minterms into cubes. So, every cube will correspond to one product on and what we get will be the minimized sum of products expression for the given function; this is the basic approach behind the Karnaugh map method.

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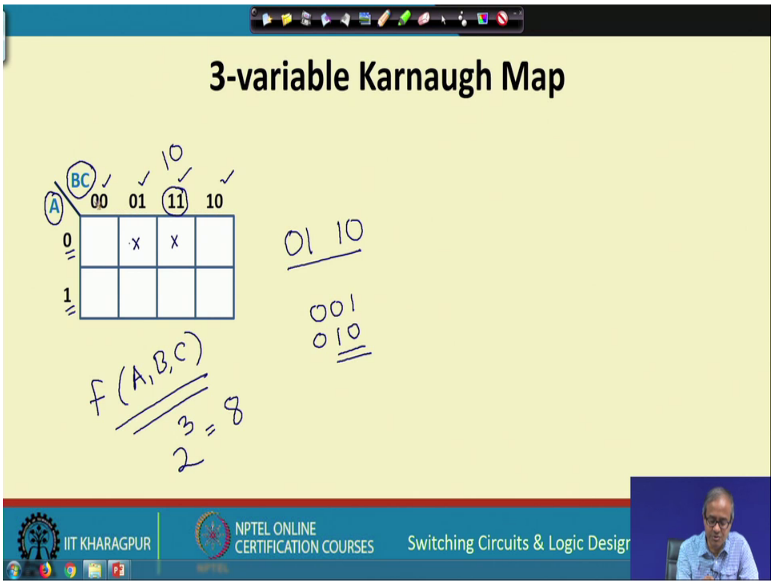

Let us take a simple example to start with we consider 3 variable functions to start with later on we shall be extending to 4 variable and see how it looks like. So, in this example that I am showing here; here I am considering a function any function of 3 variables A B and C.

So, the Karnaugh map of a 3 variable function will obviously contain 2 to the power 3 minterms which is equal 8. This is organized in this way let us say 2 rows and 4 columns and I am labeling the rows and columns by the variables. Since there are 3 variables ABC; so, along the rows I am using A, along the columns let us see I am using B and C. While here I can use other way were out also, but here in this example I am assuming that A is not in the row and B and C are marked in the column. Now one thing you see the rows indicate the value of A; if the value of A is 0 it is first row, if the value of A is 1; it is a second row. Similarly with respect to BC, you see the 4 combination of values 0 0; 0 1 1 0 and 1 1 are given.

But the point to note is that you see we have not written these values in the binary value order 0 0 0 1 1 0 1 1; rather we had written this 1 1 first and then 1 0. So, why we have done this? Because we have to ensure also as I that said that 2 adjacent minterms must differ in 1 variable only ok. Suppose if I write 0 1 and just on the right of it I write 1 0, then if I consider 2 cells 2 cells one cell here one cell here, this cell corresponds to A equal to 0 B 0 and C 1 and this cell if it is 1 0; then it will be correspond to a 0 B 1 and C

0 which means in one case it is 0 0 1, in other case it is 0 1 0. So, you see the differing in 2 variables.

So, we cannot do this; so, we write or label the columns in some kind of grey code count order 0 0 0 1; 1 1 0. So, that they vary in one bit position across columns not only that another interesting point to note is that you see you can assume that the right side and the left side also are neighbors; how?

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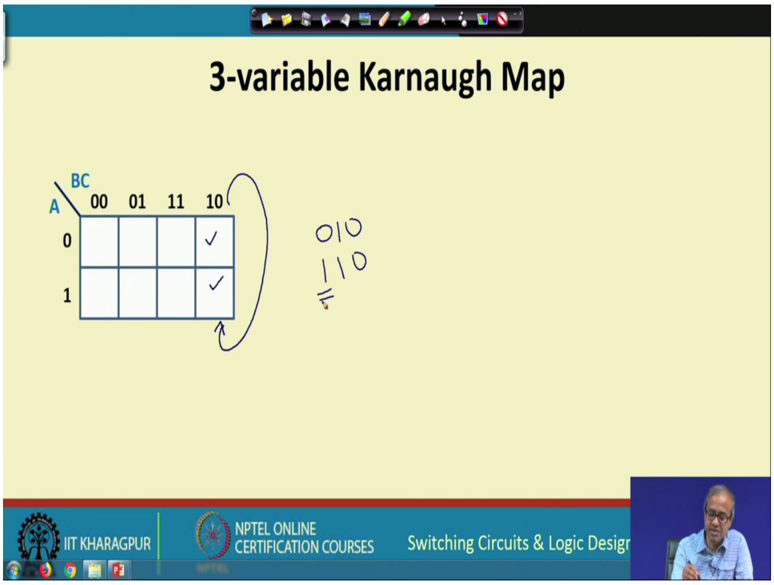

You see this is 1 0 and the left most column is 0 0; they are also differing in one position; so, right and left will also be considered as neighbors. Similarly top and bottom will also be considered as neighbors, while here in the rows we have A only. So, it will always differ in one position; suppose this and this it will correspond to 0 1 0; the first one. The second one correspond to 1 1 0; they will differ in one position right. So, this is how we make the labels and the corresponding decimal numbers corresponding to the values of A B and C.

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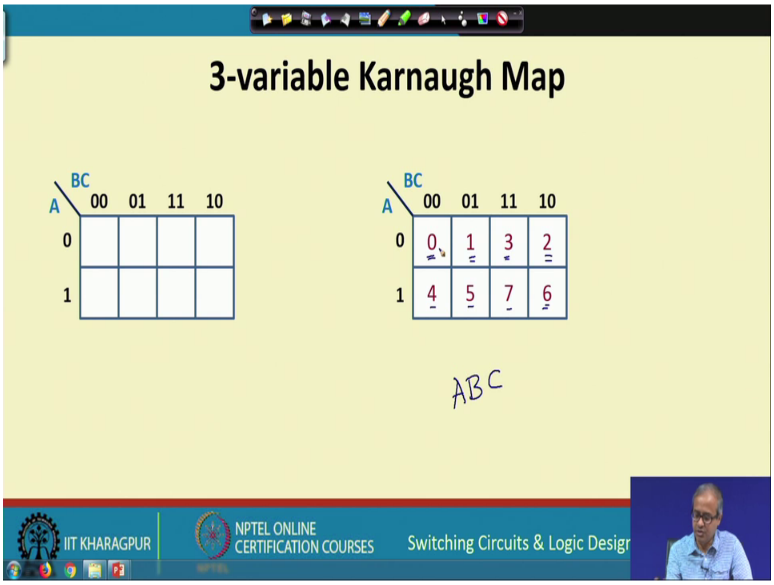

So if I write down the decimal values of A B C like in this order then they will be written like this 0 0 0 is 0 0 0 1 is 1 0 1; 0 is 2 0 1 1 is 3, similarly 4 5 6 7. So, when you use a Karnaugh map; you should be remembering these assignments of the decimal equivalent of the minterm value 0, 1, 2, 3, 4, 5, 6, 7 ok.

Now, let us take some examples of actual minimization how we can use this to minimize functions?

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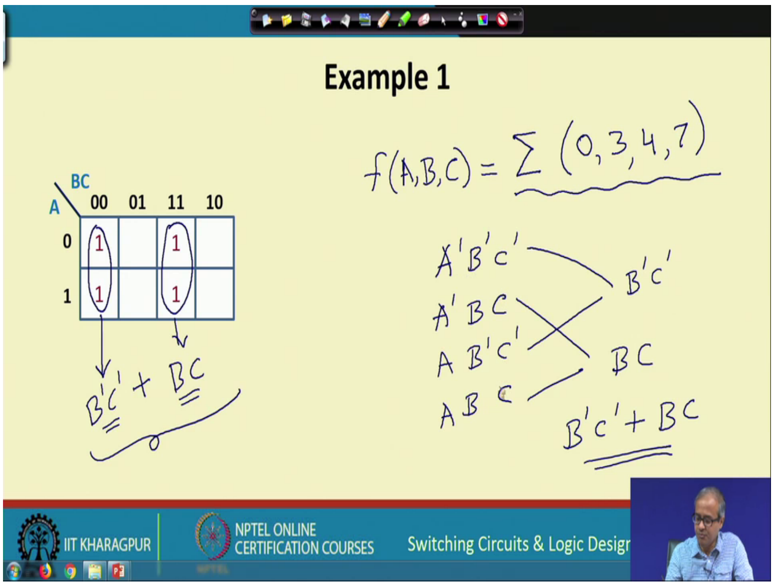

Let us start with a very simple example here. So, you see here I have shown a function of 3 variables; we write it like this and say a function of 3 variables ABC there are 4 ones meaning there are 4 true minterms.

So, we write it like this sigma notation with this means these are the true minterms. So, what are the values? 0 0 0 and decimal this means 0 0 1 1, it means 3 1 0 0; that means, 4 and 1 1 1 it means 7. So, this problem of the function I can also express like this it is a sum of the minterms, the combination there are 4 true minterms corresponding to the decimal value 0 3 4 and 7. Now one thing you see before you go in to the Karnaugh map 0 means what? All 0 0 0 which means A bar B bar C bar; 3 means 0 1 1 which means A bar B and C; 4 means 1 0 0 A B bar C bar and 7 means 1 1 1 A B C.

Now, if we use normal algebraic methods then you can see that the first and the third one B B bar and C bar is common if you take it common you get A bar plus A, which is equal to 1. So, you get only B bar C bar. Similarly second and forth if you combine BC is common and A and A bar will go you get BC. So, the minimized form will be according to algebraic technique; you will get B bar C bar or BC. Let us verify how we can get this same thing from Karnaugh map?

Now see I have shown the true minterms here let us try to form the cubes the rule of cube formation is as I said the number of cell should be some power of 2; they should be adjacent to each other and it you make it as large as possible ok. So, here you can see one cube can be like this, one cube can be like this you cannot make them any bigger right.

Because this adjacent cells are empty; now when you make this cubes say for this cube you see it is spanning both the rows. So, A 0, A 1 both it is covering; so, A is getting cancelled out you have BC 0 0. So, this corresponds to B bar C bar and this one again A cancels out; 0 and 1 and it corresponds to 1 1 which is B and C. So, these 2 cubes corresponds to these 2 product terms.

So, the sum of product expression in minimized form will be B bar C bar or BC ok. So, in Karnaugh map you get the same value, but you do not have to make an algebraic manipulation like here; directly you can get right. Let us take some more examples, let us take an example like this see here there are 5 true minterms.

So, instead of going in algebraic thing let me let me straight away come to this map method.

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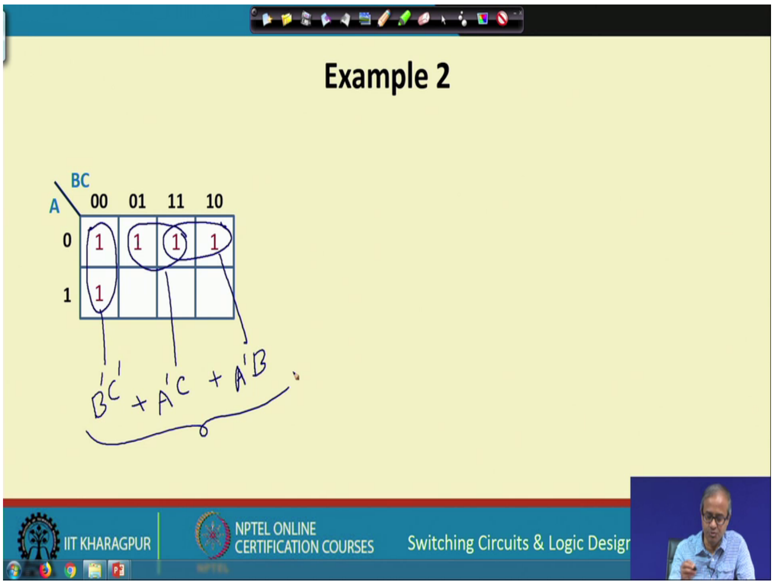

So, you see that if you want to make the cubes; I can make a cube like this I can make a cube like this every minterm must be covered by at least one cube; so, this is uncovered. So, maybe I can make another cube like this suppose I do it like. So, all the minterms are covered and what will be the expression for the first cube? This will correspond to B bar C bar, for the 7 cube you see this is in the first row first row means A bar A is 0 and it is covering 0 1 and 1 1. So, the first one gets cancelled out second one remains; it means c remains this is A bar C and this one is A bar C cancels out B remains A bar B.

So, there are 3 terms ok, but the point is notice that this is not minimal because I could have made the cube even larger this is just an example I have shown, but this is not the right process. So, if you want to follow the right process the cubes will be this is one cube.

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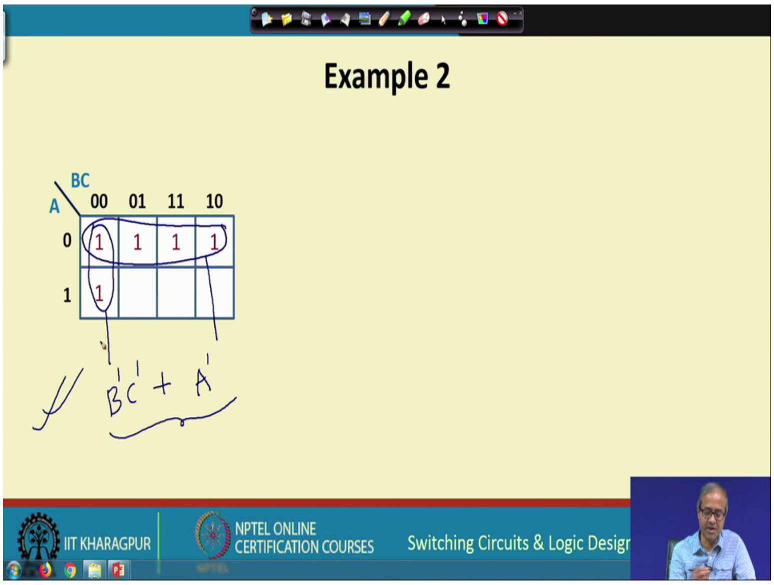

And this should be one cube all 4 cells together because as I told you; you can combine any power of 2 number of cells as long as they are adjacent. So, if you do this the first one will correspond to again B bar C bar, second one will correspond to it is a first row. So, there will be A bar and it is covering all four; so, B and C all cancels out. So, only A bar remains. So the expression is B bar and C bar or A this is the minimized expression.

So, directly from the Karnaugh map you can get this right; let us take another example while here again let us see one cube you can notice like this these 4.

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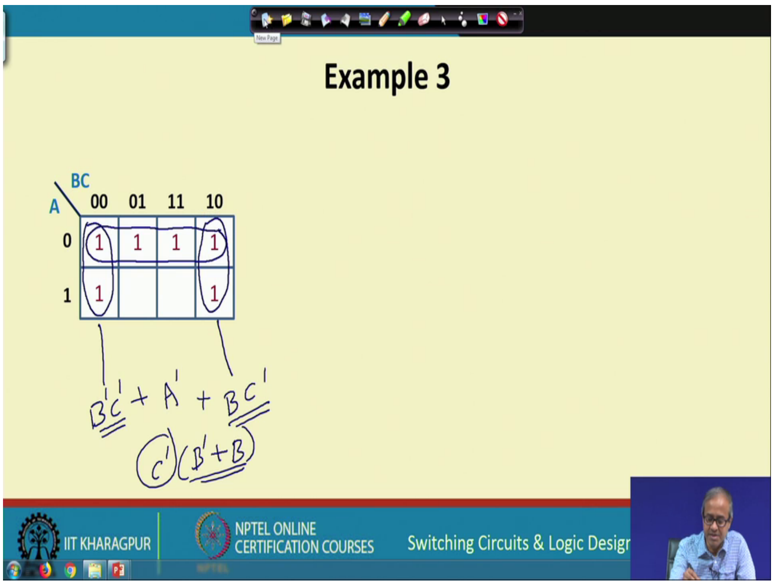

Now you may tell that will I can see one cube here I can see one cube here; suppose I do this suppose I do this then this first cube will be corresponding to A gets cancelled out 0 0, this is again B bar C bar; the last cube again A cancels out 1 0. So, it will be B and C bar and this big one will be only A bar because BC both cancels out it becomes like this 3 terms. But what I am saying that this is not minimized as you can see; this B bar C bar and BC bar can be further minimized, algebraically you take C bar common then B vanishes right.

If you take C bar common; it will become B bar or B this will become 1; so, only C bar will remain. So, this is not the correct way of cube formation again.

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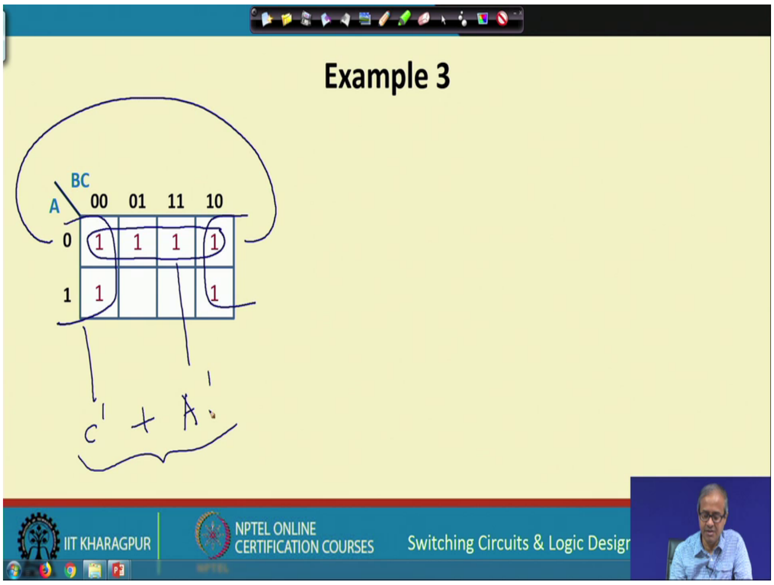

The correct cube will b; this is of course, as large as possible you can make, but you remember that I said that the right most stage and the left most stage are they are adjacent. So, you make a cube like this these 2 fellows here and these 2 fellows here; these 4 together their adjacent, right.

So, if you do this then this cell A cancels out and 0 0 and 1 0. So, B cancels out and C is 0 C bar only C bar and this one is only A bar. So, it is C bar or A bar this is the minimized form right. So, you should notice or note that when you create this cube; so, your cube should be as large as possible, if you try to make them as big as possible; then only you will get the minimized form.

But you see the advantage I do not have to go to any algebraic technique just pictorially you from the cubes and directly write down the product terms from there ok.

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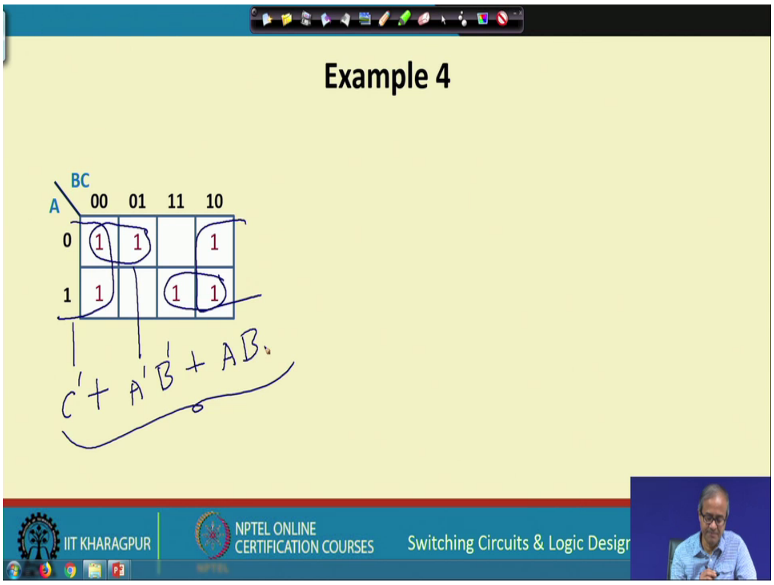

Let us take some more examples see here I have an example. So, here again I can form a cube with these 2 and these 2 to a cube of 4 these two; 1s are still remaining. So, this one has only neighbor with this and this one has only neighbor with this I cannot do any further ok. So, here it will be corresponding to this one; it will be C bar as before this smaller cube you will be A bar and 0 0 and 0 1; C cancels out B bar and these 2 it is A equal to 1 which means A and 1 1 1 0; C cancels out B is 1.

So, B this is the minimized expression this cannot be minimized any further right ok.

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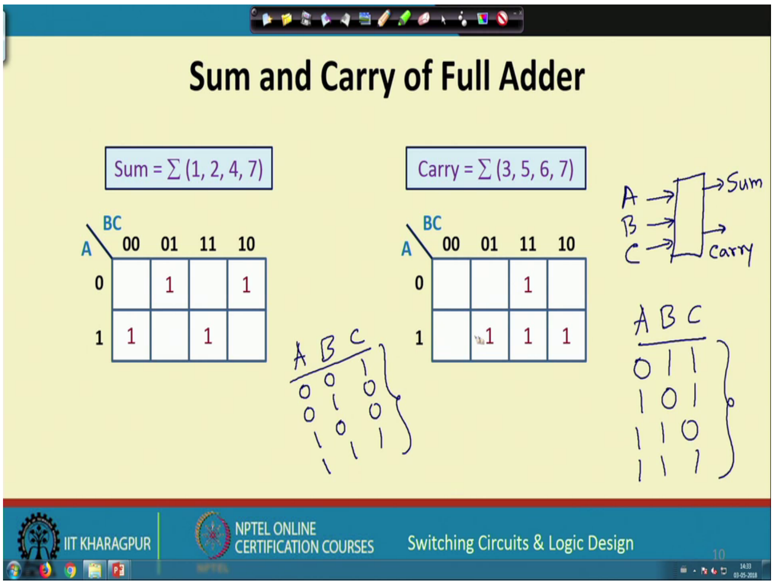

Let us continue with our examples; let us now look at a circuit which you are already seen earlier we talked about a full adder. So, you recall what a full adder is? A full adder is a circuit which takes 3 inputs to binary numbers and B and one carry input let us call it C and in the output we get a sum and we get another carry right.

So, earlier we had shown the truth table of a full adder. So, if you check the truth table if you look at the sum if we consider the values of A B and C. So, when will the sum be 1; sum will be equal to 1 only for the minterms 1 2 4 and 7; 1 means 0 0 1, 2 means 0 1 0; 4 means 1 0 0; 7 means 1 1 1.

So, only under this 4 conditions sum will be 1, this we have depicted in the truth table like this; 0 0 1 0 0 1 this one; 0 1 0 0 1 0 this 1 1 0 0 1 0 0 this one and 1 1 1 1 1 right. Similarly for a carry if you look that when carry will be generated, carry will be generated when the minterms are 3 5 6 or 7 3 is 0 1 1; 5 is 1 0 1; 6 is 1 1 0 and 7 is 1 1 1. So, this we have depicted in this Karnaugh map 0 and 1 1, 1 0 1, 1 1 0 and 1 1 1 right.

Now if we try to minimize you see for some there are no 2 adjacent to 1s which can be combined. So, this is peculiar scenario where no minimization is possible; so, each true minterm will be a separate product term.

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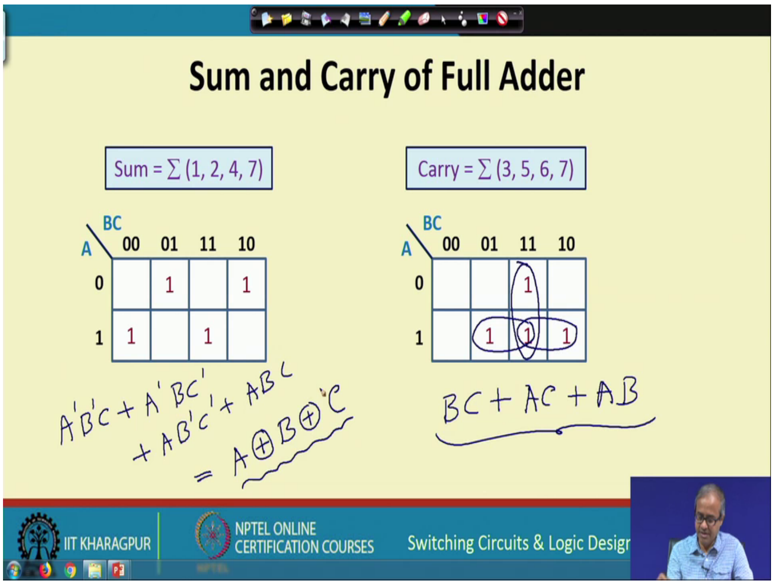

So, if we straight away write down for 0 0 1 will be A bar B bar C; second one will be 0 1 0 A bar B C bar then plus it will be one 0 0 A B bar C bar and the last one will be 1 1 ABC. This will be the form which cannot be minimized; now you please note that this is a very peculiar case of a function which is nothing, but the exclusive OR function of this 3 variables.

Now exclusive or is a kind of function which cannot be minimized any further in the sum of products form. So, this shows like that ok, but carry you can minimize carry you see there are distribution ones are like that you can make one cube like this, you can make one cube like this you can make one cube like this. You see some of the true minterms might get covered by more than one cubes no problem but; but you should ensure that all the true minterms are covered.

So, here what will be the function? Test of one with this one A is cancelled only BC this is BC plus this one; A is 1 and 0 1 1 1. So, it is C and the last 2 again A is 1 and 1 1 1 0 it is B this is the minimized expression for carry ok. So, for a full adder you can try this minimization procedure and get the minimum possible realizations. So, with this we come to the end of the lecture, where we have basically introduced the Karnaugh approach and using examples we illustrated how we can minimize a 3 variable function.

Now in the next lecture, we shall be extending this expression to cover 4 variable functions. So, we shall be talking about how a 4 variable Karnaugh map looks like and how we can carry out minimization using that.

Thank you.