Switching Circuits and Logic Design
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Lecture - 11
Switching Algebra

In this lecture, we shall be starting our discussion on Switching Algebra and Switching Expressions; so, the title of this lecturer Switching Algebra. So, here we shall basically look at how we can represent so, called switching expression. See switching expressions are ways in which you can express the functionality of a circuit in a mathematical way.

Now once you can express it in a mathematical way, there will be the number of ways to manipulate them, minimize them, optimize them so that the final circuit implementation of realization will be smaller or cheaper. So, let us look into this switching algebra.

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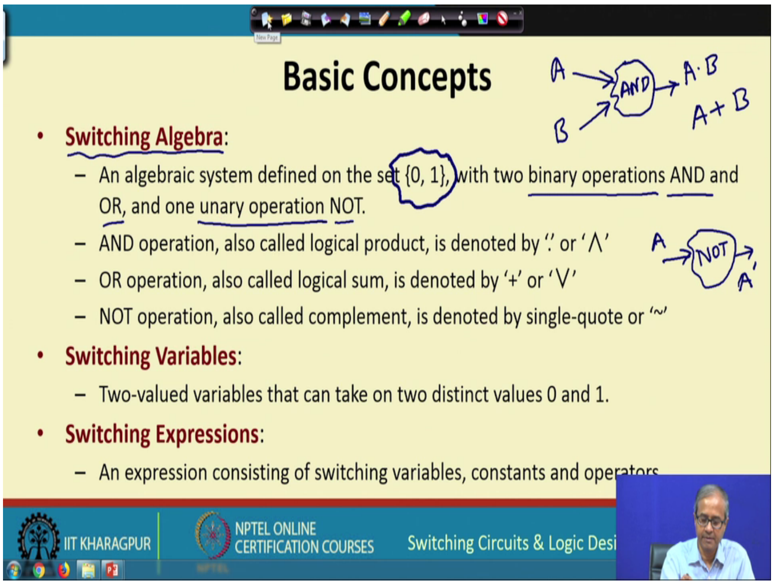

We start with some basic definitions; what is a switching algebra? You see a algebra is a mathematical structure; I am not going into the formal definition of an algebra what is an algebra. But I am mentioning the essential idea in switching algebra, you see switching algebra is supposed to be mathematical formalism which we are using to express or represent the functionalities of digital circuits, gates, binary, input and output 0 1. So, this is the basic formalism that you going to express in a mathematical way this is what we referred to as switching algebra. So, what are the basic components of a switching algebra? Let us see that first.

First is the numbers that we are working on 0 and 1. So, this algebra is defined on the set 0 and 1 everything is either 0 and 1; nothing outside that and there are 2 binary operations AND and OR and one unary operation NOT. So, what is meant by a binary operation? While an operation is said to be binary, if there are 2 inputs and 1 output binary means 2 if there are 2 inputs and 1 output I call it a binary let us say and we talked about a 2 input and gate.

Suppose I have a AND operation I have 2 inputs A and B and I have the output and A and B this is the binary operation. Similarly I can have OR the OR is also a binary operation output will be A or B what do you think of NOT? In case of NOT there is a single input right if you talk about NOT; NOT is a single input A and B and it generates as a output NOT of A. So, there is a single input which is called unary; unary means 1 there is a single input fine. So, the AND operation how to represent it?

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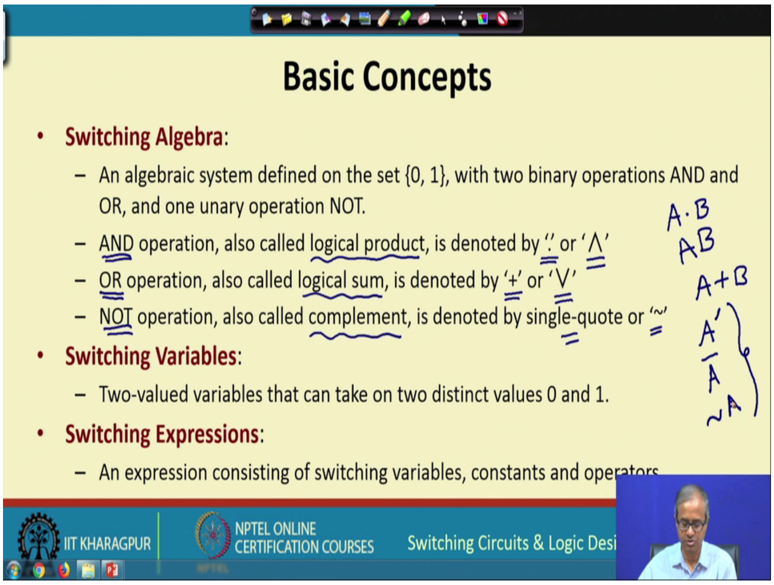

This and operation is typically represented by a dot or this symbol pointed at and let me tell you sometimes for convenience; we can also omit the dot like we can say A dot B which means AND of A and B. Sometimes we simply write A B; we do not write the dot where the dot is implied if you write A A B means it is and of A and B dot is there fine. Similarly, we have the OR operator and is sometimes called the logical product OR is sometimes called logical sum and it is denoted by the symbol plus or this reverse pointed head.

So, as I said you can express OR as A plus B this plus is the OR operator; NOT operation finally, it is also called complement is denoted by single quote. Single quote means A quote; A dash means not sometimes not mentioning also express sometimes as A bar; bar also means NOT there are various ways to represent NOT or sometimes we can write this a this also means NOT of a there are various symbols which are used to express this operations ok. So, this all represents the same thing NOT either single quote or bar or this tilde operation before this symbol NOT. So, you see switching algebra consists of this few things the numbers 0 and 1; 2 binary operations AND and OR and one unary operator NOT.

So, there is nothing outside this the basic mathematical formalism says AND, OR and NOT these are the 3 basic fundamental operations. Well we talked about other kind of gates also we talked about NAND, NOR exclusive OR, but those are not fundamental gates; those are derived gates you can say and or not are the basic ok.

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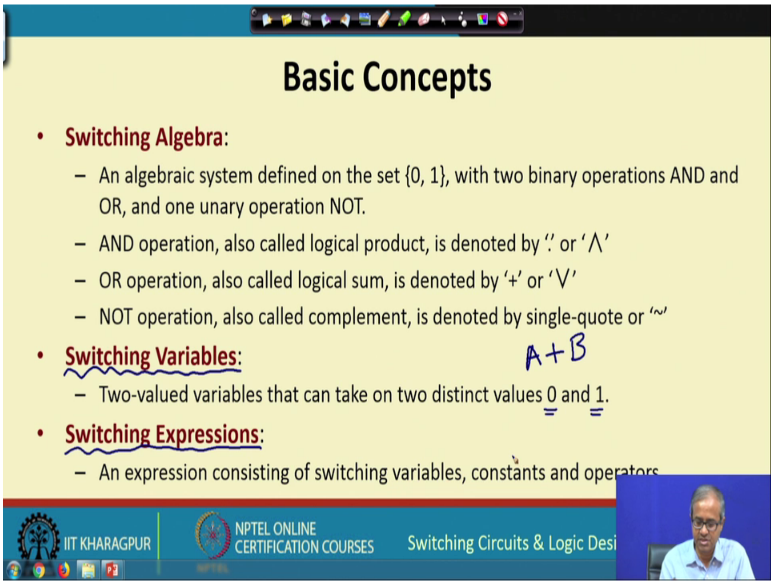

Now we talked about switching algebra; now, now any algebra there is a concept of a variables. So, when you learned algebra in your school there you talked about the variables A plus B A B C D or XYZ you operate on the variables right.

So, here also whenever you define this kind of a switching algebra, you should deal with variables these are switching variables. Like when; when I write something like A plus B, this A and B they denote switching variables. The point to note is that because we are talking about switching algebra this A and B variables can only take 2 values 0 or 1 nothing outside that. And switching expression is a general definition, it is an expression consisting of switching variables constants and operators what does that mean?

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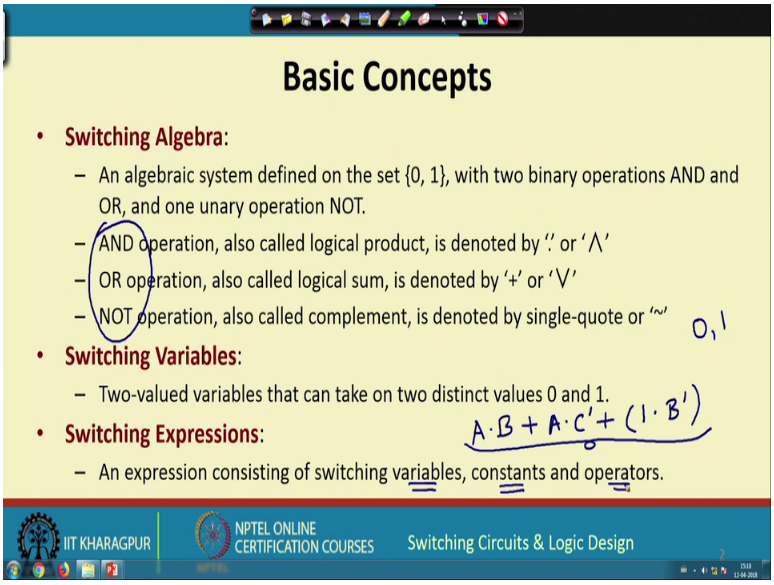

I can write an expression like this A and B plus A and C not plus 1 and B bar you see there are variables, there are constant one is a constant and operators AND OR NOT these are examples of switching expression.

Now, you observe in a switching expression basically you have nothing other than and, or and not because these are the 3 fundamental operators and, or and not well of course, you can use some brackets or parentheses and you can also use the constants 0 and 1 the expression if you require. So, there can be variables any number of variables, there can be constants and one of these 3 operators. So, the idea is very simple here switching expression.

Now we shall be talking about a number of rules of switching expression; now the point to note is that suppose I give you a switching expression how to prove it?

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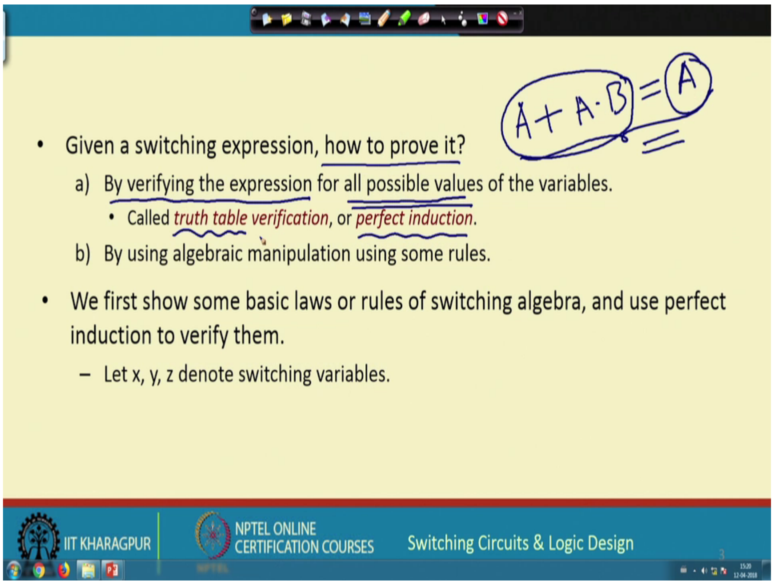

See I say let us say I give an expression I say A plus A and B; I say this is equal to A well how too say the whether this is true or false whether this is correct or this is wrong ok.

This is switching expression, this is a switching expression, this is switching expression. So, and this is a equality equal to this; so, how to prove this? You see to prove this that can be multiple ways, but the first one is a simpler method, but no tds, but right now we shall be talking about this only. We prove by verifying the expression for all possible values of the input variables this is called truth table or perfect induction method of verification ok.

See we talked about truth table also earlier when we looked at the different gates and or not. We said that I can have a truth table where I have all possible input values listed and the expected output value; that is what my truth table was. Basically here for verification I say that let us construct the truth table; left hand side let us calculate and find out what will be the output values right hand side.

Similarly, let us calculate the output values and see whether they match if they match then the 2 functions are the same they 2 expressions are the same they are verified alright. And the alternative method that we shall be learning and we will be using a little later is we can use some kind of algebraic manipulation.

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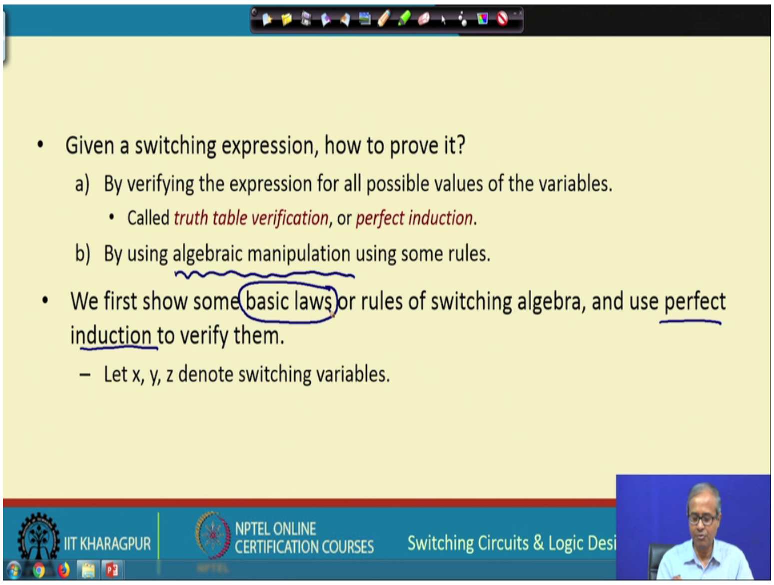

And then we try to prove and for algebraic manipulation; you need to know some basic laws of switching algebra, basic laws of basic rules. So, we shall first what to do here is that we shall look at some of these basic laws and this basic laws, we shall try to verify by using perfect induction, later on we shall be trying to apply these basic laws to use algebraic manipulation method for verifying other expressions; step by step let us proceed. And in the examples that we have in that we shall show we shall be assuming that x, y, z; they denote switching variables ok.

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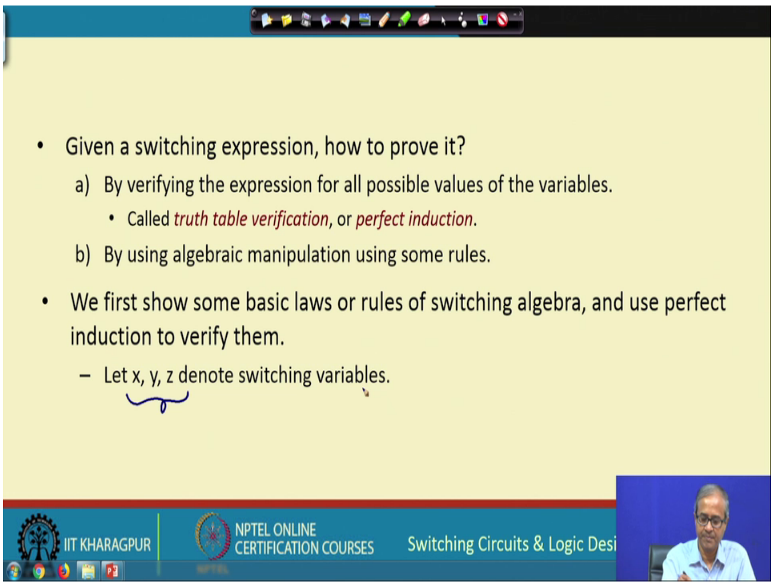

Let us see basic laws of switching algebra.

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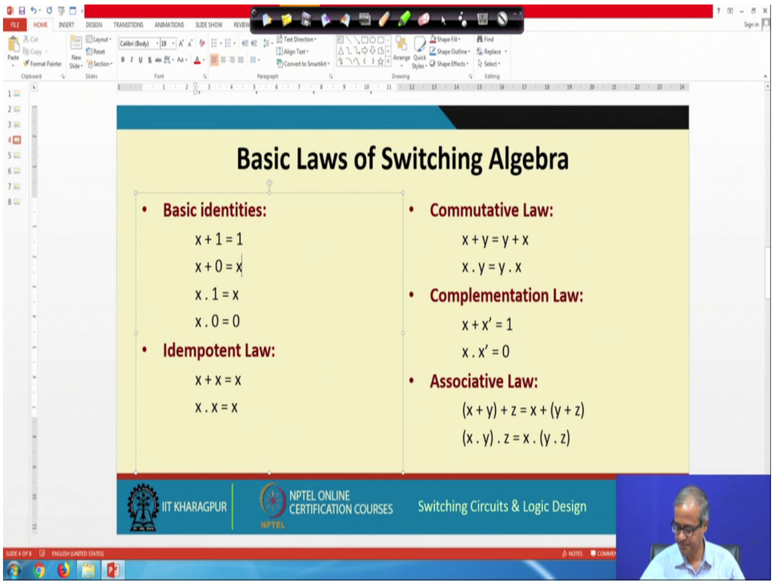

Let us look at them one by one; basic identities basic identities are go like this x or 1 is 1, x or 0 is 0, x and 1 is x and 0 is 0. You see I said that you can prove them by the method of perfect induction; it is really easy let us look at the first one. So, if we try to construct a truth table; so, here there is a single variable only x. So, x can be either 0 or x can be 1; so, I am listing 2 things I am listing x plus 1 because left hand side is x plus 1 plus is what or you look at the truth table of an OR gate for truth table of the OR gate is what was you think at the input is 0 and 0 output is 0 0 and 1 output is 1 1 and 0 output is 1 1 and 1 also output is 1.

So, here the second input is always 1; 0 and 1 the output is 1 and 1 and 1; the output is 1. So, you see that x plus 1 is always 1; so, we can always write it is equal to 1. Similarly we can verify the other (Refer Time: 13:35) same way um. So, this is quite easy to verify like this, but you should be remembering these identities that that if you if you or something with 1 the value of the expression becomes 1, but if you or something with 0 here there is a small mistake this should be x; let us correct this out fine let us correct this out this should be x alright fine this x ok.

So, let us proceed the next set of rule is called the idempotent law; what does idempotent law say? It says that is same variable if you do either and or; or and with the same variable the value remains same does not change.

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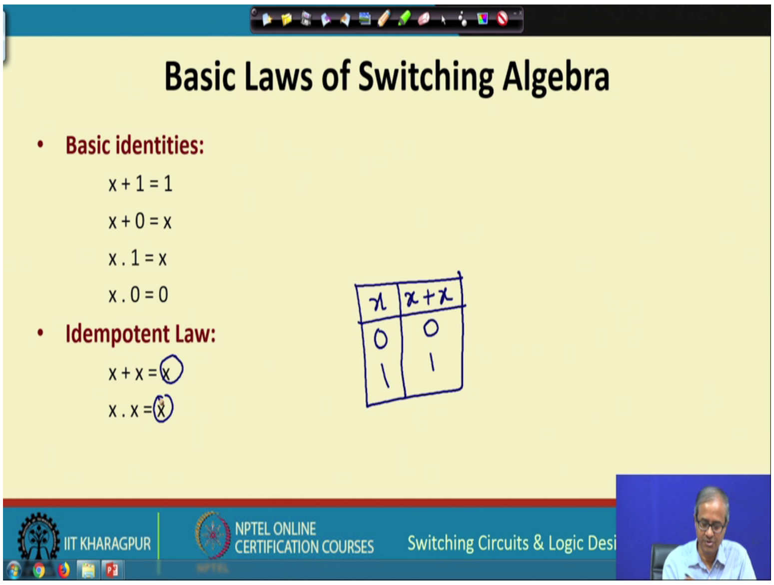

So, again you try to construct such a truth table and see; suppose the value of x is can be 0 or 1. Let us say the first one what will be the x or x? 0 or 0 to recall or get 0 or 0 is 0 and 1 1 or 1 is 1. So, you see whatever is the value of x; x plus x remains the same it is 0 it is 0 it is 1, it is 1; no change. So, the right hand side will also be x; similarly if you do and you see the same thing 0 and 0 is 0 1 and 1 is 1 same thing this is also the x ok; these are fairly simple.

Next these are fairly means obvious, but again there is some rules because you need to have some laws or rules which you can use to derive more complex expressions. It says that the order of the variable does not matter when you take and or; x plus y and y plus x they are the same x dot y y dot x and are the same.

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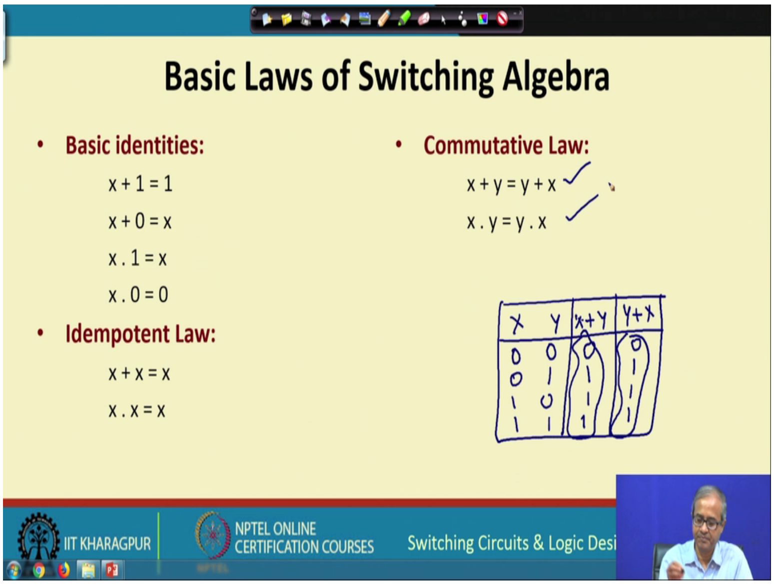

So, here again you can very easily prove by the method of perfect induction; you can have x and y these are the inputs. There will be 4 input combinations you can compute x plus y and you can compute y plus x; x plus y means sorry 0 plus 0 0 or 0 plus 1 is 1 1 plus 0 is 1, 1 plus 1 is 1, but if you do the other way round the definition does not change 0 or 0 is 0; 1 or 0 is again 1 0 or 1 is again 1 1 or 1 is again one you see these 2 are identical.

So, this is true and for and also you can similarly prove they are the same right. So, these are the laws you need to remember because later on when you do algebraic manipulation, you will be using this. Next is complementation; complementation means you take a variable and its compliment x and x bar; if you take or the result would be 1, if you take and result will be 0.

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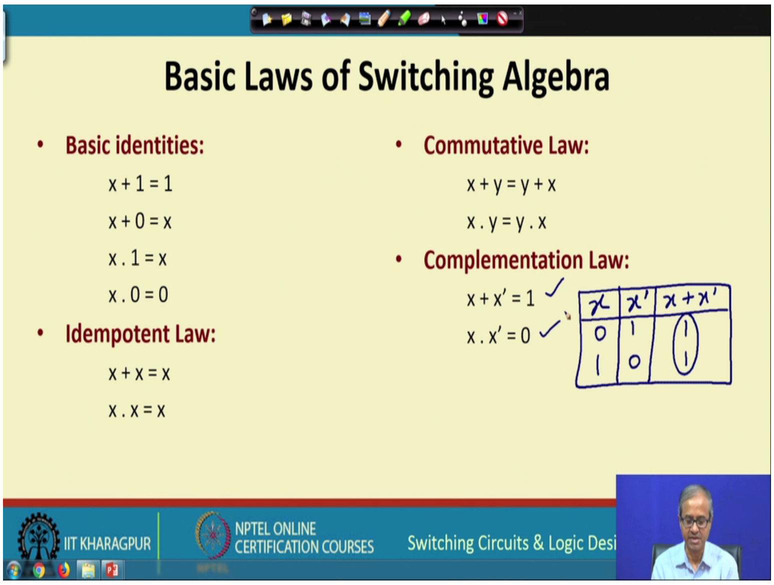

Well; this is also fairly obvious you see if you take a variable x x can be 0 or 1. So, what is x? Dash not it means not of this 1 and 0; so, if I take let us say x or x dash or of this and this 0 and 1 what will be? it will be 1 or 0 also 1 you see this is always 1 this is 1, but if you take and 0 and 1 is 0 1 and 0 is also 0 both will be 0 this is complementation.

And lastly associative; associative says that when you have more than 2 variables a anyone to carry out or or and. So, it does not matter that in which order in which do the or and and.

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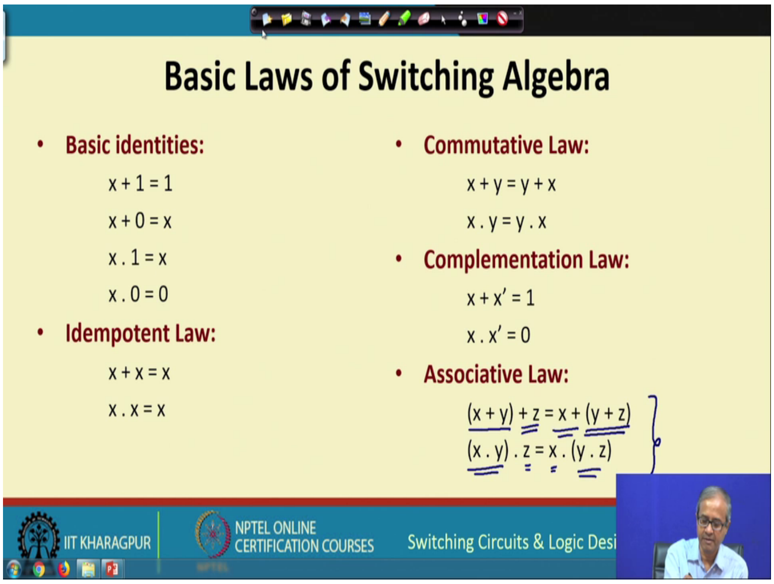

For example, you can either take x or y first and then or it with z or you can do y or z first and then or it to with x.

Similarly, for and this I suggest you can try a to construct the truth table and verify this yourself using method of perfect induction. So, here there will be 3 inputs x y and z and you can use them to check for all possible combination or whether the left hand side and right hand side are the same amount fine; I will leave it as an exercise for you some more laws.

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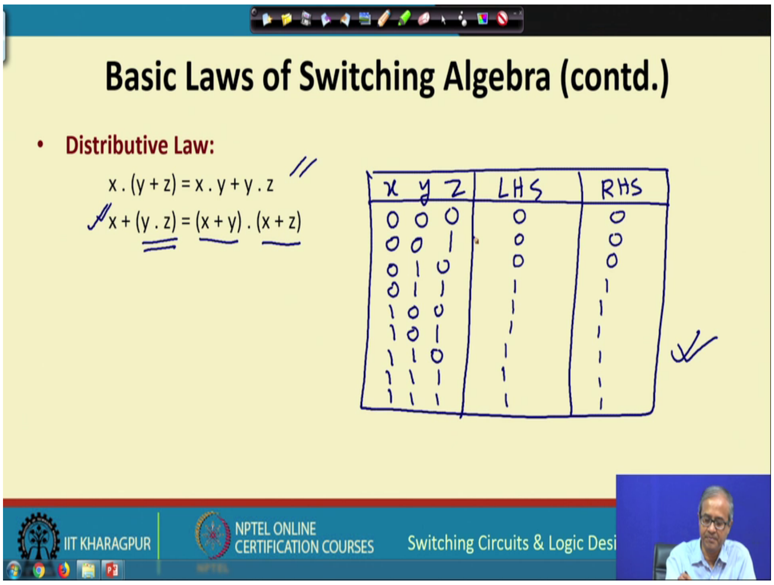

Distributive law; you see now you are coming to slightly complex rules. Distributive law looks very similar to the algebra that we know that you have studied in school x and y plus z is x and y plus x and z just like multiplication and addition we know.

But the second expression is slightly different this is not what you have studied in school, but now plus is or and dot is and not addition and multiplication x plus y dot z means x plus y dot x plus z. Let us show this one on this let us try to verify x y and z. So, x y and z there can be 8 possible combinations 5, 6 7 and 8 these are the 8 possible combinations. Now the second one let us try to calculate the second one I am trying to do let me try to calculate the left hand side and the right hand side.

So, you can step by step see first we are doing y and z then we are oring it with x; let us do it y and z 0 is 0 or with 0 0 0 and 1 is 0 or with 0 again 0; 1 and 0 is 0 or with 0 here also 0 1 and 1 is 1 or with 0 1 0 and 0 is 0 or 1 1 0 and 1 and is 0 or with 1 anything or with 1 is 1 this 0 or with 1 1 and this will be also be 1 and 1 this is left hand side. Right hand you see x or y x or z and of that x or y 0 0 x or z 0 0 or of that is 0 not or and of that 0 x or y 0 x or z 1 0 and 1 is 0 x or y 1 x or z 0 and 0 x or y 1 x or z also 1 and of it is 1 x or y 1 and 0 1 x or z also 1 and 1; this you can check the others will also be 1 ok.

So, in this way you can verify this method of perfect induction right; similarly first one is easier to verify same way you can also check this alright these are so, called distributive laws.

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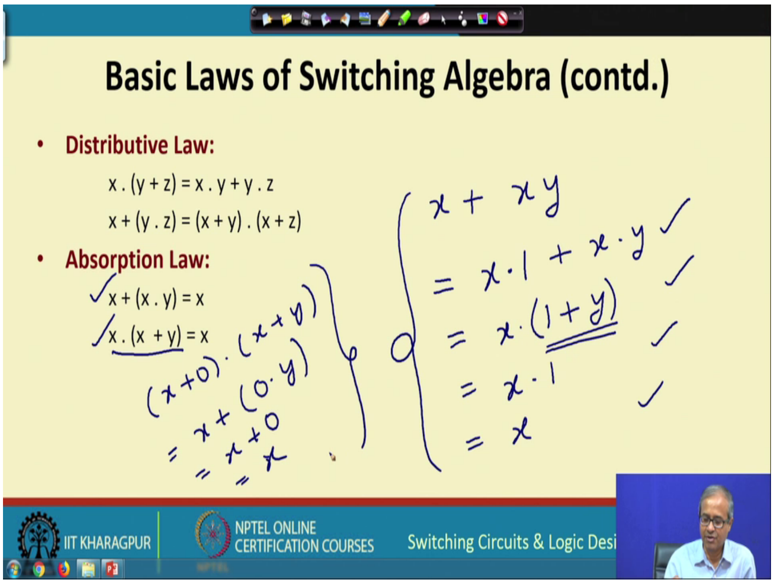

Then absorption law; absorption law says if you have an expression like x plus x dot y this will become equal to x; x dot x plus y this will also become equal to x. You see you can try to prove it by the method of perfect induction just like the way I have said, but you can also try it slightly in a different way. Like you have to see the first one this says x or well I am not writing the bracket, let us say xy I am dropping that dot x dot y. So, what I can write, this is equal to x well this rule already you know x means x and 1 plus x and y; x equal write as a x and 1.

And now you have already studied the associatively distributive law; you can take x common 1 add y. And we already know 1 or y means 1 x and 1 x and 1 is x; you see these rules you have already studied so, far. By systematic application of these rules I can prove this; this is prove using algebraic manipulation. Similarly, I can prove the next one also same way like this one for example, I can write x and x plus y I can write x or 0 and x or y; x I can write as x or 0. Again you can apply that in distributive law, this will be x plus 0 and y 0 and y is 0 x or 0 this is again x proved this also proved. So, this if you know how to apply this rules in a systematic way you can also arrive at these rules.

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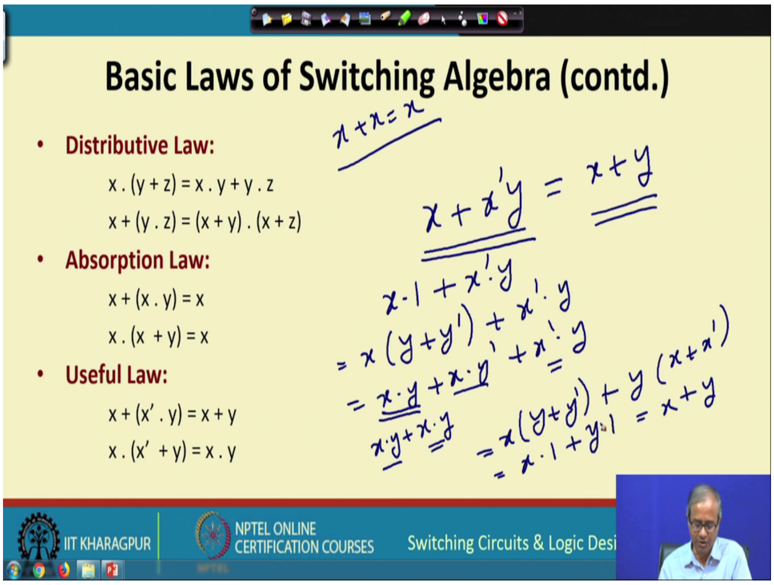

And there is another this will useful or does not have name like that this is also quite useful in minimization; it says x plus x bar y x plus x bar y; I am not writing in the bracket equal to x plus y while. Now here also you can prove this in a number of different ways you can try out, well I am just giving a hint that how you can proceed like x what you can write? This means x and 1 plus x bar y.

And this 1 you can write as y or y dash this is also rule you are studied y or y bar is 1 plus x bar y, we multiply this out x y plus x y bar plus x bar y. Now you can combine these 2 and you can see that whether you can reduce this; there is a way in which you can do that. Like for example, this x y this x y you can write as x y plus x y because there was a rule you will recalled x plus x equal to x ok.

So, anything plus anything were replaced by anything plus anything. So, now, you have 4 terms let us combine this and this in this to if I have take x common it becomes y or y bar, then let us take these 2 if I take y common; it becomes x or x bar now y or y bar is one x or x bar is also 1. So, it is x and 1 is x y y and 1 is y; so, there is a right hand side. You see you can again apply this rule system of course, perfect induction is fine you can put the perfect induction, but this is a more means algebraic or formal way of proving a thing.

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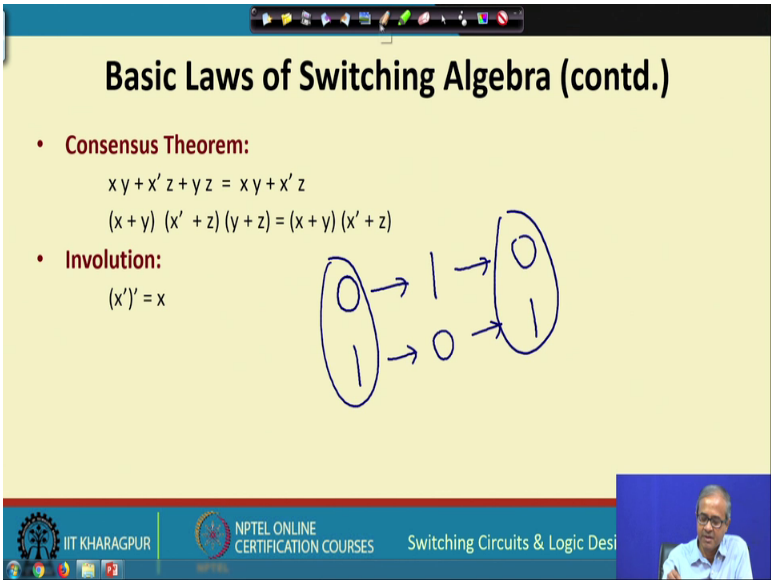

Let us look at a couple mode rules with a little more complex. Well again I leave it exercise for you to prove it using a method of perfect induction; this is called consensus theorem. What is consensus theorem? Let us see consensus theorem says if you have an expression like this x y y z and x bar z in between, then what it says this y z you can draw. This is the same as x y or x bar z; same thing if you replace just with and or if you just interchange, if you have x plus y if you have x bar z you also y plus z, this y plus z you can drop just these 2 these are equivalent.

So, consensus theorem is this if you have 3 such terms; one of the term you can straight away drop this you can prove try and prove ok. Then involution is fairly simple; not of not is the same as the variable because when you do a not of 0; not of 0 becomes one if you do not again it again become 0.

Similarly, one not becomes 0 do a not again it will again becomes 1; so, these two are the same. So, actually we have seen a number of rules that you can have as the basis for carrying out algebraic manipulation and minimizations ok. And we shall be seeing more rules later and see that how we can use it for actually functional manipulation in various ways ok.

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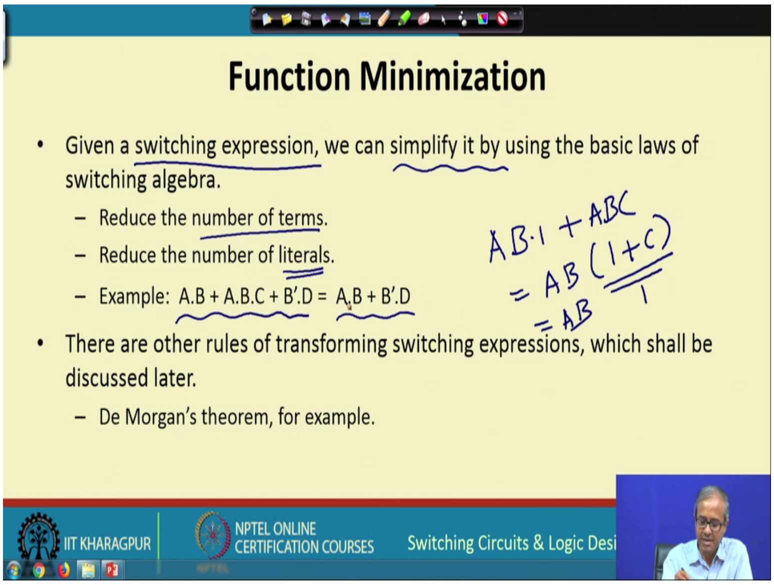

With this actually we just talk a little bit about function minimization and close today. Function minimization is something that we shall be taking up next; what function minimization says that we have a switching expression, how we can simplify it we already know the basic laws we saw many such laws.

When I say simplify it; we want to reduce the number of terms, we also want to reduce a number of literals. Literal means this A B C these are literals B bar these are called literals and term is A B A B C B plus B bar D there are 3 terms. So, there is an example these 3 if you minimize it becomes this; because AB and ABC if you take AB as AB 1, this is AB and ABC. And if you take AB common it becomes 1 plus C and 1 plus C means 1 AB and 1 is AB. So, it just becomes AB. There are other ways of transforming expressions also this we shall be studying in our next lecture; one such very important theory is called De Morgan's theorem this we shall see.

So, with this way come to the end of this lecture, we shall be continuing with this kind of algebraic manipulation in our next lecture or so.

Thank you.