Introduction to Soft Computing
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Lecture – 04
Fuzzy operations

So, we have learned about fuzzy set. Fuzzy set is the basic element in fuzzy system. Today we are going to learn about the different operations on fuzzy sets.

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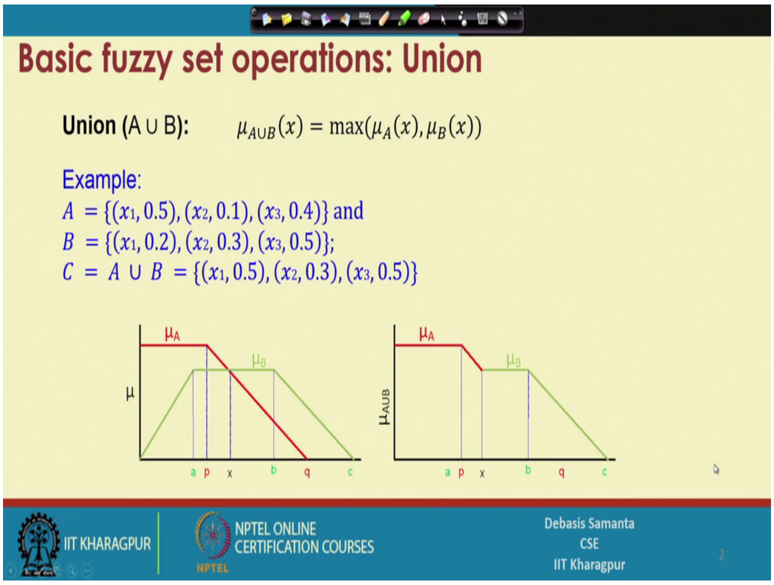

So, the different operations on the fuzzy set in many ways similar to the operation those are application to the crisp set and this is the normal that we have learned in set theory, the operation those are applicable they are in crisp set such as union intersection complement all those things also are applicable to the fuzzy sets, but the definition of all these operations are with different implications. So, we learn 1 by 1 the operations on the fuzzy set.

So, let us first discuss about the union operation of 2 fuzzy set. Now here A and B are 2 fuzzy set suppose. So, union of 2 fuzzy sets is denoted by the symbol cap this is the usual symbol that is used they are in crisp set. So, A cap B denotes the union of 2 fuzzy sets A and B. Now the union operation whenever it is performed on the 2 fuzzy set it will basically give the membership functions for each elements which are belongs to the union of 2 fuzzy sets, the membership values for different elements in the union of the 2 fuzzy set is defined by this expression.

So, if we see this expression. So, union operation for any element x which belongs to the union of 2 fuzzy set A and B is basically max of mu A x and mu B x, where mu A x denotes the membership values for an element x belongs to fuzzy set A and mu B x denotes the membership value of the any element A x belongs to fuzzy set B.

Now, let us have an example say here A is a fuzzy set which is defined with the different membership values for different element shown here, similarly B is the another fuzzy set. Now the union of 2 fuzzy set is denoted as C which is A cap B is basically this is the fuzzy sets. So, union operation gives another fuzzy set, now here we can see how we have obtain the membership values for the different elements which belongs to the sets C so here C. So, here 0.5 it is obtain as the max of these 2 value 0.5 and 0.2 it is as per the definition. Similarly 0.3 it is basically max of the 2 values membership values they are in the set A and B and likewise. So, this way we can obtain the union operation of the 2 fuzzy sets.

Now, the same thing can be better explained with the help of a graph. So, here we see the graphical representation of 2 fuzzy sets A and B, the graphical representation means it is basically a representation of the membership function how they change a change with the different values of elements belongs to the 2 sets. Say here the 2 sets with the universe of discourse. So, this is basically the universe of discourse of the 2 sets and the fuzzy set a is defined by it is membership function for the different element, which is denoted here, this is the membership element, membership function for the fuzzy set A. Similarly this denotes the membership element membership function for the fuzzy set A similarly this denotes the membership elements for the fuzzy set B.

Now as the union operation is basically max of the 2 values. So, upto this portion so this is the B and this is A. So, the max is this 1. So, these basically the union upto this part upto this part and then so union of these and these basically the max. So, it is this 1 this part is the rest of the value upto this one. So, this way we obtain the membership function of the union of the 2 fuzzy set which is represented by this graph.

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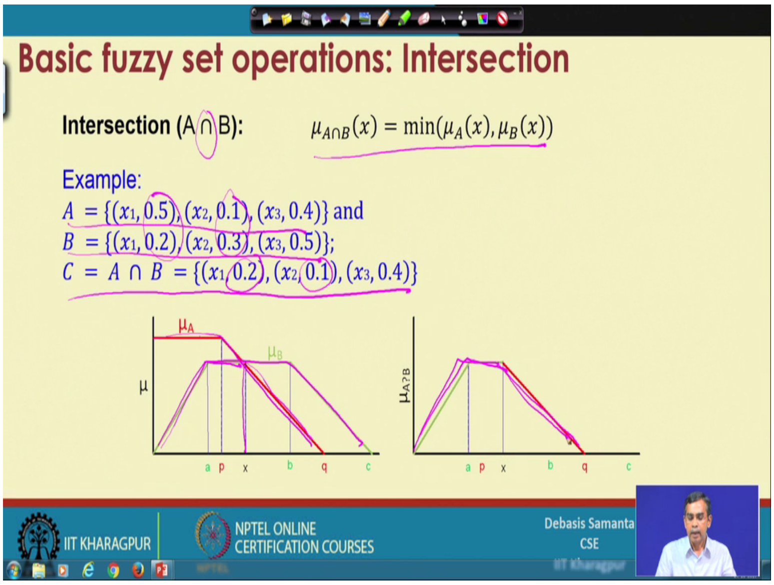

Now let us discuss the intersection operation. Intersection operation of 2 fuzzy sets is denoted by this symbol it is cap. So, here and the membership functions of the resultant fuzzy set A cap B is denoted by this expression and you can say in case of union it was max whereas, in case of intersection we have to take the minimum of the 2 values of the membership for which the A and B belongs.

Now, as an example A is the another set and B is the another set and the intersection of the 2 set is represented here, now here whenever we go for intersection for the element x 1 then we have to take the minimum. So, we take the minimum from here. So, it is the minimum. So, here 0.1 which is in A and 0.3 for x 2, which is in B and in the union operation, where the x 2 has the value minimum this 1 so 0.1, this way we can obtain the intersection of 2 fuzzy sets.

Now, the same thing again can be drawn graphically. So, this is the fuzzy sets A and this is the fuzzy sets B and we have to take the minimum of the 2. So, for upto these part minimum of A and B it is basically this one. So, we can obtain this part and for the rest minimum of this 1 is this one. So, we can take this one. So, this graph basically shows the membership functions of the union of 2 fuzzy sets A and B.

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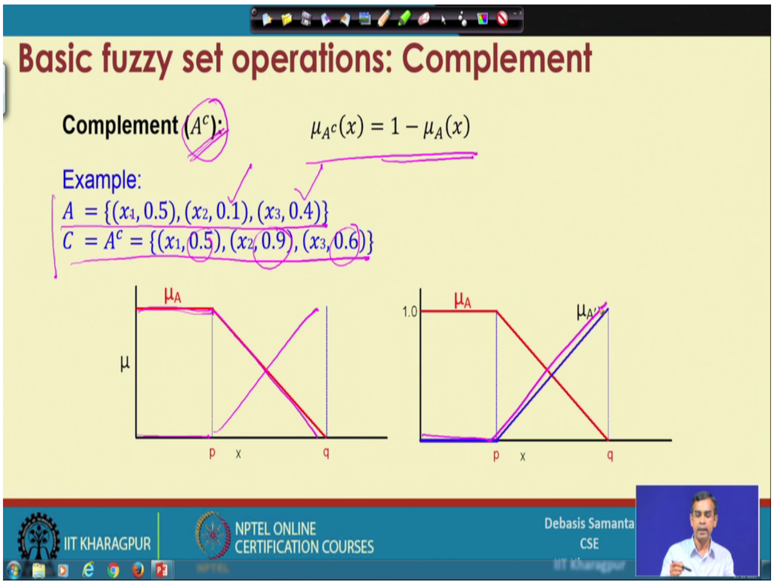

Now another operation on the fuzzy set is called the complement the union operation and intersection operation are the binary operation, because they needs 2 fuzzy sets whereas, the complement operation is an unary operation which is applicable to only 1 fuzzy set. The complement operation on any fuzzy set A is represented by the symbol A superscript C. So, it is basically a complement of the fuzzy set A and the resultant value of the membership functions is defined by this expression mu A C x is; that means, is a membership function of the resultant fuzzy set is c and it is denoted as 1 minus mu A x square mu A x denotes the membership function for the fuzzy set A.

Now, if this is an example. So, A is the fuzzy set and then it is compliment A C is like this. So, for x 1 we have to take the this formula 1 minus mu x. So, it is 0.5. So, for the 0.1 it is 0.9 for 0.4 it is 0.6. So, the complement operation is straight forward the same thing again can be shown graphically. So, this is the fuzzy set for this is the fuzzy set A for the membership function is like this now it is complement. So, the complement value of these is basically this 1 and complement value of these basically this one. So, the resultant value of the membership function for the complement A is basically this one. So, this way the complement operation can be obtained.

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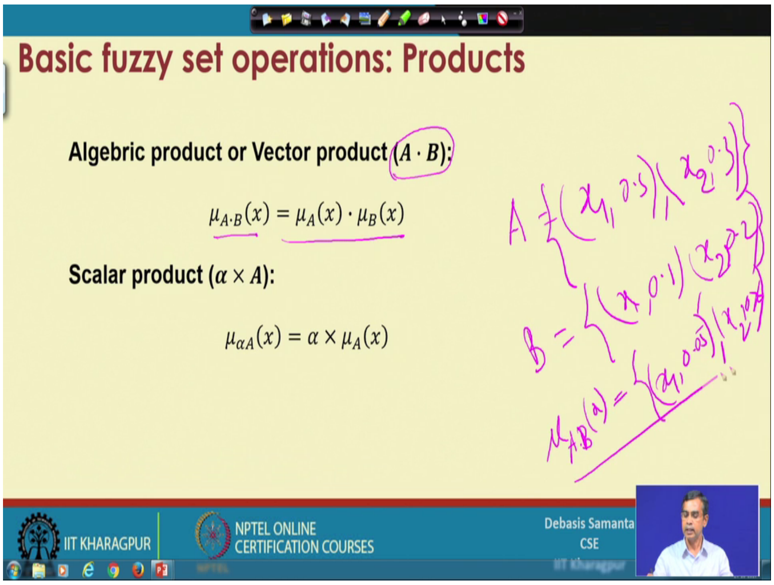

So, these are the very simple operations union intersection and complement; there are few more operations the first operation in this context is called algebraic product or vector product. Now algebraic product or vector product is denoted by A dot symbol algebraic product or vector product is denoted by the dot symbol, A dot B where a is A fuzzy set and B is another fuzzy set. So, the membership function for the resultant fuzzy set is denoted by this and it is obtained using this formula it is basically simple as a product of the values of the membership functions belongs to the fuzzy set A and B.

Now if A is a fuzzy set x 1 0.5 and x 2 0.3 this is the fuzzy set and b x 1 0.1 x 2 0.2 then the mu A dot b x for C that can be obtained. So, x 1 and it is basically product 0.5 and 0.1. So, it will give you 0.0 5. Similarly x 2 it will give you 0.0 6 and so on. So, this way the product can be obtained.

So, this is the vector product now like vector product is a scalar product where alpha is a constant alpha is a constant usually value in between 0 and 1 both inclusive. So, the scalar product of a vector A is denoted by these formula mu alpha A x for alpha a is a basically scalar product of the fuzzy set A and it is new membership value is defined as is a product of alpha and the membership values of the function a x.

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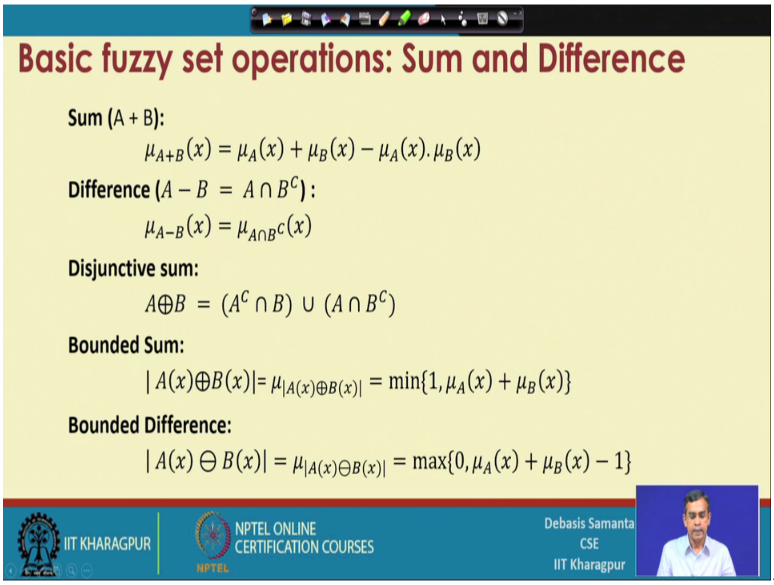

Now, there are few more operations 1 by 1 let us discuss them. So, the first operation is sum of the 2 fuzzy set A and B, it is denoted by the sum of the 2 fuzzy set A and B is denoted by A plus B plus operation and it is new membership function is denoted by this expression it is easy to evaluate, if we know the mu A x for the fuzzy set A and mu B x for the fuzzy set B, then mu A plus B for the fuzzy set A plus B obtain using this formula.

Now, the difference operation on 2 fuzzy set A and B is denoted by this notation A minus B, which is equivalent as the A intersection of B, complement and it can be obtained using this expression mu A minus B; that means, membership value of the difference of the 2 fuzzy set A and B is same as the membership values of A intersection B complement x. So, if we can calculate. So, better idea is that we can calculate the intersection of A and B complements and then the mu can be obtained easily from there.

Now, there are disjunctive sum it is denoted by this expression A xor B like and it can be obtained using this 1. So, A C intersection B union A intersection B complements. So, this means that if we know these operations first and then this operation then we will be able to calculate A xor B. Now this is the disjunctive sum next is bounded sum bounded sum expression is basically denoted by this where A and B are the 2 fuzzy sets and the membership values of the resultant fuzzy set it is denoted by this it is basically obtained using this formula. So, it is basically take the minimum of 1 and the sum of the membership values of A and the membership values of B fuzzy set B.

So, this is bounded sum and another is bounded difference the bounded difference is expressed by this notation and it is membership value can be obtained using this formula it is the maximum of the values 0 and mu x plus mu bx minus 1. So, we can take maximum of these 2 values for each element x then we can obtain the fuzzy set membership values of the elements which belongs to the bounded difference.

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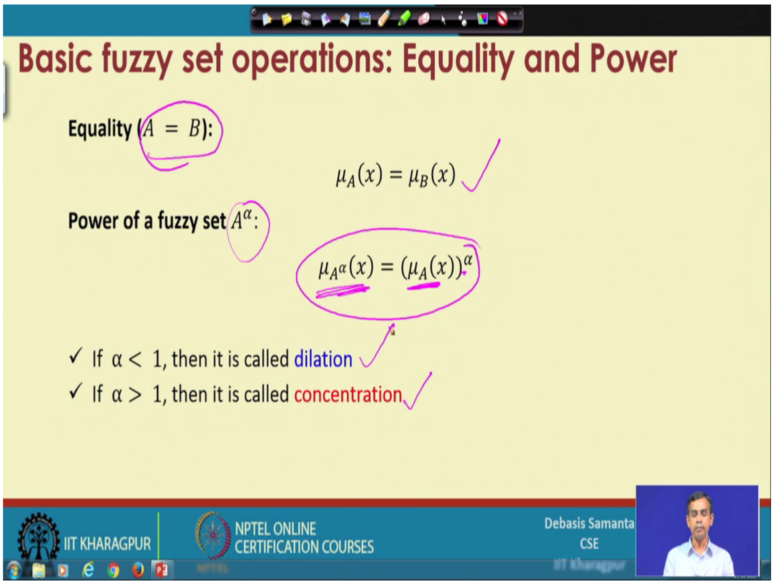

Now equality and power these are the other 2 fuzzy sets we say the 2 fuzzy set can be 2 fuzzy sets A and B they are equal and it is denoted by it is denoted by it is denoted by this expression A equals B. So, 2 fuzzy sets are equal if for all elements belong to the set has the same membership value. So, which is represented by this expression, now power of A fuzzy set which is denoted as A alpha where alpha is A constant any value, then the resultant fuzzy sets whose membership value can be obtained using this expression where this is the membership values of the resultant fuzzy sets and mu x is the original fuzzy sets and here mu x to the power alpha.

So, it is A exponential operation that can be applied for each elements belongs to the set x then we can obtain the resultant membership values belongs to the set A alpha, now we can note if alpha is greater than 1 then it is called the concentration and if alpha is less than 1 it is called the dilation. So, these are the 2 operations that is possible for A and B right.

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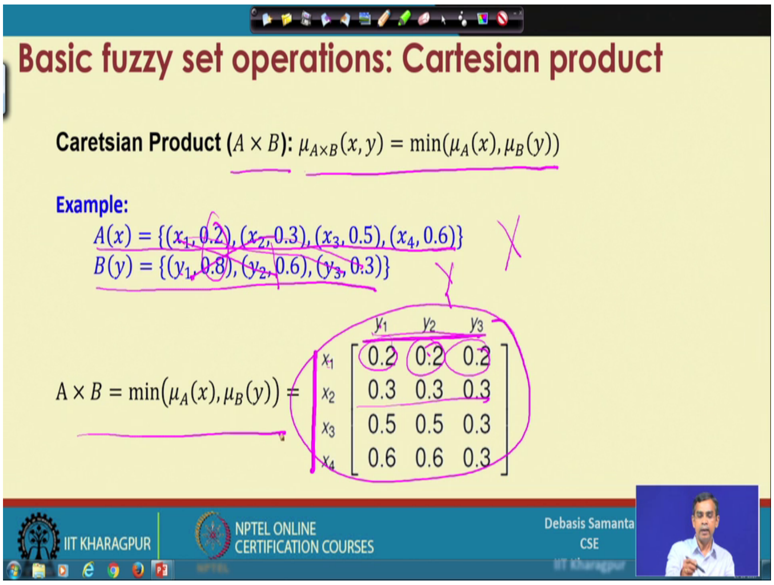

Now, another operation which is very much frequent operation in fuzzy logic it is called the Cartesian product, the Cartesian product of 2 fuzzy seta A and B is denoted by this notation denote this notation and it is membership function is denoted by this notation. Now we can note the operation it is basically mean of A x and by now here again product whenever we say the product it is basically is A product for all elements now for an example suppose x is the A is the fuzzy set, which is defined over A universe of discourse A x and B is the another fuzzy set which is defined over another discourse Y.

Then the product A cross B can be defined using. So, here x 1 and y 1 and then take the minimum so it is basically 0.2 and 0.8 so 0.2. So, these can be better represented by means of A matrix. So, all the elements which belongs to the set A can be represented here and all the elements which belongs to the set B can be represented here and then x 1 and y 1. So, x 1 y 1 and take the minimum in between x 1 and y 2 x 1 and y 2 is we are taking the minimum of 0.2 and 0.6. So, it is 0.2 then x 1 and y 3 0.2 minimum of 0.2 0.2 and 0.3 and it is 0.3.

Similarly, for x 2 y 1, x 2 y 2, and x 3 y 3, so this element will be obtained. So, the Cartesian product can be expressed with the help of this kind of matrix and it is called the relation matrix regarding the relation matrix we will learn many things in A later stage. So, this is the operation that can be performed as A Cartesian product and 1 thing you can notice the union, intersection or whatever the operation that we have discussed they basically applicable on 2 fuzzy sets A and B and all the fuzzy sets A and B they are defined over the same universe of discourse whereas, the Cartesian product can be applied to the fuzzy set they can be over the 2 of the same universe of discourse.

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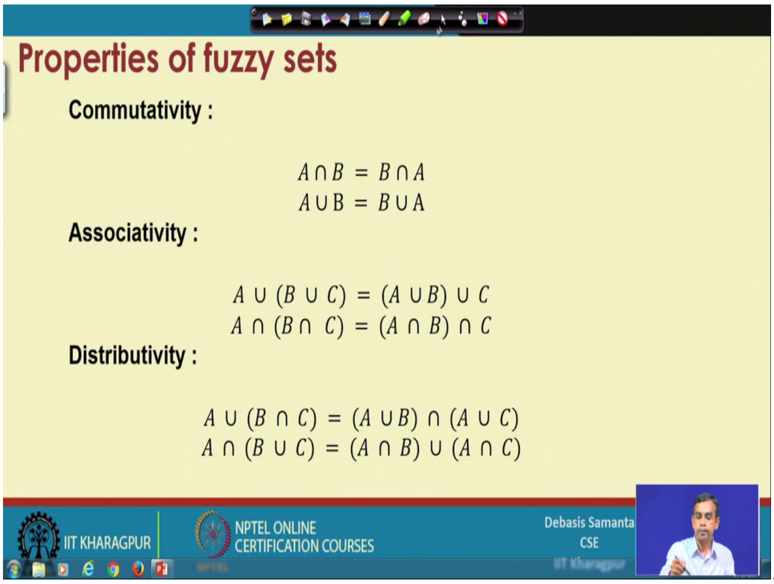

Now the operations those we have discussed they follow certain properties the commutative property that it is basically A intersection B is same as B intersection A this is called the commutative product commutative property over the intersection and this is the commutative property over the union operation. Likewise it is called the associativity; that means, this and this are equivalent or these and these are equivalent the distributivity property also if A union V intersection C is same as A union B and intersection of A union C. So, it is basically A is distributed over B and A is distributed over C likewise for this operation also.

So, whenever the many operations are involved and they can be applied on 2 fuzzy sets then they satisfy these are the properties it is there are some other properties also it can be applicable there on the fuzzy sets.

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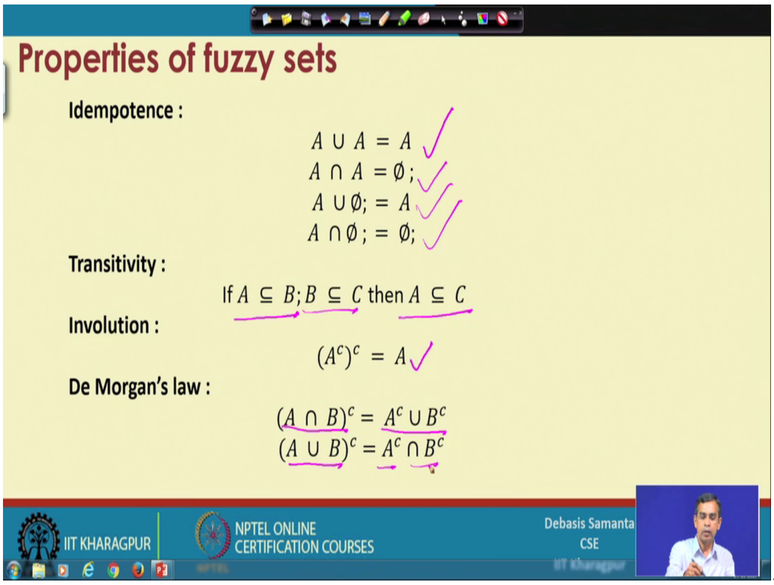

The other property like idempotence; that means, if A is A fuzzy set and if we take the union of the same in it will give you the same fuzzy set A. Similarly A and intersection of A it will give you the null fuzzy set phi and the other thing is simply understandable.

Now, transitivity is basically if A is A subset of B and B is A subset of C then we can say that A is A subset of C. So, this is the similar operation that is applicable for the crisp set now involution these operation be is if we take the complement of the complement of the same fuzzy set then it will return the original fuzzy set. The de morgans law we know which is there in Boolean logic as well as the crisp logic. So, it is basically A intersection B and if we take the complement it is same as A complement union B complement, alternatively if A union B take the complement it is equivalent to A complement intersection B complement.

So, these are the properties those are whole good or fuzzy sets these properties are very much useful whenever we want to perform many operations on the different fuzzy sets and 1 thing we can note.

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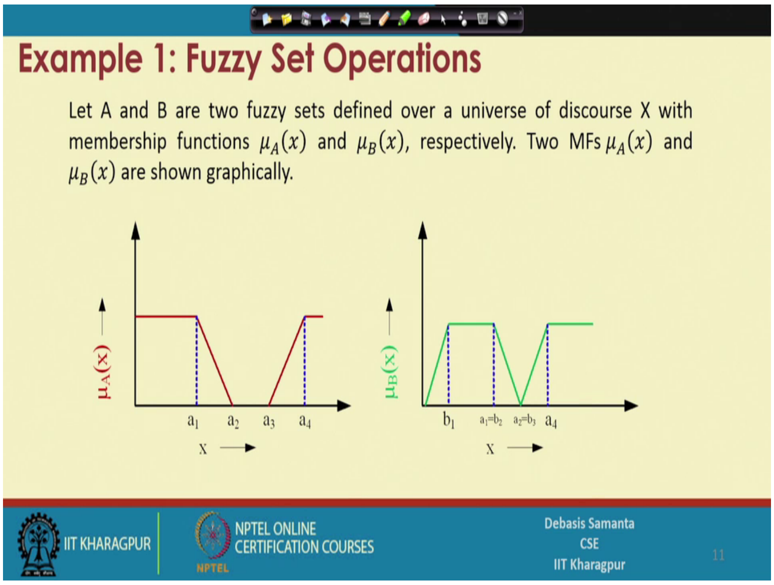

The operation that we have discussed about it basically if 2 fuzzy sets are given then how we can obtain another fuzzy set. So, if A and B is known to you then we can obtain the another fuzzy set C, now if we know C then we can help we can use another operation to find another. So, those operations are basically produce many other fuzzy sets from the given fuzzy sets.

Now I would like to elaborate the fuzzy set operation more clearly with some figures is A basically call the graphical representation. So, sometime you not to understand the fuzzy operation graphical representation of the 2 things is more useful and there are many tools which basically follow the graphical way of representing fuzzy set and then performing their resultant operation.

Now, here first example here this is the membership function of. So, this this figures shows the fuzzy set A whose membership function is shown by this graph and another fuzzy set B whose membership function is shown using this graph. Now we have to find many operation like say union like say intersection whatever it is there now in order to find the union or intersection or complement of course, complement is not required here only say union or intersection let us approach.

Then best idea what is we can draw the 2 graphs on the same graph now for example, if we draw the graph of the 2 graphs.

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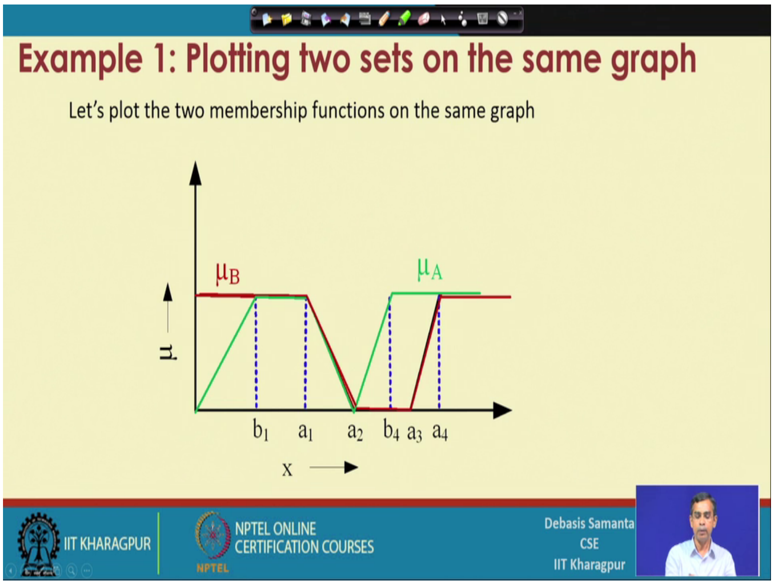

The resultant graph will be obtained like this, now once A resultant figure is available then we can easily identify if the union or it is the intersection of the fuzzy sets.

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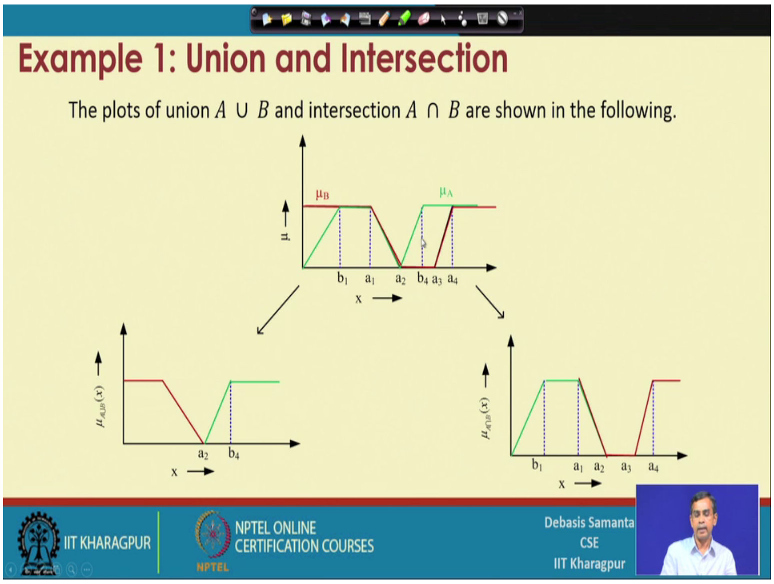

So, here if this is the graphs of the 2 fuzzy sets on the same plot then we can have the union union operation, it is basically union operation union operation we take the max of this one. So, upto this part this is the max part and then these for the next part. So, this why this is the union resultant this one.

Similarly, for the intersection we take the mean these and then these. So, basically this is the graph for the intersection. So, what I want to say that if we can plot the membership function of 2 graphical representations then from there we can obtain the resultant fuzzy sets that is also graphically.

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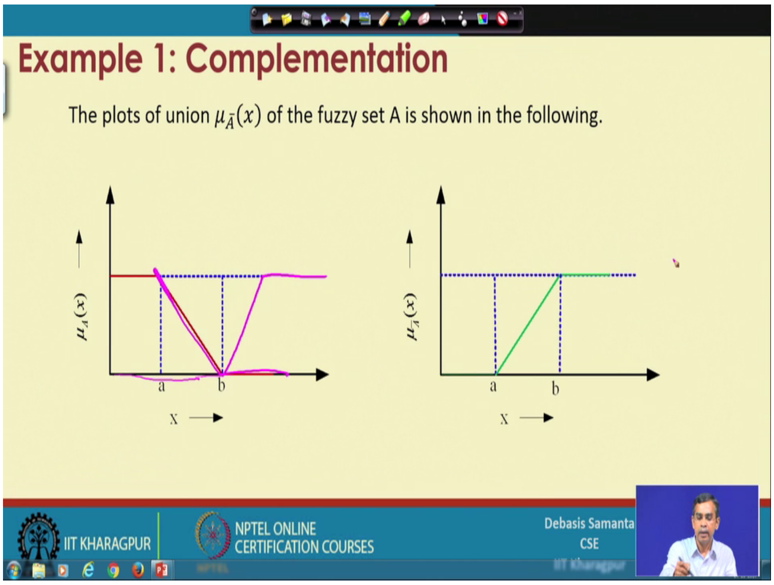

Now, here again another example so this is the fuzzy sets A and it is complement is basically complement will be this 1 this 1 is the complement. So, it is basically shown it is here. So, graphical way of representing there are operation is sometime more useful to understand. So, we are discussed it.

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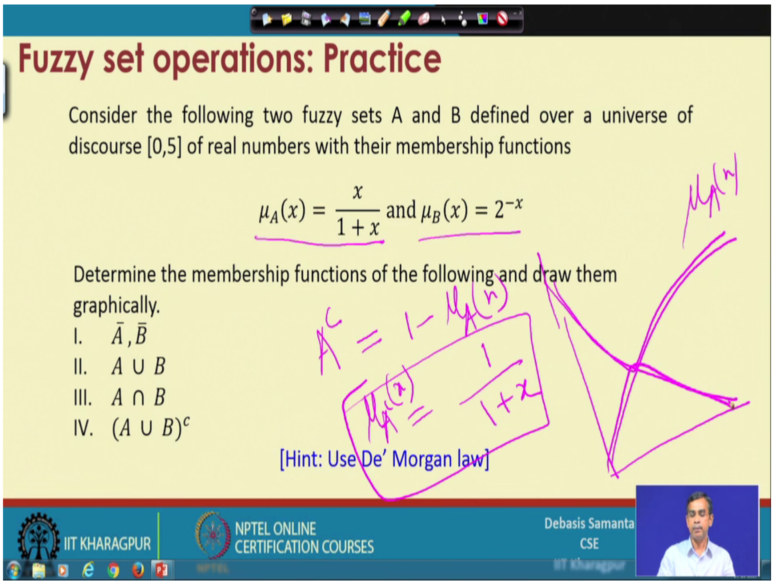

Now we will discuss about few example. So, that we can clear about our idea sometimes with A graphical representation we can express the operation, sometime also mathematically you can express the operation.

For example say suppose A is A fuzzy set A is A fuzzy set whose membership function is defined by this expression, we can plot A graph so graphical representation can be obtained accordingly. Similarly another fuzzy set B which is shown here now if I if we want to know A complements. So, A complement this 1 this is equals to 1 minus mu A x. So, this means is basically 1 by 1 plus x. So, this basically we can say this is mu A complement x this is the membership function of the resultant fuzzy set.

Now, likewise mu B x and it is complement can be obtained and graphically if we plot then mu ax can be plotted like this. So, it is mu ax and mu B x that can be obtained like this the mu B x then the union of the 2 fuzzy sets A and B is basically this is the union and if the intersection then intersection can be obtained by this one. So, graphically both can be obtained as well as mathematically it can be obtained.

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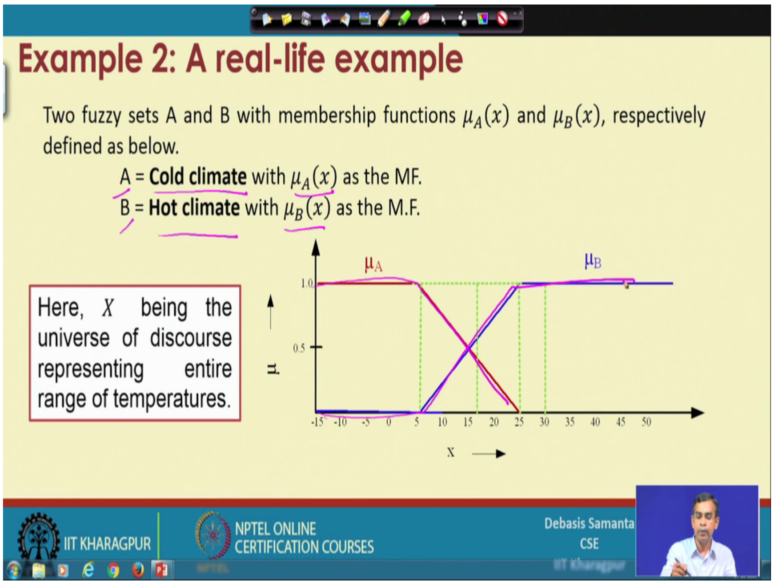

Now, the idea about this fuzzy operations is more meaningful now we know every fuzzy set is basically expressed by A certain meaning this is called the linguistic H, now for example, say suppose A is A fuzzy set and B is another fuzzy set A basically the cold climate and B is the hot climate representing with A membership function mu x and mu B x. So, that 2 fuzzy sets can be representing the graphically using this set this is the A and this is the set B.

Now, there are. So, operations whenever we perform on A and B has the meaningful representation that can be obtained like this say.

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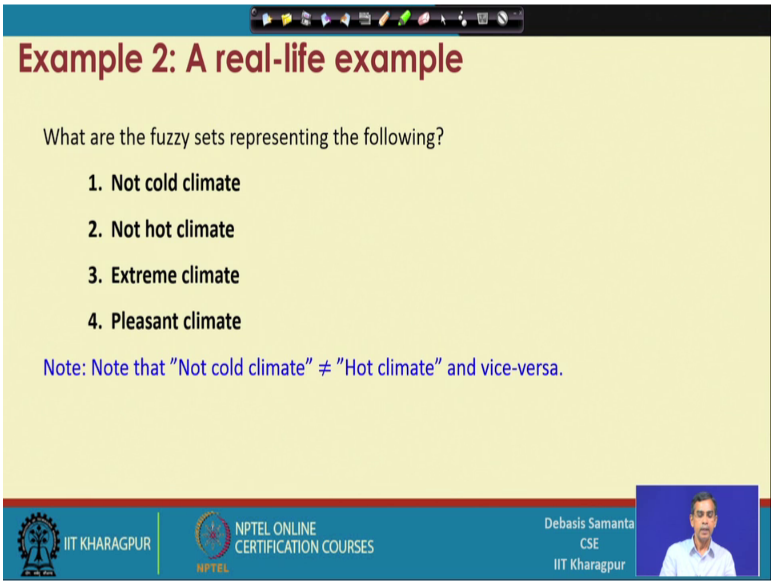

If we know the fuzzy set cold climate or hot climate then we can know the fuzzy sets not cold climate, it is basically complement of the cold climate or not hot climate is basically complement operation of the hot climate extreme climate is basically is the operation of both; that means, it is union and pleasant climate is basically intersection.

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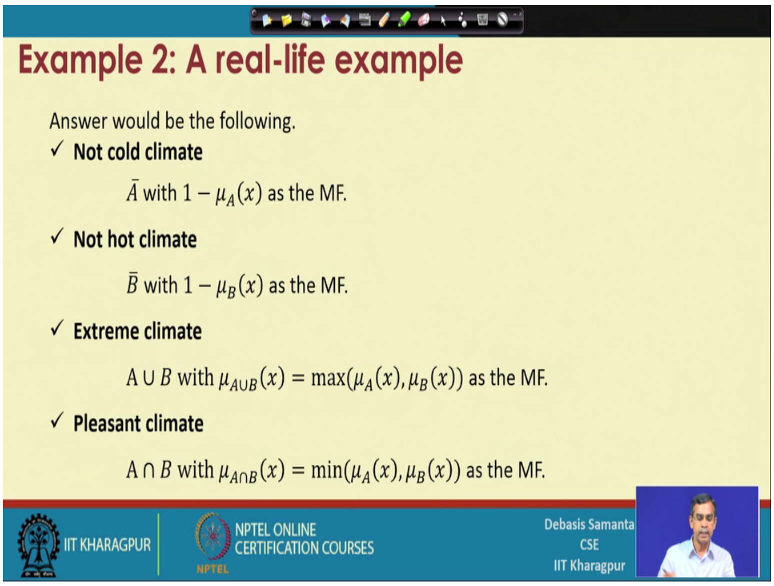

So, this is represented here by this. So, not cold climate is this 1 not hot climate is A complement B and extreme climate is union and then pleasant climate is this 1, now graphically the same thing can be shown also here.

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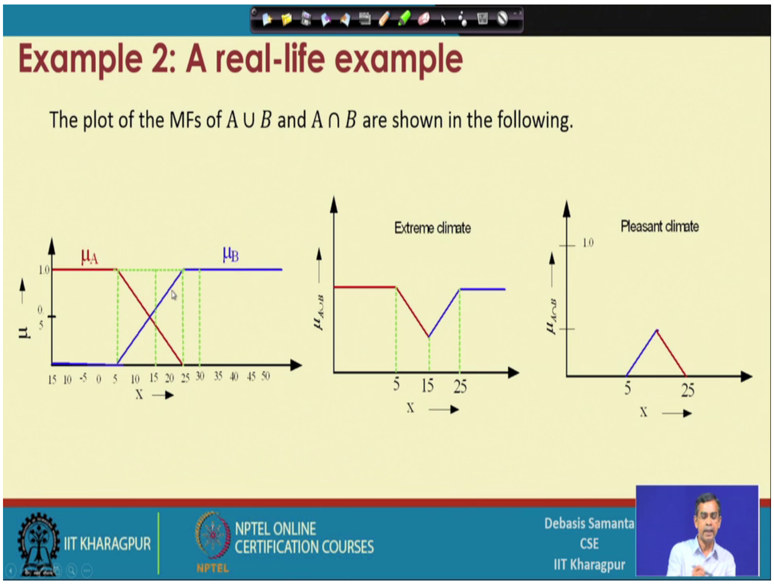

If so these are the 2 fuzzy sets A B the extreme climate that can be obtained by the plot this is the resultant graph of the fuzzy set extreme climate and this is the resultant fuzzy set of the pleasant climate.

Now, this example basically shows you that how the different operation is meaningful in the context of fuzzy sets now let us stop it here. So, these are the different fuzzy set operation and we will discuss the other things in the next lecture.

Thank you.