Introduction to Soft Computing
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Lecture - 03

Fuzzy membership functions (Contd.) and Defining Membership functions

So, we are discussing about some notations and terminologies that is required to understand the concept of fuzzy logic. So, few terminology we have discussed in the last lecture and today we will continue the same discussion, we will discussed few more terminology and so first is called the fuzzy singleton.

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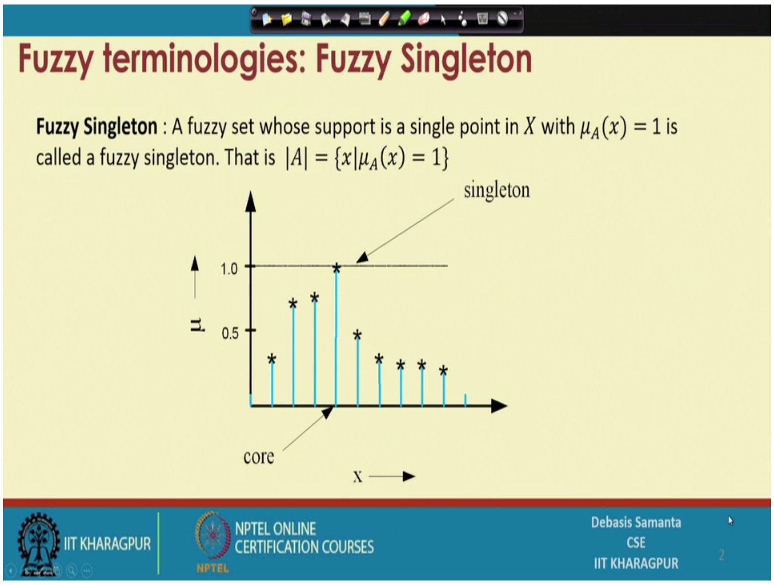

So, if a fuzzy set consists of only one element whose membership value is exactly one then such a fuzzy set is called the fuzzy singleton. For example, here this is the one fuzzy sets all the elements having the different membership values, but there is an element with membership value is this one is basically one then it is called the fuzzy singleton.

So, fuzzy singleton is like this and now, we will discuss about another two important terms it is called the alpha cut and then strong alpha cut.

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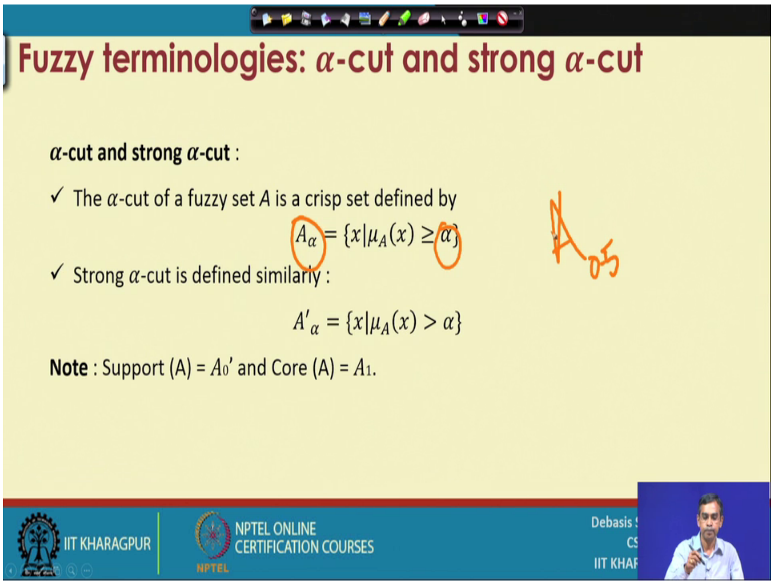

The alpha cut of a fuzzy set it is denoted as A alpha, A suffix alpha it is denoted as alpha A suffix alpha the alpha cut is basically the crisp set x set of element say x such that, the membership values of this element is greater than or equals to alpha where alpha is a predefined values and need not to say that alpha is basically a value in between 0 and 1 both include ship.

So, for example, A 0.5 if I say like this; that means, it is basically the crossover points, the set of all crossover points that belongs to the set A. Likewise the strong alpha cut the difference between the 2 is basically greater than equals and in case of strong alpha cut it is basically greater than symbol otherwise the they are the same.

So, we can easily understand that a support that we have discussed about is at A 0 complements is basically complements and complement means other than the elements which belongs to 0 is A complement will discuss about the complement is just like a set complement you know inverse actually and similarly we also can say core A same as A 1 from the previous discussion that we know. So, it is like this. So, core A is basically the alpha cut where alpha equals to 1.

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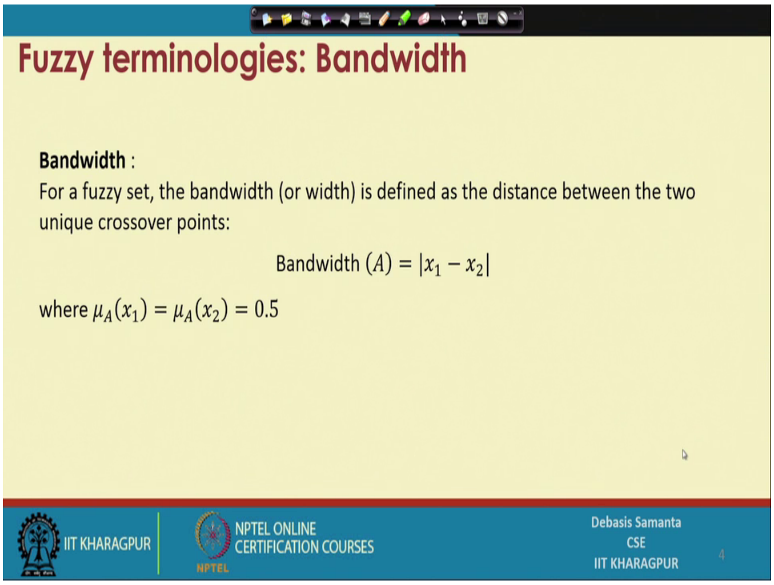

Now, we can define another term is called the bandwidth, bandwidth of a fuzzy set a it is basically the difference between the two values of the element namely x 1 and x 2 such that x 1 and x 2 both are the two crossover points. Obviously, if there is if fuzzy set which contains more than two elements at the crossover point then that two extreme crossover point can be used to decide their bandwidth. So, bandwidth is basically the difference between the two extreme crossover points x 1 and x 2 like this.

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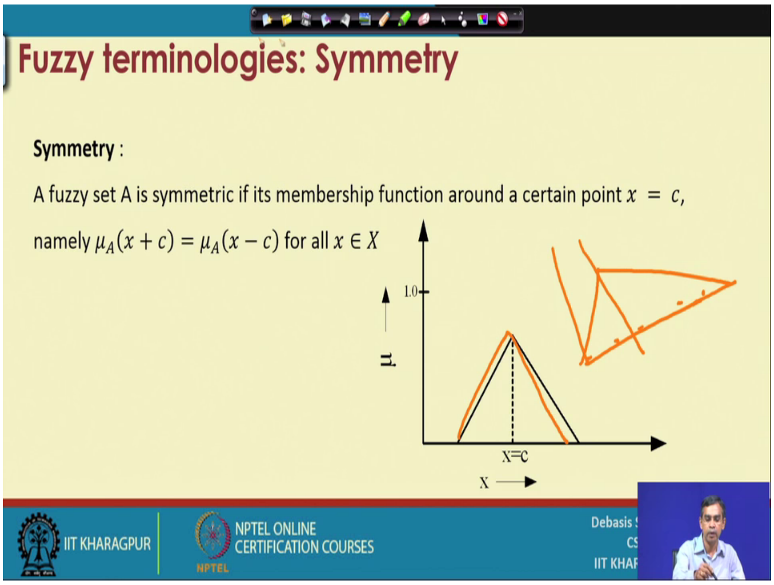

Now, we will discuss again a fuzzy set as a symmetric or asymmetric we define a fuzzy set as symmetric with respect to one element x equals to c such that all membership values for all the elements in this region has the same and there is a corresponding membership corresponding elements having the same values. Alternatively or mathematically we can say that if the two elements x plus c and x minus c have the same values for all element x that belongs to this set then we can say such a fuzzy set as the crisp set.

In other word crisp set is basically symmetry in form, but if we draw on then this then this is not a crisp set because here some elements and all elements may not have the same values like this one. So, this is the concept of symmetric fuzzy set.

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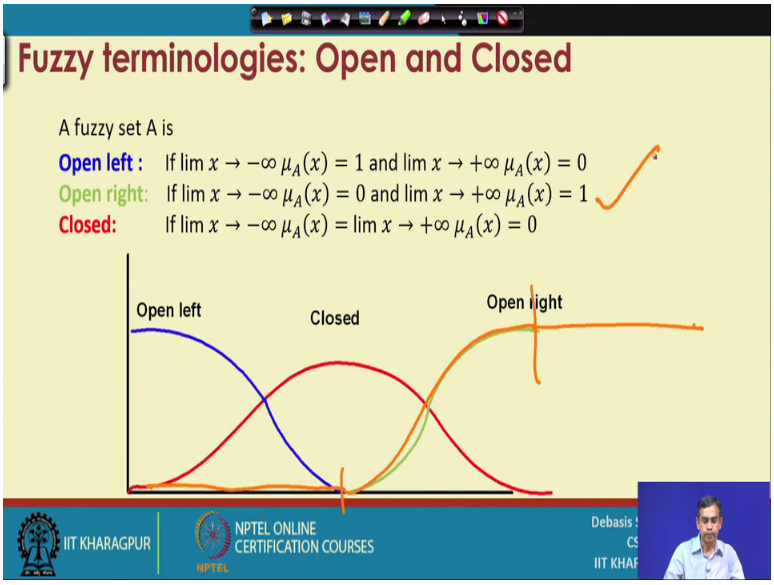

The next is basically open and closed fuzzy sets. Now, here this is the one description of fuzzy set we can say open left. Now, we say the open left a fuzzy set is if it satisfy this definition. Now, we can say for the open left all elements which is beyond the x tends to minus infinity this side is equal to 1 because it is basically one and limit x tends to plus infinity after this point is basically 0. So, there are two extreme limits this one where all elements is 0 and here all elements is one then such a fuzzy set along within this portion is called open.

Similarly, we can write open right here, here all elements which x tends to infinity is basically one and here all elements which x tends to minus infinity is 0. So, this definition it is called the open right. So, open left and open right. Similarly the closed if we say for all element x tends to minus infinity and all element x tends to plus infinity if there the value is 0 then all this is basically called a fuzzy sets and this type of fuzzy set is called the closed. So, there are maybe three different form of a fuzzy set are there open left or open right or closed. So, any fuzzy sets can be belongs to this category only either open left open right or closed.

Now, one thing just we want to clarify it is here and is there any link between fuzzy and probability. Now, there may be certain what is called the link or relation because if we see the fuzzy membership values is in between 0 and 1 both include ship.

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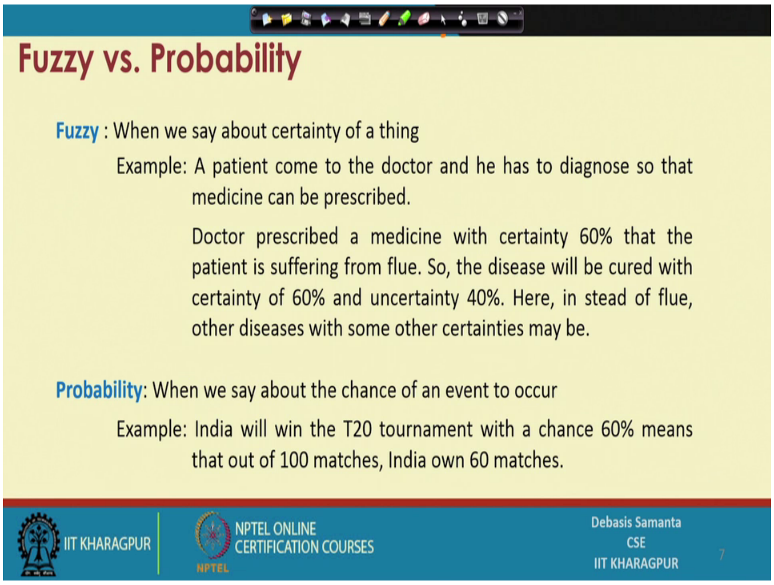

Similarly, if we say the probability value of something it also has the value in between 0 and 1 like this one.

So, for the values are concerned both fuzzy and probability is synonymous, but there is a difference again all these 0.1 or some decimal in between 0 and 1 alternatively can be expressed in the form of a percentage. So, 60 percent is equal to 0.6 like this one.

So, anyway, that whether it is expressing the form of a decimal 0 point something or it is a percentage there is basically relation between the two things in that way, but there is a clear cut difference between the two, I want to clarify this I mean difference between the two with an examples. So, first example suppose a patient when come to the doctor and doctor carefully diagnose the patients and prescribe the medicine then what exactly the thing it happens is that doctor prescribes a medicine with certain certainty, let this certainty be 60 percent this means that the patient is suffering from the flue and for that he is sure about this is that 60 percent. In other words, that disease for which he prescribed the medicine will be cured with certainty of 60 percent and there is again uncertainty 40 percent. So, this is a concept that is related to the fuzzy actually it is the certainty or clarity or the guarantee like.

On the other hand if it is the probability then we can say that probability is also 60 percent or sort of things or 0.6 like.

For example, India suppose we will win the T-20 tournament with a chance 60 percent if I say. So, it means that we have certain statistics or some previous experience that out of hundred matches India won 60 matches. So, 60 percent in this case and 60 percent in the previous case has the 2 different significance. So, in the first case in case of doctor patient scenario it is basically the certainty and in the second case the 60 percent is based on the previous experience. So, this certainty versus experience this basically defined the fuzzy versus a probability.

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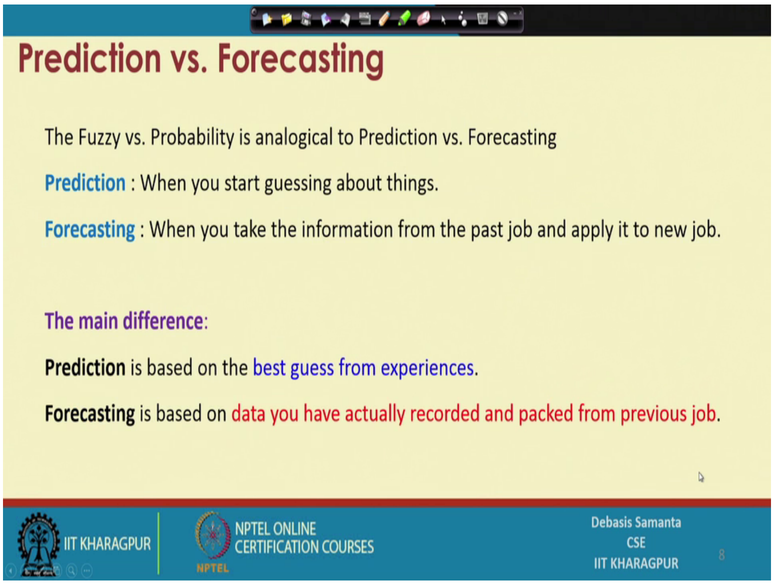

Now, likewise this fuzzy versus probability there is another relation that is also related to this fuzzy and probability also it is just in the form of prediction versus product forecasting. So, fuzzy versus probability is also in many way analogical to prediction versus forecasting.

Now, we can say the prediction when you start guessing about something. So, it is a guess, fuzzy means it is a guessing power. On the other hand forecasting means if we can say something based on the previous information or based on our previous experience, it is the forecasting, so prediction. In other words the prediction is based on the based guesses from the experts that basically would add it and forecasting is based on the data which you are already have in your mind and based on the processing of the data you can you can tell something. So, this is a forecasting. So, if prediction is related to fuzzy then we can say the forecasting is related to the other; that means, its probability. So, these are the things actually. So, sometimes we little bit get confused whatever the fuzzy versus probability or like this one.

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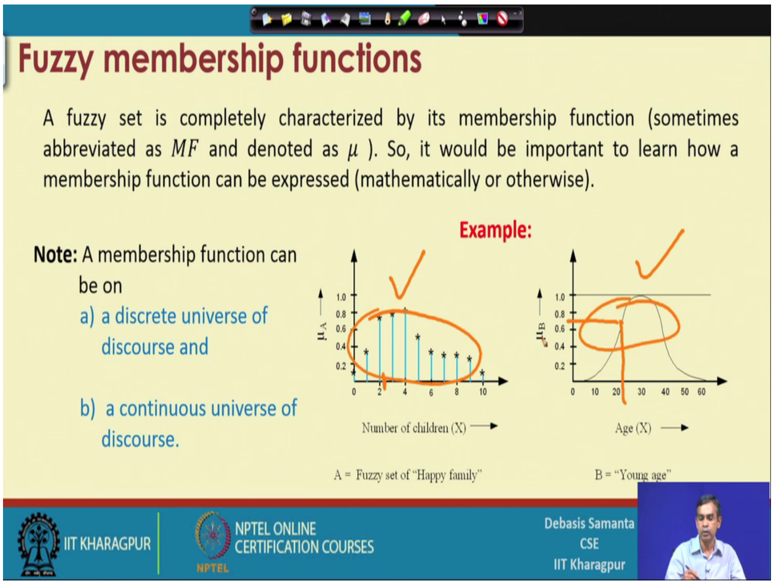

Now, next our point of discussion is basically fuzzy membership function. We have some idea about it and one thing I want to again mention it here a fuzzy set can be described better in the form of a graph that mean it is a graph versus all elements and their membership values. So, we can define in the form of a set theoretic form or in the form of a graph. So, in this slides we can see this is the one fuzzy sets and this is also another fuzzy sets.

Now, the difference between the two fuzzy sets here is that here the elements which belongs to these fuzzy sets are having discrete values. So, there will be no element within between 2 and 3 like. So, there will be no element there. But here is all elements in between the range 0 to 60 are belongs to this one. So, it is the elements say 21 and it is the fuzzy set is here.

We have also discussed about that the membership values can be discrete value also. So, here it looks at all values are possible. So, here also the continuous values for all membership values and here need not to say it is also the continuous values of all membership values. So, that we have already discussed in the previous lectures that the membership function can be on a discrete universe of discourse or can be a continuous universe of discourse, the membership value can be again discrete values or it can be again continuous values. So, whatever be the values it is there we have to express, maybe it is mathematically or using some graphical representation. So, these are the two examples where we had give the two fuzzy sets in the form of a graph.

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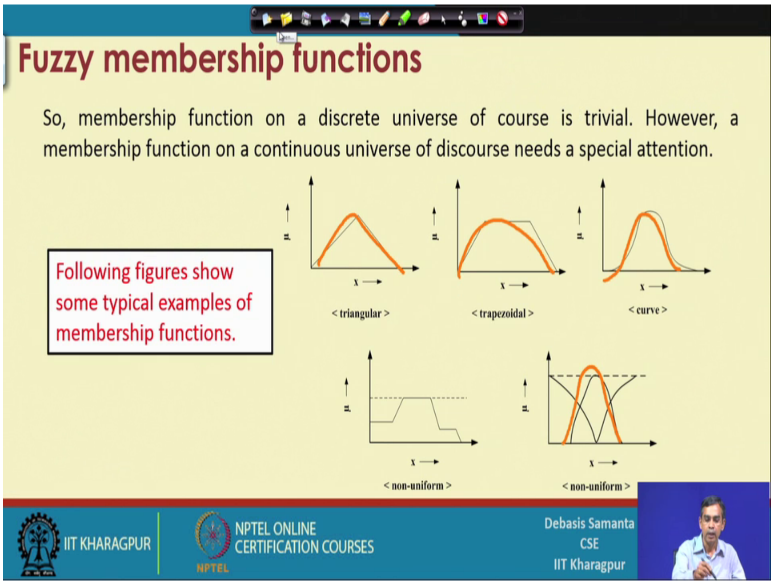

Now, I want to give more graphical representation of the standard some fuzzy sets. So, these figures basically show some typical examples of the fuzzy sets and they are defined in terms of the membership function actually and that this is the general looks that usually a fuzzy sets takes and in the first this is the one fuzzy set it is having the membership function in the form of a triangular shape.

So, this is another fuzzy sets whose membership function is expressed in the form of a trapezoidal shape. This is another membership function I did basically curve it is look like a bell curve. So, it is called belled function belled membership function like and it is the one membership function which does not have any a specific shape it is arbitrary shape is called the non uniform fuzzy sets and this is also an example it is also a non uniform fuzzy sets; however, it has some special meaning. So, this is the one fuzzy set it is called the open left, this is another fuzzy set is called the open right and this basically called the fuzzy set closed. In this sense it is also a closed, these are all the closed fuzzy set actually.

So, these are the all closed or open whatever I told you that either fuzzy sets can be open or open left, open right or is a closed anyway. So, these are is the typical form of the fuzzy set which usually consider in our fuzzy system, in our fuzzy theorem and another point is that how such a fuzzy set can be better described in some mathematical notation so that we can process them in our future fuzzy system design.

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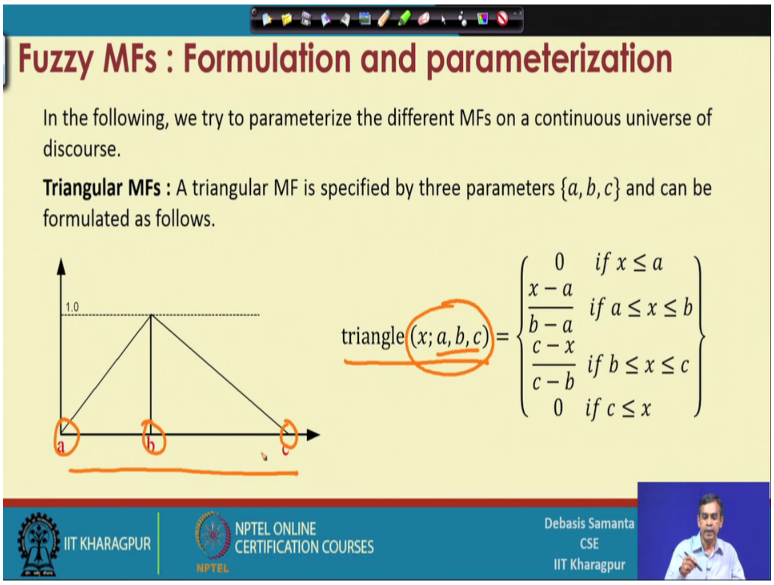

I am going to discuss these things how a membership functions can be better mathematically described and then that mathematical specification can be used to process in subsequent requirement. So, in this direction first let us discuss about this is the one fuzzy sets or a membership function and this membership function as I told you it is a triangular membership function. So, usually if this is can be described mathematically using this triangle and x is an element and this membership function can be defined by means of three parameters abc. So, three parameters are the three what is called a meaningful direction here and in terms of these three parameters the membership function can be described mathematically reading like this. So, it is clear that if x less than or equals to 0 a, it is like this then this is 0 and in between a and b the membership function can be defined by this, this is basically slope whatever it is.

Similarly, in between this one, this is another slope it can be defined like this and so for the element x greater than 0 this one it is basically 0. So, what you can say whatever the graphical representation which looks like this it can be described mathematically using this form. So, this is a mathematical expression that a fuzzy set can be defined and definitely this fuzzy set is defined over a continuous universe of this course.

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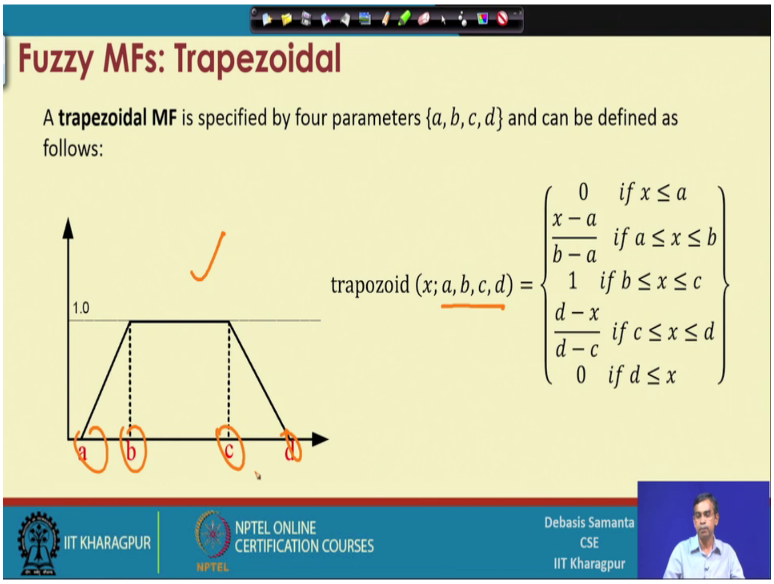

Now, this is the triangular membership function using the same concept same idea we can define the other membership function for example, in this slides we can show the trapezoid the membership function; however, unlike triangular membership function here we need the 4 different parameters they are called abcd. So, these are the parameters are like here and in terms of these 4 parameters we can define this membership function like the if x less than equals to a it is 0 and if x is in between a and b that means, in this one. So, it is basically this one which can be expressed by this form and in between b and c this is a x. So, it is basically 1 and in between c and d this is the slope which can be described by this one and if x is greater than d then this is 0.

So, with this concept can be describes in a mathematical manner this membership function now. So, this is a trapezoidal membership function triangular and trapezoidal are the two most frequently used membership function, in fuzzy system in order to design a fuzzy system.

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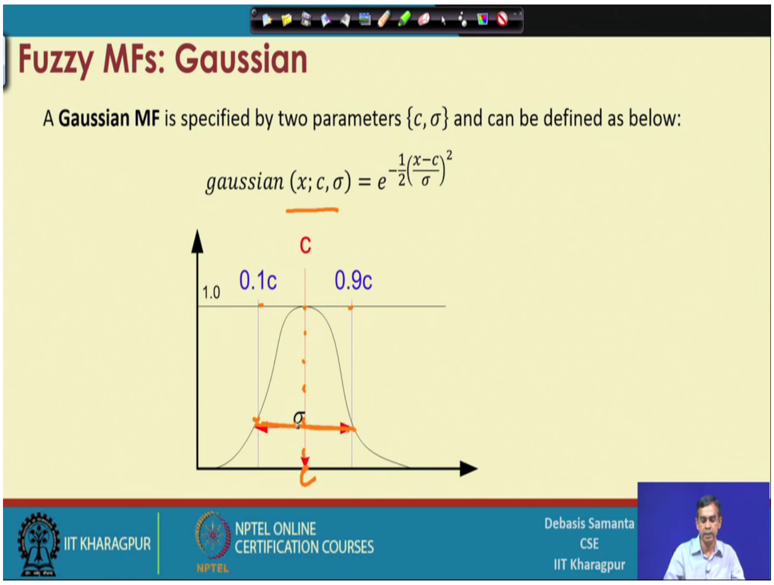

The another important membership function that is more popular in fuzzy in order to describe a fuzzy system and this is called the Gaussian membership function.

A Gaussian membership function typically takes in terms of two parameters c and sigma. So, is c basically is the middle point of this it is called the centroid or media mean and sigma is defined is a range between these two in between 0.1 c and 0.9 c if c is defined here then this 0.1 c and 0.9 c. So, this range is basically sigma.

Anyway, so if we define two parameters like c and sigma then this membership function can be better expressing the form of a mathematic notation using this one, so this basically the formula for Gaussian distribution, that is why it is called the Gaussian form. If we plot this form for a given values of c and sigma and for different values of x then the graph will look like this, as this graph is looks like a bell shaped. So, it is called the bell shape membership function also.

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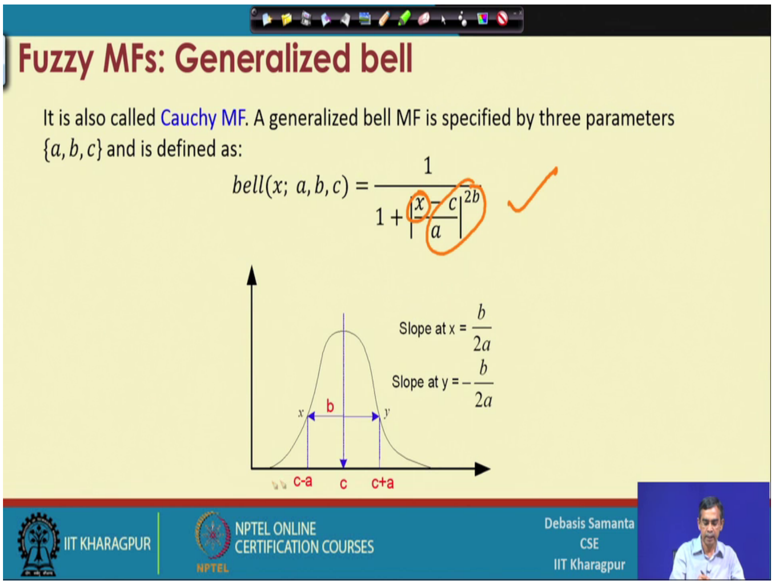

So, this is the one popular membership function like this is the membership and there are many more and another is called the Cauchy membership function. So, it is just like a Gaussian membership function, but this membership function defined in terms of three parameters abc and using these three parameter how a membership function is defined it is expressed here and it has two characteristics here that it considered two points for the slope. So, slope at this point it is called the c minus a and another is c plus a the slope of this point is basically b by 2 a and at this point is minus b by 2 a.

So, if we define a b and c then all these values their slope and point can be defined and accordingly the membership function can be defined. So, the membership function that can be defined using Cauchy membership function is this and if we plot this membership function for a given the values of abc then the graph will look like which is shown here.

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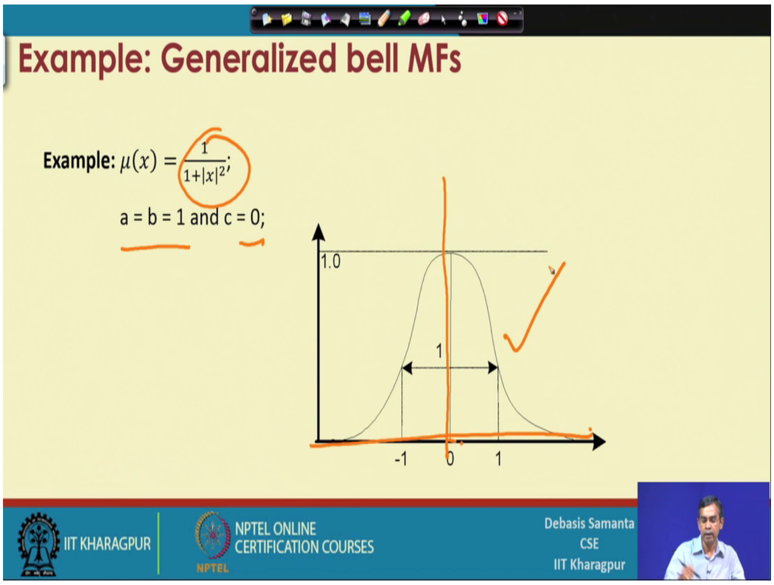

So, this is another popular Cauchy membership, Cauchy membership function or it is sometimes is called the generalized bell.

Now, this is a typical example of generalized bell for particular value of abc where a b equals to 1 and c equals to 0; that means, it is basically c equals to 0 and these are the function and if we plot these things you essentially if we plot this one then it can give back up like this. So, we can plot it and then you can have this curve. So, this is basically graphical representation of this one.

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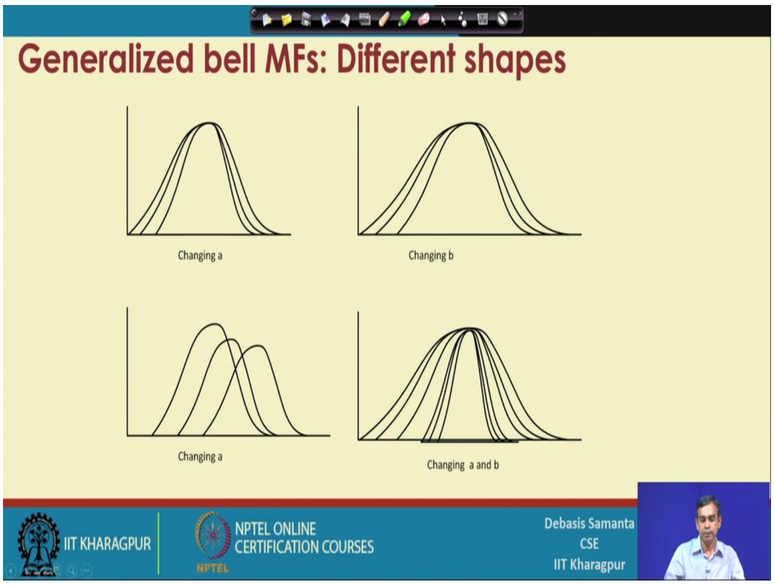

So, there are few more membership function. Anyway before going to these things for the for the different values of abc and whatever it is there. So, this membership function will look the difference shape and hence the different membership function or the different fuzzy elements can be defined. So, this is basically the idea about the membership function can be expressed in the form of a graph as well as in the form of some mathematical notation.

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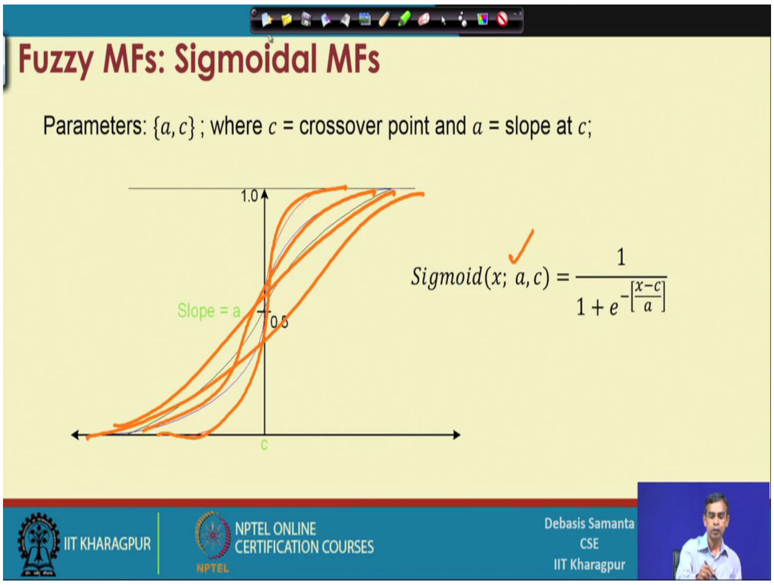

Now, this is another very popular the membership function it is called the sigmoidal membership function a typical look of the sigmoidal function is shown here in this form it is basically indeed from the s. So, that is why it is called the sigmoidal membership function just like a s. This kind of the membership function defined in terms of two parameters a and c, where c denotes the point it is like this and a is any arbitrary value it is basically called the slope of this point at a.

Now, if we take this kind of values then the sigmoid function can be plotted and if we follow this expression. So, if this is the expression if we follow then graph can be obtained for this expression for different values of it is shown like here. So, we can see here one example if x tends to infinity as you show it is basically, in this way is basically closed right and in this case it is basically open right because for all value the x which is greater than tends to infinity the membership value is 1 and few here for all value the x tends to minus infinity the membership value is 0.

So, sigmoid is typically like this and for the different values of the different curve will be obtained for one values of the curve will be like, for another value of a the curve will be like, for another is there and so on. So, if we change the values of the different pattern of the curve will be obtained. So, what I want to mention here is that this is an important function which can have the different values of a and c and the different membership functions can be obtain and hence the different fuzzy sets the different form of the fuzzy set look of the fuzzy set also can be obtained. So, this is another membership function that is very much popular in the design of fuzzy set system.

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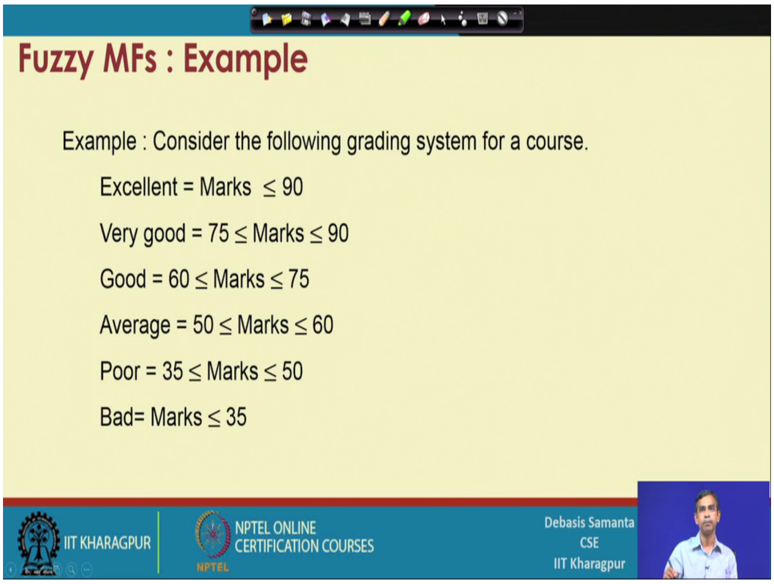

Now, we come we can discuss on example. So, that how these membership functions can be used, we are discussing about one idea about that how the grading in the form of a crisp.

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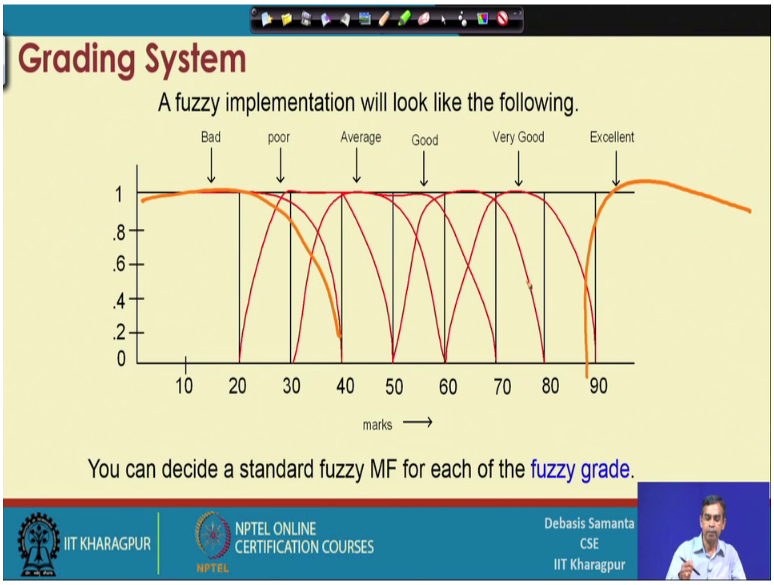

So, these are the crisp formulation of the grading and here the same thing which can be discussing the form of a fuzzy, another fuzzy representation. So, for example, the grade this one is one and grade is this one. So, these are the two different form. So, as we say. So, these can be discussed with some function and this kind of membership function can be discussed by means of say Cauchy mf or generalized bell like. So, basically this membership function can be used to different what is called the fuzzy sets in the different range. So, this is a one fuzzy set in one range, this is another fuzzy sets and this is another fuzzy sets like.

So, we can use the same formulation for defining membership function in this case other than these sets or other than these sets, these two sets can be defined either using open left or closed left with another membership function, but all the other membership function in between the range and they can be defined in terms of some graph or in terms of some mathematical notation say Cauchy membership function like and then they are basically called the different fuzzy set. For example, if we define one membership function in this form we can say that these are bad fuzzy sets where the universe of discourse is this one and the elements belong to this is this one within this element up to this is the membership elements see value is one and after this thing membership value is decided by this what is called the function.

Similarly, the another membership function like this one it can be defined in terms of this is a universe of discourse the elements is in between for all elements it is 0 for all other element is 0 and in between this the membership value will be decided by the type of the curve. So, these are the basically the way which we can use to define the membership function for the different elements.

So, we have understood about the different membership functions and their mathematical representation.

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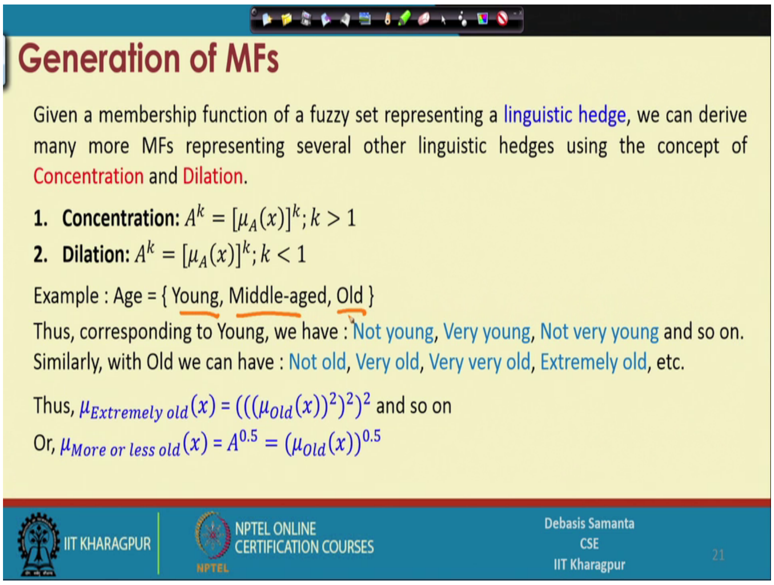

We will quickly cover few more concept about the membership function the two concept it is called the concentration and the dilation. So, the idea is that if A is a fuzzy set given to you right then we can define another fuzzy set then let this fuzzy set be A k where k is some value that may be greater than equals to greater than 1, such that this mu A x to the power k is the new membership values for the element x which is belongs to the set A k. So, this means that if A is known with mu A x is a membership value then we can derive another fuzzy set A k with membership value this one and for k greater than 1 if it is like this then it is called the concentration.

Similarly, if it is k less than 1 the same thing if whole goods then it is called the dilation. So, this is a very important concept that from a given fuzzy set we can find more fuzzy sets many more fuzzy sets, this fuzzy sets can be obtained by simply using some mathematical notation like this concentration or dilation.

Now, you can recall each fuzzy set basically used to express something say high temperature or low pressure or sweet appeal whatever it is there. So, such things in fuzzy concept it is called the linguistic heads; that means, high temperature the linguist head it is that temperature is high, but temperature is high; that means, the different temperature is high with the different membership values.

Similarly, if we say the pressure is low; that means, the different pressure can be termed as low and then same pressure can be termed as belongs to low pressure or high pressure. So, here pressure low pressure high they are called the linguistic heads. Likewise there are many linguistic heads for example, related to our age. We can define related to our age say young, middle aged and old. So, these are the three fuzzy sets if you can define then from these three fuzzy sets, we can easily defined another fuzzy set likes a very young, not very young or like this one similarly if the old is a fuzzy set very old, very very old, extremely old all these fuzzy sets.

Now, the question is that how these kind of fuzzy sets can be obtained if a fuzzy set is available already with us. Here is an example we can use or you can use the concept of concentration and dilation to do these things. Now, here for example, mu extremely old say if we know that mu x is a fuzzy set which is defined for the old fuzzy set and x is any elements belong to the fuzzy set old then any elements belongs to the fuzzy set extremely old having the membership value mu extremely old x that can be obtain like this. So, it is basically mu old x that is the original fuzzy sets having the membership value and we can take x square x square x square. So, this is basically gives that very old, this is very very old and this is basically extremely old. So, these are the different linguistics can be obtain and here we have used the concept of concentration.

Now, similarly if more or less old if x mu is defined for the old fuzzy sets then more or less for k value say 0.5 we can define this one. So, it is called the dilation. Now, graphically the same thing can be plotted nicely we can say it. Here is an example.

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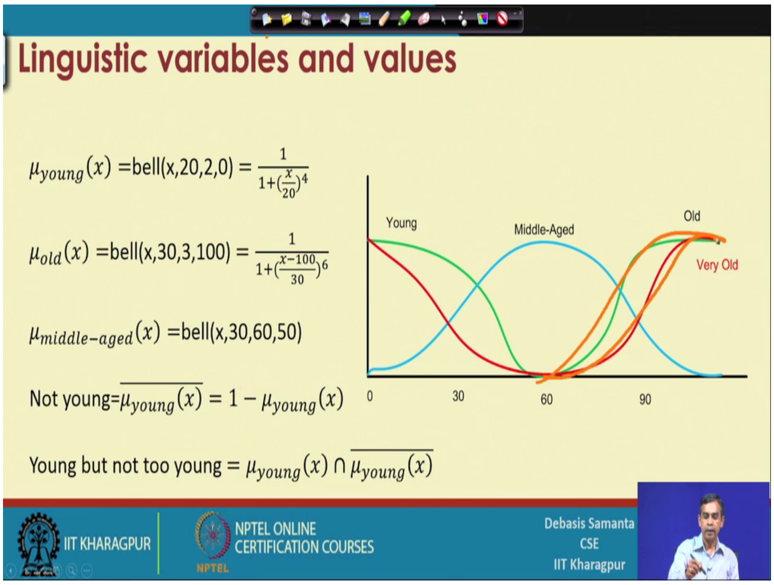

So, we can write it. So, if suppose this is the fuzzy set young then by applying the dilation we can say the very young we can define like this another fuzzy set. Similarly if this is the fuzzy set old then using the contention we can define another fuzzy set a very old like this one.

So, the different fuzzy sets can be obtained different fuzzy sets can be obtained from existing fuzzy sets if we follow some concentration and dilation formula. Here is few example again mu young x is one fuzzy set defining membership function fuzzy set young defining each membership function as mu young x and it is supposed defined by means of bell shaped curve that we have discussed and this is the curve, then mu old also another fuzzy set which is defined by another bell shaped curve which is like this one.

Now, given these two we can define the other not young. So, not young can be defined one minus mu young x. So, this is the another fuzzy sets the value or membership values belongs to another fuzzy set let the name of the fuzzy set be not young. So, this can be derived from the fuzzy set young. So, as another example young, but not too young we can define this kind of concept. So, mu young x and it is complement of this one. So, it is young and it is not young. So, young, but not young can be defined by this one.

Now, regarding this kind of formulation we will discuss in details in our next lectures. So, what I want to say here that given a fuzzy set we can derive another fuzzy set easily using some mathematical computation and formulation. So, these are the things it is there we have discussed about the concept of fuzzy sets first and then we learned about what is the difference between crisp set, versus fuzzy sets. Subsequently we learned about the different membership function that is with which we can define fuzzy sets and then different properties in it and how the membership function can be better mathematically expressed also we have learned it and we will discuss about the different fuzzy set operation in our next lecture.

Thank you.