Introduction to Soft Computing
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Lecture – 09

Defuzzification techniques (Part-I)

Now we have discuss about the different elements that can be derived. So, either these elements is a fuzzy set or is a fuzzy rule or is a relation or proposition or some inferences whatever it is there. Now whatever the elements that is there they expressed in the form of a fuzzy concept right, but the idea is that our we can understand instead of fuzzy rather the crisps, whenever we have to use this fuzzy value then they should be converted into the crisps value. So, in today actually we will discuss about how a fuzzy value can be converted to the crisps value that is this discussion basically involved a lot of technique to be discussed. So, we will take maybe 2 lectures 2 I mean to discuss the whole topics.

So, first we can consider the first part of the discussion. So, we say the discussion defuzzification techniques part 1.

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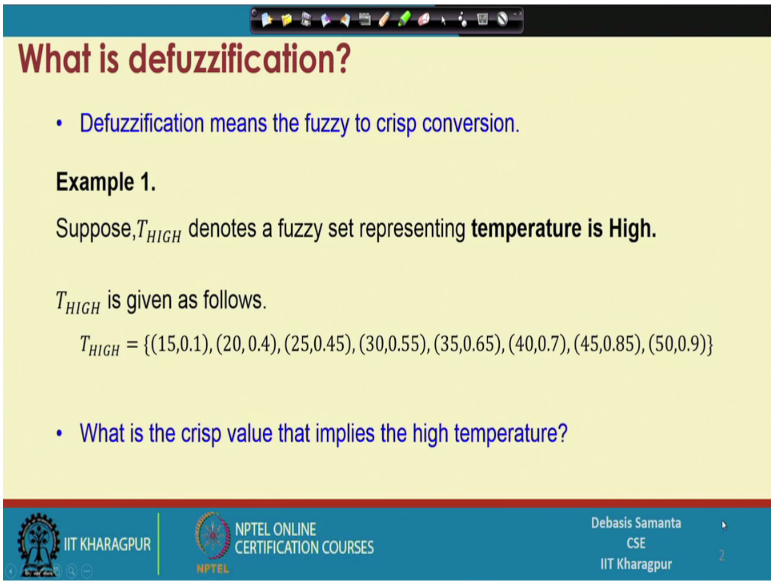

And. So, let us see how it is basically the concept of defuzzification it is actually. As I told you defuzzification means from a fuzzy value to a crisps value.

Now, as an example I can say T high is a fuzzy set the temperature is high representing and these fuzzy sets take these form. So, our objective is that, if this is the fuzzy set then exactly what is the high temperature in the concept of crisps value? That means, what is the crisps value that implies high temperature.

So, 1 way if I ask you, you can give that 50.09 is the high temperature because it has the highest what is called the high value of the membership degree of membership. But some people can say that I can take either very low value or high value or there are medium value; that means, this one; that means, thin one. So, let me 2 value and then take the minimum or average of the 2 value. So, this one whatever it is there or a middle value 1 also can be taken as the high temperature.

Now, these are the basically guessing and in let see in fuzzy theory, how such a crisp value can be calculated rather with more mathematical tuning.

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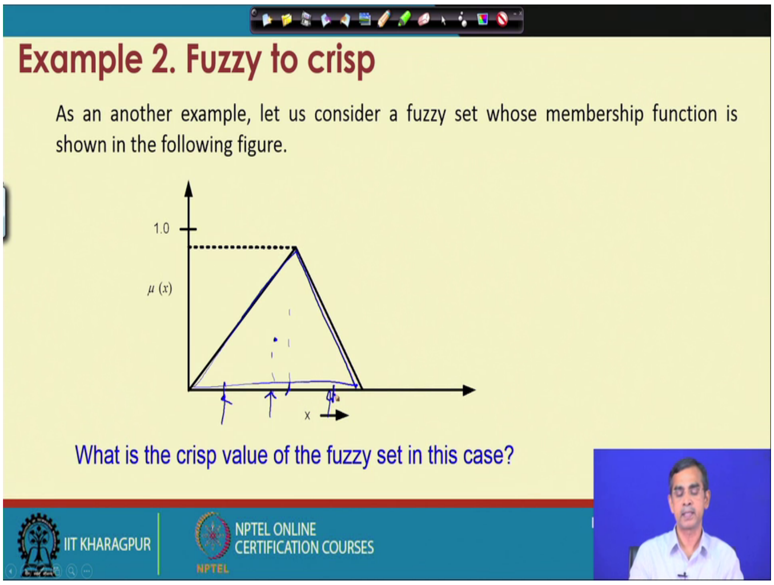

Another example say suppose this is the 1 fuzzy set is given in the form of a graph. So, these basically describe a fuzzy set what with these continuous values of element x, then what is the crisps value. That means, whether crisps value this one or this one or this one or it is somewhere this one. So, crisps value is this one or this one or this one which one it is basically.

So, let us see how the fuzzy mathematics or fuzzy logic provides as a way to calculate the crisps value for such element like a fuzzy set shown in the form of a graph.

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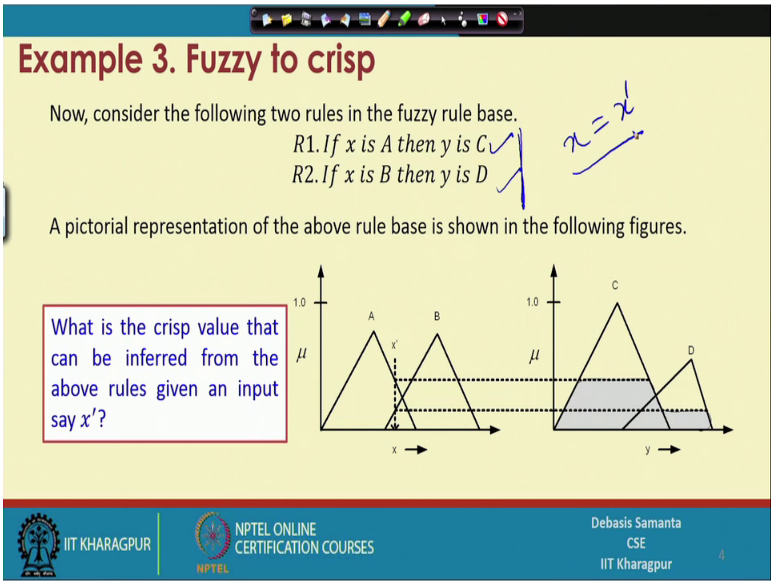

Now another example, here also we have to obtain the crisps value here these example is more complex suppose. So, idea is that these are the 2 rules is given these are rule right then what is the output. If x is A then y is C and if x is B then y is D, then if the these 2 s rules are given then for x is equals to say some element x dash; that means, that x dash is A then y is C and x dash is B then y is D. then what is the final crisps conclusion from the 2 rules that is fireable to x is equals to x dash.

So, this rule can be computed again using the defuzzification techniques, with the broad idea of the defuzzification technique is shown here. So, suppose this is the fuzzy set A defined over the same universe of discourse x and this is the B defined over the universe of this course x. And y is C; that means, C fuzzy set D fuzzy sets are defined by this and this one and they are defined over the another universe of discourse y.

Now, for some element x dash right, this is the x dash. So, we want to calculate the rule strength actually this is called the rule strength or the rule value. So, here basically if we see x dash is fireable with this membership value mu x dash here and a mu bx x dash is here. So, if we draw a line and then this one. So, this basically if x is a then, y is c basically have this train up to this one. So, this one if x thus is someone here then it will go there or above there, it will go there or it is here there.

So, depending on the values x dash. So, the portion that will cover it is there. Now these portion, basically tell that what is the value of this rule? Likewise if x dash is B and then y is D. So, if we draw a line then we intersect this portion. So, these basically rules 10 for this. Now taking the taking the merging of the 2, so that 2 rules have the value this set at area. So, this is the set at area basically is the fuzzy value given this 2 rule. Now given this is the fuzzy value when how we can obtain the crisp value. So, this is the task and we can solve this kind of problem using the concept of defuzzification technique.

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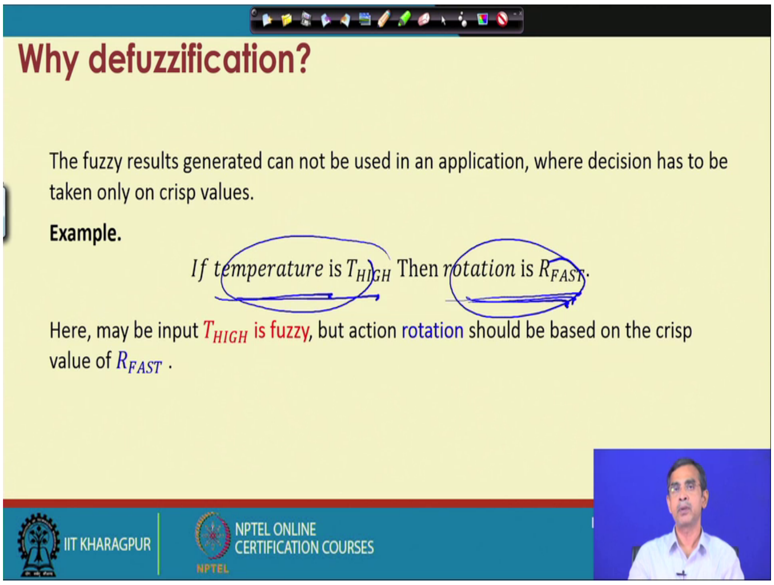

Now, why defuzzification or really it is very important or not that is also impo. So, as I told you defuzzification require, because some application where it can take if the conclusion is available in the form of a crisps value. For example, if you want to develop on what is call the ac controller air conditioner call the controller. So, that controller basically controls depending on the temperature. So, the controller is design like this if temperature is high, then rotation is fast.

Now, temperature is high for a particular temperature is 40 degree then rotation is fast; that means, we have go for fast rotation. Now these value should pass to the controller in such way that controller can understand a particular value only that is this is the rotation say 30 rotation per seconds. So, then it will calculate. So, that if this is fuzzy and this is fuzzy then finally, we should have what exactly the defuzzified value or crisps value for this one. So, it is the usual this is the one example that I have placed here so that if something is fuzzily available, input or output whatever it is there finally, we need the fuzzy finally, we need the crisp value to be used for some final application.

Now, so this is basically the model that is followed in fuzzy system design as I told you here.

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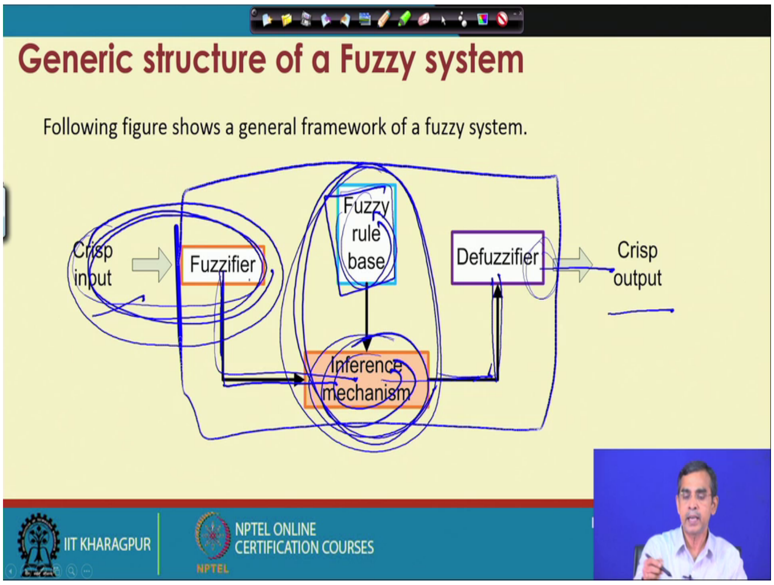

So, basically this is the fuzzy system that we want to design. Now fuzzy system takes the crisps input and then crisps output because we can think only in the terms of crisps value, and also we can conclude something if the crisp value is known to us. So, we are habituate with the crisp value rather than fuzzy value, but the fuzzy system will take only the fuzzy values.

So, the first step is that the crisp value should be converted to the fuzzy value. So, this is a fuzzified input, then from the fuzzified input it will go to the inference mechanism so; that means, if this is the input and they are the fuzzy rule base; that means, a set of rules we have already discussed about how the rules can be store in the form of a matrix. So, it is basically set of matrix given there. Now there is a technique regarding the inference mechanism we will learn about it later on and if the fuzzy input is given to this inference mechanism inference mechanism we will discuss about the fuzzy rule base, then we will obtain another fuzzy output.

Now, this fuzzy output needs to be defuzzified and then the defuzzified result can be passed through the outside of the fuzzy system as a crisps output. So, basically is a fuzzifier and defuzzifier and then fuzzy rule base and inference mechanism, these are the 4 basic components or 4 basic task that is involved in order to developed a fuzzy system. Now the fuzzifier as you know, the fuzzifier is basically is a task of the fuzzy designer fuzzy engineer, who basically can convert crisp input to fuzzy input. So, it is basically in the form of a fuzzy system development, fuzzy engineer has to given idea how a crisp input can be stored in the fuzzy form.

And then these are the basically fuzzy rule base system and then the inference mechanism can results the defuzzy. These ate the basically different fuzzy operations related the fuzzy sets related to the fuzzy rules, fuzzy proposition and then fuzzy implications finally, the different inferences can be obtained and these inference from the inferences we will be able to defuzzified it so the crisp output. So, now, we are discussing basically how the fuzzy elements whether it is in the form of set or it is in the form of a relation matrix can be defuzzified, that the crisp result can be obtained.

So, let us see what are the different techniques are available so for the defuzzification techniques concern.

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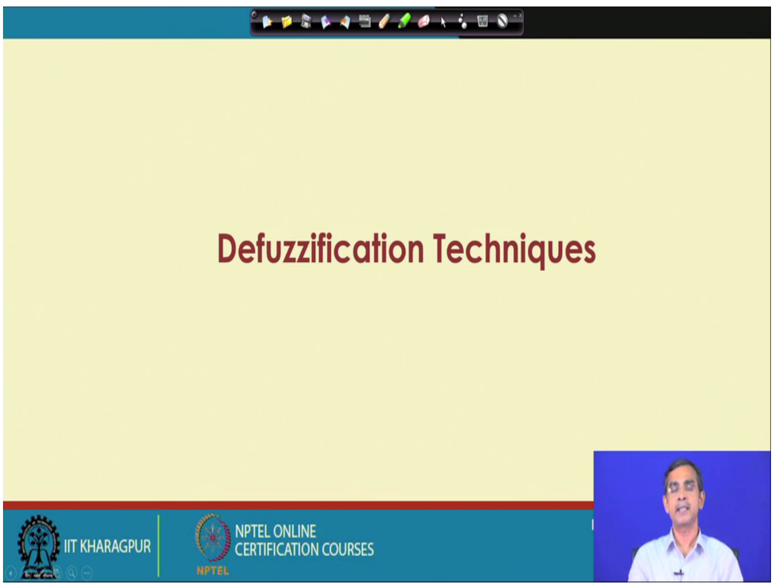

So, for the defuzzification techniques is concerned many methods are known we can broadly categorized the different methods, we have categorized 4 the first method is called lambda cut method.

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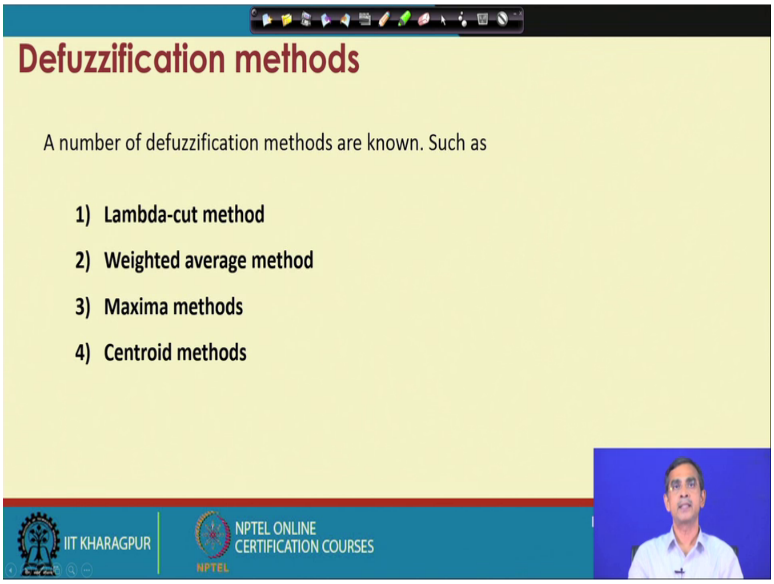

The second method is weighted average method, and the next is maxima method and finally, is the centroid methods.

Now, these are the different methods the way a fuzzy value can be converted to its corresponding to its crisp value. So, different methods has the their own merits as well as our limitation, we learn about it whenever we discuss the different methods one by one.

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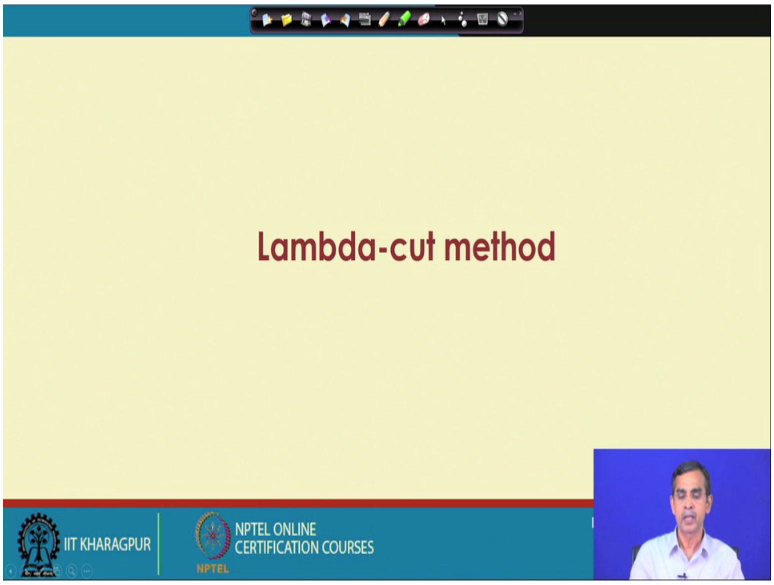

Now first we will discuss about the lambda cut method

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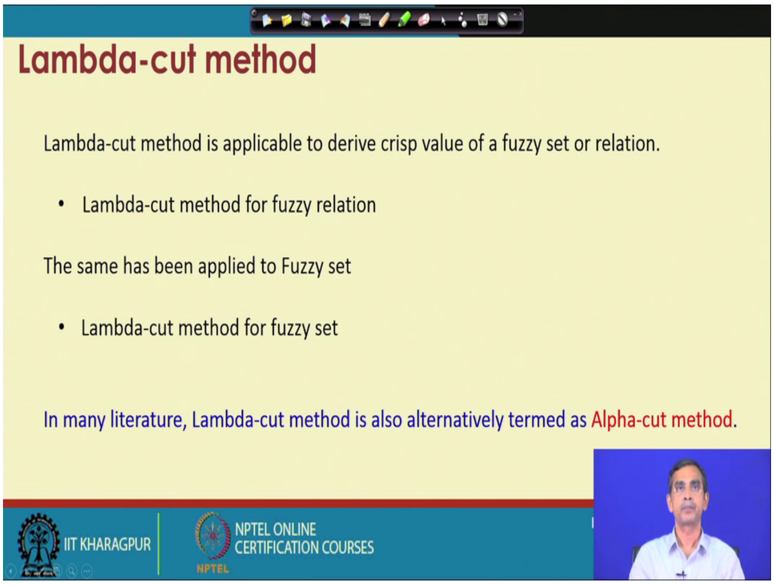

Now, lambda cut method is a very popular method already known in the context of crisp theory crisp algebra; that means, in the context of set algebra or Boolean algebra, the same thing can be extended and the context of fuzzy theory. So, lambda cut method for fuzzy relation and lambda cut method for fuzzy set.

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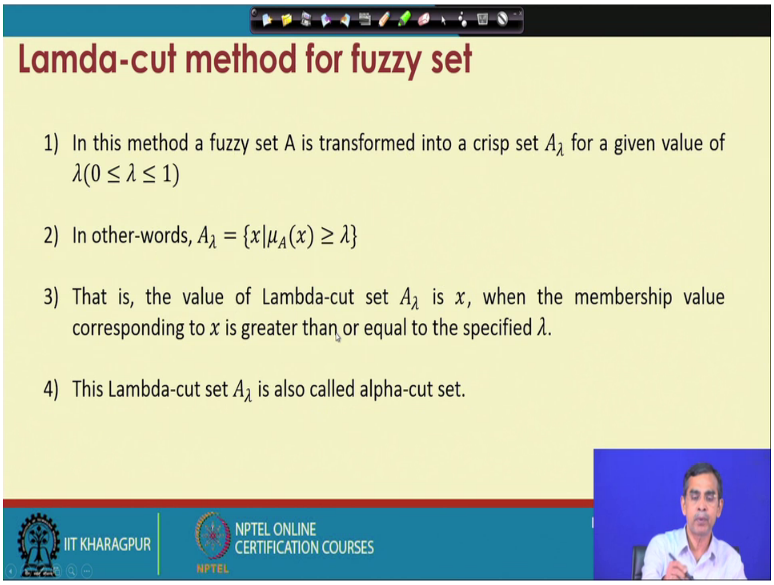

We will discuss about the lambda cut method for fuzzy set and then lambda cut method for fuzzy relation.

Now, let us first discuss about the lambda cut method for fuzzy sets so that from a fuzzy set to crisp set can be obtained. So, idea it is. So, in this method the fuzzy designer should choose 1 value, the value is called lambda that is why the method is called lambda cut method. So, lambda value should be in the range 0 to 1 now. So, then the lambda cut method it is basically we have discussed when we discussed about the fuzzy terminology, that is basically A lambda is a alpha cut like is. So, it is a alpha actually alpha is lambda here. So, A lambda and A lambda is basically a crisp set which can be obtained that it includes all the elements x such that mu Ax is greater than or equals to lambda. That means, from the given fuzzy set we can find a crisp set so that the elements whose membership value is greater than or equals to lambda.

So, this is the value of lambda set and. So, this is also depend on the A lambda. For the different values of lambda definitely A lambda will be different now. So, A lambda is also alternatively called alpha cut set, because we have discuss about the cut set of a fuzzy set. So, it is a alpha cut set. Now let us see one example so that we can discuss about this lambda cut method.

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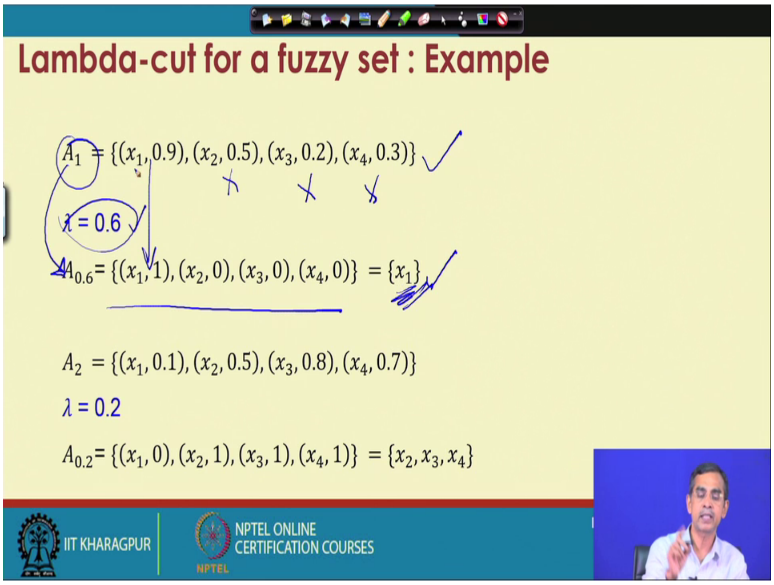

Yeah. So, these example say. So, A 1 is a fuzzy set and let us consider lambda is 0.6. So, it is A lambda that is the lambda cut according to the value of lambda these one. So, it will basically take all the elements, which whose membership value is greater than or equals to 0.6.

So, this way x 1 is qualified this is not qualified, this is not qualified this is not qualified this is not qualified. So, we can convert these fuzzy sets into the this is the crisp set or more general form the crisp set can be express these one, because those are the 0 they should not be there and those are the element one these one. So, A 1 if this is the fuzzy set and then if we follow certain value of lambda, then the crisp set can be obtain and this is the crisp set. That means, crisp set temperature is high these one, then the high temperature will be x temperature like this one. So, it is the example like this.

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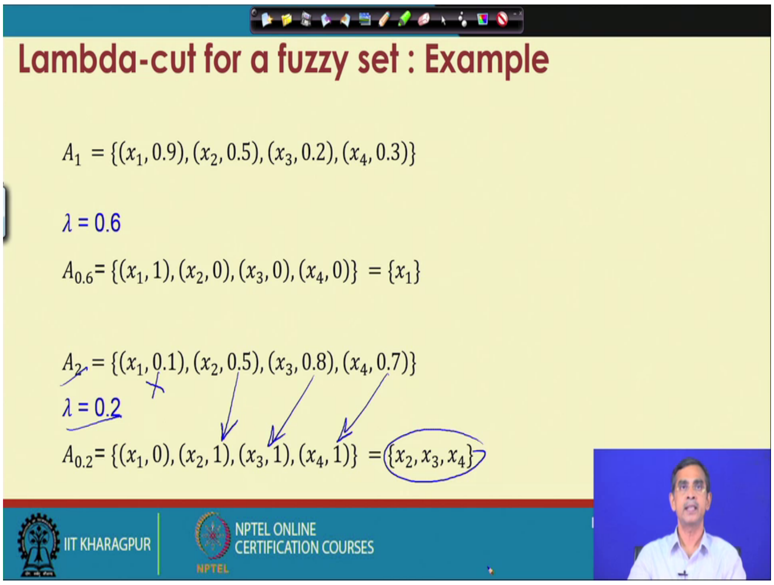

Now, as a second example. So, suppose this is a another fuzzy set A 2 and we take lambda it also 0.2; that means, you will take those elements whose membership value is greater than or equals to 0.2. So, it is not qualified this is qualified, this is also qualified and this is also qualified. These means that the A 2 given the fuzzy sets and lambda which is specified a 0.2 the crisp set that can be obtained this is the crisp set. Now you can say that. So, lambda cut gives you a crisp set, crisp set can be an all or it can have 1 or more elements in the set.

Now, in case of one or more element, they are basically all the equivalence are as the in the in the context of crisp that if this is the fuzzy then this is the crisp value either it is x 2 x 3 or x 4 or you can take the mean of the 3 values to conclude precisely that this is the crisp result. So, it is the idea about the lambda cut method and this is the in the context of fuzzy sets.

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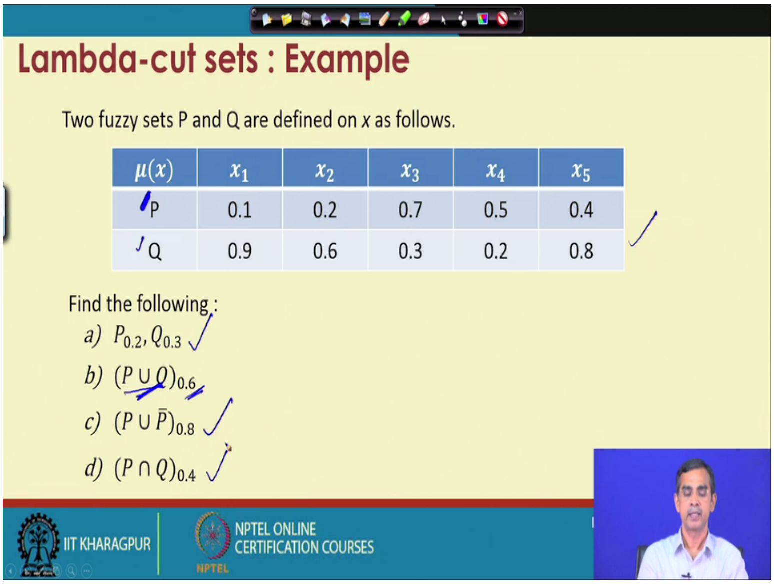

Now, the same idea can be calculated for the other complex fuzzy forms, here as an example. So, suppose you have given 2 fuzzy sets P and Q with the membership value it is shown here in this table then definitely will be able to calculate these things without previous knowledge.

Now, if we have to calculate P union Q and lambda is 0.6. So, the idea is that we first obtain the fuzzy union of these to fuzzy sets P Q and then for the resultant fuzzy sets we can apply 0.6 then we can obtained the result of this p union Q for this 0.6 lambda set similarly these one and these one also can be calculated . So, idea can be extended to the complex Rst formulation relating to 2 or more fuzzy sets as well.

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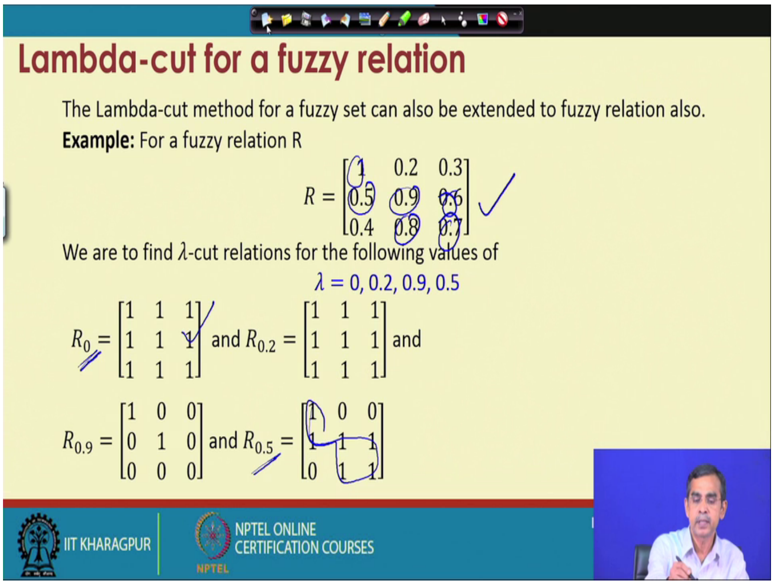

Now, we can again extend the same concept of lambda cut set to fuzzy relation or in a relation matrix. So, idea it is like this. A relation matrix we know it is suppose for an example a relation matrix is given here it is take this form. Now again a relation matrix a if a the lambda cut for a fuzzy relation can be specified with respect to a specific value of lambda, I am giving here 3 examples 4 examples. So, if lambda equals to 0 then how the crisp relation can be obtained. So, if lambda equals to 0 that means, will take only those entries whose basically value greater than 0. So that means, in this case if. So, this is R lambda, lambda equals to 0 and basically all entries are qualified. So, this is the crisp relation matrix.

Now, again say suppose 0.5 if we take; that means, we can take only those entries whose value is greater than 0.5. So, these one, these one, these one, these one, these one and these one. So, these way these these these are the different values that is there in the crisp relation. So, this is a fuzzy relation and these are the different crisp relation. As you know the fuzzy relation can have entries in between 0 and 1 both in to (Refer Time: 18:09) crisp relation is basically have these one and so it is. So, is a crisp. So, is a fuzzy. So, fuzzy to crisp can be calculated using the lambda cut method they like this one.

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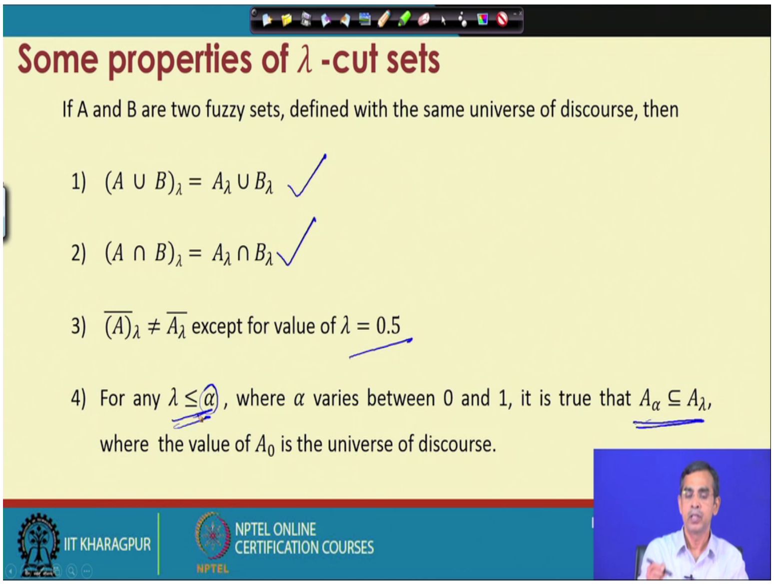

Now, the same thing can be applied to other things also there are some properties whole good. So, for the lambda cut set is their, these property needs to be understood right. So, basically this is the one property that if A union B and their lambda cut is taken this is basically equivalent to take the lambda cut of A and lambda cut of B and taking their union. So, this is basically 1 rule also applicable this rule also just it for union it is for intersection and one thing is that A lambda their compliment, and then compliment A lambda they are not generally equal, but their only equal for a equals 0.5. And for any lambda which is less than alpha, alpha is another value in between 0 and 1 we can check we can prove that a alpha is always a proper subset of A lambda if this in equality holds.

So, these are the properties it basically holds good, for the lambda cut technique is concerned.

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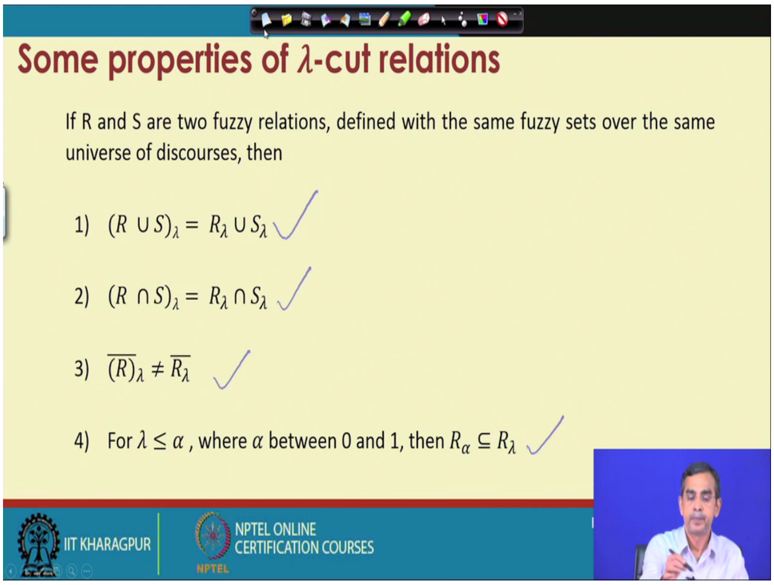

Similarly, for the relation these are the properties also satisfied, this property is for union and this is for intersection and this is for the compliment and this is the subset of this relation.

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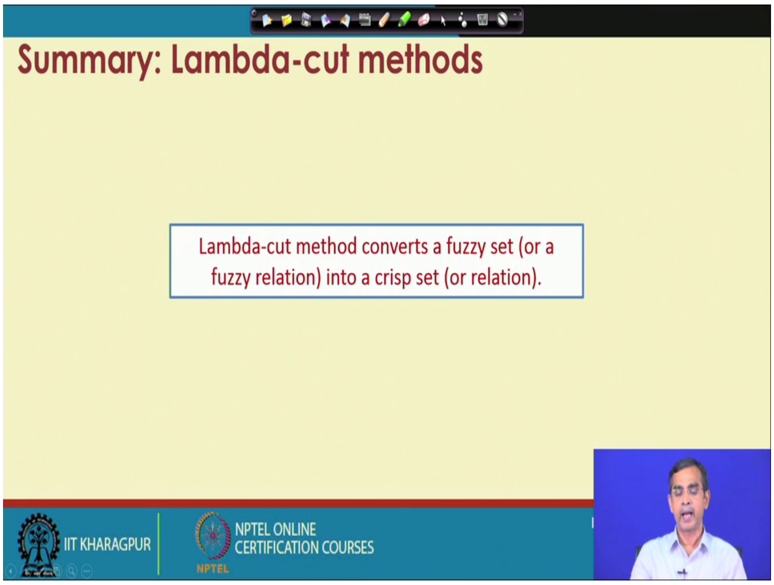

So, this is the lambda cut method in general and we can conclude one thing regarding the lambda cut method is that, lambda cut method converse a fuzzy set for a or for a fuzzy relation into crisp set or crisp relation, basically fuzzy set to fuzzy crisp set or a fuzzy relation to crisp relation.

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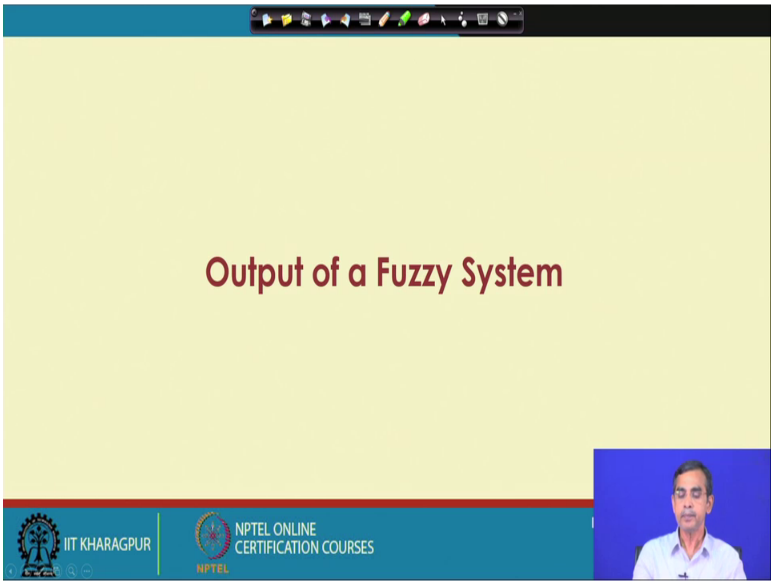

Now, before going to discuss the other technique, I will discuss about output of a fuzzy system; how basically the output of a fuzzy system can be concluded or can be calculated given many fuzzy elements or more precisely we will discuss about many fuzzy rules are there.

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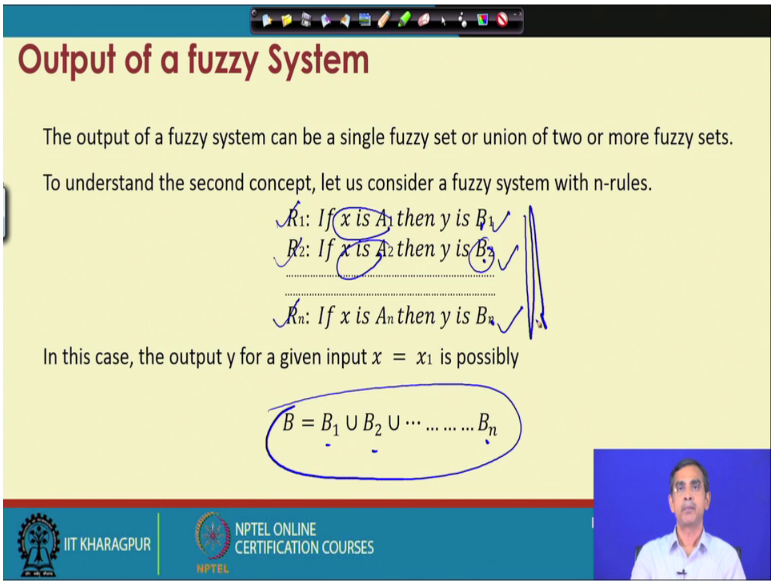

So, if more rules are given then how you can conclude the what is the result is there; that means, for a given input say suppose the n rules are applied. So, for a input R 1 is also satisfied R 2 is also 2, R n is also 2. So, n number of rules are true for a particular input then what is the output.

So, output is basically. So, if it is a rules satisfied giving B 1, B 2, Bn then output can be like this that it is basically union of B 1, B 2, Bn, but not exactly the union it is basically the rules strength related to x is 1 that B1. So, it is not the entire B1, but a part of the B 1 x is A 2 not the entire B2, but a part of the B 2. So, basically fuzzily it is part of B 1 part of B 2 part of Bn. So, is a part of B 1 part of B 2 part of B n and if we take the union of all these parts then finally, it will gives the total what is call the parts that all these rules basically signify for the output.

Now, here is the idea about that how these kind of calculation is possible and then they are corresponding a crisp value.

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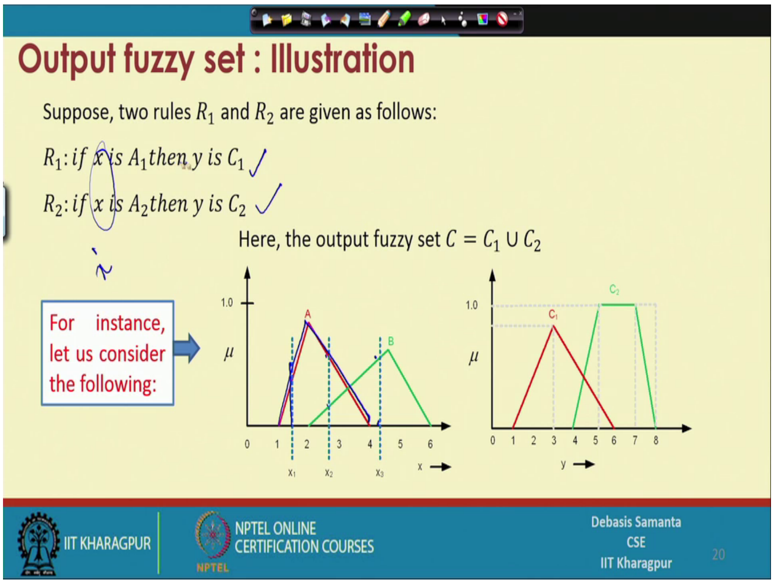

So, these idea can be mathematically better x described, and I just discuss in terms of 2 rules the same idea can be explained more than 2 rules as well. Now here is an example we can take it clearly. So, suppose 2 rules are given here the 2 rules is if x is A 1 then y is C 1 and another rule is if x is A 2 then y is C 2.

Now, for an input say x equals to some x dash we have to calculate what is the rule strength. Now here you can see I just. So, graphically I can calculate it or graphically I can display it. So, it is basically the fuzzy set A and this is the fuzzy set B and here the C

1 and C 2 are the 2 fuzzy sets C 1 and C 2, suppose they are defined over the same discourse x and this the rule C 1 C 2 fuzzy set defined over the discourse y and these are the graphical representation of their sets now. So, if ax equal to x 1; that means, it qualify this one. If f equals to x 2 then it fire both A and B and if x equals to x it fires only set B, but not set A.

Now, so, the 3 different situations I have mentioned here if for a given value only if x is A then y is C it is fired, but this B is not fired only one. So, in that case if we draw a line parallel to these.

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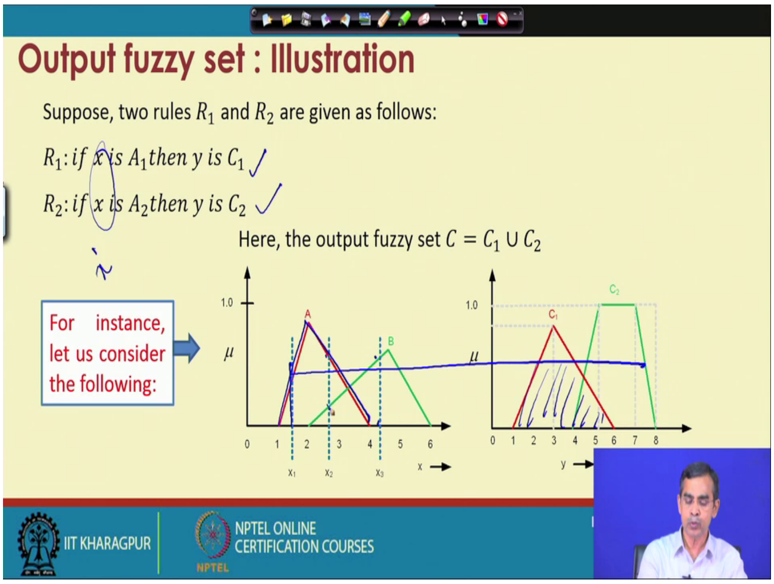

So, as it is only. So, related to C 1. So, this is the part of the set C 1 that is basically the rule strength of this one. Now again if x 2, if x is 2 when it satisfied both part.

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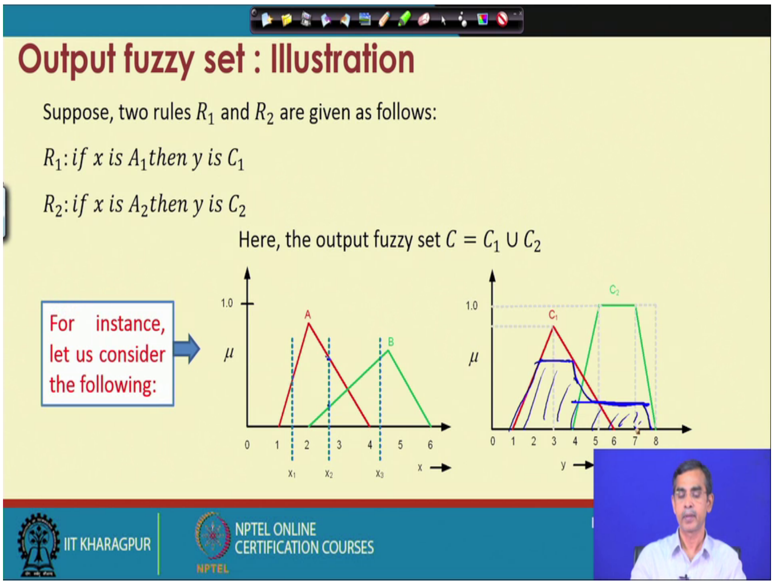

So, if we consider these part then this is the C and if we take this part when it is this one.

So, the rules strength is basically the total part is these one. So, this is the basically the resultant fuzzy set. Now the next is x 3 is basically only these on a does not satisfy these means these rule does not fire, but these rule fire in this case, and then we can see this part then it is basically these part. So, what this rule the rule strength will be these one; that means, this is the fuzzy output.

So, you have some idea about that for the given rules rule or set of rules, have the fuzzy output will be there. So, geometrically a fuzzy output is basically they the geometrical shape and these basically a portion like this portion or is a curve like. So, this is also a fuzzy set I mean like have look like this. So, you have to obtain the crisp value from this fuzzy sets like. Now let us see how such a things can be extended in more general way, I can the extend the same idea around again here.

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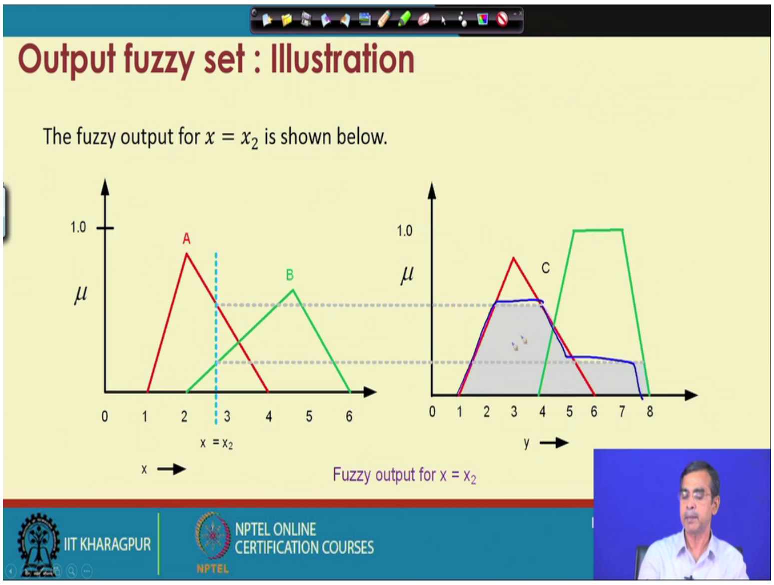

So, as I told you if x equals to x 1. So, this is the only part that is cover; that means, if x is a then y is c the fuzzy output is this one is the fuzzy output in this sense.

Now, if the as is another example if x is 2 then the fuzzy output will be these one. So, this is the fuzzy description of the output and the second case is if x equals to x 3.

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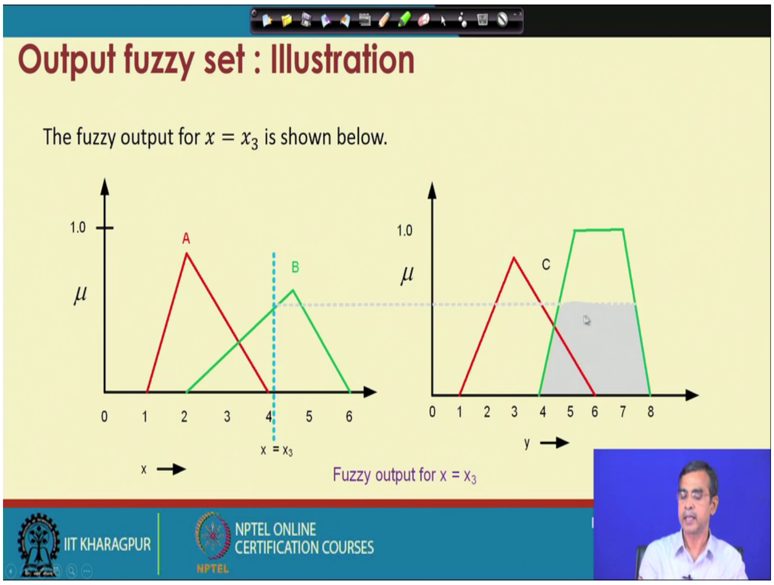

Then this is the fuzzy output. So, this is the fuzzy output fine. So, we have discussed about the fuzzy defuzzification technique, and particularly we discuss in this topics the lambda cut method and then how the output of a fuzzy system can be calculated. Will discuss the second part of the defuzzification technique particularly if a different geometrical set is given to us a which represents a fuzzy output, how the crisp value can be calculated that will be discuss from our next lecture.

Thank you.