Switching Circuits and Logic Design
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Lecture - 13
Properties of Switching Functions

So, in this lecture we shall be discussing about some of the properties of Switching Functions and Switching Expressions Properties of Switching Functions. Now, recall earlier we talked about the notions of literal switching function, switching expressions and so on. And we looked at a number of a rules basic rules using which you can manipulate such switching expressions. Now, since our ultimate target is to design and implement some functions using the basic gates AND, OR, NOT, NAND etcetera.

So, let us try to move in that direction. So, in this lecture we shall be first introducing some of the basic concepts of switching expressions and, then we shall see how we can use them in designing's circuits or functions fine.

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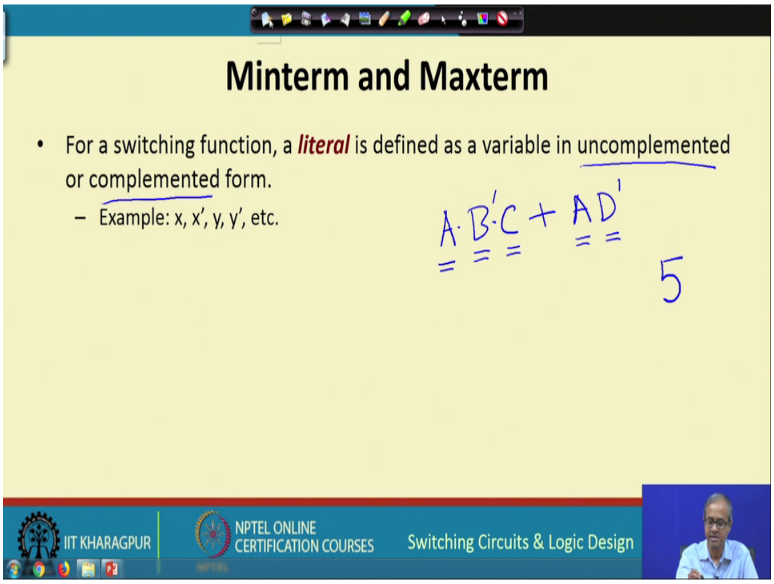

The first thing we talk about is something called minterm and maxterm, but before trying to define what minterm and maxterm is let us define a literal, what is a literal well as this definition says, literal is nothing but a variable which can appear in either uncomplimented form or in complemented form.

So, suppose I have an expression let us say A B bar C or A D bar. Now, here examples of literals are this A B bar C A and D bar, there are a total of 5 literals in this expression ok, literal is a single variable either appearing in uncomplimented form, or a not complimented from. So, here are some examples are given X X bar Y Y bar just like that.

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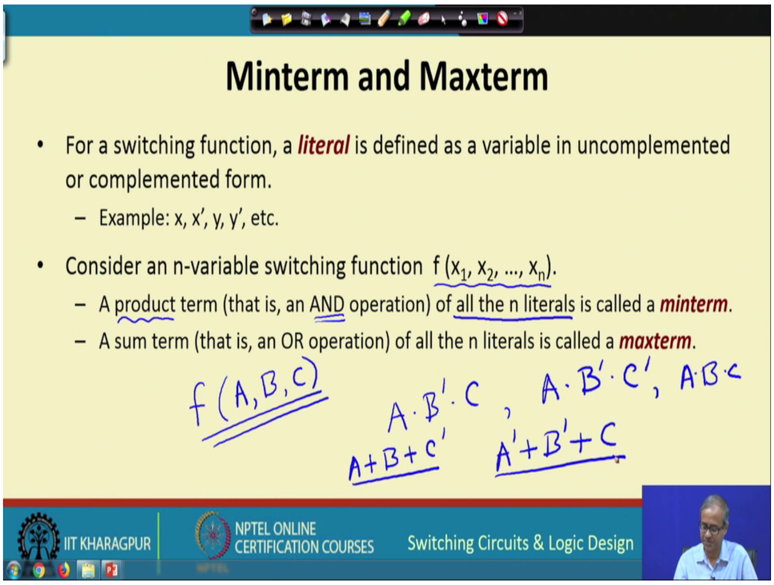

Now, let us try to see what is the minterm and a maxterm. Now, that we know literal considered and n variable switching function f, where the input variables are let us say X 1 to X n.

So, any product term product term means an end of some of these literals, well in again complemented or uncomplement form because, literal definition is that. So, product term that contains all the n literals it is called the minterm. Let us consider a function let us say f of 3 variables A B and C in this function some examples of minterm will be A bar C. Let us say another example will be A B bar C bar and so on, A B C. So, all the literals should be appearing all the variables literals for those should be appearing, they are called minterms

Similarly, maxterms refer to the all operations, say again for this 3 variable function, if I write sum term let us say A plus B plus C bar, this is a maxterm, then similarly A bar plus B bar plus C, this is also a maxterm ok. These are the basic definitions.

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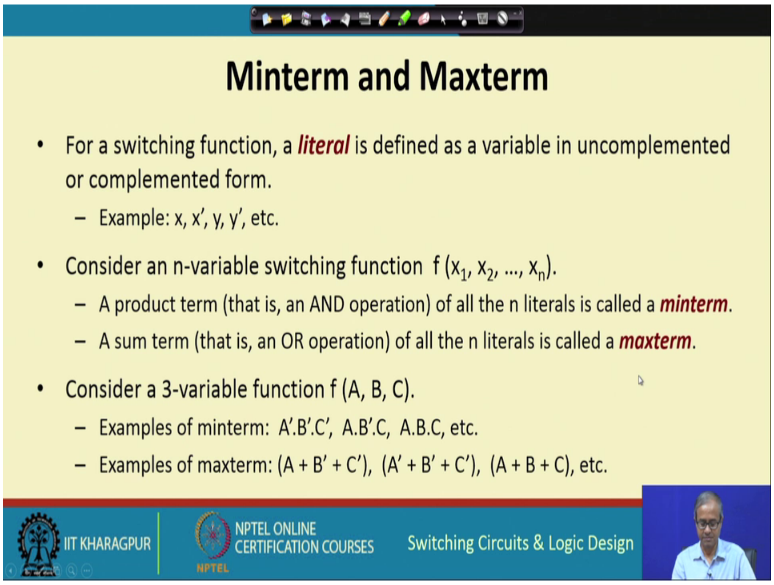

So, some examples are shown here in the slide also, for the three variable function f of A B C, A B C are the input variables some examples of minterms at this. You see if some of the variable is missing for example, A B that will not be a minterm ok. A minterm must contain all the variables, either in complemented or in uncomplemented form. Similarly, maxterms will refer to the sum of the literals, corresponding to all the variables ok, these are minterms and maxterms.

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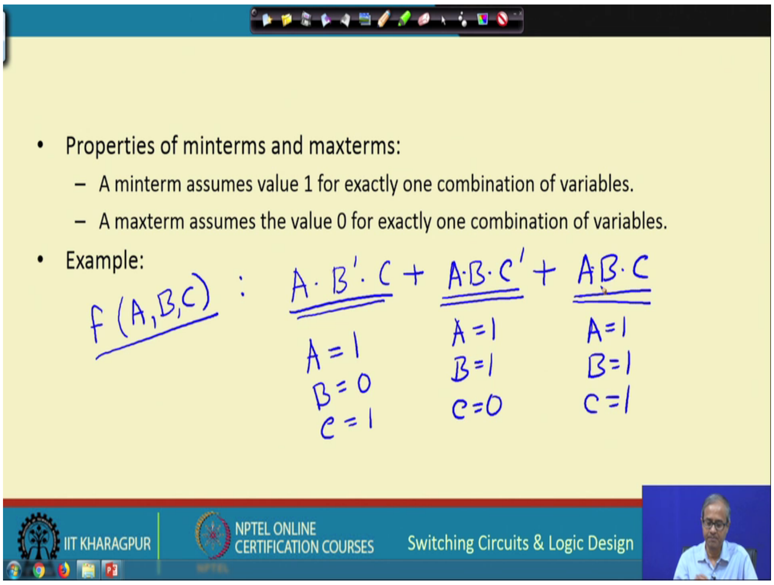

So, some properties of minterms and maxterms are well minterm will assume the value 1, for exactly one combination of variable. Maxterm assumes the value 0 for exactly one combination. let us see what you mean by this taken example, a function of three variables A B C. Let us first look at the so, called sum of products expression, where let us say my function is like this A B bar C, or A B C bar or A B C.

So, according to definition these are minterms, there are three minterms. So, what I say is that a minterm assumes the value 1 for exactly one combination of variables, you consider the first minterm, only when the input variable A is 1 B is 0 and C is 1. The first minterm will be having a value of 1.

Because A B bar means 0 bar is 1 C is 1 1 and 1 and 1 it will be 1. Similarly for the second one we required A equal to 1, B equal to 1 and C equal to 0 and for the third one we require, A equal to 1, B equal to 1, C equal to 1. So, for exactly one combination of the input variables particular minterm will assume the value of 1 ok.

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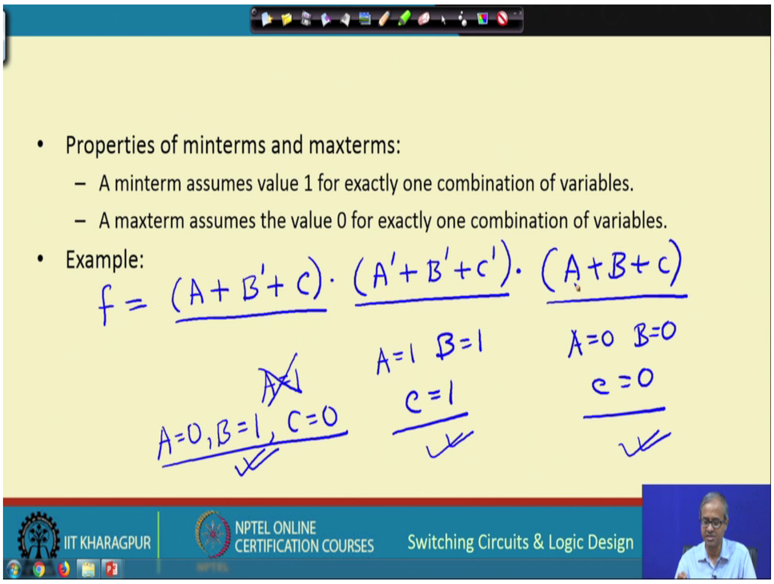

Similarly, for a maxterm the condition is slightly different. Again let us say take a function again of three variables. So, I am writing it in terms of maxterms, let us say one maxterm is A plus B bar plus C and, it is a A bar plus B bar plus C bar. And let us say A plus B plus C. So, there are 3 maxterms. Now, let us taken example let us say if we follow our previous approach, you try to see for which combination this will be 1, you see a this is the odd function. So, either A 1 or B is 0 or C 1.

Let us say if A is 1, then the first maxterm will also be 1, because A is there A or 1 or anything is 1. Similarly the last one here also A or is there this will also be 1. So, just by taking A equal to 1 will not server purpose, rather what will do we will see for what combination this minterms will be 0's see A or B bar or C, it means that if A is 0, if B is 1 and, if C is 0. Then all these three things literals will be 0's 0 or 0 or 0 the first maxterm will be 0, or for the second one let us say A is 1 B is 1 and also C is 1. And for the third one A is 0, or B is 0, or and C is 0.

So, if we have these conditions, this condition says that that when this input combinations is there this maxterm will be 0. So, what I am I can say is that you see this statement. Now, maxterm assumes the value 0 for exactly one combination of variables, for this exactly one combination of variable the first maxterm will be 0. For exactly this combination second one will be 0 and, for this combination the third one will be 0 right. So, I just remember this.

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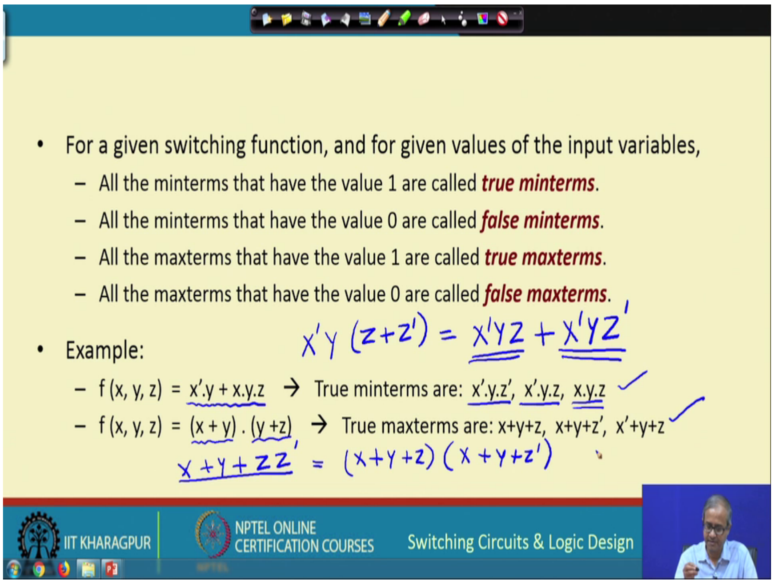

Now, this minterms and maxterms can be classified as either true or false. So, I can say some minterms as true minterms and some minterms of false minterms. So, how are they defined this definition is made with respect to some given values of input variables, for some specified given value of the inputs.

If I, evaluate the values some of the minterms will be having the value 1, some of the minterms will be having the value 0. So, accordingly I call them either true minterm, or false minterm. Similarly I can evaluate the maxterms some of them will be 1, some of them will be 0 and, accordingly I call them true maxterms, or false maxterms this is how the definition goes, let us look at some examples.

Let us take a simple switching function here f of three variables X Y Z X bar Y or plus X Y Z, you see this X Y Z is a minterm all right all three variables are there, but X bar Y is not a minterm. So, what we do? This X bar Y I multiplied by Z plus Z multiply means end because, Z plus Z bar is 1 so, anything and 1 will be the same thing. So, if I do this this is nothing, but X bar Y Z, or X bar Y Z bar. So, effectively this X bar Y this product term is corresponding to two different minterms, X bar Y Z and X bar Y Z bar this is what is mentioned here right.

Similarly, if you take a product of sum there are sum products sum X plus Y, Y plus Z and product of that product of sum terms. So, here again these are not maxterms, because all the variables are not present. Let us say X plus Y so, what you can do you can write it as X plus Y plus Z Z bar, Z Z bar is 0 so, anything or 0 is 0. Now, you have this now we apply distributed law, this will be equivalent to X plus Y plus Z and X plus Y plus Z bar.

Similarly you can do for the second term. So, in this case the true maxterm will be X plus Y plus Z X plus Y plus Z bar and the third one will be coming from Y Y plus Z X bar plus Y Z, this xyz will be common to both ok. This is how you can calculate which minterm, or maxterm are true the remaining ones will be false fine.

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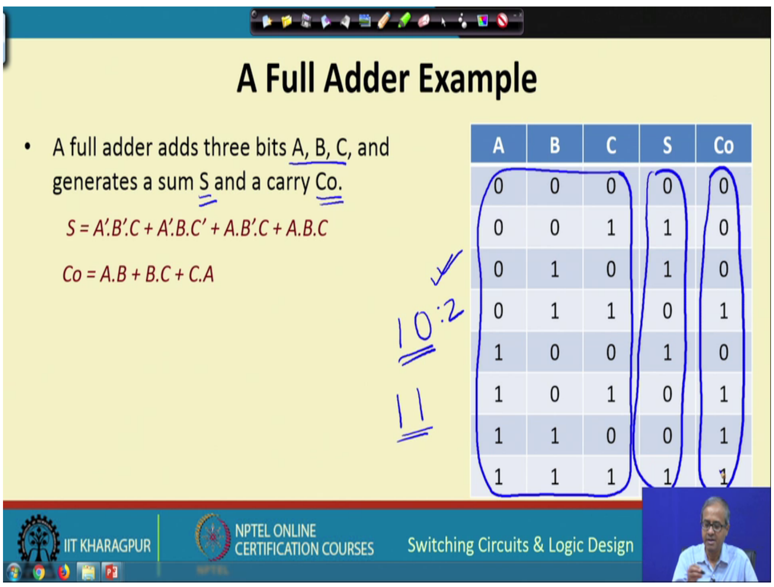

Now, let us try to take a small real example and with the help of that example let us see, how this concept of minterm and maxterm goes. The example we take is that of a 3 bit adder, well we already have learnt earlier, how to carry out binary addition. 3 bit adder is sometimes known as full adder there are 3 inputs A B and C. And as the output you will be having a sum 1 bit and a carry that will also be 1 bit. So, let us see how a full adder is defined, as it said it will be having 3 bits A B C as input, it will be having a sum output and a carry output. And here we are showing the truth table of the full adder.

So, you see for all combinations of A B and C we have listed, what will be the sum and what will be the carry ok. So, if you add all 0's the sum is 0 no carry 0 and 0, if you add 0 0 and 1 the sum is 2, but no carry. Similarly 0 1 0 or 1 0 0 there is a single 1 sum will be 1 no carry, but if there are two 1's 0 1 1 1 0 1 or 1 1 0. If there are two 1's then sum will be 0 and carry will be 1 because, carry and sum 1 0 in decimal means 2. And lastly if I add 1 and 1 which is 3, 3 means 1 1 in binary right. So, sum will be 1 also carry will be 1.

Now, you see this again we shall be coming back later, the way we can just express, or write down the expression for this sum and carry are very simple. First look at the sum, you look at the column for sum you find out that for which rows, this sum is 1 there are 4 such rows. Then you look at the corresponding input combinations, these are the corresponding input combinations for which this sum will be 1 ok. So, if it is 0 you use that variable in complemented form A bar, if it is 1 you use it in un complemented form C, that is a first process 0 0 1 so, you write A bar B bar C.

Second one says 0 and 0 A bar B C bar 1 0 0 A bar C and finally, 1 1 1 A B C right. This is how we can write down the expression for this sum switching expression directly from the truth table, this is very simple right you see which are the 1's and then write down, we shall again be coming back to this later as at said ok.

Now, let us come to the carry, look at the carry function again for the carry also you see, there are four 1's 1 2 3 and 4. So, if you follow that same principal what will be the expression 0 is 0 1 1.

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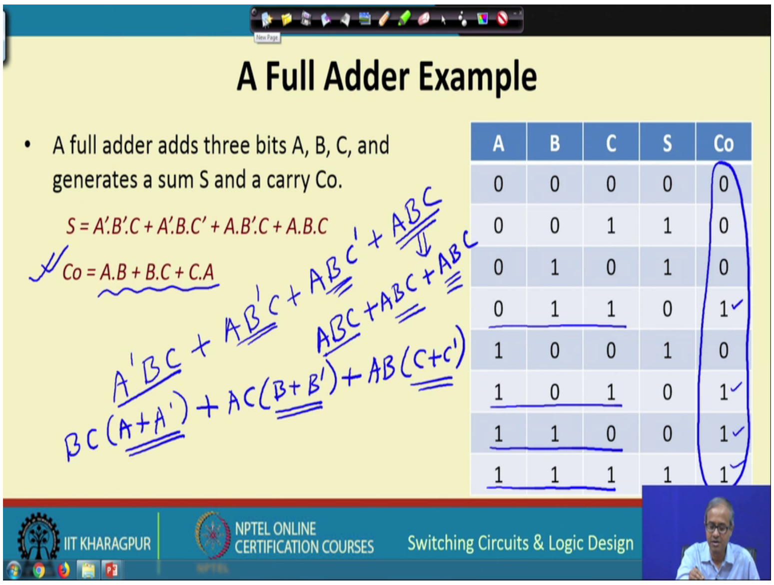

So, it is a bar B C plus second 1 is 1 0 1 A B bar C plus, third one is 1 1 0 A B C bar and 1 1 1 plus A B C. So, I took the same example earlier once, this A B C I am writing as A B C, or A B C, or A B C, I can write like this there is a rule of switching algebra which allows me to write something as same thing plus same thing. And I combine A B C A bar BC with the first one, this with this and this with this. This A bar BC and A B C if I take BC common, it becomes A or A bar A from here and A bar from here, from the second two if I take A C common, I get B from here and B bar from here. And lastly from this and this, if I take A B common I get C plus C bar.

Now, A plus A bar is 1 B plus B bar is 1 C plus C bar is 1. So, I get finally, A B or B C or C A. So, what I am showing here is the same function, but in a minimized form A B, or B C, or C A ok. This is what a full adder is.

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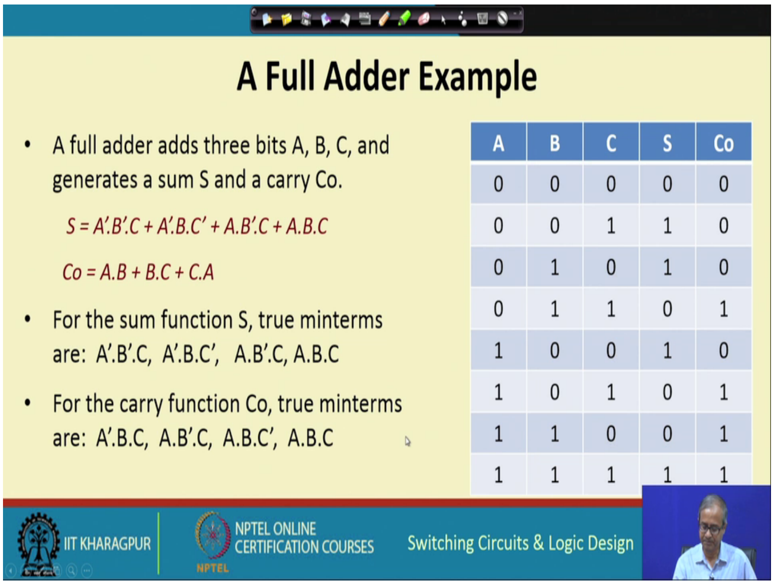

So, talking about the true minterms for this sum there are 4. So, the true minterms will be these 4, corresponding to the rows where sum is 1. Similarly for the carry function the 4 true minterms will be 0 1 1 1 0 1 1 1 0 and 1 1 1 that corresponds to this.

So, the similar way you can just find out the maxterms ok, say for the 1 maxterm will be A bar plus B bar plus sorry C, second one it will be A bar plus B plus C bar this one will be a plus B bar plus C bar and so on fine.

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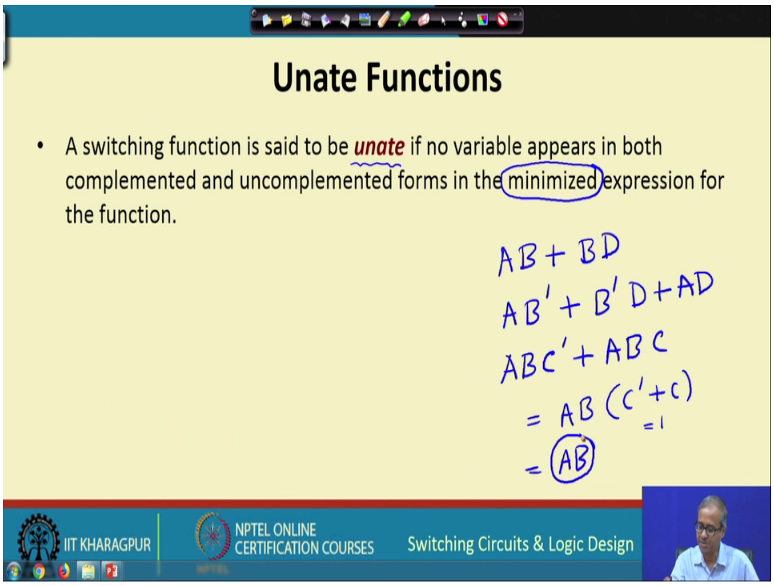

Now, let us introduce some more definitions, there are something called unit function, you see means we had defined what is the switching expression, we talked about literals, literal is a variable which can appear in complemented form also in uncomplemented form. We say function is unit, if for every variable let us say for a for a variable A will A will means A will appear, only in complimented form, or in uncomplemented form not in both that will be called the unit function.

So, let us see the definition switching function is called unit, if no variable appears in both complimented and uncomplemented form. Just you note this term minimized expression well, I will take some examples. Suppose switching function A B plus B D. This is unit because everything appears in uncomplemented form take another example, A B bar plus B bar D, this is also unit let us take another term let us say A D. So, a appears in uncomplemented form, B appears only in complemented form, D appears only in uncomplemented form, this is also unit.

Now, you consider an function like this, A B C bar or A B C. Now, you will say that well this is not the unit because C is appearing in both complimented and uncomplement form, but I will argue that this expression is not minimized. So, if I take A B common, there will be C bar or C and C will cancel out this is equal to 1. So, this will be only A B and A B is unit right. So, this minimized this term is important.

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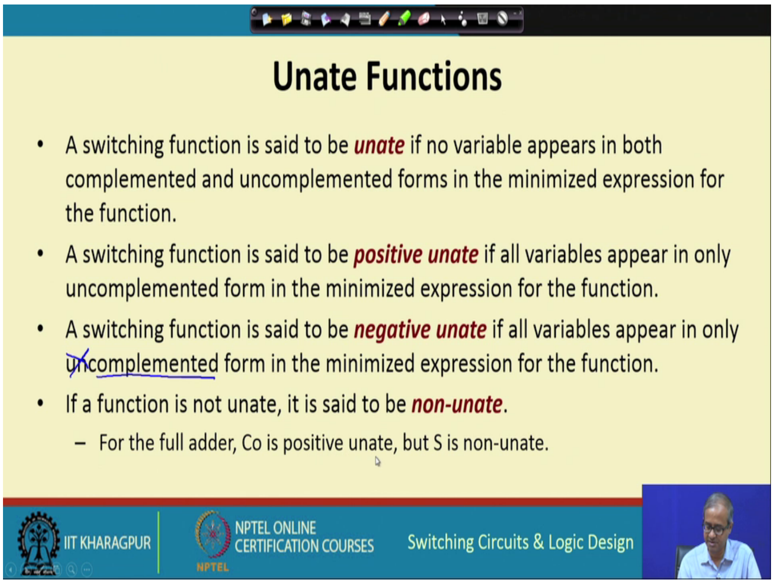

There are two special categories of unate function. So, I say a function is called positive unite, if all variables appear in only uncomplemented form, there is no complemented literals. Similarly I can defined negative unite, if all variables appear in only ok, this will be complimented sorry, this is a type of this should be complimented.

So, if it appears only in complemented form, then it is called negative unate right. And if it is not unite then it is called non-unite. So, for the full adder function the carry is the positive unit, if you can check A B or B C or C A, but for sum the variables appear in both complimented and uncomplemented form, that will be non unate fine.

Now, let us talk about some unique way of representing functions, you see we said that a function, or a switching function is given to you, you can apply the rules to carry out minimization. Now, minimization can be carried out in a number of ways well, I mean if you can figure out which rule to apply may be we are able to minimize it very well, but it may also a possible that we have minimize it to some extent, but then we have missed out some rules.

So, we have a reduced expression, but that may not be the smallest possible expression ok. So, there is the question is it possible to have some kind of a unique representation of a function, well any unique representation is sometimes called canonical representation. So, we are talking about canonical representations of functions let us see that.

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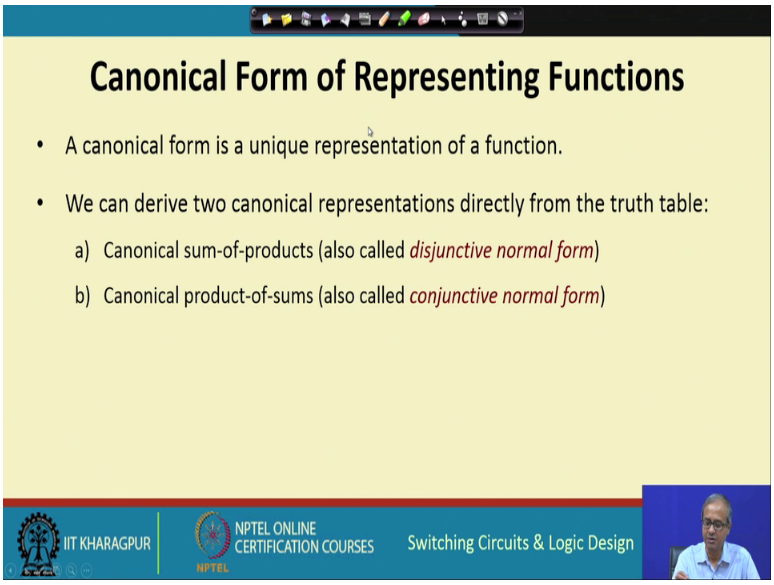

Canonical representation of switching function canonical forms, now as I said canonical form is nothing, but some unique way of representing a function. Given a truth table from the truth table we can directly arrive at two different canonical forms, one is called canonical sum of products, which is sometimes also known as disjunctive normal form, or as a product of sums product of sum term which is called conjunctive normal form.

You see normal form is a term which is used in propositional calculus, where which refers to some kind of canonical representation, some kind of a unique form of representation ok.

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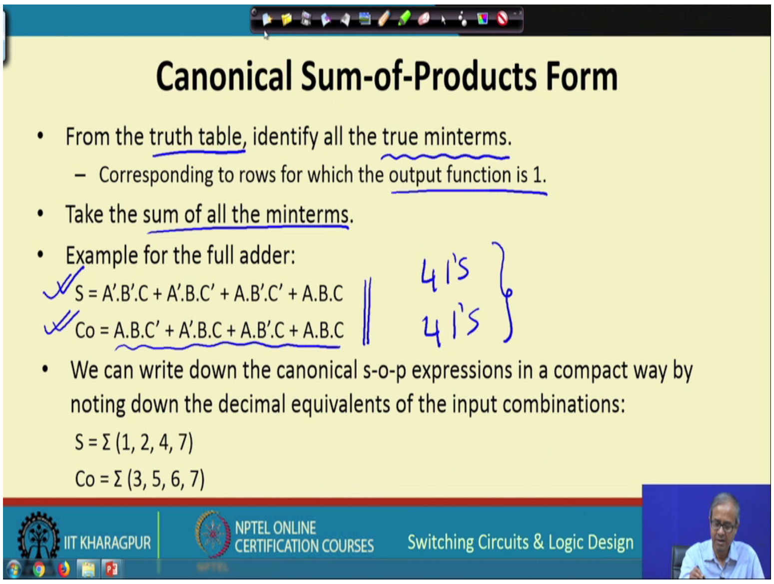

Let us first look at canonical sum of product. So, what you are saying is that, you have the truth table from the truth table you identify all the true minterms, true minterms means the rows for which the output is one. Take this sum of all the minterms. So, this we have already showed an example earlier for the full adder, for the full adder sum will be like this and carry will be like this. You take all the corresponding rows for which the sum is 1 there are 4, carry is 1 there also there are 4.

Now, what you have saying that this is nothing, but the canonical representation, you see I mentioned that the scary function can be minimized, but here we are not minimizing. If you are not minimizing from the truth table we have seen, that there are 4 1's in the sum and, there are 4 1's in the carry. So, if I list out all the 4 like this, this will; obviously, be a unique representation. There is only for it is not 3 or 5, there will only be 4 such minterms right. So, if I express a function as a sum of the minterms the true minterms, then it will; obviously, B A unique representation.

Now, we can write such canonical expression in a short form as a minterm, using the sigma or summation notation, well I will explain it with the help of the truth table that how it is written let us check this.

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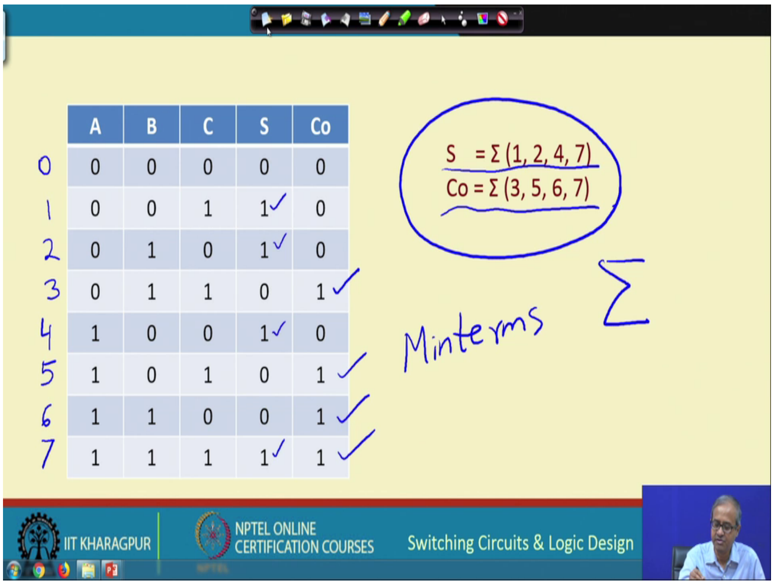

Let us consider again this truth table for the full adder, what we do is for every row of the input, I write down the decimal equivalent of the binary the input 0 0 0 is 0 0 0 1 is 1 0 1 0 is 2 0 1 1 3 1 0 0 is 4, 1 0 1 is 5 1 1 0 1 1 1 0 2 7, for this sum you list out which rows are 1, it is 1 2 4 and 7 so, I write it like this summation, or sigma notation sigma means sum 1 2 4 7.

Similarly, carry 3 5 6 7 carry is 3 5 6 7. So, this is a compact way of representation of canonical sum of products, this 1 2 4 7 or 3 5 6 7, they all represent nothing, but sum minterms of the function, they represent 1 row of the true table right. This is one way we and which you can represent.

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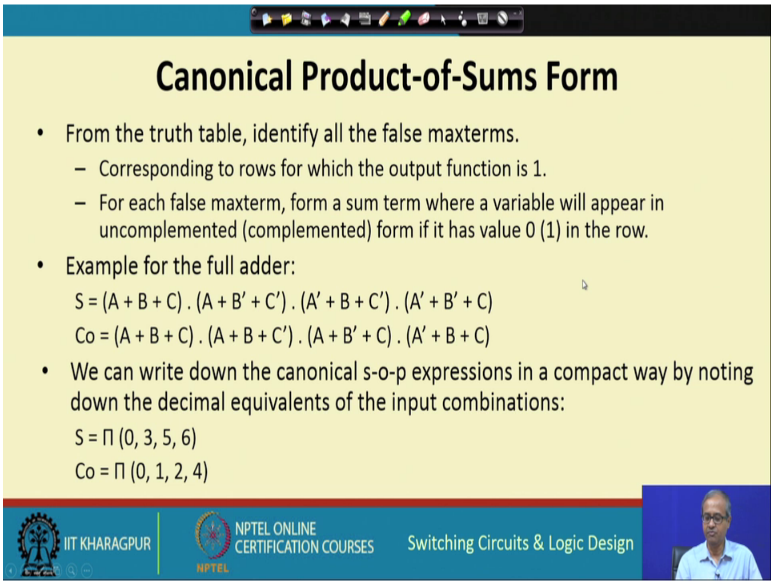

Similarly, we can have canonical product of sum form. So, what is the rule here again rules are very similar, the only difference is you identify the false minterms, earlier we had identify the true minterms, you find out the rows for which the outputs are 0 ok, false minterm means this will be 0 not 1 0 right.

Then for every false minterm you form a sum term, but now the sum term are reversed in polarity that means, if it is 0 in the truth table, you use and in uncomplemented form, if it is 1, we use it in complemented form, I will take that example of the full adder and explain right. So, the expression for the full adder are shown let me go to next slide show, the example and then again come back. So, the truth table is shown here right.

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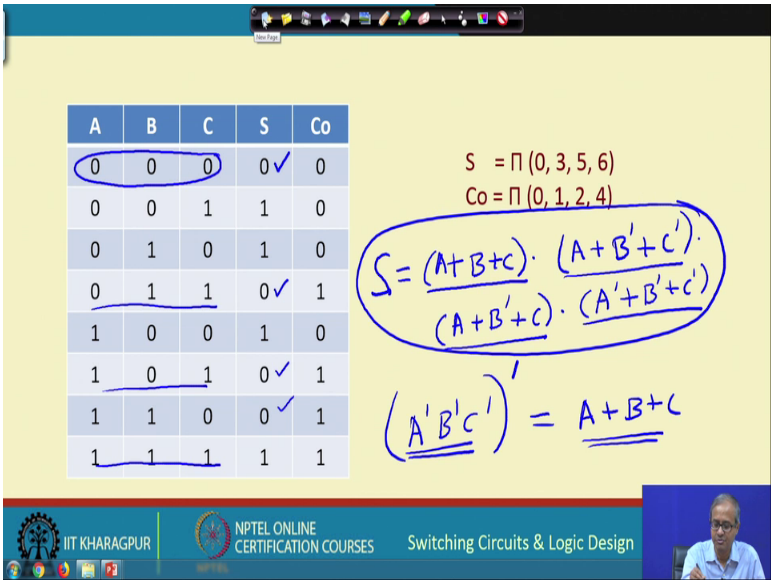

So, what you are saying is like this suppose I want to write down sorry, the expression for sum, I want to write down the expression for sum. So, what I do here is that here, I look at the false maxterms. And we see it is 0 0 0, if it is 0 I use it in uncomplemented form. So, here it will be A plus B plus C second term 0 1 1, it will be A or B bar or C bar and third one is 1 0 1 A B bar or C and lastly 1 1 1, it will be A bar plus B bar or C bar. So, what is this actually mean you see, here we are talking about the false minterms right, but when we comple false minterm means what that when A bar B bar C bar, this condition is true A 0, B 0, C 0 then sum is 1.

But if either A is 1, or B is 1, or C is 1 like a I write it like this, then this will be 1 right. So, it is not A bar B bar C bar see just applied De Morgan's law A bar B bar C are not of that is nothing, but A or B or C. So, we are actually applying the reverse 0 0 0 is 0 means not 0 0 0 is 1 not 0 1 1 is 1 not 1 0 1 1 not 1 1 1 is 1. So, this and this and this and this, this will represent my function sum.

Similarly for the carry so, I can do the same thing. So, if you go back to the previous slide. So, here we have written down the expression for the sum and also the carry, there are obtain in a very similar way, sum I have shown so, you can see that the carry also can be obtained.

Now, again when you write this down again from the truth table, in a short form you can write it in this way in a product notation using the pi notation, so again by noting down the decimal equivalents.

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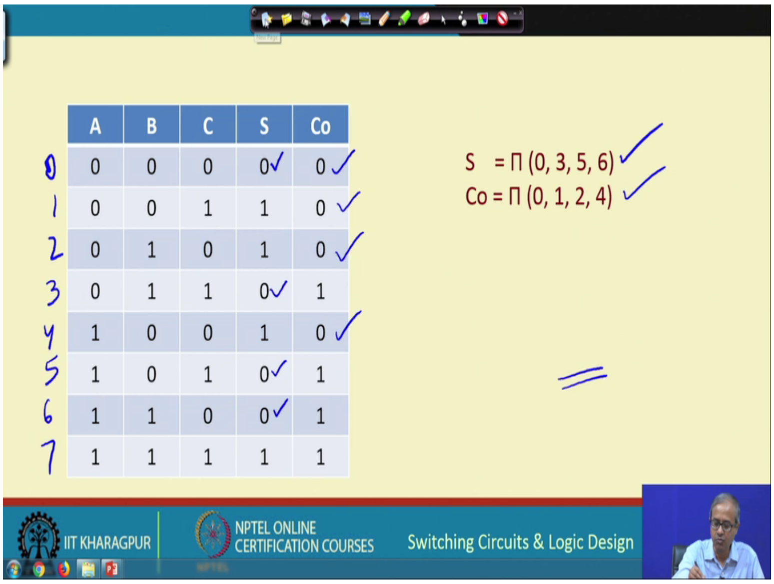

So, let us see from the truth table once again here, if you again note down the decimal equivalents 2 2 3 4 5 6 and 7. So, sum will be 0 for 0 3 5 and 7 6 0 3 5 6 and carry will be 0 for 0 1 2 and 4 0 1 2 4. So, you see this is a very simple way, in which starting from the truth table I can directly write down sum of product, or a product term sum expression, which is canonical; canonical, means it consist of either a sum of sum minterms, or it consists of a product of sum maxterms.

This is as at said is called canonical representation. So, now, you know that from a function specification in the form of a truth table, how to write down the expression, either in sum of product, or in product of sum later on we shall see, how we can manipulate this forms in various different ways, ok.

So, with this we come to the end of this lecture. So, we shall be continuing with our discussion in the next lecture.

Thank you.