Introduction to Soft Computing
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Lecture - 07
Fuzzy implications

So far we have learnt Fuzzy set. Fuzzy set is the basic building box for the development of fuzzy system. Now in the context of fuzzy sets we have learned different operations so that from two or more fuzzy sets how the other fuzzy sets can be obtain and then you have also learned about the relations between two fuzzy sets to get another, a fuzzy elements.

Today we are going to learn about fuzzy implication. Now we know that there is a proposition for a fuzzy set A and which we denote as x is A. So, there is a relation among the propositions. Now such a relation can be better described with the help of fuzzy implication. So, today we are going to learn about fuzzy implication and the different operations or the computation techniques so that the fuzzy implication can be calculated.

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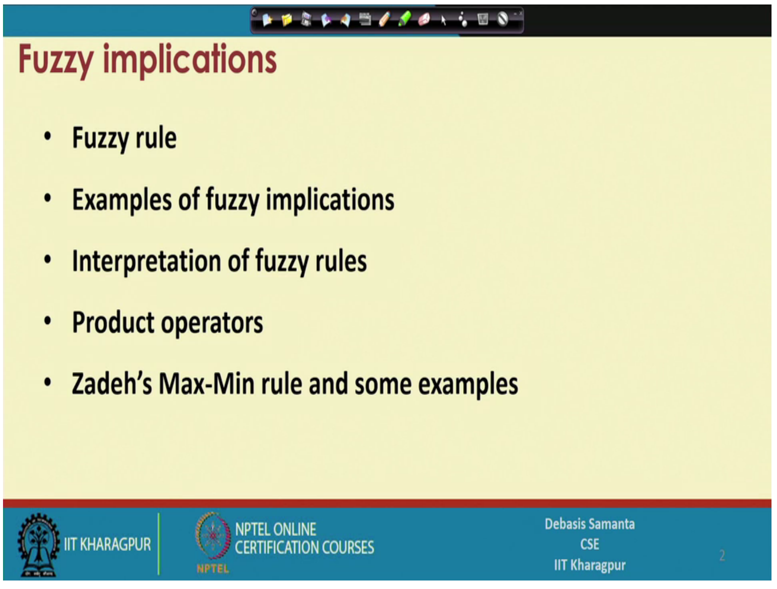

So, today basically we will discuss about fuzzy rules this is the another name of the fuzzy implication. In fact, and some examples and then interpretation of these and then some operations by which the fuzzy implication can be computed and one operations which is extensively used in fuzzy system development it is called the Zadeh's Max-Min rule we will discuss in details about a Zadeh's Max-Min and we will illustrate the Max-Min rule composition technique with some example.

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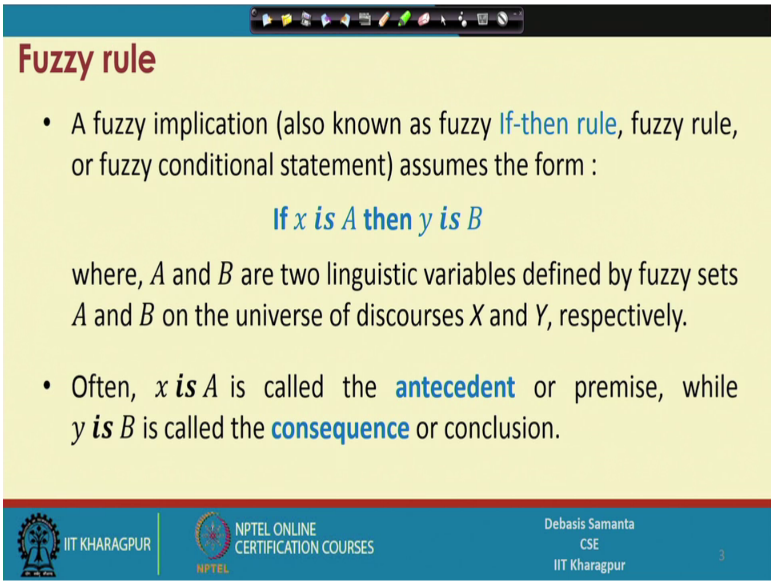

So, now let us come to the fuzzy implication that is also called fuzzy rule, it is also sometime called as if then rule, sometime it is called a fuzzy conditional statement. Now such a fuzzy implication is basically is a relation between one proposition to another proposition. For example, see this is one example of a fuzzy proposition fuzzy implication and here x is A as we know it denotes a proposition; proposition regarding and element x in A set A and another proposition y is B. So, it basically gives a relation among that 2 this is the proposition and this is a proposition and then relation in the form of if then. So, that is why it is called if then implication or if then rule.

Now, it is it is necessary to calculate what it does mean; that means, if x is A then y is B what value it does return to us. So, we will see that if x is A then y is B can be expressed in the form of a relation matrix, that mean the fuzzy implication can be stored in the form of a matrix and it is important to calculate the different entries in the matrix. We probably you can recall if x is A then y is B we represent such a implication or rule in the form of a Cartesian product, but Cartesian product is basically the very basic things in this relation there are many things that can be considered. So, in order to represent a fuzzy implication that will learn it.

Now, in this context so fuzzy implication takes in a general form as I told you if x is A then y is B here the first part this is called antecedent or it is called the premise and then the general form of a fuzzy rule.

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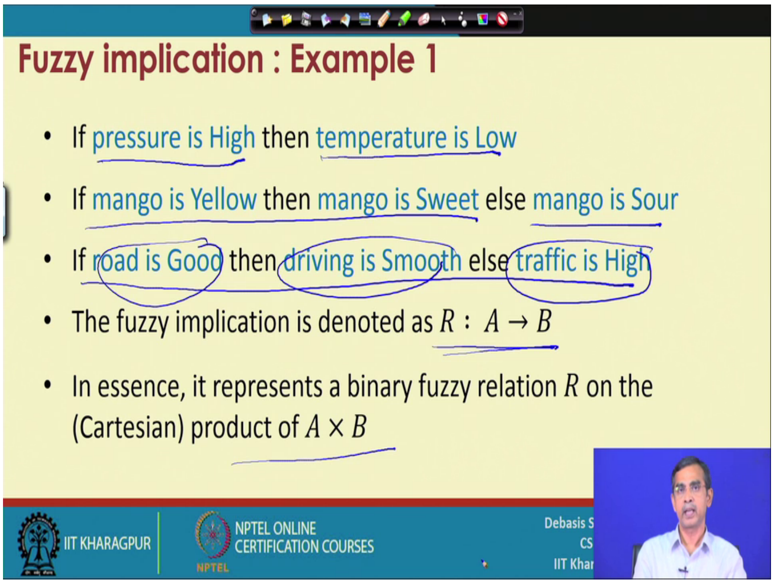

And we are to calculate the fuzzy rule basically the different membership values that can be that can be applicable to such a rule, anyway that will be discussing details in due time.

Now, I can put some examples. So, that we can understand how fuzzy implication or fuzzy rule looks like here I have given some example if pressure is high then temperature is low. So, as you know. So, pressure is high is a another what is called this is the premise or antecedent and this is the consequence. In fact, these are the 2 propositions.

As another example if mango is yellow then mango is sweet else mango it is basically x phased in if then else. So, if then else is another form of course, but sometime we can represent everything in the form of a only if then and this is a unification method usually followed in a fuzzy system. So, anyway, but if then else also another structure that can be represented only in terms of if then else that will be we will discuss it about later on.

As another example this is another if road is good then driving is smooth traffic is high. So, here basically relation among 3 prepositions road is good, driving is smooth and then traffic is high. So, the relation takes the form as I have discussed here with in terms of few examples. So, for that notation of representing a fuzzy implication is concerned it is usually represented using this form. So, R R represents a fuzzy rule and if it is basically if x is A then y is B it is represented a implies B. So, that is why it is also called fuzzy implication and I told you such a fuzzy implication is basically a relation from the fuzzy set A to fuzzy set B and broadly it can be expressed in the form of a Cartesian product. So, it is in a broad sense it is a Cartesian product actually, but there are some methods by this relation can be calculated in a more mathematical way that will be a more formal way like that and we will discuss. And we are going to learn this the method by which the fuzzy relation can be calculated.

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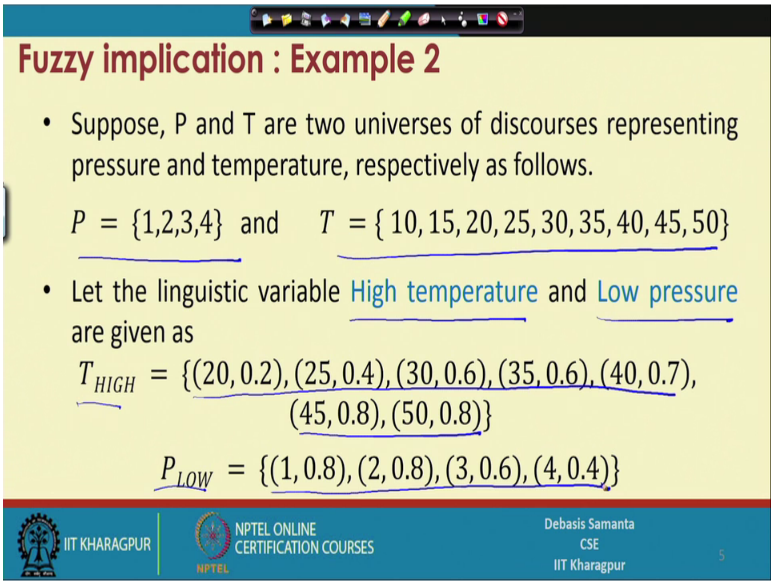

Now let us see one another example. So, that we can understand the fuzzy implication better in this example let us consider 2 fuzzy sets P and T. So, this the P and T these are the basically denotes that pressure in some scale and temperature in some scale we can consider pressure and the temperature are the 2 universe of discourse relate, I am giving the value of pressure that is possible and temperature that is possible.

Now, let us consider the 2 ling fuzzy sets they are expressed in terms of linguistic variable called the high temperature and low pressure. So, these are the 2 fuzzy sets and the 2 fuzzy sets say high temperature we denote it as T high and the degree of membership values for the different element is shown here like likewise. So, the low pressure it is denoted as P low and is represented in the form of a fuzzy set which is shown here.

So, given these are the 2 fuzzy sets now let us see how the fuzzy implication can be can be expressed.

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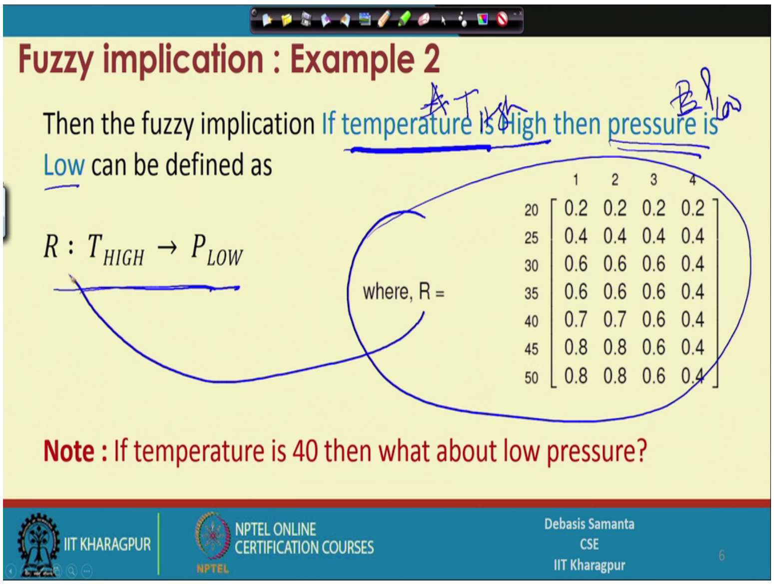

Now, so fuzzy implication fuzzy implication from the 2 premi 2 prepositions say temperature is high then pressure is low. So, here the proposition is temperature is high then pressure is low. So, it is basically this is the one we can say this is a temperature is high and the pressure is B say B. So, it is basically we can express these in terms of like this one sorry it is not high it is a we have denoted a T high and this is B here P low.

So, that illation or we can say implication between the 2 can be expressed we using a short form T high to P low, now using simple Cartesian product that we have already discussed in the previous lecture we can obtain the relation matrix; that means, give me a showing these it look like this. So, this is a relation matrix r showing this relation R so, using the Cartesian product that we know.

Now, 1 point 1 interesting point is that here if temperature is high then pressure is low now say suppose temperature is forty then what about the pressure. So, it is basically if temperature is 40 then it basically shows the pressure.

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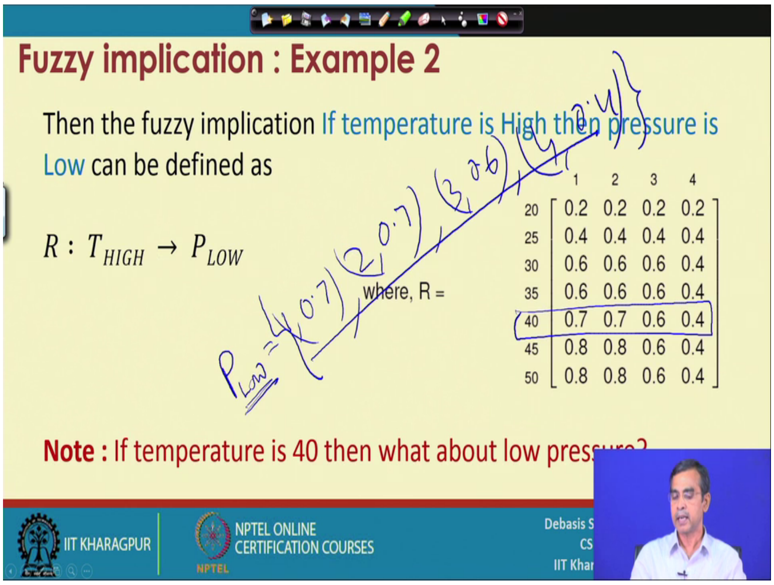

Now, you can see what temperature 40 the pressure. So, I can say P low here P low this is basically can be expressed in term of a fuzzy set 1 0.7 then 2 0.7 then 3 0.6 and 4, 0.4. Now, so this is basically P low provided that provided that temperature high as 40. So, this basically is the fuzzy set. So, answer is like this. So, this rule gives an answer about the pressure that is pressure as low for a given temperature this one.

So, this is basically one imp implication or purpose of the fuzzy set that we will use in in our fuzzy system development. So, these two examples can be helpful for us to understand how the fuzzy implication works for us.

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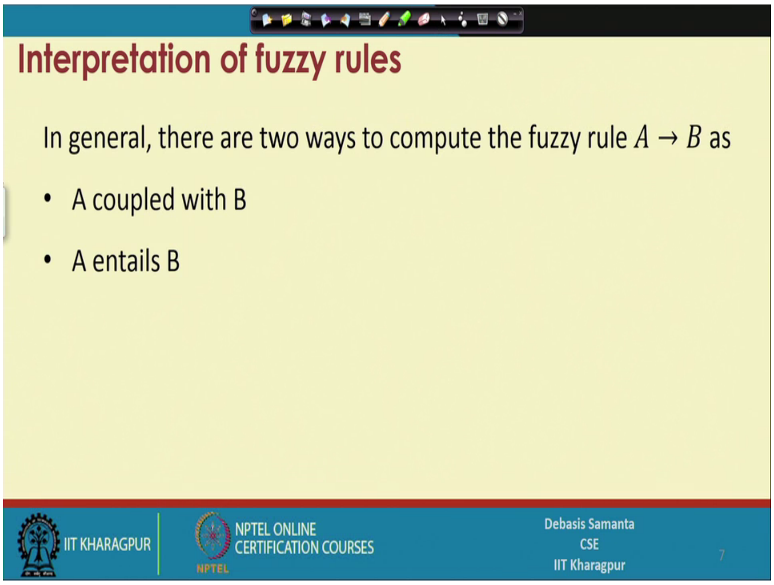

So, now, what is the important concept here it is that here in this calculation of these relation R, we have used min max simply a min a formula right if T high P low. So, the Cartesian product and taking the min we obtain these 2 these are the different entries, but in a more what is called the sense more practical manner there are many other calculations are involved so that this relation matrix can be calculated.

Now, whatever the methods are there they can be broadly classified into 2 broad categories one is called a coupled with B and A entails B. Now these are the basically different techniques or different principles to obtain the relation matrix. So, these not necessarily give the same result, because different method different principle or different interpretation give the different results actually, but all those results work in a different context of course, but we can use any one method to I mean calculate our relation matrix. So, it is depends on the fuzzy designer fuzzy engineer who wants to use in it is system that which method he should consider.

Anyway, let us see what are the different methods are there which belongs to a coupled with B and A entails B.

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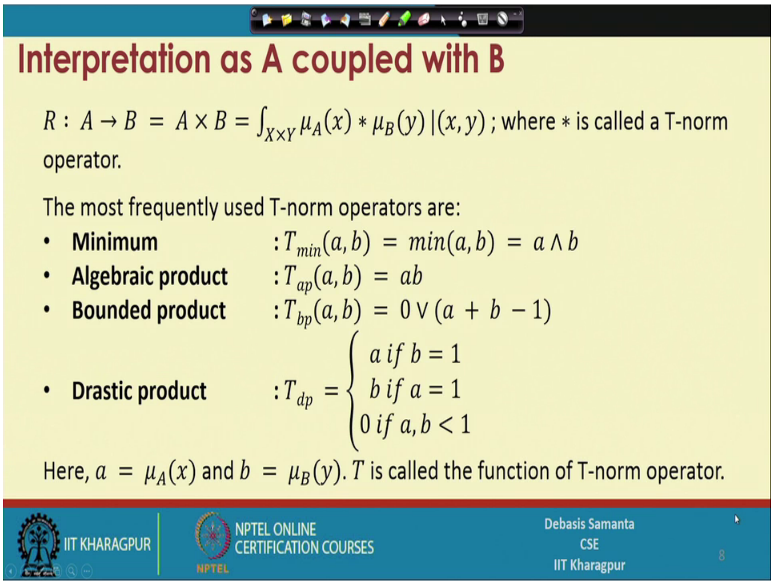

Now, let us first start with a coupled with B. So, a coupled with B are usually the relation R in this form and it is basically A cross B instead of only min operation that we have already discussed. So, it basically takes the general form which is expressed here. So, the relation matrix it is basically it is a mu a x star it is a operator is an operator among the 2 membership values mu ax and mu by for all x y that is there in the relation. So, here this star is called specifically it is called an operator it is called the T norm operator.

Now, let us see what are the operations that this T norm operator can signify here we see 4 different operations that can be can be explain or that can be applied. So, for the T norm operators are concerns. So, basically operators is symbol star it can be applied true compute either these or these or these one. So, different operator differ same operators can perform different calculations or different results depending on what type of operation that we are fixing for a relation in a relation.

Now, the first operation the T norm operator it is called the minimum and if you see T min a b this is the form this is a min a b. So, if A cross B and if it is the min of mu a and mu B that is basically the general Cartesian product we have learn so far. So, T min that is a minimum T norm operator is our the usual Cartesian product that we have already learned.

Now, there is other it is called the algebraic product, algebraic product is expressed is a product of the 2 membership values. Now like algebraic product it is called the bounded product bounded product is defined using this expression it basically takes the value of it is a maximum of 0 or this 1. So, it will take the value.

Drastic product it is denoted as T d p. So, star can be consider T d p and if we follow this one in a if b equals to 1 it will take the value b if a equals to 1 and it will take the value 0, if a b both less than 1. So, what I can say is that for the different entries we can have the different value if we follow the different operations according the T norm operator. So, if we follow min then it will give another relation matrix, if we follow algebraic product it will give another relation matrix bounded product another. And if we follow this kind of operation then it will give another relation. So, relation matrix will vary if we follow the different operations. So, if a particular operation is followed it will give a unique value. So, it is the concept that is the coupled with B the operation.

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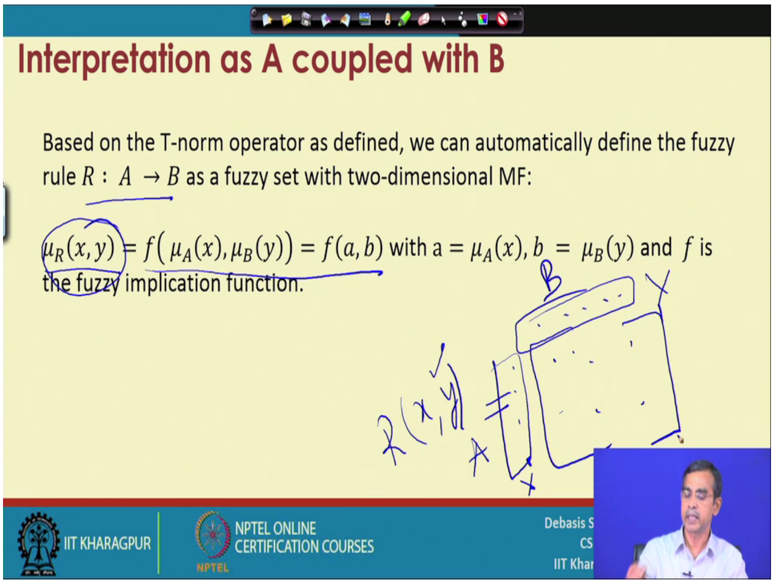

Now, so based on this T norm operators right we can automatically defined the fuzzy rule that is R A B, we can define a fuzzy rule R A B. So, this fuzzy rule can be as we told you that this fuzzy rule can be expressed in terms of a 2 dimensional membership function, a 2 dimensional membership function is basically mu R x y this is basically the value this one and representation of these 2 dimensional membership function is basically takes in the form of a matrix and usually this is related term as R x y. So, this matrix is this 1, where x is any element in this direction and y is any element in this direction.

So, x an element particularly belongs to a particular universe of discourse and y is an element belongs to a particular discourse. So, it is basically universe of discourse of y and these are the element of discourse x. So, these entry whatever it is there it can be calculated applying the 2 fuzzy sets A and B that is from the 2 universe of discourse x and y and following T norm operator.

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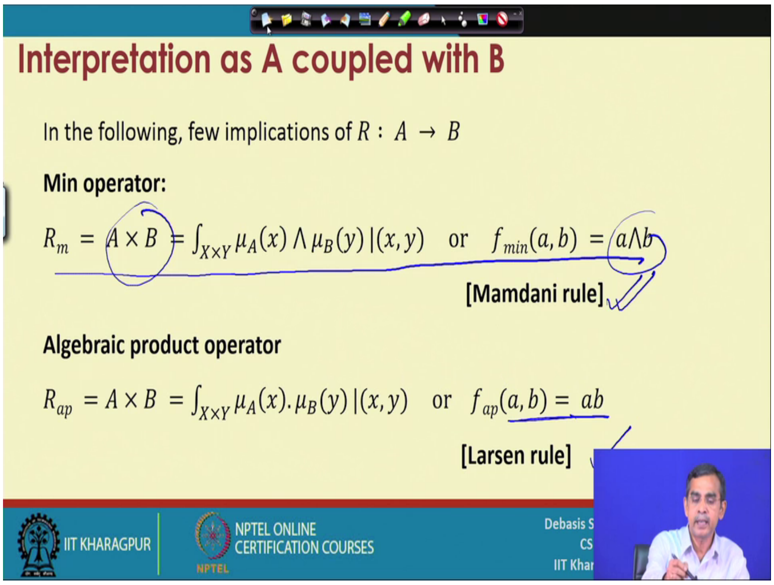

So, this is the basic concept of calculating the fuzzy relation calculation of fuzzy rules. Now interpretation of a coupled with B now I will come to the interpretation so min operator as we have discussed already. So, min operator T min it basically takes this form now such a rule is popularly known as Mamdani rule if we follow T min and In fact, the Cartesian product that we have discussed in last lectures we follow this A cross B as a min and then this one it basically follow the Mamdani rule there.

Now, algebraic product operator just we have discussed about just simply a product then it basically call the Larsen rule. So, the difference scientist who has proposed these rules according to name all those I mean rules name like Mamdani rule and then Larsen rule.

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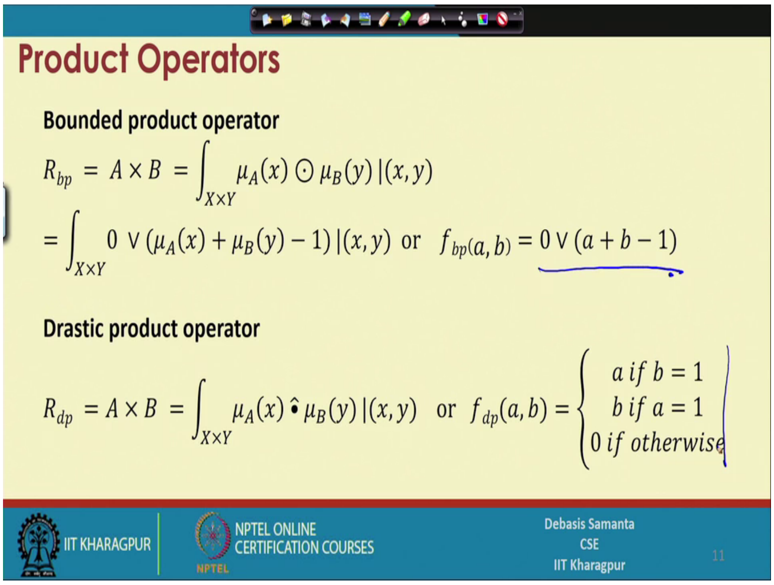

There are some other rules also bounded product operator it is just simply a there is no rule as such specific like Mamdani rule or Larsen rule it is just expressed using this formula using this formula. In this formula and drastic product operator as we have discussed using this one actually no specific in I mean rule or the name we assign to this kind of things are there they are basically the different way to calculate the fuzzy implication.

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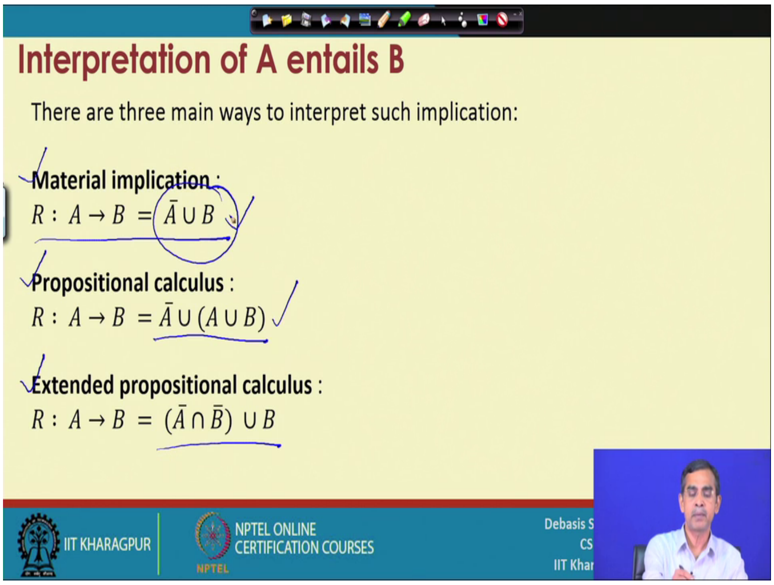

Now, I will come to another type it is called A entails B, now A entails B takes 3 forms. In fact, and the 3 forms are called material implication, the propositional calculus form and another is called extended propositional calculation calculus form.

Now, according the material implication a rule a fuzzy rule is describes in the form like this 1. So, fuzzy rule is here is basically a complement union B. So, we know how to perform this operation a complement union B. So, this rule can be obtain like this, now the proposition can be calculus takes slightly different from which is shown here it is basically a complement union and then a union B also take this form and the extended proposition calculus take this form.

Now, what we can learn from all these thing all these discussion is that. So, the different way of calculating the relation matrix that is all whether these things are equivalent not that is a another question, what I say that these may be this is and this is not necessary the equivalent this means that if you follow these and if we follow these they are not necessary give the same relation matrix. The different in a relation matrix interpretation may be different and then purpose or the application may be different. So, that is the thing and regarding application it is not necessary; it is not the right time to discuss when we discuss about it is a application when we discuss about this meaning that what are the different context the different rule that can be the different way the rule can be calculated is applicable.

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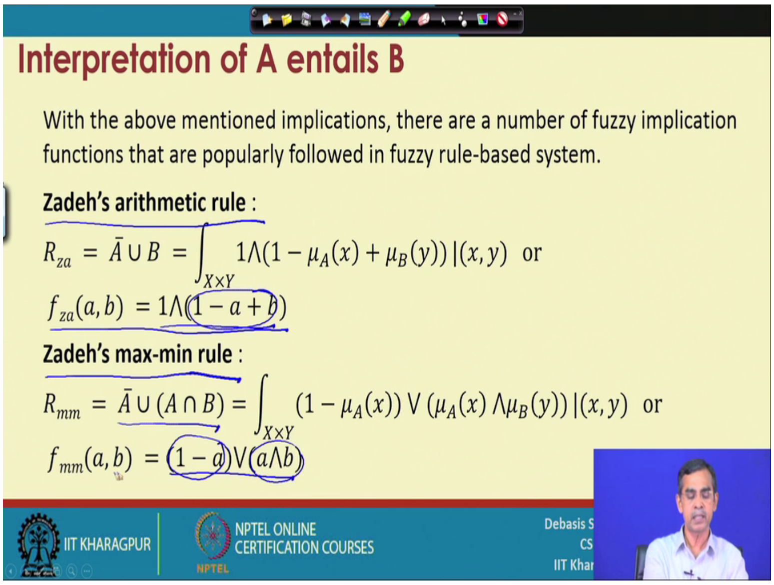

Now, so A entails be needs certain more discussion about how they work for us, now I will go one by one to discuss each and everything. So, the first rule it is called the Zadeh's arithmetic rule that we have discuss a material implication. In fact, so this Zadeh's implication is basically the material implication it is according to the Zadeh's name it is called the Zadeh's arithmetic rule and Zadeh's arithmetic rule. That means, if the 2 values A and B 2 membership values A and B give given then the relation entries for these 2 values can be obtained according to this formula is a basically 1 and; that means, it is basically min of 1 and the min of the result of this 1.

Now, another implication that is basically the proposition calculus proposition calculus this rule is again proposed by Zadeh's and it is called the Zadeh's Max-Min rule and it is notation is like this. So, it basically express in this form it is basically max of 1 minus A and then A and B. So, max of 1 minus A and A and B so this way. So, this basically the method or calculation to find a entry for 2 different values there and this 1.

So, these this is basically the 2 rules which is in the techniques of called A entails B.

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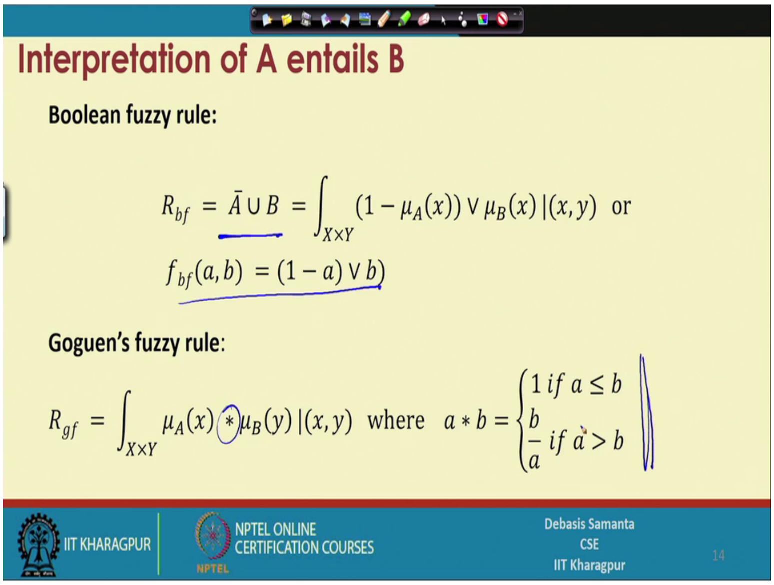

And there is another rules also a belongs to A entails B it is called the Booleans fuzzy rule Boolean fuzzy rule takes these form we can see this is already expressed the material implication according the Zadeh's min rule and here arithmetic min rule, but here in this context Boolean fuzzy rule they have given the different interpretation 1 minus A or B.

Similarly, the Goguen's fuzzy rule is a operation is like this and it is defined by this formula this expression. So, this is also another way to calculate the relation matrix.

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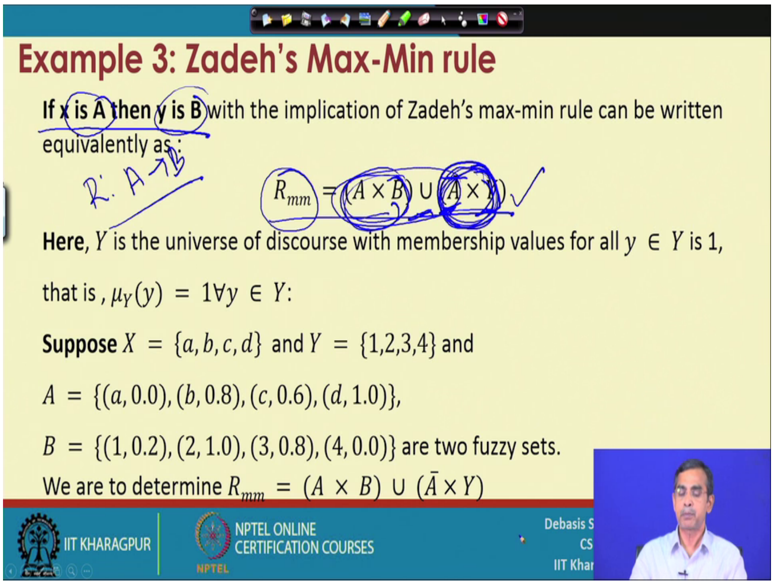

Now, here I want to discuss more elaborately about Zadeh's Max-Min rule this is the 1, what is called the method which is frequently used or more fuzzy engineer prefers this rule to calculate their relation matrix regarding a rule computation.

Now, let us see what exactly the Max-Min rule it is basically Zadeh's Max-Min rule that just now we have discussed it, but it takes slightly different form I will tell you why this form is anyway. So, this is basically to calculate the rule if x is A then y is B. So, these also can be say the R from A to B where A this is a proposition related to A and proposition related to b. So, this is the rule R A B.

Now, so far the relation matrix is concerned and this can be better expressed in the form A cross B union then a complement and Y, where Y; Y is the universe of discourse for the set B and A is a fuzzy set B is another fuzzy set. So, if we use this formula then we will be able to calculate the relation matrix which basically represents the rule like R A B where if x is A then y is B.

Now, you can recall when we discussed about Zadeh's Max-Min rule we wrote that it is basically R mm A bar union A cross B only this part, but here to make it generalized we follow this one right now the because if A cross B is basically can be store this result in the form of a 2 dimensional matrix and then union a; however, a is a 1 dimensional matrix. So, it is not possible to apply the union operation on these and these 1.

Now, in order to make it applicable so a another cross Cartesian product is applied over the A cross and Y. So, then it gives a relation matrix it gives a relation matrix 2 matrix are of same size and therefore, union operation can be applied; however, if we use this one result can be a bit different, but that is absolutely not an issue because. So, far the certain fuzzy is concerned this result is acceptable.

Now, so this is the concept regarding the Zadeh's Max-Min rule, now let us elaborate this Max-Min rule technique to compute the rule matrix using an example. So, this example I would like to refer here say x is A universe of discourse y is another universe of discourse this universe of discourse contains 4 element a,b,c,d and it contains 4 elements 1, 2, 3, 4.

Now, let us consider these are the 2 fuzzy sets which are defined over X and Y respectively. So, A is defined over X and B is defined over Y, now we want to calculate the relation matrix that means if x is A then Y is B form and this basically in this form. So, you want to calculate this 1.

Now, let us see how this value can be calculated. So, the idea is very simple we have to calculate first A cross B and then A cross Y and then take union. So, the relation matrix a can be calculated now here is an idea about the details calculation.

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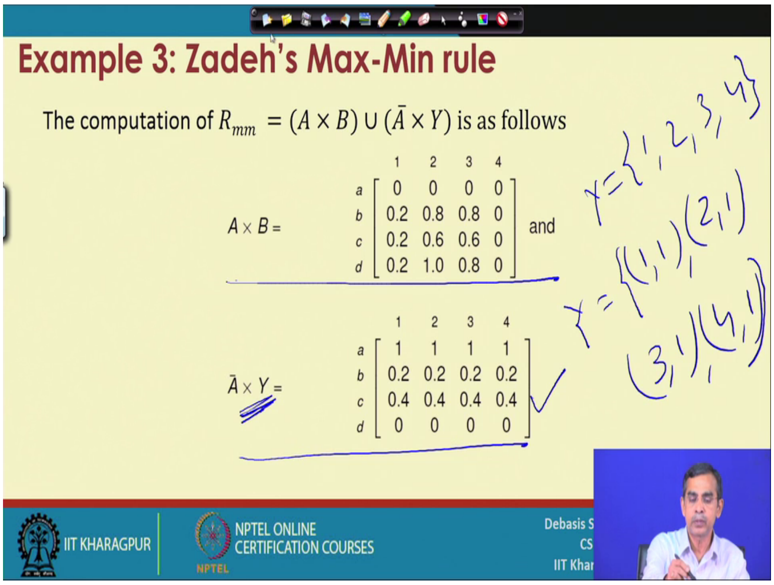

So, that we can understand about it so you see so given the 2 2 fuzzy sets A and B you can check that A cross B give this 1 and A cross y takes this form now here whenever we consider Y. So, Y is basically for all the discourse element that is belongs to the universe of discourse with their membership value one. So, Y is basically 1 2 3 and 4 and when we compute this result we take that Y this is equals to 1 1, because in the fuzzy form 2 1 3 1 and 4 1.

Then we can take the product Cartesian product a complement and then Y taking the min of these 1 and this kind of matrix can be obtained. I hope you have understood these things whether how these can be calculated once A cross B and A cross Y is known to us we will able to obtain.

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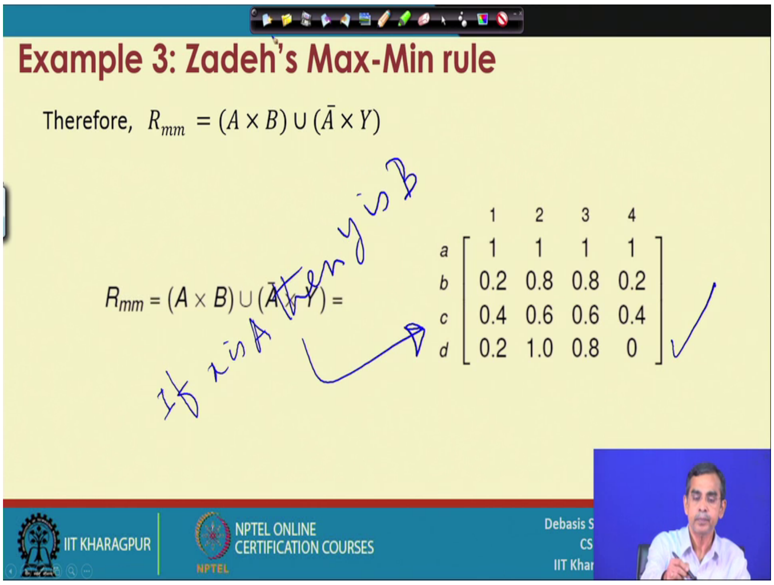

The final matrix that is basically a relation matrix and the relation matrix will takes this form. So, this basically is a relation matrix represents a rule that rule as I told you that rule is if x is A then y is B. So, this basically stored this basically can be represented in the form of a relation matrix.

Now, usually we follow in our illustration in subsequent lectures we follow generally Zadeh's Max-Min rule, otherwise you can practice or you can compute another matrix another relation matrix following other operation likes a T norm operator or some other a coupled with B operation whatever it is.

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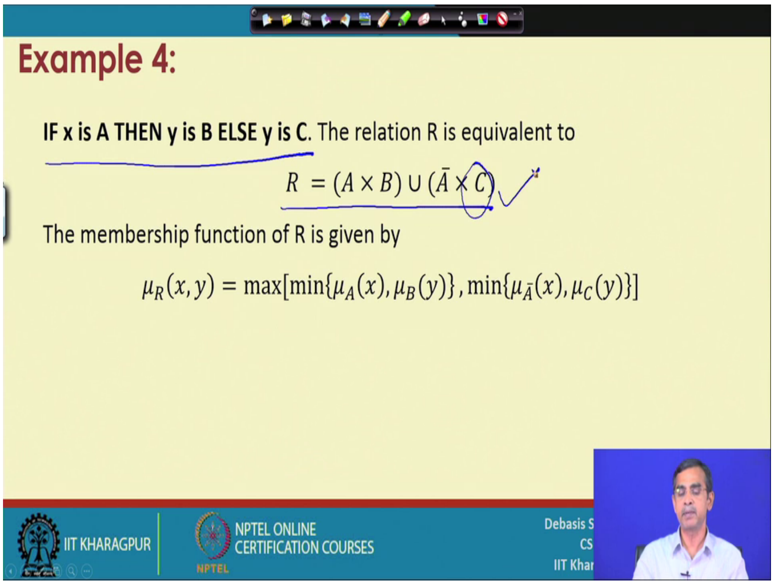

Now, so this basically gives a calculation about a rule if x is A then y is B, now the same Zadeh's Max-Min rule can be extended to another type of rule here I have mentioned the else part right if x is A then y is B else y is C. So, if it is like then it can be expressed using Zadeh's Max-Min rule composition it is like this one, we can note that this part that is a instead of y we use C. So, this is the o a that can be calculated and the rest of the things is very similar to the previous calculation here basically this 1. So, this else part is extra else part is extra if else C is added then we can add this 1 in this Max-Min composition.

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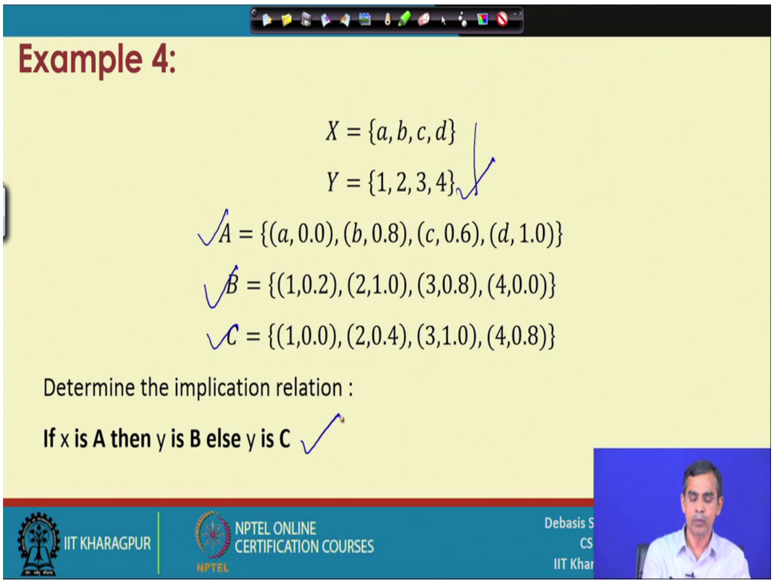

Now, these are example that is basically to calculate this 1. So, let us see here X and Y are the 2 universe of discourse defined over the A fuzzy set B fuzzy set is defined over X and Y respectively C is another fuzzy set defined over this universe of discourse.

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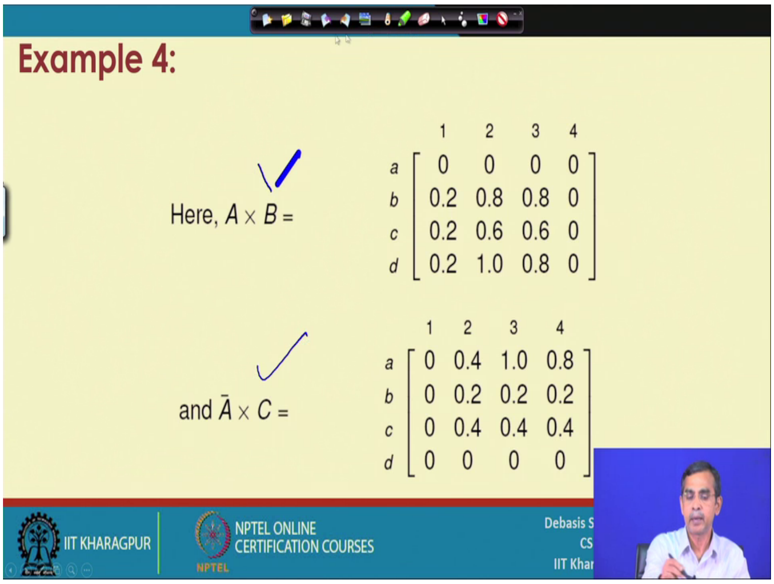

Now, this is the rule we can calculate the rule using the Zadeh's Max-Min composition here the A cross B calculation and here A cross C calculation. And finally the rule R this basically giving if x is A then Y is B else Z is C.

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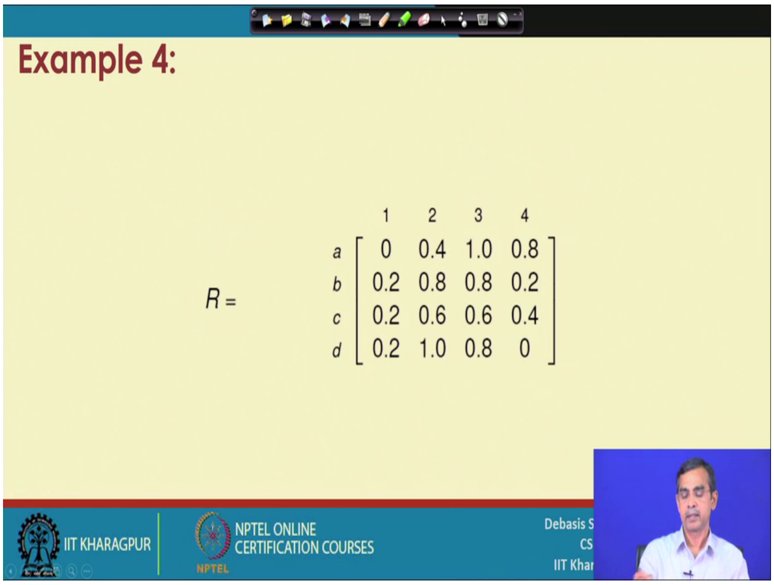

So, this is the rule that we can consider for this rule camp computation.

With this let us conclude here about this fuzzy rule calculation. We have learn many method out of which we want to limit our what is called the process in order to Zadeh's Max-Min method.

Thank you.