Switching Circuits and Logic Design
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Lecture - 12
Algebraic Manipulation

So, continuing with our last lectures discussion, we shall be talking about Algebraic Manipulation in the present lecture. We already seen some examples of algebraic manipulation that using the basic rules that we had introduced we had learnt, we can apply them in a systematic way to try and prove sum given expressions left hand side and right hand side are given, we try to show that their equivalent ok.

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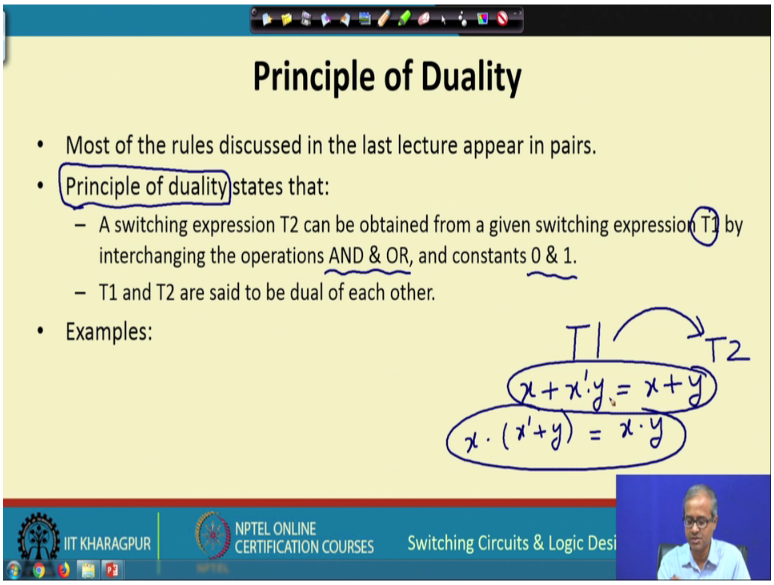

So, let us start with this, the first thing that we talk about is something called principle of duality.

Well, knowingly or unknowingly we have already got introduced to this principle. So, what does this principle of duality say, principle of duality says that well suppose I have a switching expression T 1 given to me. So, I have this T 1 given to me it says, I can transform this expression or this identity into another expression T 2 by following some rules. So, what are the rules? I interchange the and or operations, I interchange 0 and 1 right like well we have already seen some rules, let us very quickly recall some you see we talked about a rule if you recall, x or x or y equal to x plus y.

This was one of the identities that we got introduced in the last lecture. Now, suppose if we apply the principle of duality, left hand side we replace dot and plus this is dot. So, what the left hand side will become it will become x dot x bar this dot will become plus right, hand side it will become plus will become dot. So, if this is true, this will also be true this is what principle of duality says right.

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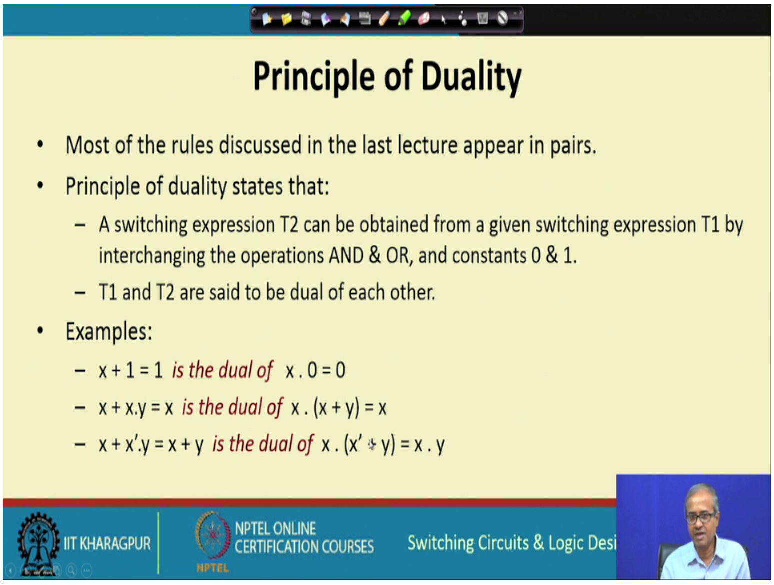

So, some examples x dot 0 x and 0 is 0 already known. So, just following this rule dot will become plus 0 will become 1 0 will become 1 this is also true.

This is the dual of this. Similarly this was one of the rule dot replaced by plus plus replaced by dot, this was another rule, this was given this I just mentioned this rule. So, you see because of duality, when we presented the rule in the last lecture most of the rules are coming in pairs, pair of rules one is the dual of the other ok. So, if we prove one the principle of duality says the other will also be proved. So, you need not have to prove both, you prove 1 the other will follow from that this is what principle of duality is.

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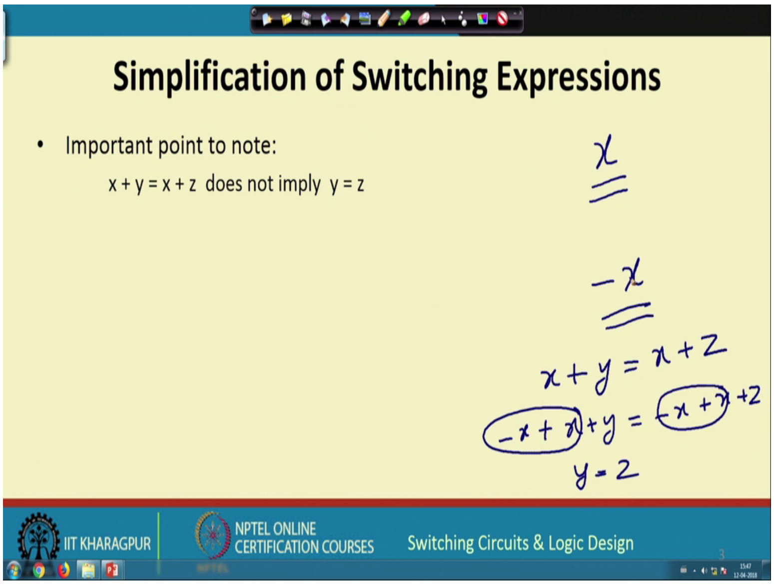

Now, talking about simplification of switching expression, just you need to remember one thing, this is in contradiction to our conventional algebra.

So, when we carry out conventional algebra, the thing the algebra that you studied in your school it says, that if I have x plus y equal to x plus z, then I can cancel x from both sides I can say y equal to z, why was that if it was because the inverse of a number exists sorry. Suppose when I have a number x in algebra the inverse of this number minus x in our conventional algebra. So, when we can add an expression like x plus y equal to x plus z. So, what we did? We can add this same thing on both sides, we can add minus x to this side, we can add minus x also to the other side.

And these two got cancelled out these two get cancelled out hence, we could write y equal to z, but in switching algebra the inverse of a variable does not exist, there is no concept of minus of a variable ok, because of that you cannot say if x plus y equal to x plus z you cannot say y equal to z.

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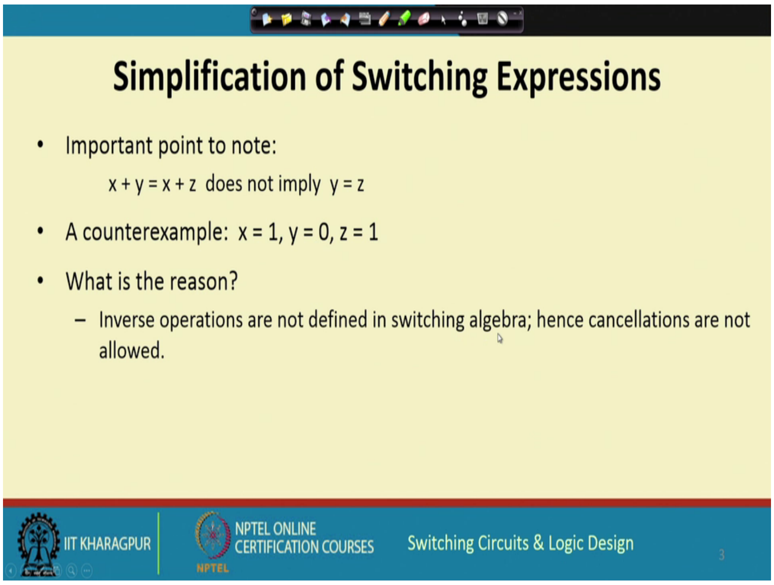

Just a simple counterexample let us say x is 1 y is 0 and z is 1. So, what is x or y, x or y is 1 or 0 is 1, what is x or z, x or z is also 1 plus 1 or 1 is 1. So, this is true, but does it imply y equal to z you see y and z are different, you cannot simply cancel x like that from here ok.

So, in switching expression minimization or simplification you should not cancel variables like this. You should strictly limit yourself to applying the rules that you have learnt and nothing outside that ok. You forget what algebra you studied in your school do not try to apply here, if you apply here your result or deduction will be wrong ok, this way you have to remember. This is what I mentioned the main reason is that inverse operations are not defined switching algebra, you cannot cancel variables just like.

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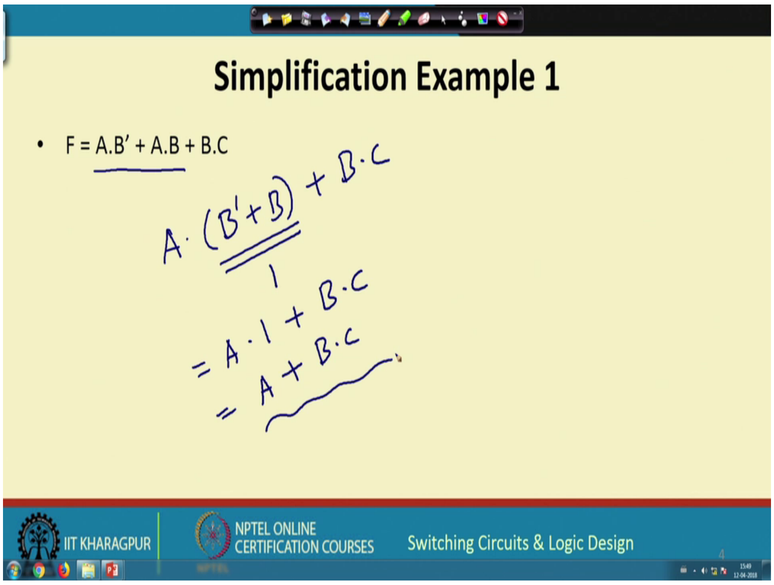

Let us work out some examples very simple examples. Suppose, I have expression A B bar A B plus B C which you want to simplify.

Very simple I can apply distributive law on the first two, I can take a common B bar or B B C remains and B bar or B I have an identity this means 1. So, this is A and 1 plus B and C A and 1 I have another rule this is only A, A or B C. So, this expression get is reduced to A or B C right. Let us take some more examples.

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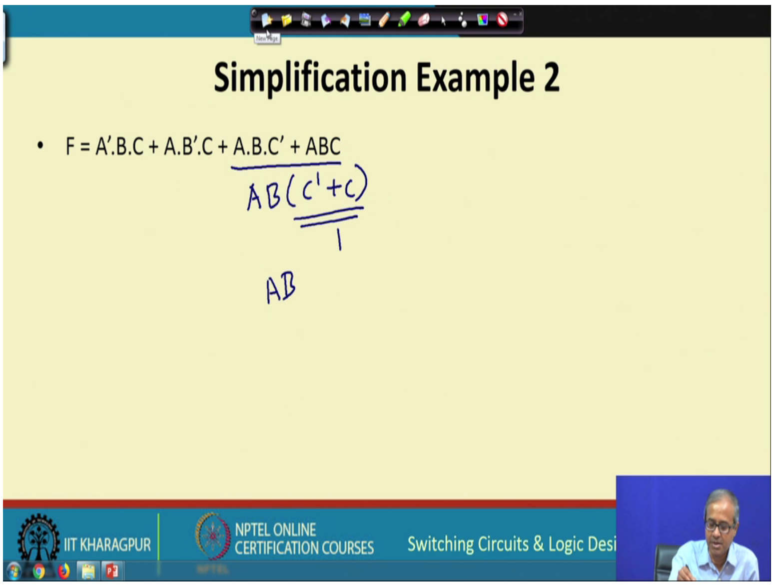

Let us take an example like this, which is slightly more complex in the sense that you want to minimize it you see, let me try it first like this. You look at this expression you see this A B C got an A B C has a B in common.

So, if you take A B common you get C bar or C C bar or C is 1 A B and 1 this is only A B, but can you simplify it any further. So, you do not see any apparent rules, but there are another rule, but let me try it in a slightly different way that is of course, another rule you can apply, but let me try it in a slightly different way.

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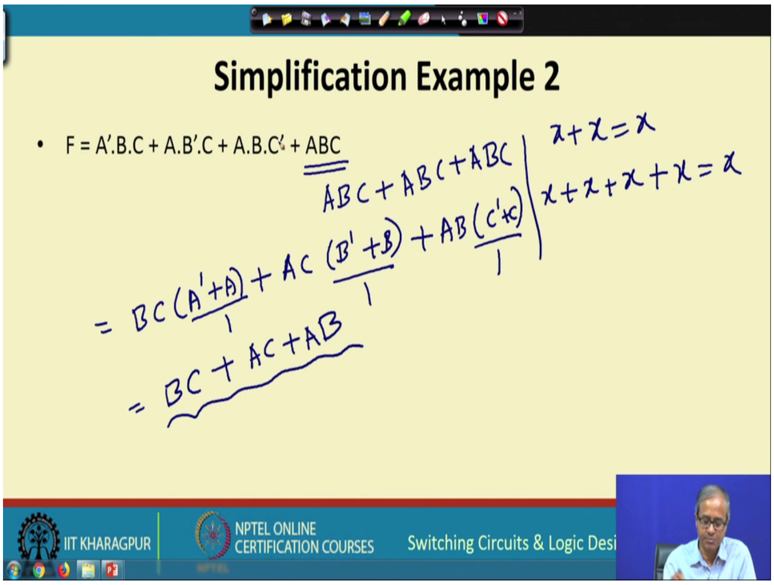

So, what I am saying is that while, you had a rule remember x or y equal to x. So, if you have x you can create any number of times you can also, repeat this once more x plus I mean any number of times this will be equal to x 0 a 0 a 0 a 0 is 0 1 or 1 or 1 or 1 is 1. So, what I am doing is that, this ABC I am writing as ABC or ABC or ABC because, there are three terms I will group one of them with one each. So, the original expression A bar B C and A B C I take 1. So, B C I am taking common A bar or A. Second A B bar C and this second A B C ac I am taking common last of course, A B you take common C bar or C.

So, this is 1 this is 1 this is 1 BC and 1 is BC this will be AC, this will be AB, this will be your minimized expression right. So, there are multiply ways to proceed this is one way ok. This will be the minimized expression corresponding to this. Let us take another example again.

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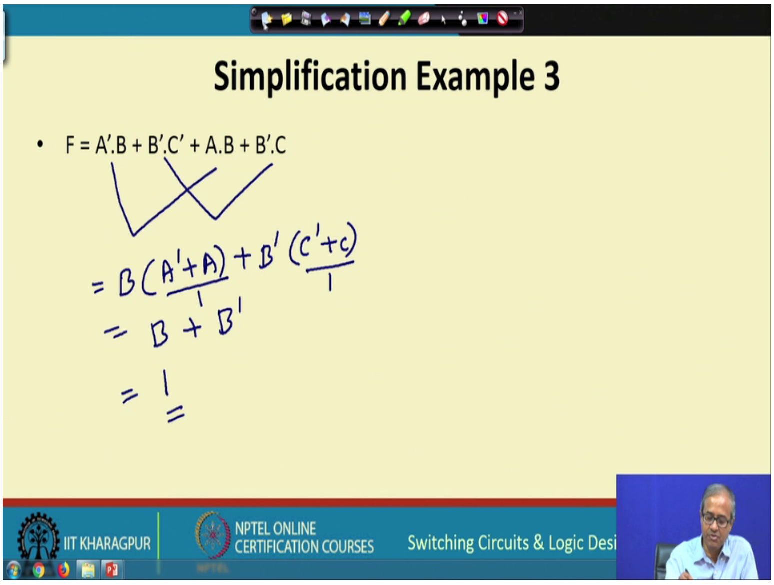

Let us take this, this again you can see here, that what are the things you can combine, these two you can combine, these two you can combine. You see A bar B and A B, if you take B common it will become A bar or A. The last two, if you take B bar common it will be C bar or C.

So, again A bar or A is 1 C bar or C is 1 B and 1 will be B, B bar and 1 will be B bar and B or B bar is 1. So, it minimizes to only 1 A constant expression right. So, in this way you can simplify any given expression using the rules, if you know how to apply.

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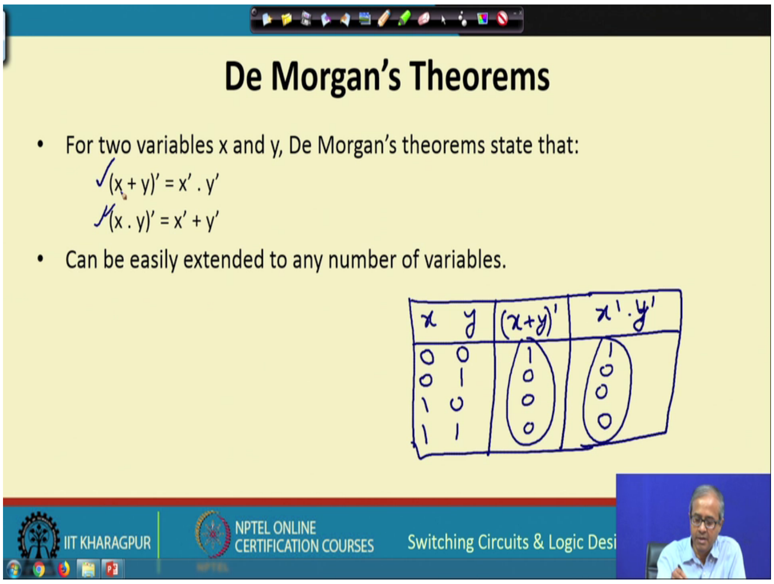

Now, this is a very important law that you can apply for transforming expressions in variety of ways; this is called De Morgan's theorem. This is the statement of De Morgan's theorem in 2 variables well you can extend it any number of variables.

It says x or y not of that is the same as x not and y not and, just the dual of this x and y not of that equal to x not or y not. Let us say let us look at the first one, we can easily prove it by perfect induction just show it x and y. So, you have the different values 0 0 0 1 1 0 and 1 1. Let us show the two different expressions x or y bar of that and also, let us show x bar and y bar.

Let us first take odd of x and y and, then not or 0 what 0 is 0 not of that is 1 0 or 1 is 1 not of that is 0, or is 1 naught or is 1 naught. Here you are taking not of both and then, do reading 1 and 1 and this one 1 and 0 and is 0 0 and 1 and is 0 0 and 0 and is 0 you see these two are same. Similarly the second one you can prove.

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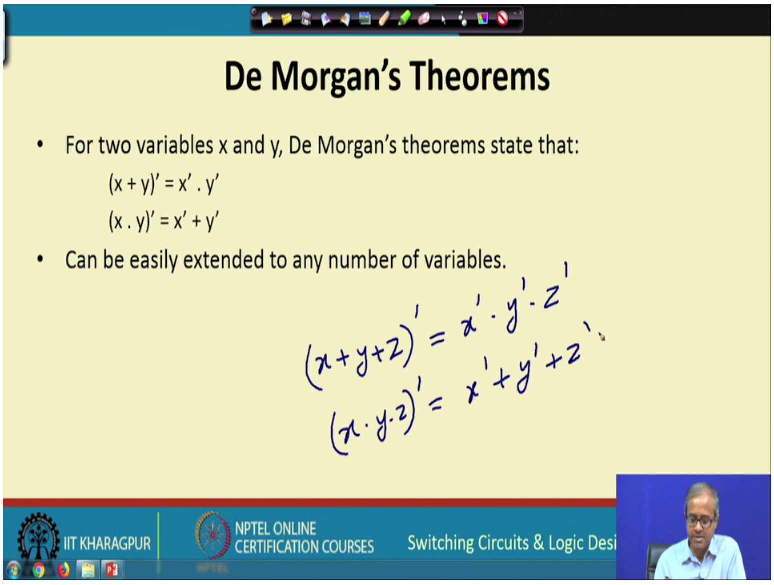

Now, when I say it can be extended to any number of variables and you are showing one for three variables you can write like this, x plus y plus z suppose there are three variables, if there is A bar this will be same as x bar and y bar and z bar. Similarly x y z bar will be equal to x bar plus y bar plus z bar right. This can be extended to any number of variables you want.

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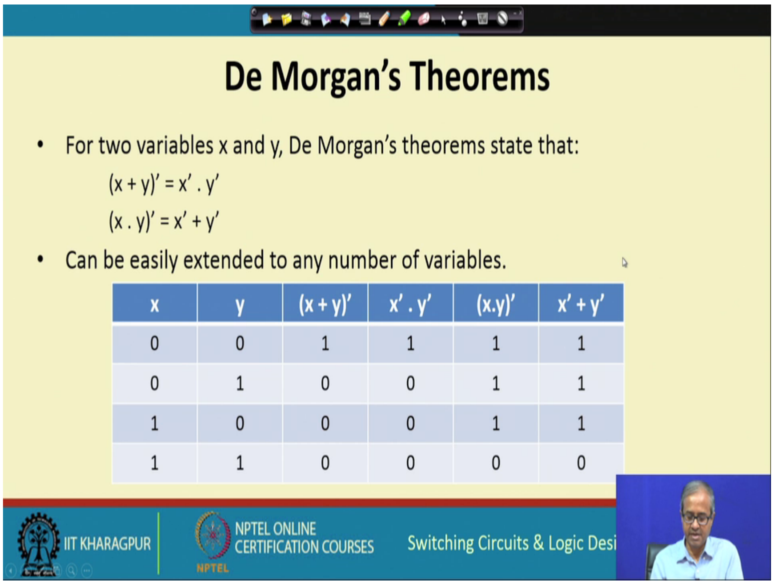

This exactly the truth table I worked out this is shown here. So, for all possible value x and y the left hand side is shown, right hand side is shown also these two are shown. So, these and these are identical for the second one this and, this are also identical right ok.

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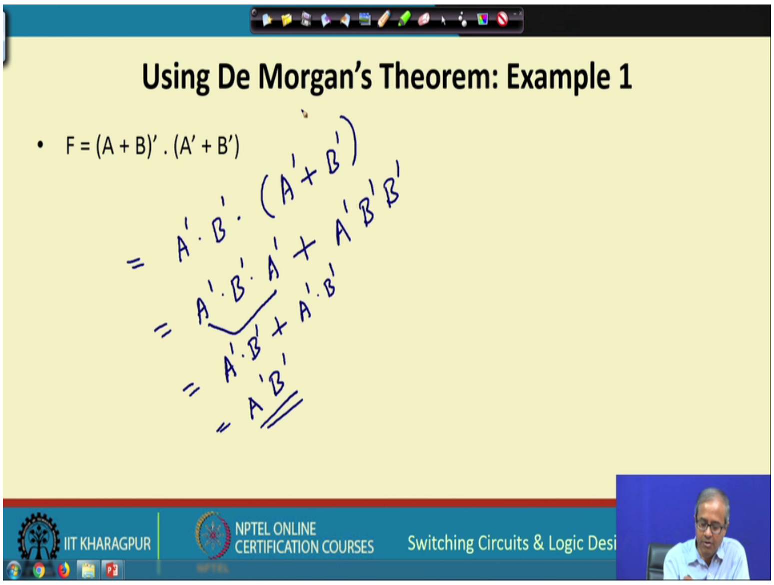

Now, let us just work out some simple example here, we are trying to do some simplification or transformation of the expressions. Now, for this kind of expression you have to use De Morgan's theorem, because you see you have an expression here A plus B bar the other one is fine. So, A plus B bar you apply De Morgan's theorem, it becomes A bar B bar, then you have this A bar or B bar.

Then you multiply it you can use the distributive law A bar B bar into this. So, A bar B bar A bar, or A bar B bar B bar this dot as I said sometimes you can drop. So, it implies you see here. this there are 2 A bars right A bar and A bar, this is associative B bar A bar you can write A bar B bar you can bring it here, then A bar A bar means only A bar. Similarly here also B bar B bar you can make it B bar this is only B bar and, something or that same thing it means that same thing A bar B bar, this is the final expression right. You see unless you new De Morgan's law, you could not arrived at this expression it would be quite difficult right. So, De Morgan's law is also very important.

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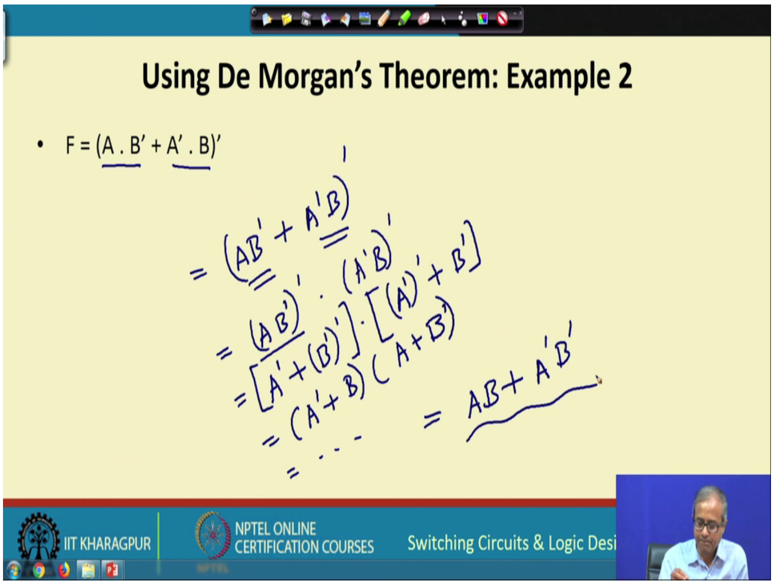

Let us take another example this is a slightly more complex example. So, this is something plus something I write it without dots it is easier, I write A B bar or A bar B whole bar. So, this is something plus something bar I apply De Morgan's law, this something dot and this something bar.

So, I apply De Morgan's law again on this it will be A bar or B bar bar and, I apply De Morgan's law on this same way, it will be A bar bar or B bar so, this becomes A bar B bar bar means only B A or B bar, this you can multiply to again. So, this way you can proceed I am showing it partially. So, the final thing if you proceed the final result there will be getting for this one, this will be equal to a B or A bar B bar this will be the final minimized expression right ok.

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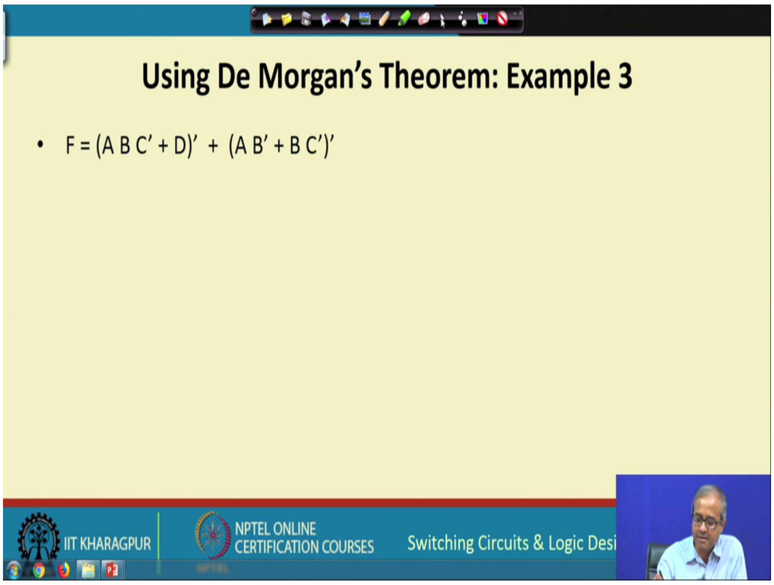

This is another expression I leave it as an exercise for you, you see there is a term here, you can apply De Morgan's law here there is another term here you can apply De Morgan's law here, or it out when then simplify again you can apply some of the rules, whatever minimization you can do you can do right. So, this way you can apply the rules wherever you can and you can carry out the minimization. This one last example let us just work out this.

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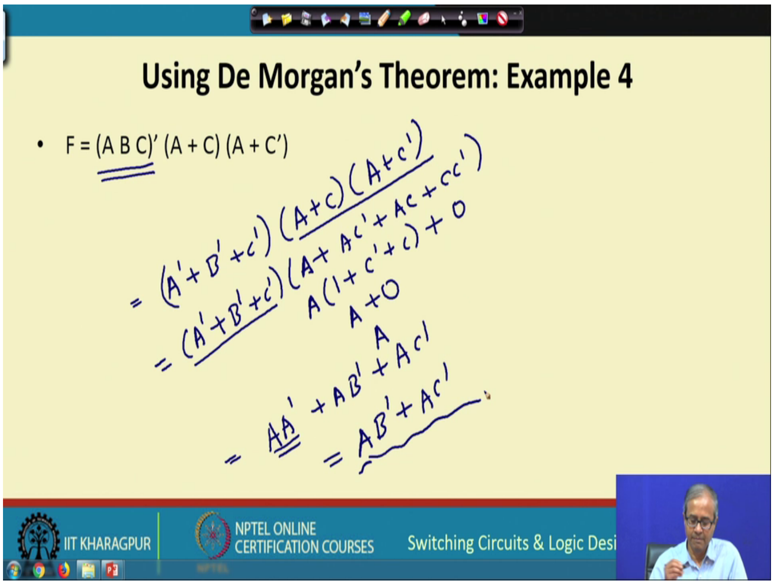

Here the first one you need De Morgan's law, it becomes A bar or B bar or C bar the other 2 are A or C A or C bar, let us multiply this out well you can use the rules of multiplication for expansion. Let us suppose I first expand multiply these two out, the first one let us leave, we do not disturb this A and A I am just doing a simplification A and A is only A, A and C bar is A C bar C and A is A C or C A whatever commutative C and C bar C C bar. Well I am only writing the second part. First part is same you see A A A in the first three you can take a common 1 or C bar, or C and C C bar is nothing but 0 and 1 or anything is 1.

So, this is A and 1 this is A or 0 this is only A this becomes only A. So, this multiplied by A and so, you just and it with this A A bar plus A B bar plus A C bar, this again is 0 so, your final expression is A B bar or A C bar, this is your minimized expression right.

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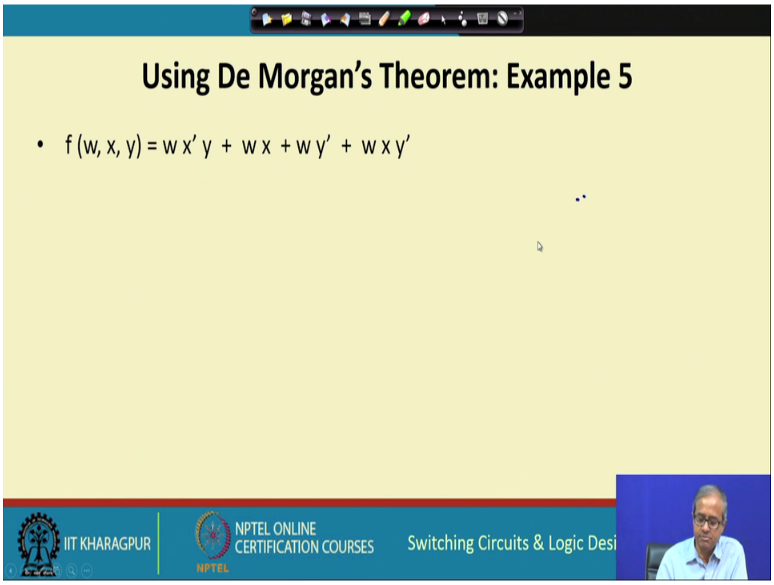

So, here we had looked at a number of rules and ways to minimize it. So, this one more expression I leave it for you to minimize and so, just try to work out ok.

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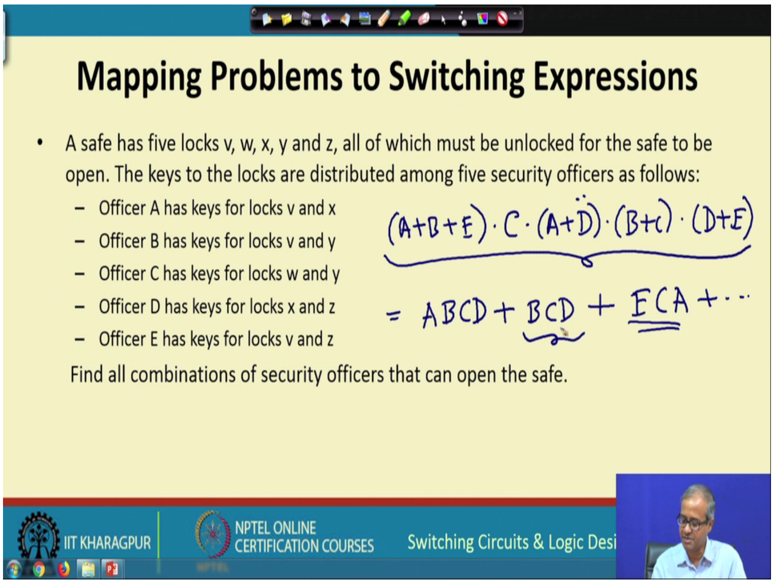

Now, let us look at some practical problems and see the how we can convert it into a switching expression, I am just giving you two examples here. Let take an example a safe has 5 locks v w x y z let say and to open this safe you have to unlock all 5 of them and, what we are assuming is that the locks of the keys are distributed among 5 security officers A B C D E like this. A has the keys for locks v and x, B has locks v and y, C has

2 each. So, we want to find out what are the combination of security officers that must be present. So, that they lock can be opened.

You see there are 5 locks I need all of them right, let us look at one by one, v who is having v, v is a B and E I write it like this A or B or E, this is the first requirement for lock v. Then w who is having w only C so, C is must C must be there then x who is having x A and D, A or D, then y y is B and or C B or C. And finally z D or E you see this is a switching expression you have created. Now, you can multiply this out I leave it an exercise for you, multiply this out let us say one of the term you will see is A B C D, I am showing you couple of terms. Another term you can see like this B C D.

Another term you can see like ECA like that and so, on this means there will be on more terms this means, that if security officers BCD are all their all three of them, then this 5 keys are available you see B is having v y, C is having w y and D is having x and z. So, all 5 are available. Similarly ECA so, if you write down expression like this you can find out all combinations of security officers that can open the safe right.

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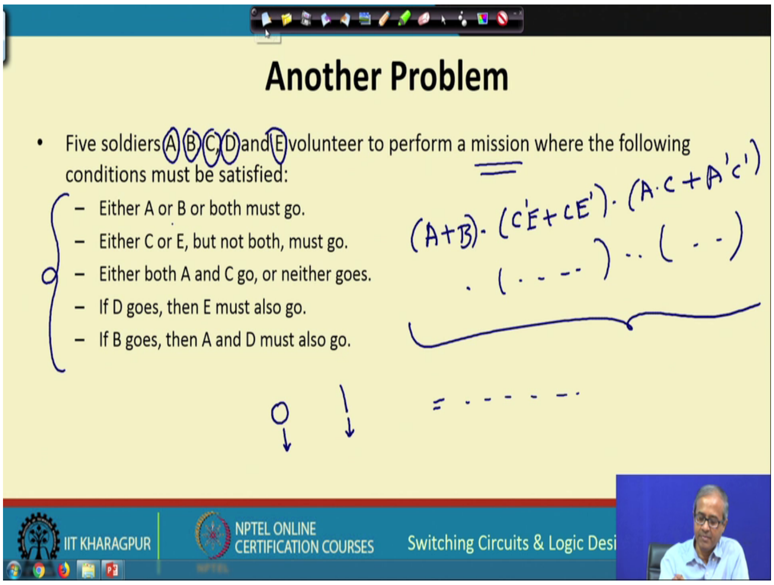

Let us take another such problem there are 5 soldiers let us say A, B, C, D, E they want to go for some secret mission some mission. And the conditions are as follows either A or B must go, or both must go either C or E but not both must go, either both A and C goes or neither goes, if D goes then E must go if B goes then A and D must also go.

You see here also the condition you can write very in a very similar way, it says either A or B or both this is nothing but the or function I can write A or B A or B or both will make the or function true, either C or E but not both; that means, not both either C or E. So, how will you write this, you will be write this as C bar E or C E bar that means, C is false E is true; that means, C is not going E is going or C is going E is not going. Either both A and C goes or neither goes this is both A and C goes or neither goes, like this you can write down the expression I will leave as an exercise for you to complete the rest ok.

This kind of problem if it is given, you can map it to a switching expression why, switching expression because, you see in this problem each of the variables A B C D E if you treat them as variables, these are binary variables either 0 or 1, 0 means this soldier is not going 1 means the soldier is going ok. So, for all such problems if you have a very well defined specification provided, you can write down such a switching expression minimize them. And, when you get the expression you can find out what sequence of soldiers, or what group of soldiers can be sent for the mission ok, I also leave this as an exercise for you can try this out ok.

So, with this we come to the end of this lecture. Now, one thing we saw in this lecture towards the end that, given a problem you can write down an expression for that well of course, the expression can be pretty large in some cases. And if I give a large expression and, if I ask you that we apply the rules that we have learnt to try and minimize it, sometimes it may not be a very easy task. You may have to work out for multiple pages and pages to minimize simplify very large expression.

So, there should be some more systematic way to approach this problem. So, we shall later on see that, there are some more systematic method for minimizing such logic expressions switching expressions, which will tell you some systematic steps to be followed and if you do that you will get some expressions which are in minimized form. So, we shall be learning all these things in due course of time.

Thank you.