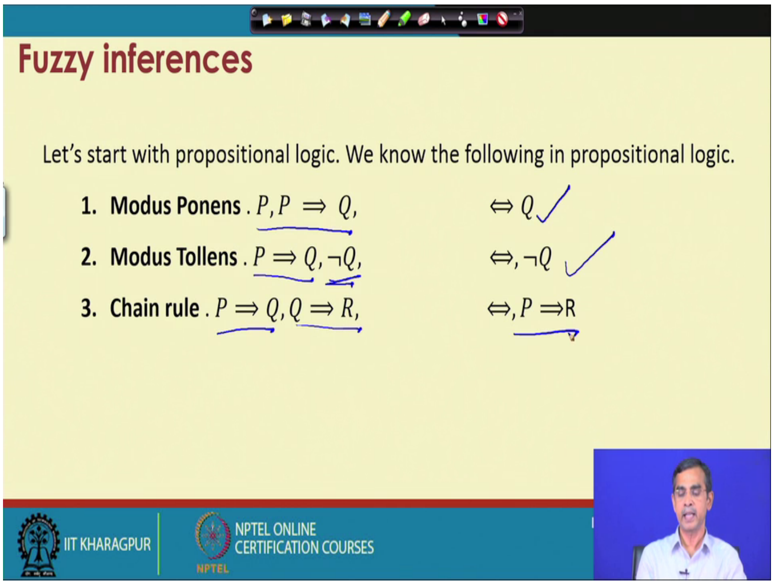
Introduction to Soft Computing
Prof. Debasis Samanta
Department of Computer Science & Engineering
Indian Institute of Technology, Kharagpur

Lecture - 08
Fuzzy Inferences

Fine, so we have learned about fuzzy implication. So, fuzzy implication basically calculates fuzzy rule and now we will discuss about how given a set of fuzzy rules, we can infer some other fuzzy rules. So, this this topic is called fuzzy inferences.

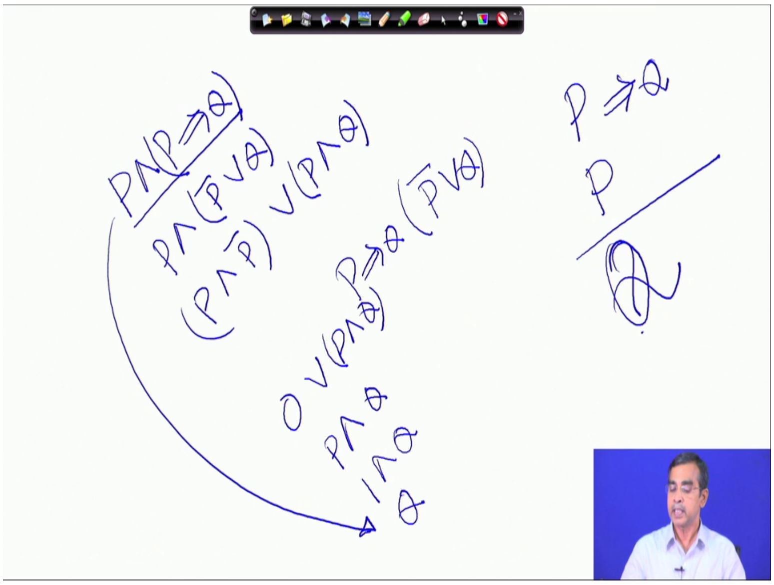
(Refer Slide Time: 00:44)



Now, in order to understand the fuzzy inferences, we need some discussion about which is very much popular there in predicate calculus of predicate logic, probably we know that operations one is called the modus ponen and modus tollen. I first discuss above the modus ponen then the modus tollen can be understood automatically. And then I will discuss about these rules. Actually these are the different what is called the logical rules that can be applied to infer some other rules actually.

So, let us see first the modus ponen. The modus ponen is a very famous rule that is known in the predicate logic. It is basically the concept about if the 2 propositions or 2 formula it is given to you how can derive another formula. So, this is in the context of predicate logic and we know predicate logic is A 2 valued logic, but our fuzzy logic is a multi-valued logic only the difference is there, but the most of the method that is the here it also applicable to fuzzy logic or it has been extended to fuzzy logic.

(Refer Slide Time: 01:58)



Now, the idea it is like this say suppose P implies Q. So, this is the one what is called the rule and P is the another rule say it is a proposition or it is a formula whatever it is there. Now the question is that so from this P implies Q and P whether we can infer something and if we if we can then how it can be inferred. Now in the context of this inference we assume that this P implies Q takes the truth value true; that means, it is always 1 or is a true. Similarly this P also takes the truth value true; that means, the 2 formula which has the truth value 2 and the another formula P these are the truth value 2 then how another formula can be derived. So, that it is truth value is true.

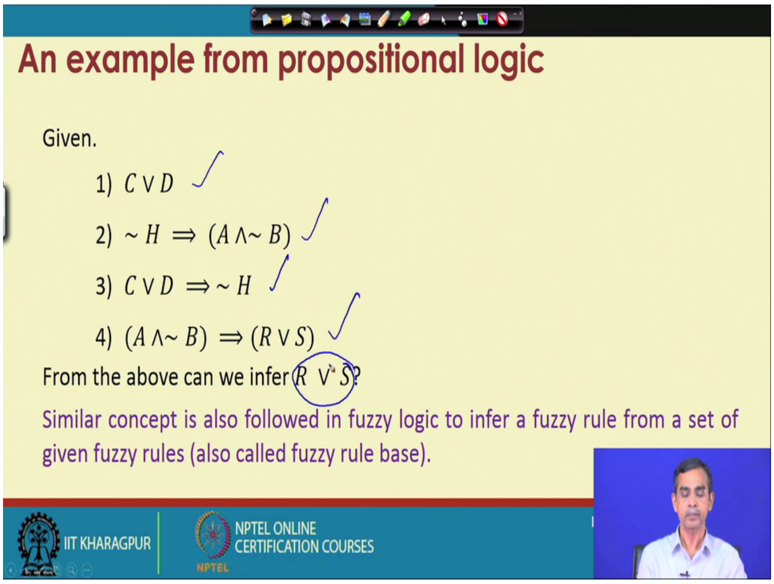
So, in case of predicate logic all formula use the truth value true and then false is automatically there. So, it is there, now let see if such A 2 rules are given to you how you can derive one rule I can tell simple algebraic manipulation. So, basically P implies Q and P I can write the 2 rule is given P and P implies Q it is like this. So, the 2 rules can be combined together to these 1.

Now, we know that P implies Q is P implies Q also alternatively P written as not P Q it is basically P implies Q. So, P implies Q also can be expressed in this form. So, using this I can write P and not P or Q and using the distribution law we can write P and not P or P and Q, now P and not P always give the results0. So, 0 this or P and Q now 0 or P and Q implies is basically P and Q.

Now, as I told you P always takes the truth value 1. So, I can write 1 and Q. So, 1 and Q means I can write it is Q. So, what I can say that if given these are the 2 premises then we can derive another premises it is called the Q. So, what I can say P implies Q if it is given and P is given then we can write it a Q. So, these basically is the basic concept that is the formula there in modus ponen, now the modus ponen it is like if P and P implies Q is known then we can conclude Q. Likewise the modus tollen says if P implies Q is true and not Q is true then we can imply not Q and this is the chain rule if P implies Q is true and Q implies R is true then we can infer P implies R.

So, these are the 3 rules that we have mention here there are many such rules in the predicate logic that is not the topic of our discussion here. So, we can use this concept to extend it into the fuzzy rule also. So, that is discussion in fact, in our next hour.

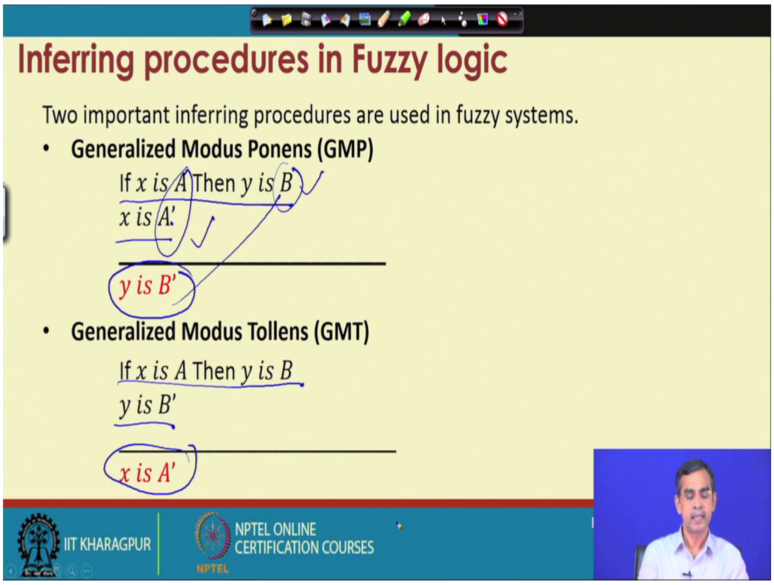
(Refer Slide Time: 05:28)



So, it is basically another example that you can follow here these are the different premises that is given to here this is there.

Now, you can try and you can easily find that from this premises we can conclude about another R or S. So, the concept is like this now the similar concept is basically applicable to the fuzzy algebra.

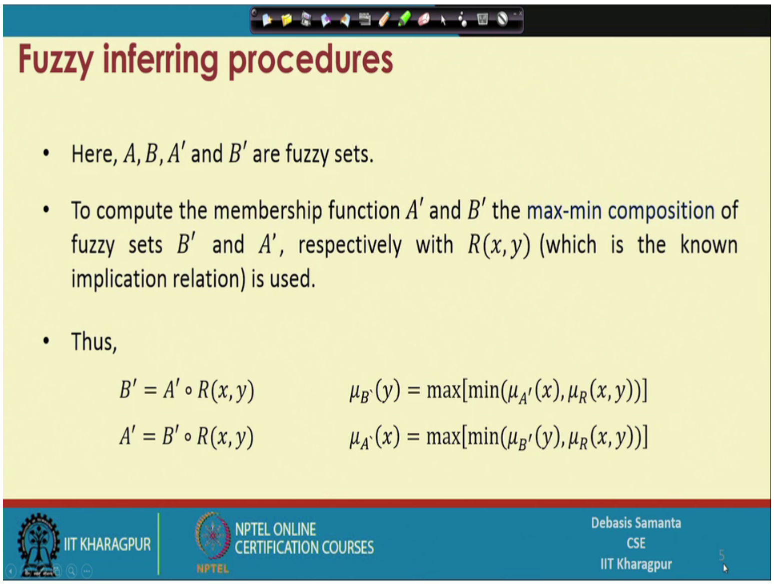
(Refer Slide Time: 05:58)



And, now we are particularly interested about the 2 such inferences formula they are called generalized modus ponen and it is another is called the generalized modus tollen or GMP and GMT. Now the generalized modus ponen is basically the form it is basically A 1 rule we have already learn about a rule if x is A then y is B and this is the another proposition x is A dash A dash is another fuzzy set, where A and B other fuzzy sets and one thing you should note that this a and A dash they should be defined over the same universe of discourse where B is the fuzzy set it defines another or similar fuzzy universe of discourse may be, ok

So, what is the idea is that GMP idea is that if this is given and this is given then we can conclude or we can infer another proposition it is there y it is called the y is B dash where B dash is the another fuzzy set and B dash is defined over the same universe of discourse as B. Now similarly if x is A then y is B given and here y is B dash is given then we can infer x is A dash. So, here x is A dash is given then we can inverse y is B dash here y is B dash is given then we can in can infer x is A dash. So, this is A 2 rules it is popularly called GMP and GMT and using these 2 rules we can infer some other fuzzy rules.

(Refer Slide Time: 07:39)

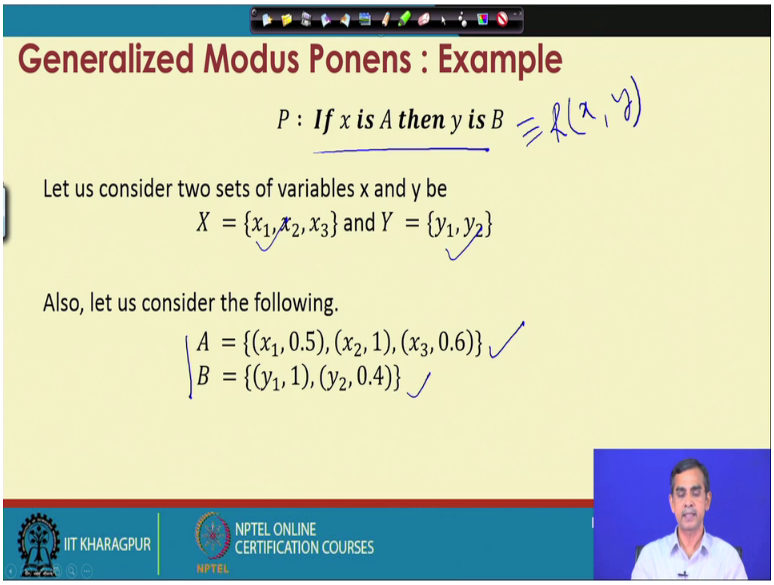


Now, let us see how these rules can be applied to our different fuzzy operations or sets.

Now, so the idea it is like this. So, here basically the input that is given to you 2 fuzzy sets a B and then either A dash and B dash and then you have to conclude either B dash and A dash from this. Now so this basically can be obtained now you can see the first rule that is there if x is A then y is B it can be expressed in the form of a relation matrix and we can represent R x y, and the next premises if x is A dash then that this relation matrix R x y and then A dash can be used and then can be obtained the B dash applying the composition formula.

So, it is basically the composition relation composition formula that we have already discuss in previous lectures. So, it is a composition operation; the composition operation takes this form. Now on the other hand if B dash is given and if rule matrix is given then we can calculate A dash using this composition formula. So, it is like this. So, this is the basic idea.

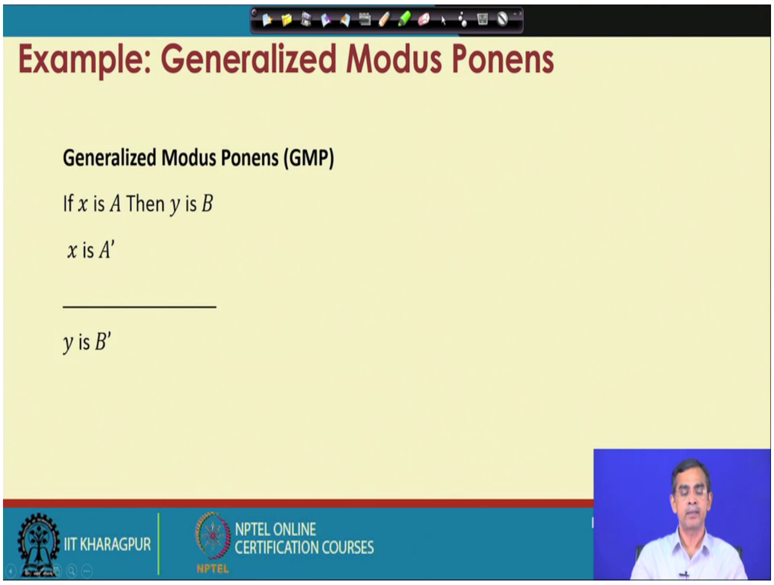
(Refer Slide Time: 08:59)



And let us see how we can use this idea to solve our some problems. So, I want to give an explain illustration about it let this is the one rule that is if x is A then y is B; that means, it can be expressed in the form of a relation matrix R x y and this relation matrix can be calculated using Zadeh's Max-Min rule like that ok.

Now, let us consider 2 fuzzy sets A and B which are defined over the universe of discourse x and B is defined over universe of discourse y and the 2 fuzzy sets are given here and here. So, these are the 2 fuzzy sets. Now given the two fuzzy sets then we will be able to obtain the; we can apply the GMP.

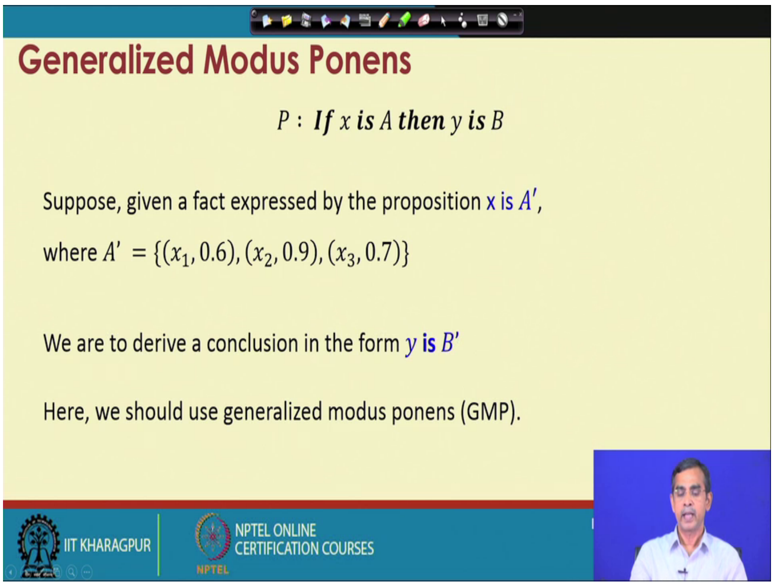
(Refer Slide Time: 09:42)



And then we can conclude about another fuzzy set B dash so in order to do these things. So, this is the GMP we can follow I just want to give an example regarding the GMP another example regarding the GMP also will be given.

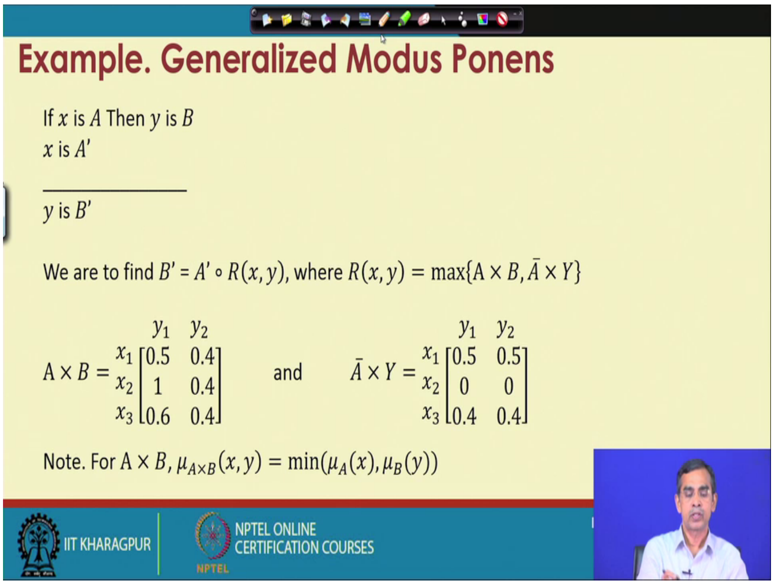
Now, so this is the GMP can be followed if this is given in the form of a R x y and x A dash is given to you then we will be able to calculate B dash, now let see one example in this direction.

(Refer Slide Time: 10:10)



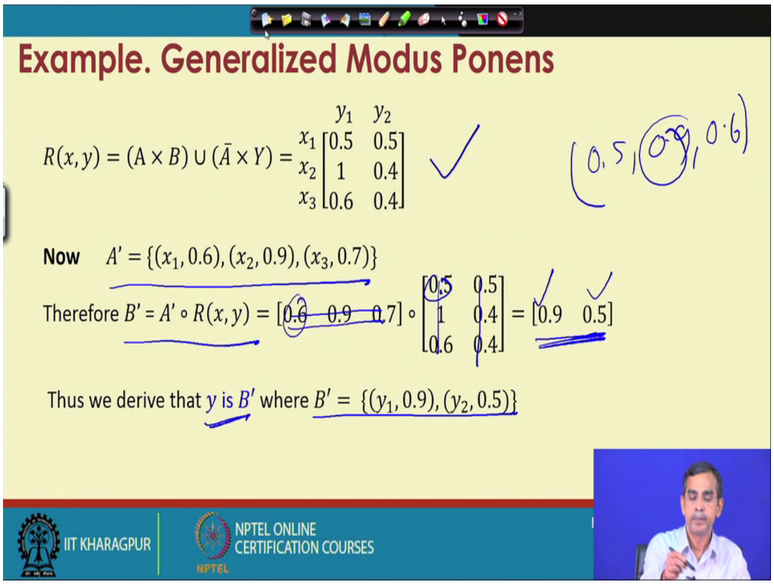
So, x is A dash x is A dash this takes this form where A dash is this one and we are to derive or we have to infer y is B dash it is like this. Now, so as it is x is A dash is given and this is the rule matrix and we have to include it. So, GMP is applicable here now let us see how the GMP can be calculate use and then the relational matrix can be obtained ok.

(Refer Slide Time: 10:35)



So, we have to use this formula as you have discussed it given that R x y is available to us using the Zadeh's Max-Min rule. Now, so if we can apply the Zadeh's Max-Min rule from the given set A and B we can be able to calculate A cross B A cross y and then finally, the rule matrix R x y.

(Refer Slide Time: 11:04)

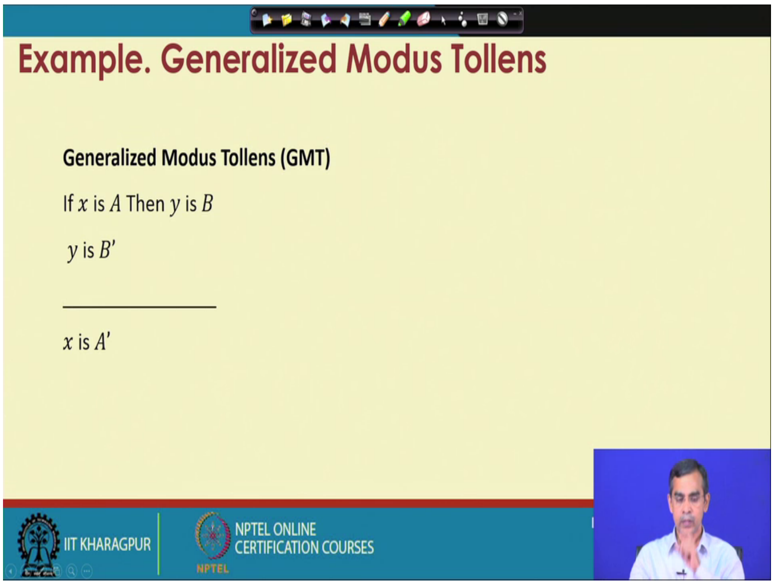


So, here is the rule matrix R x y these are rule matrix R x y that can be obtained given a B and then they are universe of discourses and this is the another input A dash is given to us then B dash can be calculated like this.

So, it is basically max that is a composition formula again this is a max-min composition formula we know. So, this means we can apply this one take these and these minimum so right. So, 0.6 and 0.5 we can take the minimum 0.5, then 0.9 and 1, we can take the minimum 0.9 0.7 and 0.6 take the minimum and then take the maximum so 0.9. So, the first entry 0.9, similar if we apply these and these we can obtain the 0.5.

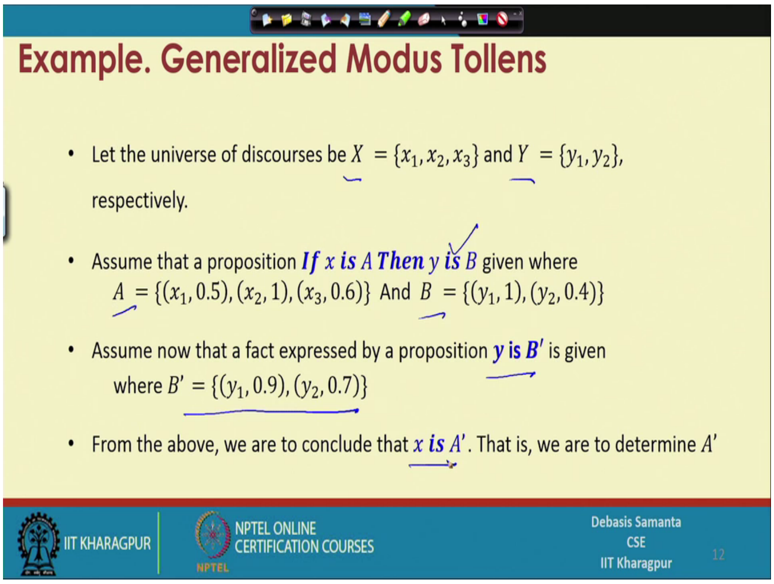
So, B dash has the membership values for it is elements 0.9 and 0.5 alternatively we can write B dash as y one 0.9 y 2 0.5. So, this is the application that we can conclude from a rule another proposition the proposition is that y is B dash.

(Refer Slide Time: 12:29)



Now, let us consider another example of GMT it is the same way if we understood the GMP then it is also similarly equally understandable easily.

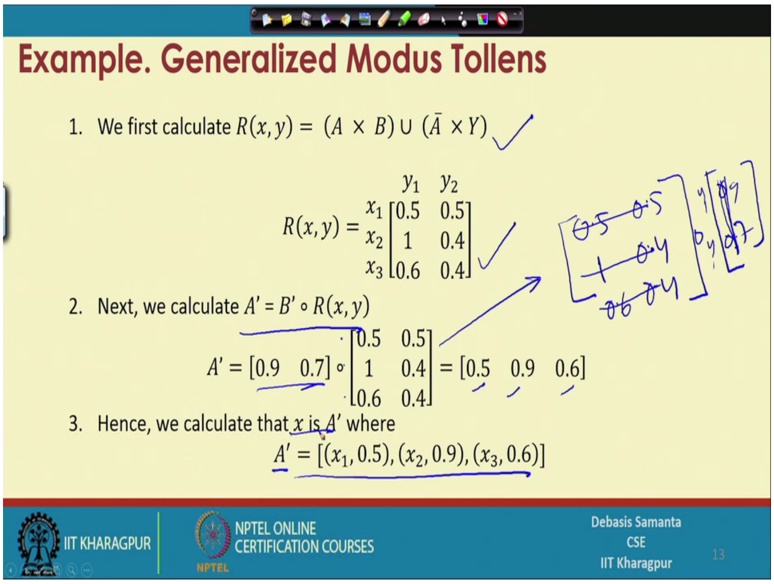
(Refer Slide Time: 12:37)



Now, again the same example I can use to illustrate the GMT. So, these are the 2 universe of discourse x and y. And this is the rule giving the relation between x is A and y is B and A and B are the 2 fuzzy sets it is given like this.

And you have given y is B dash where the B dash takes like this form and we have to compute the proposition x is A dash. So, here basically B dash is given. So, compute A dash. So, you have to apply G M.

(Refer Slide Time: 13:15)

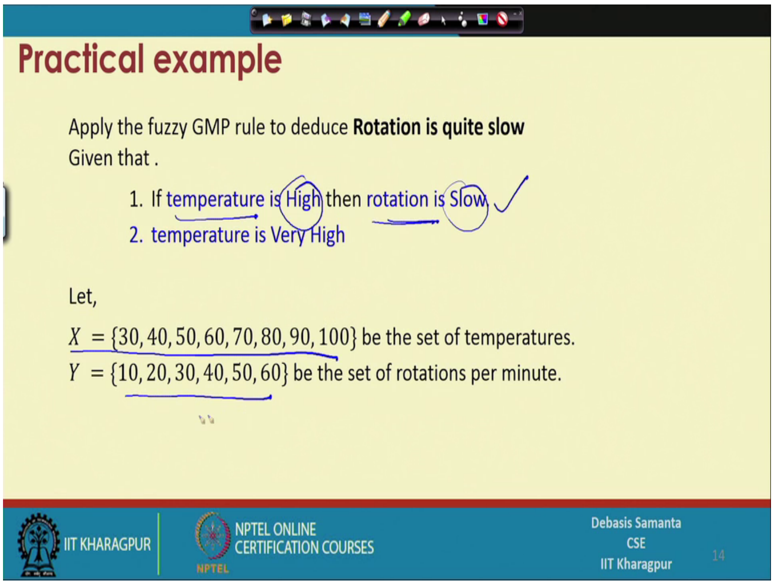


So, this is basically takes this form. So, this relation matrix can be calculated which can be obtained like this and then applying GMT we have to calculate this 1; that means, A dash is equals to B dash composition R x y. Now here now here you can check it this basically not applicable because it is the number of elements 2 and number of elements 3.

So, actually we can alternatively we can write it because A dash should have the elements x 1 x 2 x 3 here x 1 x 2 and x 3 is there. So, I can write it basically this a 0.5 0.5 1 0.4 and 0.6 and 0.4, then the composition and this composition I can write either this way or we also we can write in this way where this is basically y 1 and y 2. So, it is 0.9 and 0.7 then the different element can be obtained applying this one and this one in direction; direction using max-min composition. So, if we apply this one this one then the first element 0.5 can be obtained if we apply this one and this one the 0 9 0.9 can be obtained and this one this 1 0.6 can be obtained.

So, this way we shall be able to obtain A dash given R x y and B dash and which takes this form. So, this is basically x is A dash another proposition. So, you can infer from 2 rules or propositions another proposition the similar idea can be extended to 2 rules also we will see it shortly. Now so we have understood about the GMP and the GMT the generalized modus ponen and general modus tollen are the 2 tools.

(Refer Slide Time: 15:08)

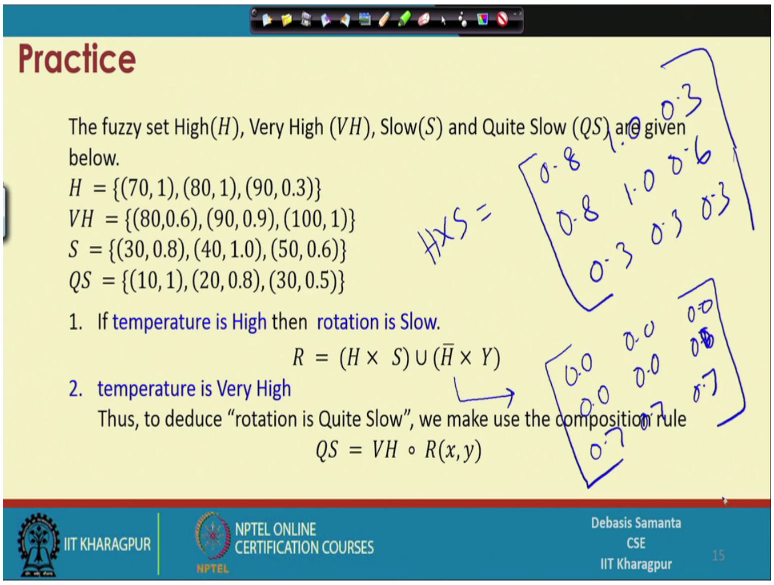


For the calculation of or for the inferences of some other rules, now here I just want to conclude with another example and let us see. So, this is an example here we have consider.

Say suppose these are the 2 rules are given there and this is the one example it is there given the 2 rules here if temperature is high and rotation is slow. Now here this high right is a basically one fuzzy sets and slow is a one fuzzy sets, this high and slow the 2 fuzzy sets are defined over 2 discourse one is the discourse of temperature, another discourse regarding the rotation. So, here basically X is the universe of discourse regarding the different temperature there is basically sets and then the rotation rotation has certain metrics. So, they are expressing this form 10 20 30 like. So, X and Y are the 2 universe of discourses representing the temperature and the pressure respectively.

Now, if I want to express the high temperature then definitely we have to define one fuzzy sets high temperature. Similarly if we define the another fuzzy set say rotation slow or slow rotation we can define this one, likewise high there may be another fuzzy set can be defined very high provided that their difference in the sense that the different membership values, for the different elements. Now let us see; what are the different fuzzy sets that we can conclude here for example, here.

(Refer Slide Time: 17:00)



So, suppose temperature with the universe of discourse as we have discussed 10 to 100 and we define the 2 fuzzy sets regarding the high temperature and very high temperature.

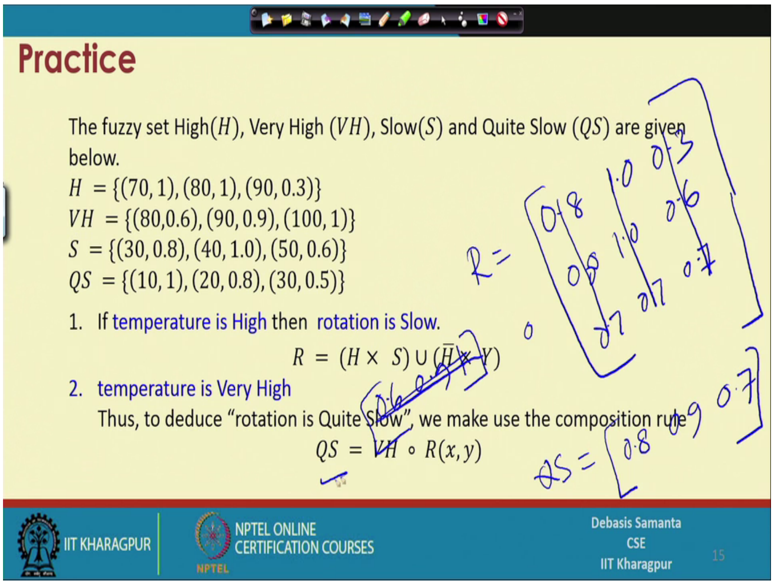
So, this is basically one fuzzy set high temperature and this is the another it is called the very high temperature and you can see that difference is there it is like either the in terms of elements and then degree of membership or both like. Now likewise regarding the rotation as the universe of discourse we define 2 fuzzy sets one is called the slow and another is quite slow, the slow fuzzy set is defined using this form and then quite slow is defined this form. Now, so the rule temperature is high then rotation is slow it can be expressed using this expression relation that is the according to Zadeh's max-min composition relation. So, R this is basically the relation matrix r showing if temperature is high then rotation is slow we can calculate these value.

And then if temperature is very high is given then we have to conclude some other in the universe of discourse slow. So, it is rotation is quite slow so; that means, this is the one premise is given and this is another premise is given and we have to derived another premises or proposition rotation is quite slow. So, we can apply GMP in this case because this is this is to given and we have to derive this 1 x is A y is B x is A dash you have to obtain y is B dash. So, we can apply GMP and this is the final formula that can be used to calculate this one, now if we take this calculation we can check that the result that can be obtained we can check that.

So, here basically H cross S and H H cross Y. So, H cross S it can be calculated if you can check it I am giving the final result. So, I advise you to check yourself. So, this is 0.8 1.0 0.3 it can be calculated 0.8 1.0 and 0.6 and 0.3 0.3 and 0.3. So, this is basically gives the calculation H cross a S. Now, similarly this also can be calculated and we can obtain another relation matrix that can be obtained as 0.0 0.0 0.0 0.0 0.0 0.0 0.0 and 0.7 0.7 0.7.

You can verify this calculation now once you know this one then we can take the mini the union operation that means taking the value.

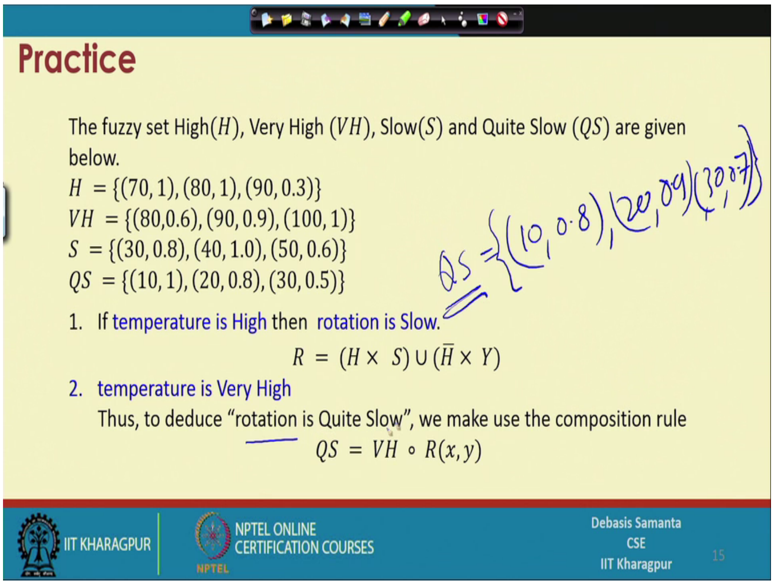
(Refer Slide Time: 20:13)



So, this relation matrix are then can be expressed r this is equals to another relation matrix you can verify this relation matrix that can be given a 0.8 1.0 0.3 0.8 1.0 0.6 0.7 0.7 0.7.

So, it is given then it is given like this one now we have to. So, this r is available now. So, we have to obtain the Q S. So, we can use this formula V H composition R x y. So, basically composition V H is basically this 1. So, you can write 0.6 0.9 and then 1. So, again the Max-Min rule can be applied and then Q S this can be obtained this and this it will give the first element you can say 0.8 then this and the we can obtain 0.9 and this and this you can obtain 0.7.

(Refer Slide Time: 21:29)



So, this finally, Q S can be calculated. So, Q S that can be obtained therefore, it is a universe of discourse this 1. So, we can write 10 0.8 then 20 0.9 30 0.7. So, this is basically the fuzzy set expressing the Q S or rotation is quite slow giving the fuzzy set it is like this.

So, this is the way that we can use the GMP, and GMT to infer fuzzy relation. Now, so far we have discussed about the fuzzy implication or inferences; rather inferences given one rule and one proposition. In the next lecture we will discuss about the fuzzy implication whenever the two or more rules is given, and then how another rules can be obtained from them.

Thank you.