Introduction to Soft Computing
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Lecture – 06
Fuzzy Relations (Contd.) & Fuzzy propositions

In the last lecture we are discussing about fuzzy relation and there are few portions are remaining. So, I will start with that remaining portion and then finally, I will go to the another concept of fuzzy it is called the fuzzy proposition. So, the fuzzy relation that we have discussed as we see it is called the binary fuzzy relation. Binary means it is the relation over the 2 fuzzy relations. So, that is why binary or the union intersection or the composition they are binary fuzzy relation other than the complement that is the intersection, it is basic the complement of the fuzzy relation is a unary fuzzy relation.

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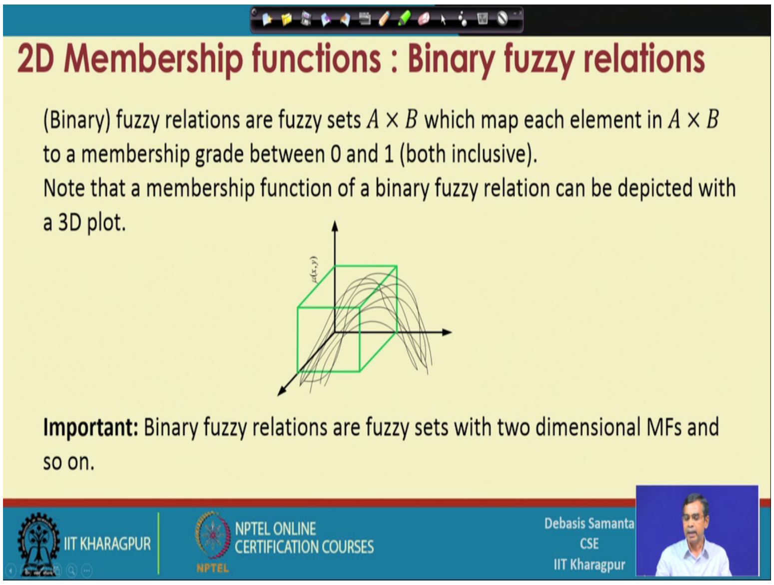

Now such a fuzzy relation can be also graphically display and that graphically it is look like. So, mu x y is basically the membership values of the relation and if we plot in a I mean a 3 D graphs then the graphically representation it will look like this is this kind of form like; that means, the membership function how varies. So, the graphical representation can be plotted with some mathematical tools like Matlab if the 2 relations given or the fuzzy the sets are given and all these thing. So, sometimes the mathematical operations and using some tools we can pictorially describe different relation or operation to understand how system works.

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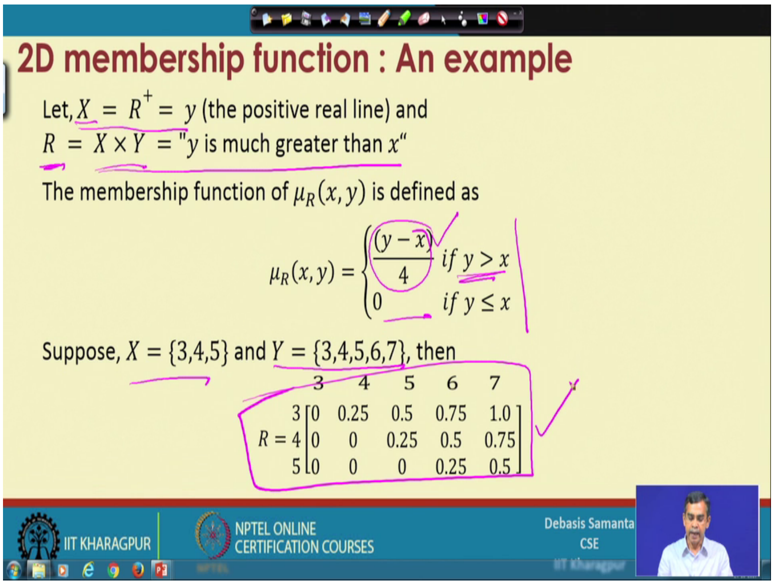

Now I want to discuss about this binary fuzzy relation with some mathematical meaning in it and here is an example that I want to show this example you can check it carefully. So, X and Y X is a fuzzy set which is defined like this over the universe of discourse R the positive it is basically positive values or positive numbers and R is the another relation which is defined over the discourse X and X cross y or Y is also another relation is defined over the a set or positive numbers. And the relation x is basically or y it is the relation is defined here that y is much greater than x the membership function of such a relation can be defined more mathematically using this formulas approach. So, is a much greater than it is discussed by these relation and if it is y is greater than x and if y less than x then it is 0.

So, this basically gives the membership value for a relation be between x and y is an order pair in the relation and it can be depicted in the form of a matrix now here an example. So, x is a set y is another set and if we follow this relation then the relation matrix that can be obtained like this one. So, this is basically is an another way of representing 2 D membership function is a binary function also other than the max min composition that we have discussed. So, there is many interpretation that can be applied, and then fuzzy relation can be obtained now after this fuzzy relation is known to us.

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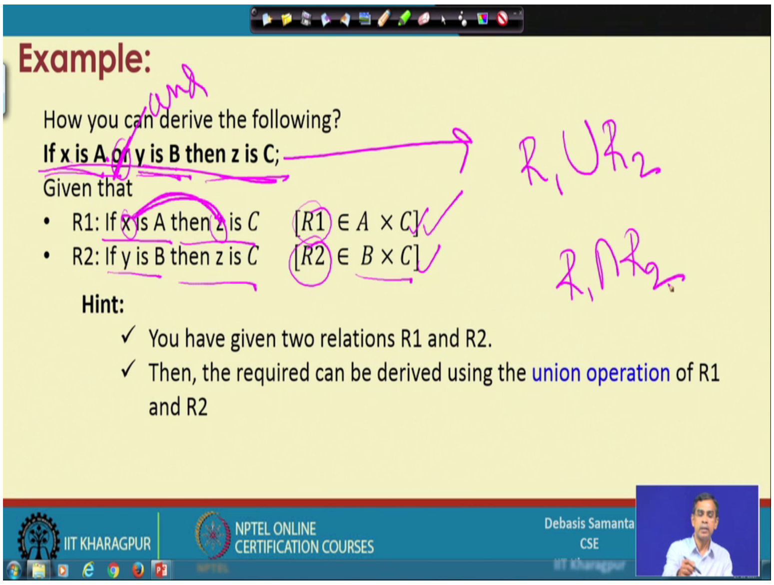

This is the another implication of the fuzzy relation. So, basically A cross B if it is a fuzzy relation as you know that is basically say the relation that if it belongs to a particular relation then what is the strength.

Now, I can say one example here see if x is A or y is B then Z is C. So, this kind of concept can be expressed by means of 2 product like see if x is a then Z is C it can be represented by means of the product Cartesian product of the 2 set A and C. So, this gives the relation R 1, similarly if y is B then Z is c it can be represented by the Cartesian product B and C and this is the relation R 2. Now A Cartesian product between the 2 fuzzy sets results A relation and A relation takes the form like this if x is a then Z is C this means that, how x and Z is related or the relationship strength between x and zee Z which belongs to the set A and set C likewise.

Now, if here this is another example if you see if 2 relations are related then what will be the result in this one for example, if x is A or y is B then z is C how it can be represented. So, it is basically if x is A this is another relation y is B then another relation. So, it gives z is the another relation. So, if the 2 relations are given to you by this form. So, this kind of expression can be obtained that this is the relation R 1 or is a R 2 so union. Similarly if I say if x is a instead of or it is and then that relation can be obtained by intersection this R 2.

So, this basically shows some application where the relationship is used and they can be applied. So, the operation that we have discussed there we can apply and then we can find other relations as well as this one. Next we will discuss about another important elements in the fuzzy system it is called the fuzzy proposition. So, exactly what is the proposition proposition basically one statement? So, statement like sun rises in the east. So, it is a proposition what is the truth value truth value means sun rises in the east it is value is basically either 0 or 1 or true or false. So, it returns true.

Similarly the many propositions can be given for example, the mango is sweet it is a proposition and it can result any value sweet yes no or may be sweet may not be sweet or these kind of things are there. So, proposition not necessarily give only 2 value either yes or no or true or false it can give any value. So, in case suppose it gives only 2 values then we say it is a binary proposition or binary proposition is also sometimes called the predicate proposition or simply the crisp proposition. Now the fuzzy proposition means that truth value; that means, the value of a proposition that is possible not initially only 2 values it can be true or more values.

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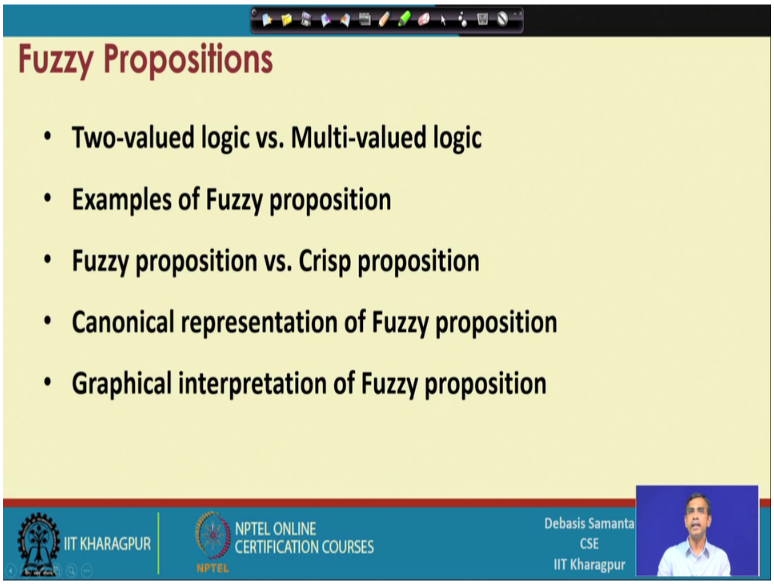

So, let us see how the fuzzy proposition can be defined and how it can be applied in our fuzzy system and what are the different operations those are possible in order to have the fuzzy proposition more meaningful implication or significance. Now fuzzy proposition is basically as so fuzzy logic. In fact, is called the multi valued logic in contrast to the

Boolean logic is called the 2 valued logic. So, this is the one difference between the crisp logic or fuzzy logic or Boolean logic or fuzzy logic. So, crisp logic or Boolean logic they are basically 2 valued logic or as the fuzzy logic is basically the multi valued logic

So, exactly this is the important difference is there now in this discussion in the coming discussion, we will first learn what is the difference between 2 valued logic and then multi valued logic or how the 2 different logics are related and then we will discuss about some examples on fuzzy proposition, and then we will try to find the difference between the fuzzy propositions and then crisp proposition mainly the Boolean proposition or predicate proposition and then we learn about how to represent a fuzzy proposition, this kind of representation is called the canonical representation and the graphical interpretation of a fuzzy proposition.

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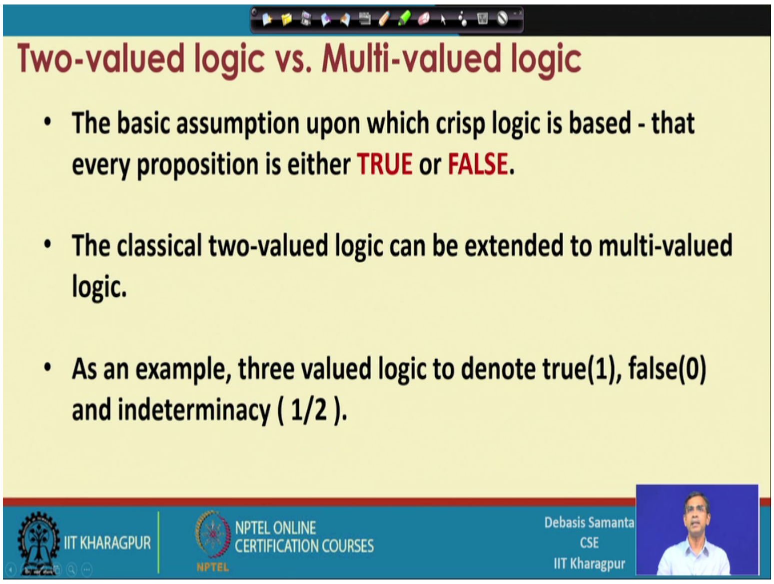

Then now let us start about the discussion about 2 valued logic versus multi valued logic as I told you 2 valued logic basically based on the 2 values of the logic the 2 values are namely true or false sometimes it is represented 1 or 0 yes or no. So, whatever it is there only 2 values now the classical 2 valued logic. In fact, whatever the concept that is applicable to 2 valued logic can be extended to multi valued logic. Now it is interesting to note how these extension can be can be take can be taken place in case of multi valued logic namely say binary logic.

Now, for this illustration we will consider multi valued logic in terms of 3 values true false and in in determinacy; that means, in between 0 and 1 it is say half; that means, a logic can be defined whose values or outcome can be in terms of any 3 value 0 half and 1.

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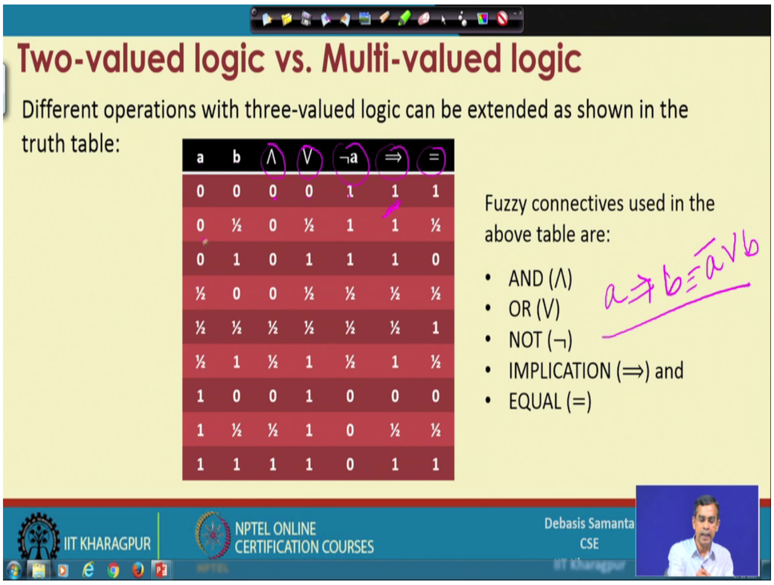

So, having this is the composition now let us see how the multiple logic can be defined using the conventional 2 valued logic operation. Now conventional 2 valued logic operation we say it is the and operation or operation and operation is very similar to the intersection operation the or operation is very similar to the union operation. And this is the operation called the complement and this is the implication operation and this is the equal operation.

Now if you know Boolean logic then you will be able to understand how the and operation of 2 Boolean variables can be applied or a complement of a Boolean variable can be can be operated. Now here for example, if a and b are the 2 values for the 2 variables a and b then it is and operation is 0 and then or operation is 0 complement is 1 and this implication is 1. So, implication is basically see a implies b is equivalent to a complement or b. So, it is the general expression this is basically same thing a implies b is equivalent to a complement or b. So, this is borrowed from the Boolean logic here for example, a complement means one or b 0. So, this is why this value is 1.

Now, let us consider. So, these are 0 0 now here 0 and half now likewise the and operation is basically 0 and half we have to take the min minimum. So, and operation is minimum. So, this operation gives the result 0 and or operation means taking the maximum 0 and half the maximum is half. So, it is half and then complement complement means a complement. So, it is this one if you take the b complement then you can understand, what is the result b complement will also result half because is 1 minus half and a implies this and can be obtained using this formula you can obtain this one and a equals b it is half this kind of things it is there.

So, these are the table that can be applied for the different operation namely and which is equivalent to intersection or union is a complementation and implication and equal are the 2 special operation that is there in case of Boolean logic. Now these are the operation that we can have that this is the I mean truth values for the 3 valued logic and 3 valued logic is very closely related to the concept of multi valued logic.

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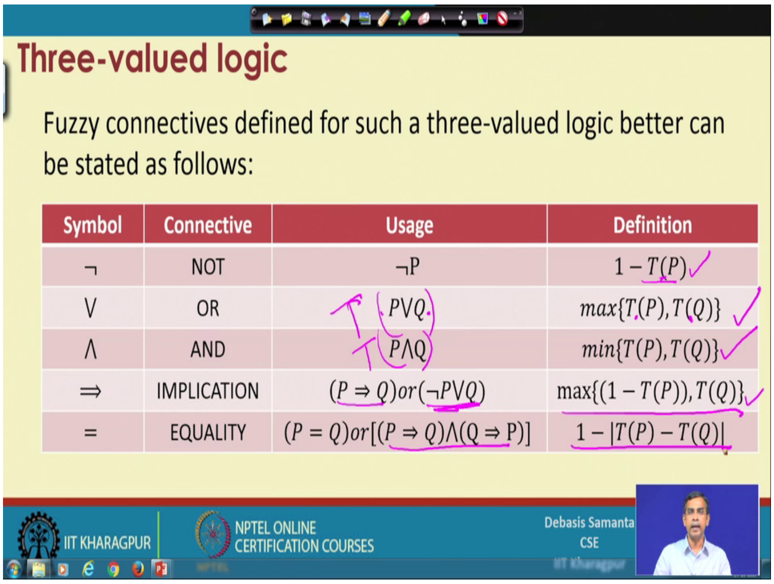

Now, I can little bit move forward to indicate the different operation that can be applied in terms of proposition now here for every proposition have some truth value.

If we denote these proposition as a T P say suppose P is a proposition suppose P is a proposition and T P denotes it is truth value then the complement of this proposition is basically one minus T P, now or if 2 proposition P and Q and their truth values T P and T Q known then P and Q the union operation of the 2 propositions can be expressed using this formula is a max; that means, maximum of the values of T P and maximum values of T Q gives the result of the this is basically T P T P truth value of the P or Q.

Similarly, 2 proposition P and Q if we apply the intersection operation which is defined by this expression then we can give these are truth value of these operation, then implication it is I told you P implies Q equivalently not P or Q this can also be alternatively can be observed that it can give this one. So, it is the max of one minus T P and T Q, now equality with a P equals Q it can be defined alternatively P implies Q and Q implies P which alternatively can be obtained using this formula.

So, what I want to say is that the propositions are there propositions may have 2 valued 3 valued or multiple valued logic and then their operations deferent operations these are the different logical operations symbols and then whatever the meaning and that can be applied and can be obtained using this kind of definition. Now let us see some example. So, that we can understand the concept of proposition more meaning More clearly say one proposition these ram is honest.

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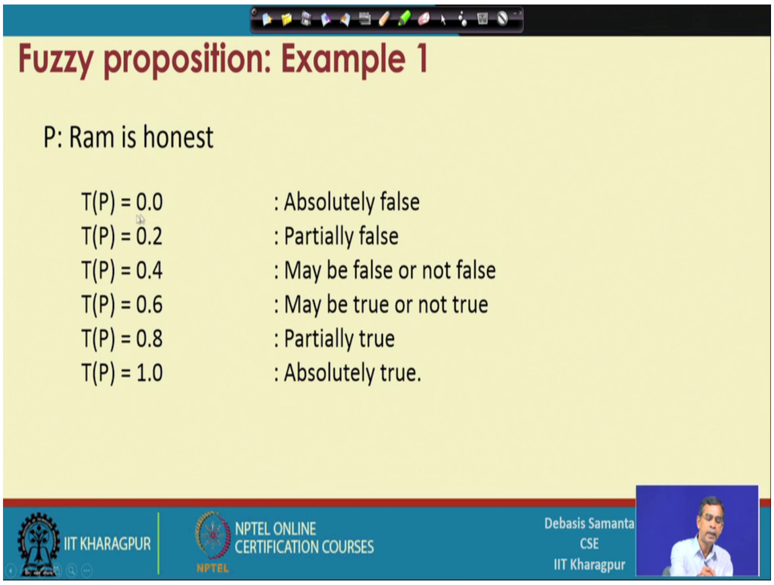

Now, Ram is honest having this proposition can have many truth value. So, T P 0.0 this is basically in terms of fuzzy logic; that means, if we draw a fuzzy sets that ram is honest then the membership function take these value it is like this.

So, according to a different a elements it can be obtained like this. So, it is basically the different truth value that is possible ram is honest T P 0.0 0.2 0.4 whatever it is there a meaning that it is absolutely false is a meaning that it is absolutely false or partially false these kind of things here so; that means, P is a proposition and this proposition may have different values that we can say the different truth values the other. So, this way it is multiple valued concepts.

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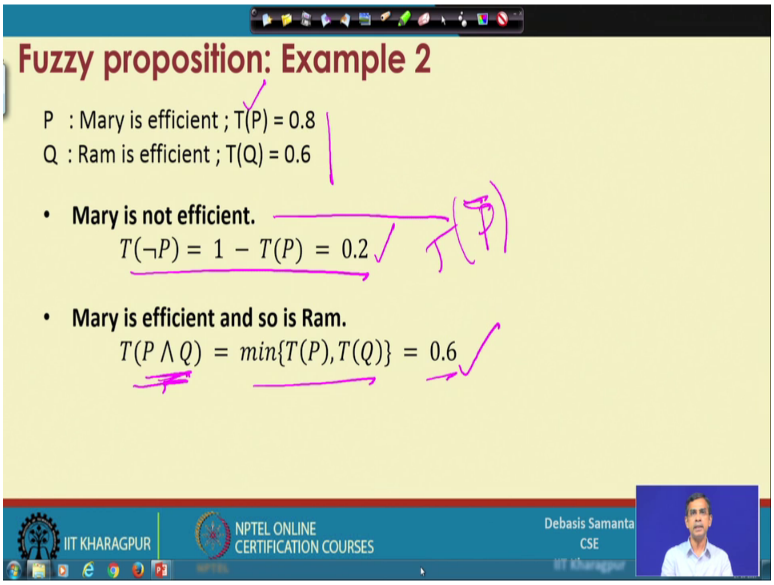

Now, let us see some idea about extending this idea. So, say ram is honest as you have discussed about it now let us consider other 2 propositions P and Q where the P is defined as Mary is efficient and another proposition Q it is defined as ram is efficient. Now this P being a proposition let it is truth value that is denoted as T P and it is 0.8 the another proposition Q having truth value T Q and is denoted as 0.6. Now these are the 2 propositions then we can see how the different operations are applicable to this proposition to give something more meaningful proposition.

Now P is the Mary is efficient we can have another proposition Mary is not efficient, this basically equivalent to P and Q are the 2 proposition as you have discussed here with this notation and Mary is not efficient this is the another proposition and this proposition I can express as a P complement or not P whatever it is so P complement. Then if Mary is not Mary is efficient given this T P then the T P of this t naught P this can be expressed by this formula 1 minus 2 T P; that means, if Mary is efficient is a proposition then Mary is not efficient another proposition naught P having the truth value 0.2.

Now, if we say the another proposition in terms of P and Q S are given propositions let the proposition be Mary is efficient and. So, is Ram; that means, it is Mary is efficient as well as ram is efficient. So, if so in terms of P and Q we can apply the and operation then we can obtain the truth values of this new proposition P and Q and that can be obtained taking the minimum values of this 1. So, it is basically 0.8 and 0.6. So, this is the results or resultant values of the new proposition P and Q and truth value is 0.6.

So, the different propositions and having the different proposition the different operation that can be applied those operations very much, similar to the operation that is there is Boolean logic that can be further extended to the multi valued logic that we have discussed using min using max, and some mathematical relation and then we can obtain the different propositions given the some elementary proposition or some basic proposition it is there.

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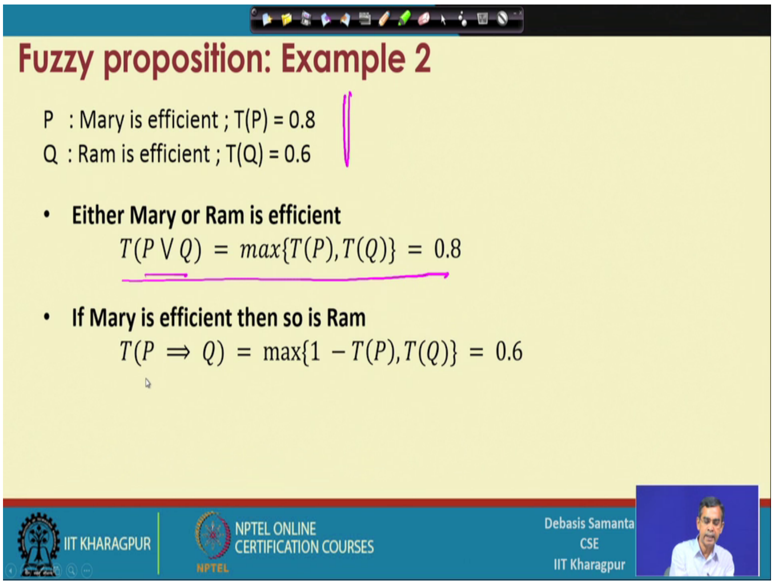

Now again let us see. So, P and Q are the 2 proposition which is discussed here and then we can have another proposition either Mary or ram is efficient.

So, if it is it is basically depressant by means of the operation P or Q; that means, Mary efficient or Ram is efficient then truth value can be obtained using the max operation another example it is here if Mary is efficient then. So, is Ram? So, the operation that is applicable to this fuzzy set is basically is basically represented here if Mary is efficient then. So, is Ram? So, we can represent this one; that means, if P is true then Q and it is a implication now truth value of this P implies Q can be obtained using this expression 1 minus T P and T Q and this value can be obtained at 0.6; that means, if Mary is efficient then. So, is ram is another proposition with truth value is this one.

So, it is basically in terms of proposition and using some proposition we can obtain some other proposition applying the operations.

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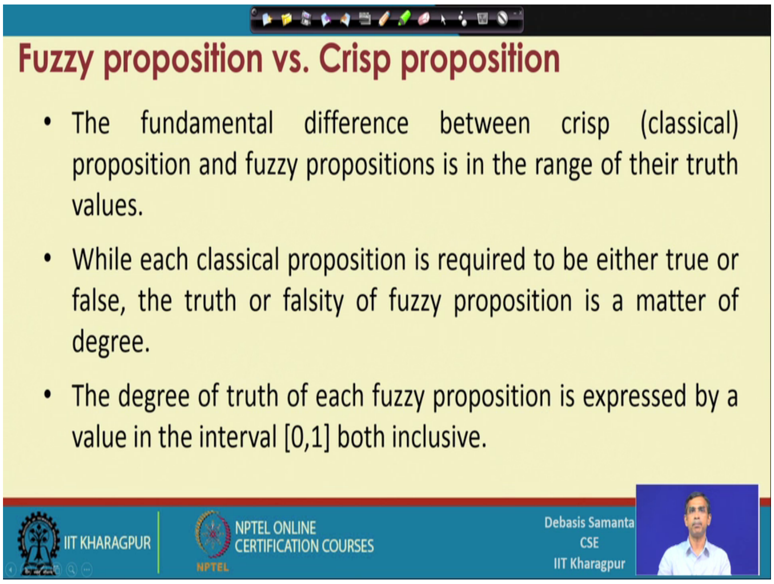

Now, on the multiple logic that we have discussed, now we have learned about the fuzzy proposition and the crisp proposition the fuzzy proposition basically is the multi valued logic proposition where the crisp proposition is basically 2 valued logic proposition. So, so is a this is the crisp value is also called the classical proposition and the fuzzy proposition is a newly defined proposition for the fuzzy logic

Now, so here we can see in case of crisp proposition this is required either true or false value to be return in case of fuzzy proposition it can return any value in between 0 and one and this value basically signifying the degree of strength in the propositions the resultant proposition.

So, degree of truth of is fuzzy proposition is expressed in the range 0 to 1 both inclusive. So, this is the difference between the fuzzy proposition and crisp proposition in case of crisp proposition the result is always in terms of 0 and one and in case of fuzzy proposition it is in between any value 0 and 1 inclusive both the things.

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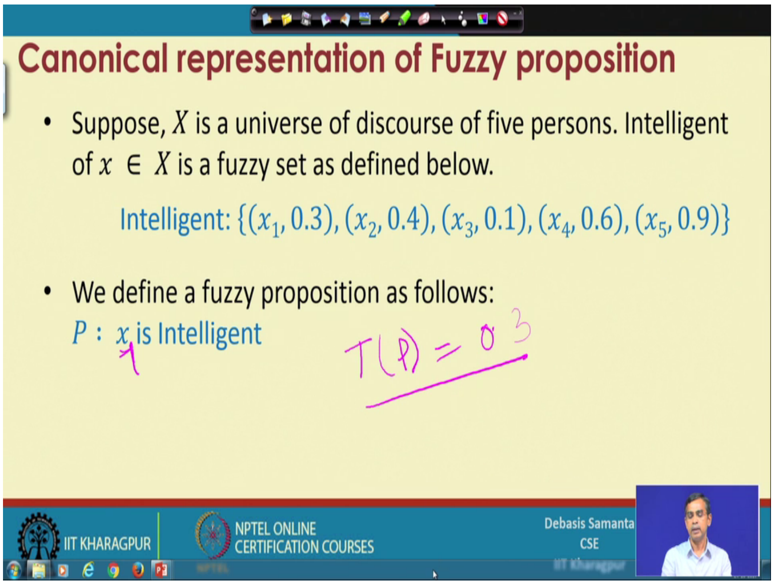

Now, we will come to the discussion about canonical representation of fuzzy proposition because whenever we learn about fuzzy logic we have to express some canonical way the universal way to do this thing let us consider the X is a universe of discourse this discourse consist of 5 persons and we want to define one fuzzy proposition that intelligent. So, X is intelligent like say ram is honest with the different value. So, intelligent is a fuzzy concept or is a fuzzy linguistic and any element X that can be belongs to this fuzzy set intelligent can be defined using this formula this is the fuzzy set.

So, now here you can check it. So, x 1 is a one person I say that he is intelligent with this degree 0.3, similarly x 4 is a another person he will be treated as a intelligent with degree 0.6. So, this is the fuzzy set defined over a discourse universe of discourse of 5 person and their membership value is represented here. Now having this representation now I can say that we can define a fuzzy proposition in terms of these fuzzy sets like this x is intelligent. Now here x is basically can be qualified any one of this one x 1 or x 2 or x 3 this one; that means, then it will return say suppose x is for an example x is x 3 then what is the T P.

So, it basically T P in case of x 1 is intelligent. So, T P will give you sorry it is ah T P x 1 is intelligent it will give you 0.3. So, this is the proposition that x one is intelligent and it basically gives the truth value that 0.3 now so this is the truth value if it is represented, Then we can say this is basically is a one way of representing a proposition and this is called the canonical representation.

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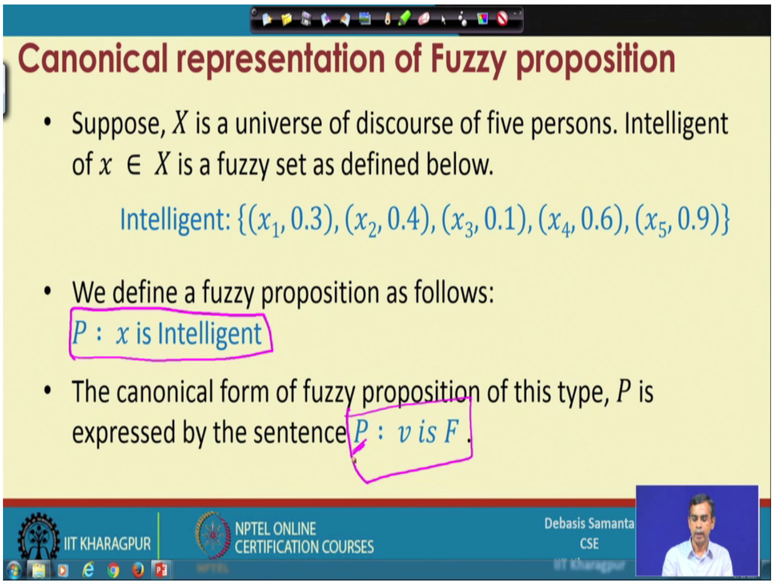

So, canonical representation in general can be expressed like this form where P is a proposition and v is an element which belongs to a fuzzy set a and it gives basically proposition that v is F and then P gives the proposition.

For example F denotes a temperature and v is say 20 degree. So, 20 degree is. So, F denotes a hot temperature. So, 20 degree is hot temperature then it basically is a proposition and the 20 degree is a hot temperature whether it is with certain value it can be any value in between 0 and 1, it is basically gives the value of this proposition. So, this is the canonical representation of the fuzzy sets and it is denoted by P as P that is v is F.

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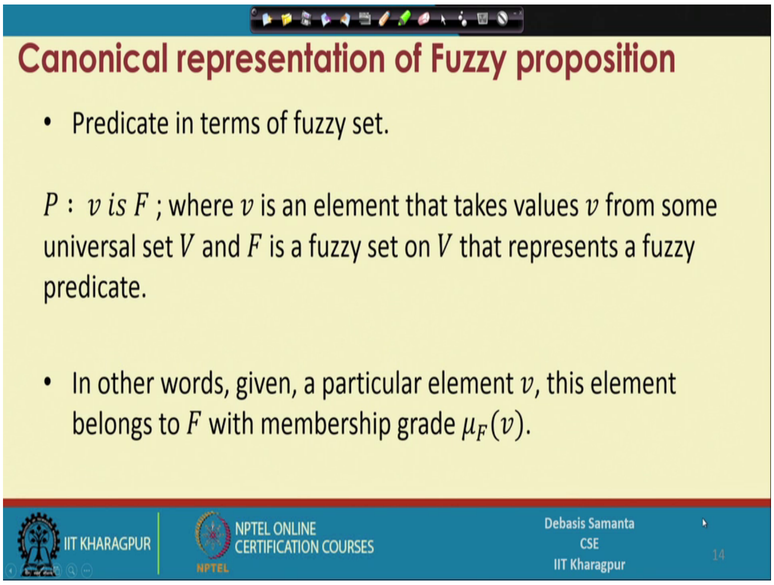

Now, this is basically other way P in in terms of proposition in fuzzy logic this P is called the predicate in terms of fuzzy sets. So, P is the predicate and this is the proposition expression in the canonical form.

So, where v is an element that takes value v from some universal or some universe of discourse the v and f is basically a fuzzy sets defined over this universe of discourse v. So, this way we can have the fuzzy fuzzy proposition or a predicates we can say.

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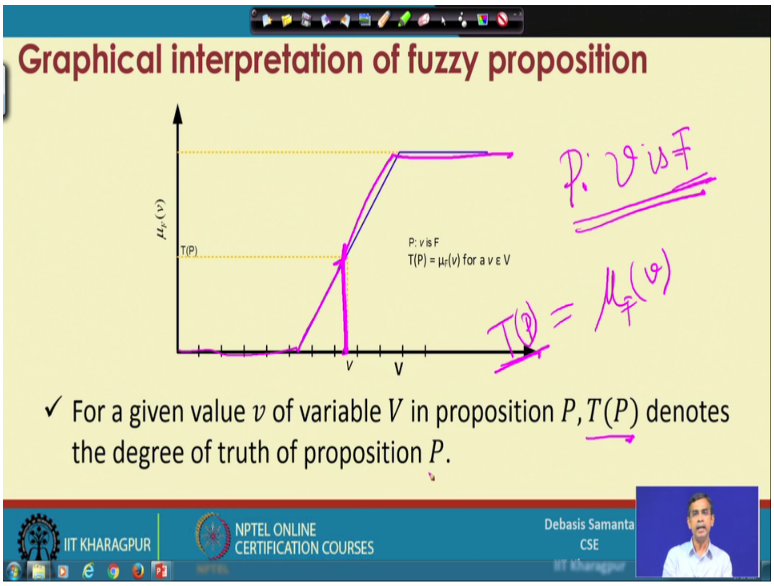

Now, in other words given an given a particular element v this element belongs to F with membership grade mu F v. So, here the idea it is here we can see we can elaborate this concept more meaningfully here and see here as I told you P is a proposition and it is denoted as v is F; that means, where F is a fuzzy set if f is a fuzzy set then definitely it will be defined by a membership function this means that their same membership function can be drawn by means of a graph.

Now, here this is the graph of the fuzzy set a; that means, how the value of the membership function varies with different elements and the different elements is defined over the discourse universe of discourse this is the v. Now for any element that belongs to this fuzzy set a let this element is v then v is F it basically gives the membership values for these v. So, it is basically mu F and v now this is basically the T P that mean truth value of this proposition.

So, it is basically we can write in another way if P denotes a proposition such that v is F then T P basically denotes the membership values for this proposition. So, for a given value v of variable belongs to set v it is in proposition P with degree of value T P it denotes the degree of that proposition and this is the graphical interpretation of the fuzzy proposition there.

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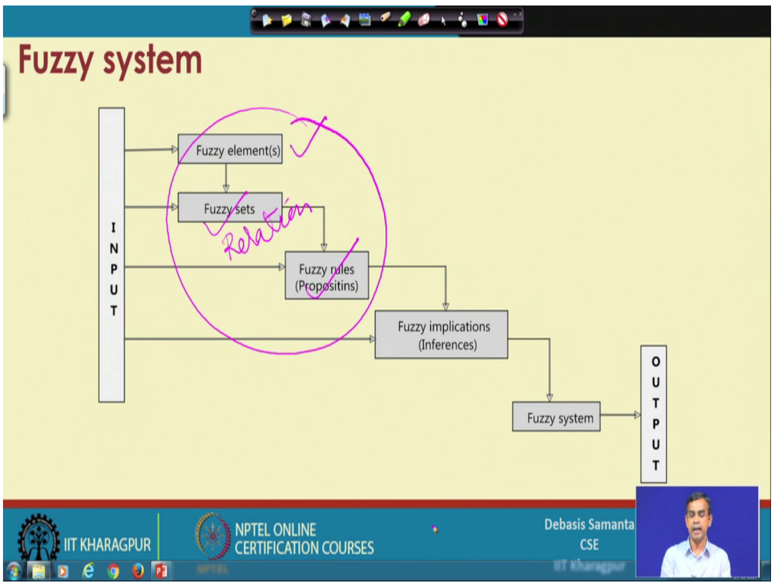

So, this is the proposition now we are we have discussed many things so far. So, fuzzy element the basic concept we have already discussed there, now fuzzy elements we have discussed and then we have learned about the fuzzy sets fuzzy proposition just we have learnt in order to learn this fuzzy proposition we have learned that the relations that mean fuzzy sets. So, these are the portion that we have learnt. So, for now our next learning objective is fuzzy implication. So, these things will be discussed in our next lecture.

Thank you.