Introduction to Soft Computing
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Lecture – 27
Non-Pareto based approaches to solve MOOPS

We have learned some concepts regarding the multi objective optimisation problem, now it is a right time to discuss about the different approaches. In the last two decades, there is a huge research is to develop the best solution solving multi objective optimisation problem and then it opens up further many other what is called the areas. So, that further research can be extended and as the number of approaches are very large.

So, it is not possible to discuss all the approaches out of the different approaches we will try to discuss the popular and then path breaking approaches or we can say they are the state of the art approaches to solve multi objective optimisation problem. Now, we will discuss the different approaches in a when they are discussed in a different chromogical sense; that means, the first approaches we will be discussed here we will be discussed first here and then the latest the approaches in this field will be discussed at the end.

So, it will take few classes of course, to cover all the approaches , now we will discuss about multi objective optimisation problem solving approach in order to make a difference between the traditional Genetic Algorithm to solve single objective optimisation solving, but using the same as a GA concept then this technique is also alternatively called MOEA. So, it is MO stand for Multi - Objective and then MOEA it is called the Evolutionary Algorithms. So, algorithms are particularly termed as evolutionary algorithm. So, it is more popularly termed as MOEA algorithms.

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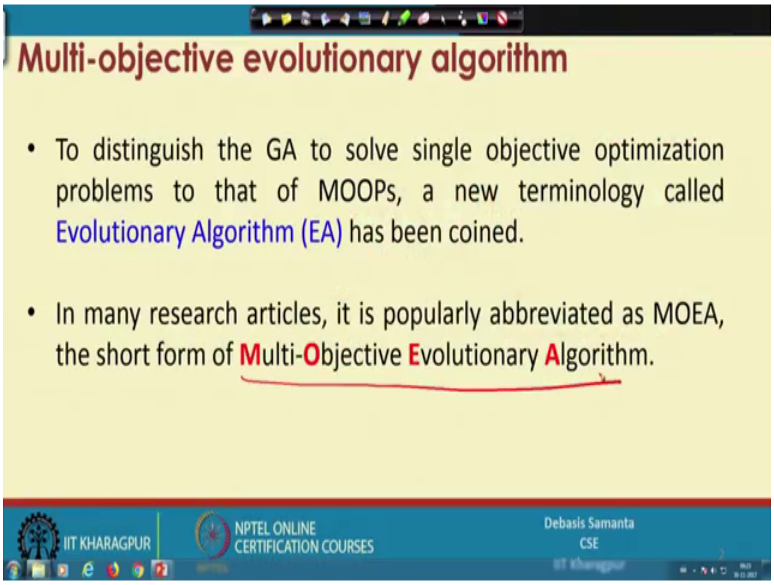

So, we are going to discuss about the different MOEA algorithms to solve multi objective optimisation problem.

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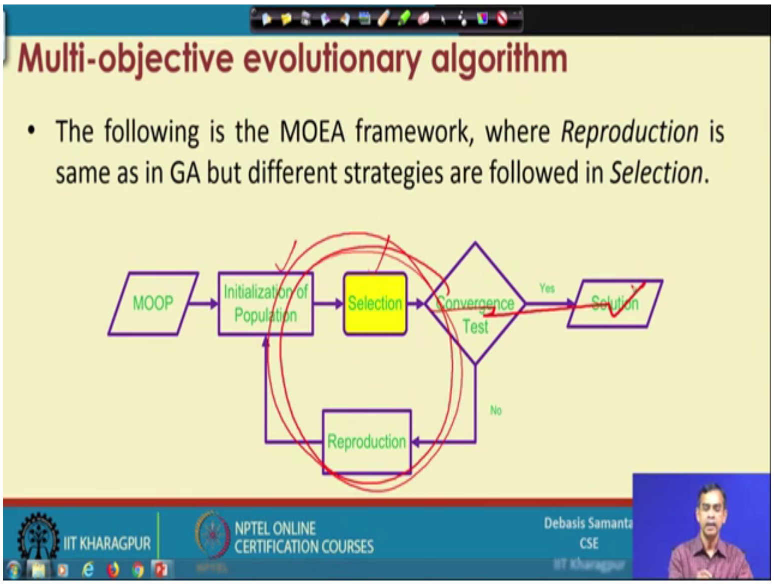

So, for the concept now we are going to discuss the concept is that using genetic algorithm framework that we have learned earlier to solve the single objective optimisation problem then how they can be used to solve multi objective optimisation problem and you can recall we are discussing about 2 approaches one is called the a priori approaches and then a posterior approaches.

So, it is the same concept it is here also we will discuss about on the top of a priori and a posterior and they can be further classify as a Non-Pareto based approach and then Pareto based approach we will discuss the classification shortly. Now let us first start to it how the simple genetic algorithm framework that is the g a framework can be applied. So, initially the attempt was to apply the GA framework to solve the multi - objective optimisation problem that is why it is called the GA framework to solve MOEA problem or it is also called MOEA framework.

Essentially the MOEA framework and GA framework are same because in case of GA framework you can understand. So, you have to create the initial popularization and then the selection and then we have to do the convergence test and then the reproduction. So, it is basically repeats of selection and reproduction until this convergence test is satisfied to find the solution. So, these are same as the GA framework here, but there is a difference the difference in the sense that ok.

So, for both GA and then MOEA framework is concern these steps are same, all the step remain same; however, the only step which is different is called the selection. This means that to solve the simple of single objective problem the selection method is followed that we have discussed when we are discussing about solving single objective optimisation problem using GA framework, but here in case of solving the multi - objective optimisation problem a totally different selection method methodology are followed and this way the GA framework is different or a MOEA framework is different than the GA framework. Now we learn about what exactly the selection strategy that is followed in MOEA and that is the different selection means is a different algorithm in fact.

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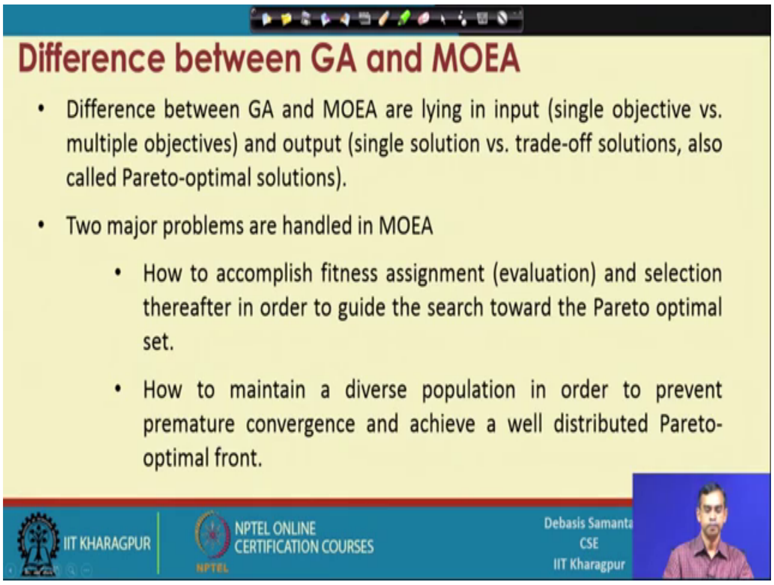

So, we have learned about the difference between the GA and MOEA. So, they are in the sense that GA is useful for solving single optimisation problems whereas, MOEA is used for solving multiple objective optimisation problem. Now so, for the input is concerned they have the same input; however, so far the output is concerned as you know single objective optimisation problem is basically gives the single solution; however, for the multi multiple objective problem they are not the single solution rather trade of solution and they are also called the Pareto optimal solution.

Now, so for the different issues are concerned in case of MOEA approach so, basically the idea is that MOEA the fitness assignment that is a evaluation then followed by the selection needs to be considered in a different manner than the single objective optimisation problem and the another issues so, for the MOEA framework is concerned is that how we can maintain the diversity in the population. So, that we can search towards the Pareto optimal front only; that means, we have to direct our searching procedure. So, that out of the entire surface the solution can leads to the Pareto optimal front. So, if we can find the Pareto optimal front and that is basically the trade of solutions to solve the multi - objective optimisation problem.

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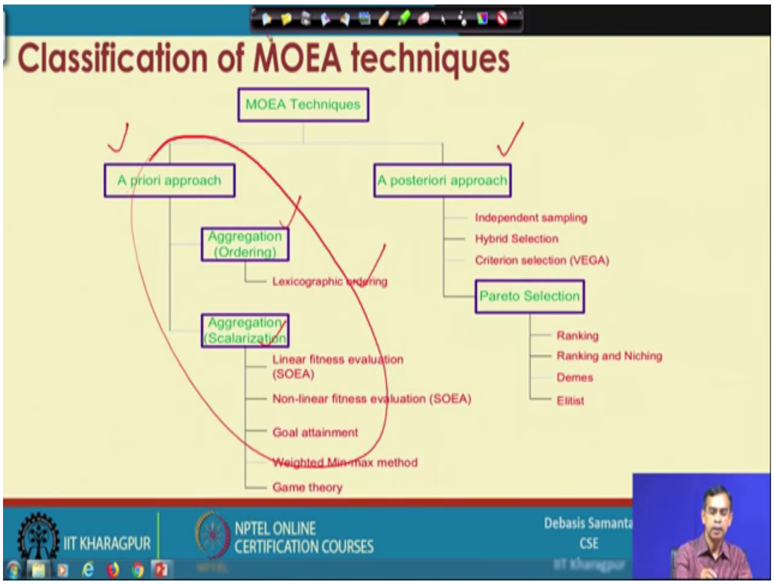

So, this is the concept between the GA this is the difference between the GA and MOEA techniques. Now, let us see what are the different approaches are there, whether I should discuss about the taxonomy of different MOEA techniques no. So, all the MOEA techniques can be broadly classified into 2 broad categories they are called a priori approach and another is called a posterior approach.

So, for the a priori approach is concerned again there are different division one is called the a priori approach based on aggregation or ordering and then another is based on scalarization. So, an example of a priori approach is ordering is called the lexicographic ordering and there are many other methods following the game theory, weighted min -max method, goal attainment method, non-linear fitness evaluation scope it is called the SOEA linear fitness evaluation it is also called SOEA.

So, these are the different methods are there so, for the a priori approach is concerned. So, this is the a priori approach approaches and then a posteriori approaches they can be of different types one is independent sampling or hybrid selection and then vector evaluated genetic algorithm it is also called VEGA and there are many other methods again a posteriori approach ranking, niching, elitist and demes, so these are the different methods are there.

So, these are the techniques as you know we have consider we have listed here only few because we cannot list all the solutions here, but these are the state of that solution. So, for the different MOEA techniques is concerned all the solutions have their own prose and own cons that will be discussed while discuss it is topics, it is techniques individually. So, these are the different techniques are there and all these techniques again there is a different classification also there.

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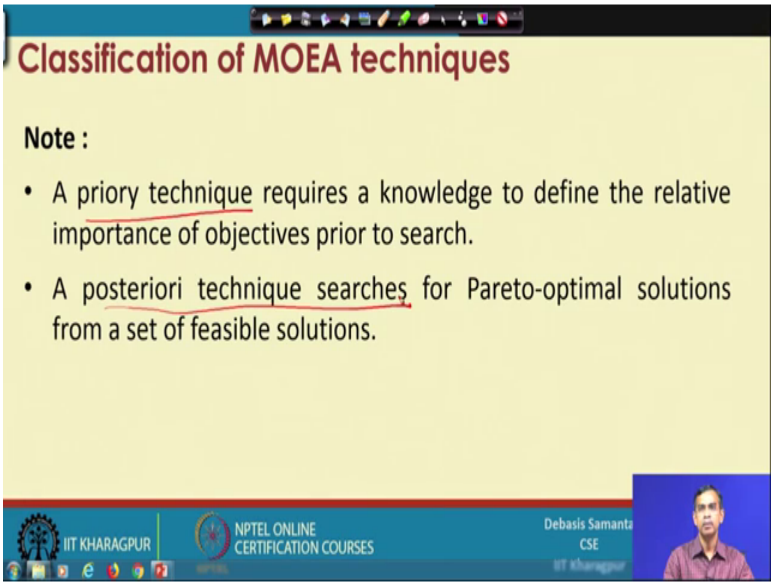

Now, a priori techniques as we have mentioned there is that it requires a knowledge to defined the relative importance of the objective vectors prior to the search then it is called the a priori approach. On the other hand in case of a posteriori technique it does not required any priory knowledge, it basically required certain techniques or certain mechanism by which it can limit the search towards the Pareto optimal front only.

So, this is the concept that is there so; obviously, the first approach; that means, a priory techniques required a lot of experience for the programmer whereas, this is computationally expensive; however, it does not require any knowledge about the solving problems or the different strategies to be adopted in order to apply a particular technique is there.

So, these are the I means major what is called the pros and cons the a priori approach is first whereas, the a posteriori technique is computationally expensive, but a priori approach requires the knowledge of the programmer where a priori technique does not required the any knowledge of the programmer. So, these are the merits and demerits in the 2 techniques there.

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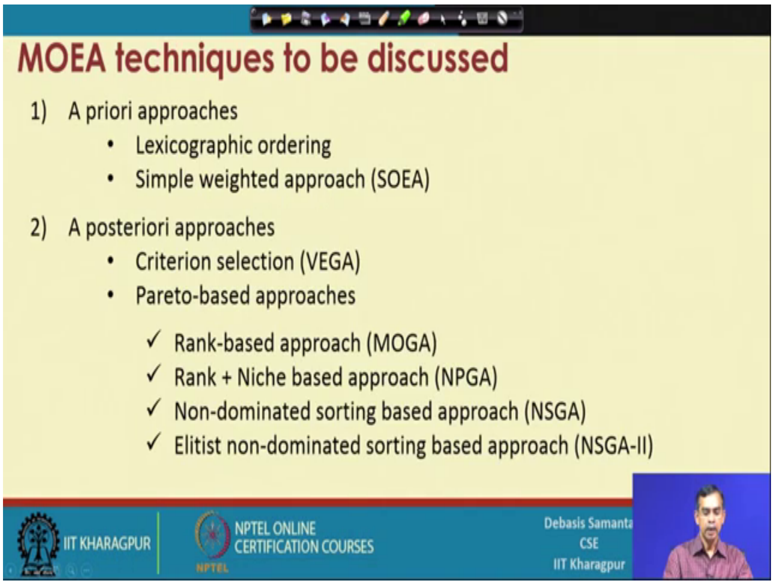

Now, again so, we have discussed about all the MOEA techniques based on a priori approach and a posteriori approach as we have listed all the techniques, which are belongs to a priori approach and a posteriori approach the different techniques that will be discussed in this course is basically so, for the a priori approach is concerned we will discuss 2 techniques not all techniques; however, so, these on.

So, for the a posterior approach is concerned we will the discuss these are the techniques will be one by one and again all these techniques whatever the techniques all these techniques again can be classified into different whether they are Pareto based or Non-Pareto based.

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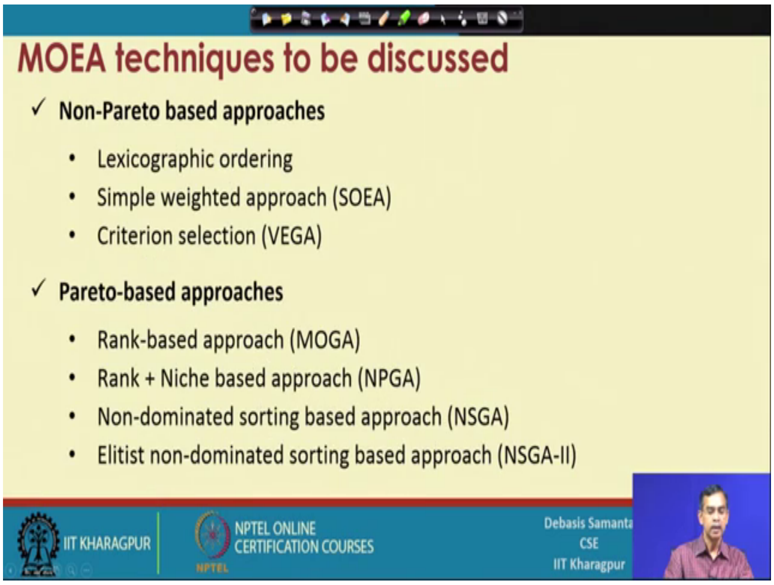

So, here is another division based on the Pareto based approach and Non-Pareto based approach all the techniques these are included here are the Non-Pareto based approaches and these are the Pareto based approaches.

So, this way the different techniques can be classified which belongs to a particular concept or principle Non-Pareto based, then Pareto based. So, first we will start about discussing about the Non-Pareto based approaches and we will discuss the first approaches in this direction is called the Lexicographic ordering.

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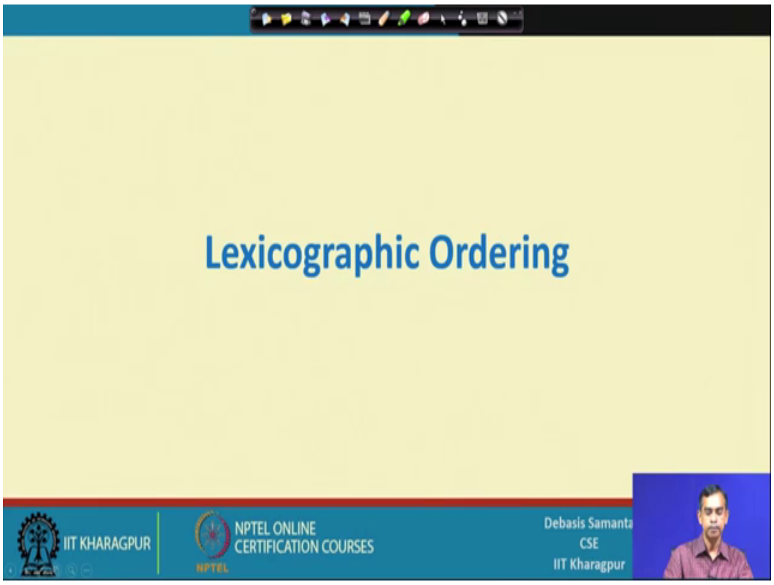

Now, it is an a priori approach as well because it requires the knowledge of the ordering of the objective vectors.

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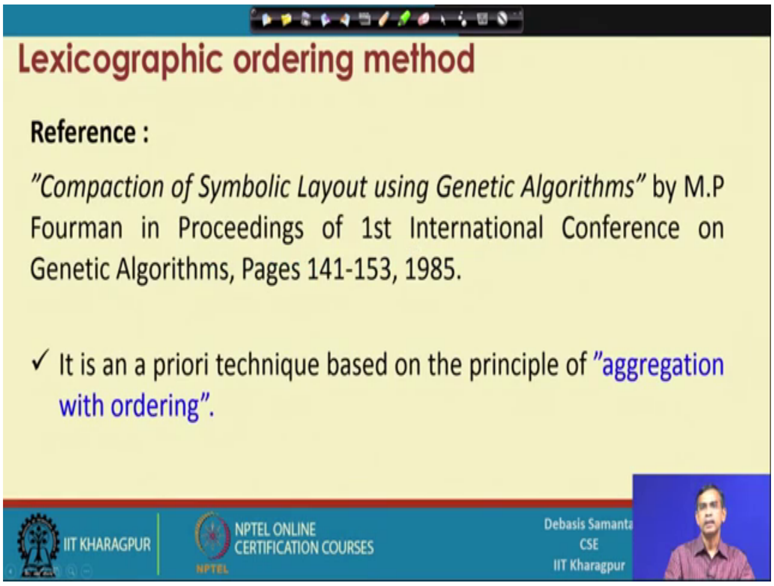

That means there are if m f n number of objective functions are there you should have a knowledge about that which objective functions are most important than the others one or like this or some relative ordering with respect to their evaluation or their importance. Now the approach the lexicographic approach first time proposed in 1985 by Fourman he introduced this concept and published a paper entitled as compaction of symbolic layout using genetic algorithm it was published first time in 1985 in the conference first international conference on genetic algorithm.

And it is a priori technique as I told you and the principle it follows is based on the aggregation with ordering now we will understand how why this principle is said so, aggregation and what is the concept of ordering, now let us discuss this concept.

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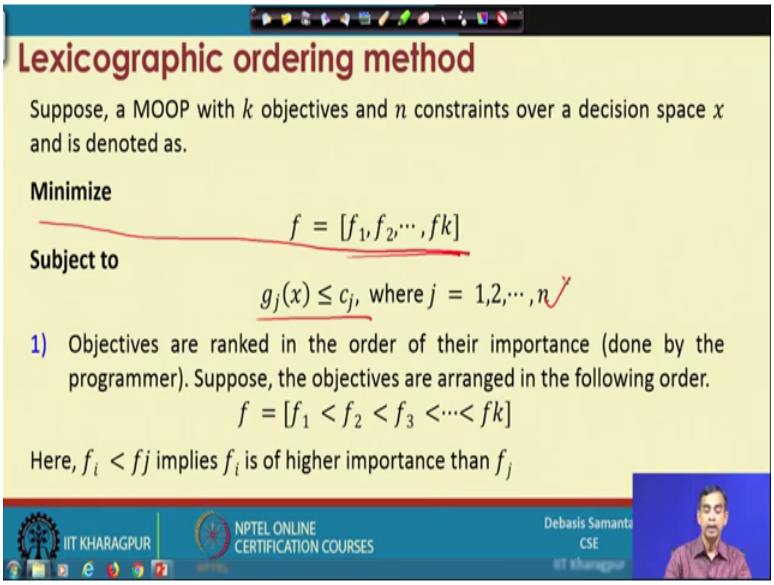

And without any loss of generality again I should say that there are k number of objective functions in this concept. So, k number of objectives can be denoted as f 1, f 2 dot dot f k and we will assume that all objective functions are to be minimise, this is not a contradiction or any problem because all the objective function can be converted into minimisation type also. So, we will consider that these are all the objectives are to be minimised and like all objective of that for any objective problem optimisation problem there are the constant.

So, let these are the constant, constant is denoted i. So, n number of constants are considered here and as I told you this lexicographic ordering technique recovered ordering of the objective vectors. So, we will consider the one objective ordering of the objective vectors are like this. So, for the importance is concerned that in this order then f 1 is first then, f 2 is then, f 3 is then, f k is there.

So, this ordering like this f 1 is most important, f 2 is less important, then f 3 is then lesser than the f 2 and so on. So, it is basically here we can say that f i less than f j implies that f i is of higher important than f j this is the concept that let us assume it and then. So, this ordering is known a priori. So, this ordering is known a priori then we can follow the idea about lexicographic ordering.

Now so, the idea or the state that is followed in lexicographic ordering is first we have to rank all the objective vectors into their importance of a importance; that means, ordering there. So, this is the first step that we have to follow it ordering is the first step once the ordering is done then our next step is a basically iterative steps.

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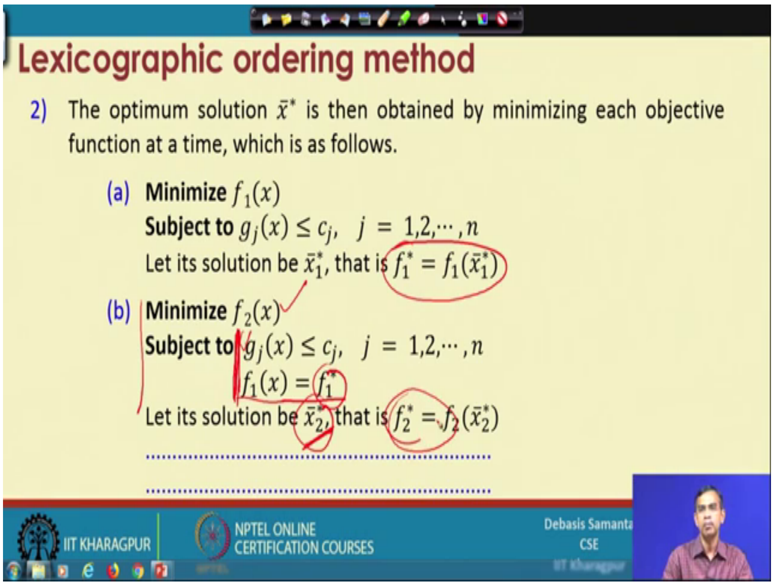

So, the first is that we have to consider only one objective function in their order. So, in this order f 1 comes first. So, we have to first minimise only f 1 x without considering the other objective functions and with the same subject the constant it is there n number of constants.

So, this can be considered as the simple or single genetic single objective optimisation problem solving and for this we can follow the simple GA framework in fact. So, these suppose so, these leads to so, when we apply using single objective optimisation problem using GA framework and then it will give solution let the solution in case gives x 1 star. So, x 1 star is a optimum solution with respect to the first objective f 1.

Now the second step we have to then find solution objecting the second objective function. So, it is in this case minimise f 2 x, but the constant will be whatever the other constant it is there it is the original problem and another constant that f 1 x is equals to f 1 star, where f 1 star is basically the solution that is often from the first step.

So, on constant will be included here in order to solve the second ranked objective function namely f 2 there and this will give a solution say x 2 star; that means, with respect to only optimising f 2 x x 2 star is the solution that is the optimum solution to with reference to f 2 x and with respect to these are the constant and let this be f 2 star. So, it is basically optimum we with respect to second objective f 2.

Now, we will repeat the same thing again, but for the next ranked objective function like f 3 and f 4 and so on.

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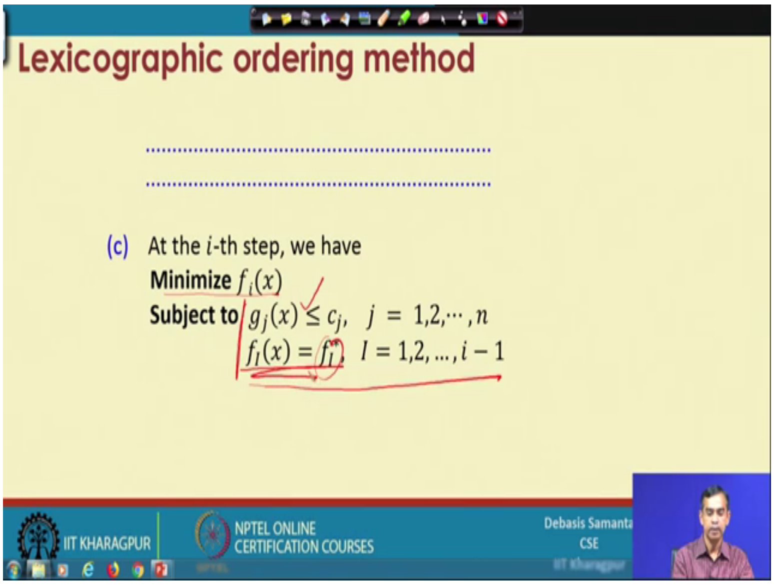

So, finally, at any i - th iteration. So, the objective is to find a solution targeting the minimisation of the i - th objective function f i x and the constant will be again increases one by one. So, it is the original constant for the all objective function and they are will be for the i - th iteration i minus 1 number of constant will be added which is basically the solutions that we have obtained with respect to f 1, f 2 dot dot f i minus 1. So, these are the solutions and it satisfy this condition so, this is the idea that is followed there.

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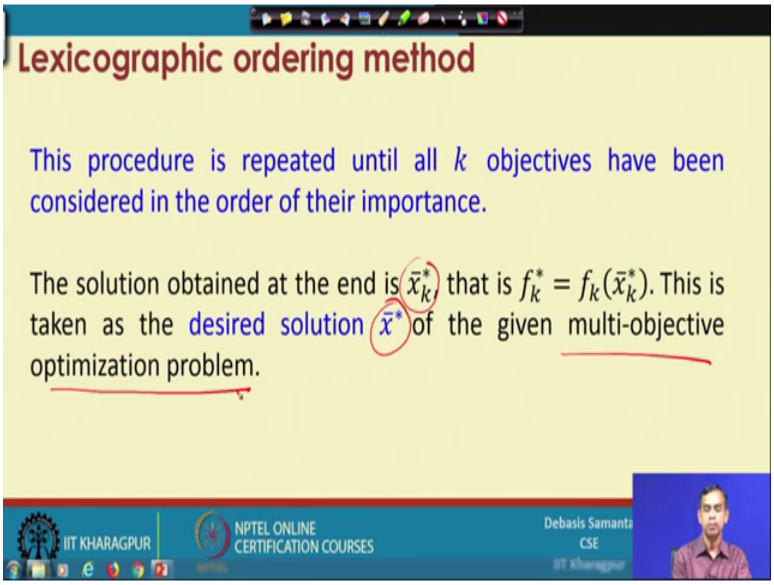

So; obviously, after the k iterations, we will find a solution let this be the solution is let this be the solution x k star right that is f k is basically solving the k th optimisation problem at the end and when the constant is all the constant that is there in the original problem plus the constant f 1 x equals to f 1 star f 2 x equals to f 2 star dot dot f k minus 1 x equals to f k minus 1 star this one. So, this solution after the, this one is basically the desirable solution and we can termed as this solution as the optimum solution.

Now, this is the solution that can be obtain after the repetition of the k objective in succession one by one according to their order of importance and this way the order of solution can be obtained as a multi objective optimisation problem and you can understand that ah. So, here the solution that can be returned by this approach lexicographic ordering is only one solution instead of many solutions that is called the trade of solutions there that is why it is also called Non-Pareto because it does not give any Pareto front or Pareto optimum solution it is an a priori approach as well as it is a Non-Pareto based approach.

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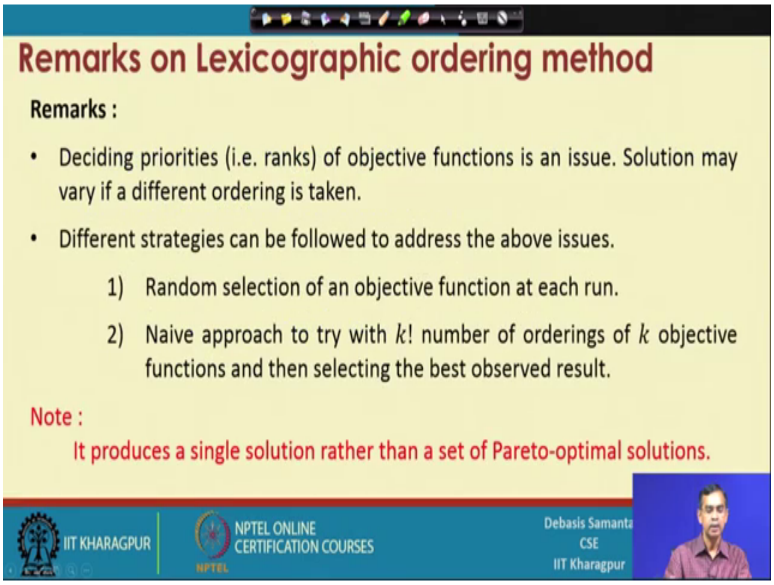

So, this is the concept of lexicographic ordering and; obviously, there are certain criticisms about this one um. So, the first criticism regarding this method is that we have to decide the priorities of all the objective functions that is there in the multiple objective optimisation problem. That is we should have a correct or the knowledge of correct or actual knowledge of the ranked of the different solution.

If we do not have any ordering of the concept of or knowledge on ordering of the any objective function then it may leads to non-erroneous result or non or you can say that non - optimal solution. Now; however, if you do not know the orderings of the solutions then there may be some strategies can be found out one is that random selection of objective function at each run, but you know there are say if k number of objective is there then finding random out of which the based is a basically is a k factorial search.

If the number of objective factor is only 2 3 4 it is possible to apply this one, but if the number of objective factor is very large then this method is cannot be applied. So, this is really one serious drawback of this of this method and as I told you if it is a objectives are in conflictive nature usually we have to find a trade of solutions that is a Pareto optimal solution, but lexicographic order gives a single solution instead of trade of solution that is more desirable than the multi - objective optimisation problem solving is concerned.

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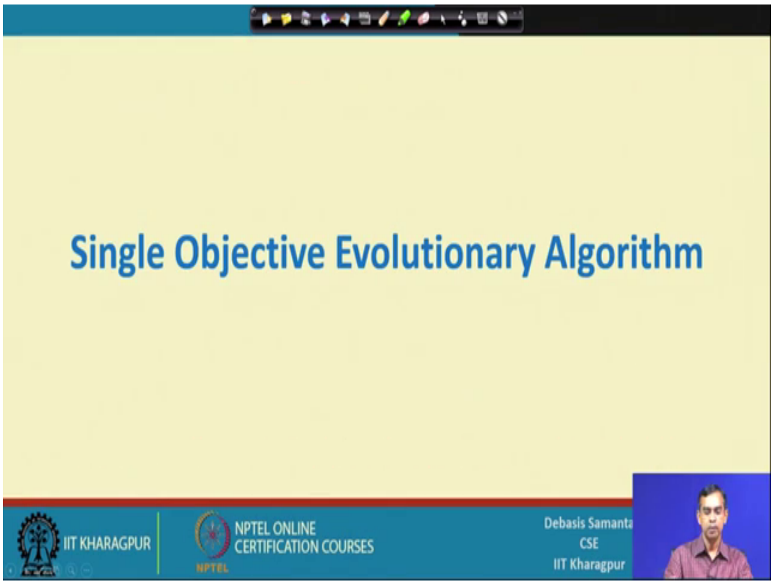

So this is the lexicographic ordering next another simplest method which also can exercised using the same as a GA framework it is called the single objective evolutionary algorithm. Now it is basically the idea is that if there are multiple objectives how we can convert this problem into a single objective optimization problem. So, this is the basic idea that is followed here in this technique and this technique is also called SOEA it is Single Objective E A Evolutionary Algorithm S O E A. So, it is SOEA approach is there.

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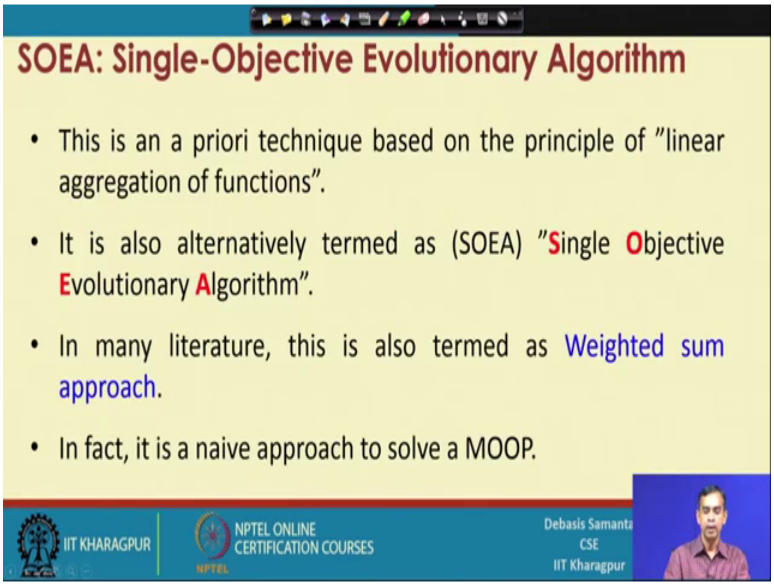

Now, so, it is also alternatively called Weighted sum approach because it basically consider some weights in order to find the in order convert the multiple objective optimization problem into a single objective optimization problem it is on basically nascent approach published as early as the lexicographic ordering solution was proposed in 1985 or so.

Now, let us see exactly, what is the technique that is followed there in case of SOEA approach.

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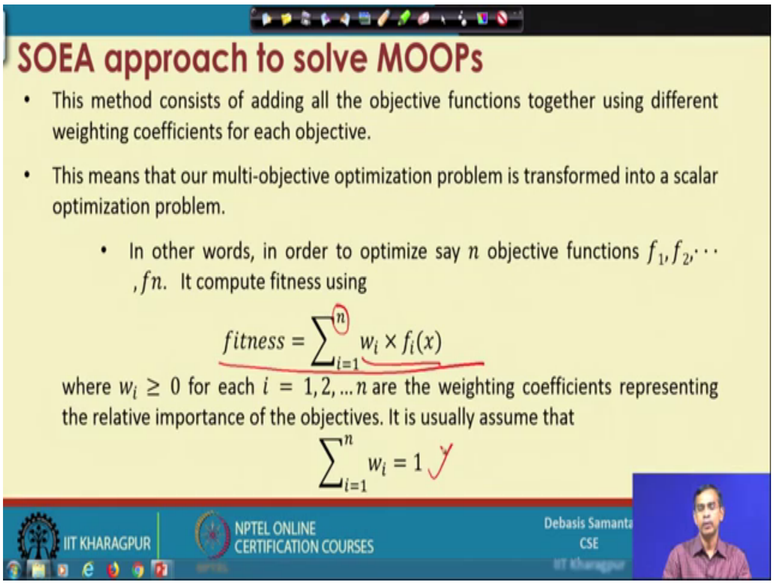

First we have to decide the weighting coefficients for each objective function. So, in case of lexicographic ordering it requires to know a prior knowledge about their ordering of the objecting function. Here is a same way of course, it is not exactly the ordering rather weights of the objective function, in other words if suppose f 1 is most important then we should a weight w 1 which has higher value than the least important less important objective functions say f 2 and then weight is w 2.

So, we have to decide if there are k number of objective functions the k number of weight coefficients weighting coefficients w 1, w 2, dot dot w k and once the k number of weighting coefficients are known to us then we will be able to formulate a fitness value; that means, the single objective there. So, this within this formula if the n objective functions are there so, is basically sum of products of weights and then their objective values.

So, it the w i and then f i x and summation of all these and there; however, all the weights are to be decide in such a way that the summation of the weighting coefficients is equals to 1. So, this is a normalized form of the weighting factors weighting values and. So, here the main important most concern about how to decide w i. So, once it is decided correctly we will be able to get the solution correctly.

So, this is the idea about the single objective evolutionary algorithm or it is called the SOEA approach to solve multiple objective optimization problem and as this is a single objective optimization problem like this here; that means, the same genetic algorithm framework can be applied here without any change, basically the same reproduction, the same selection, the same initial population creation cross over mechanism and convergences all these things can be applied here.

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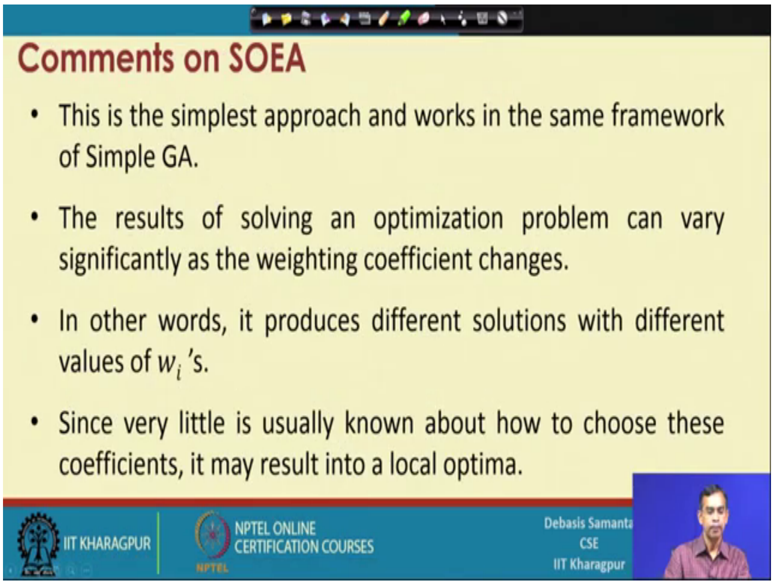

So, this is the idea about SOEA approaches and definitely SOEA approaches one is a simple most approaches because it can be used, it can be solved using the a simple g a framework; however, the results of solving an optimization problem can vary significantly if their weighting coefficients values are changes; that means, for different a different solutions can be obtained for the different values of w i. So, that is why all the weighting coefficient values are to be decided as accurate as possible to get the base solution.

So, since very little information is known to us how to choose this coefficient usually this solution may leads to a non - local solution or is a non- global solution is a local optimum solution rather. So, usually so, it is simple, but solution may not be. So, accurate it is very fast compared to other MOEA techniques there.

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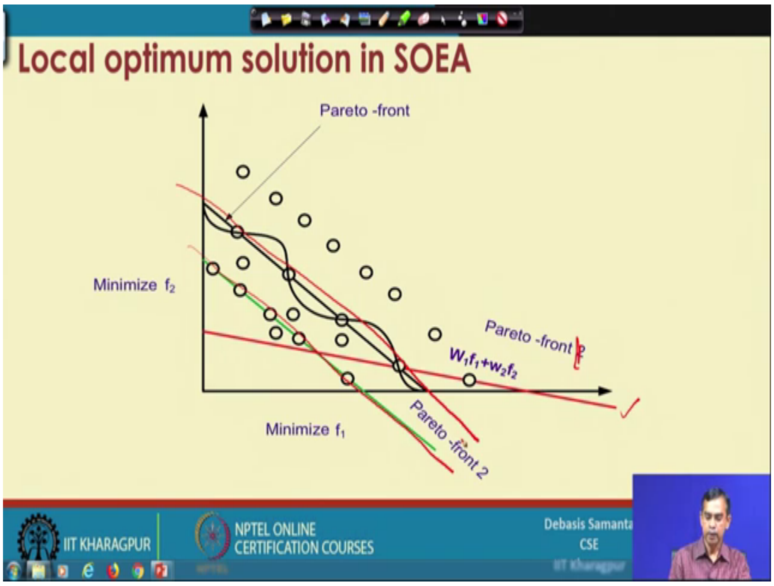

Now, just let me illustrate about how if you decide the different weighting factors and then it gives to the different solutions actually. Now here the idea it is there, so the vector or you can say single objectives that can be represented by this one. So, it is basically single objective this one and if w 1 and w 2 are the weighting coefficient and this is basically f 1 and f 2. So, it can be plotted at the linear function of w 1 f 1 plus w 2 f 2 for the unknown f 1 and f 2.

So, it is just like a and you know. So, w 1 f 1 plus w 2 f 2 in a 2 dimensional space f 1 and f 2 it basically represent a straight line. So, the idea it is there so, for different w 1 and w 2 values, w 2 objectives f 1 and f 2 the different different solutions can be obtained. Now each solution in basically, so if this the one line corresponding to one value w 1 f 1 plus w 2 f 2 then it basically gives one front like. So, this can be termed as a Pareto optimal front 1.

So, all the solutions which lies on this is basically the trade of solutions. So, for our this is concerned and definitely it will return only one value depending on the values of f 1 and f 2. So, suppose either it written this one or it written this one depending on a f 1 and f 2 and depending on f 1 f 2 or if any other solutions are there. So, essentially it try to find a Pareto optimal front, but actually it returns one single solution depending on the f 1 and f 2.

Now again this Pareto optimal front that we have that we can obtain using single objective evolutionary algorithm where is if the coefficient values are varies. For example, so, if this the Pareto optimal front 1 then another plot the different values of w 1 and w 2 another Pareto optimal front can be obtained or this is another Pareto optimal front can be obtained for the different coefficient values of f 1 and f 2 different coefficient values of w 1 and w 2.

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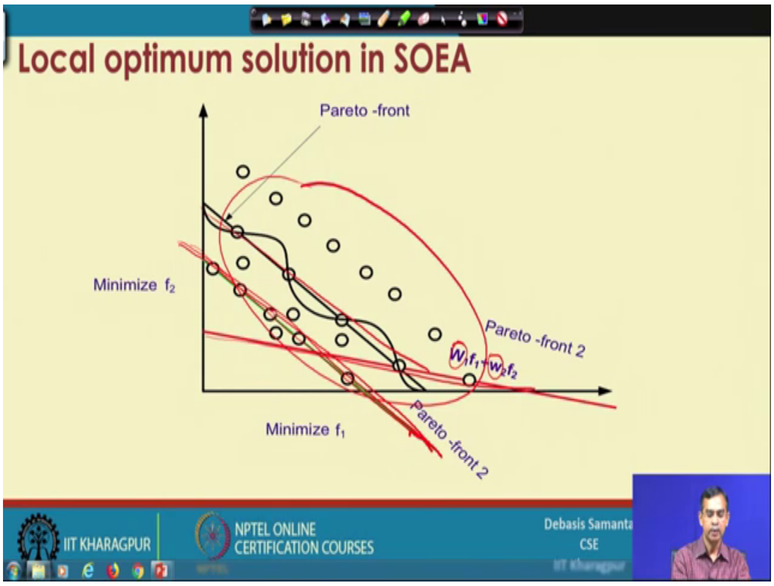

Now, so, we can say that if we do not select the w 1 and w 2 precisely then many Pareto optimal front are possible and then which solutions are the correct they are not necessary the local they are not necessary the global solution at here. So, our objective is to find the right values of w 1 and w 2. So, that if this is the solution space there it will find the solutions like this one so, that any one solution can be obtained as a solution for the multi objective optimization problem.

So, what you can say that, the usually if we do not select the coefficient values correctly then it leads to a local optimal solution, now this is our major drawback of this technique Single Objective Evolutionary Algorithm.

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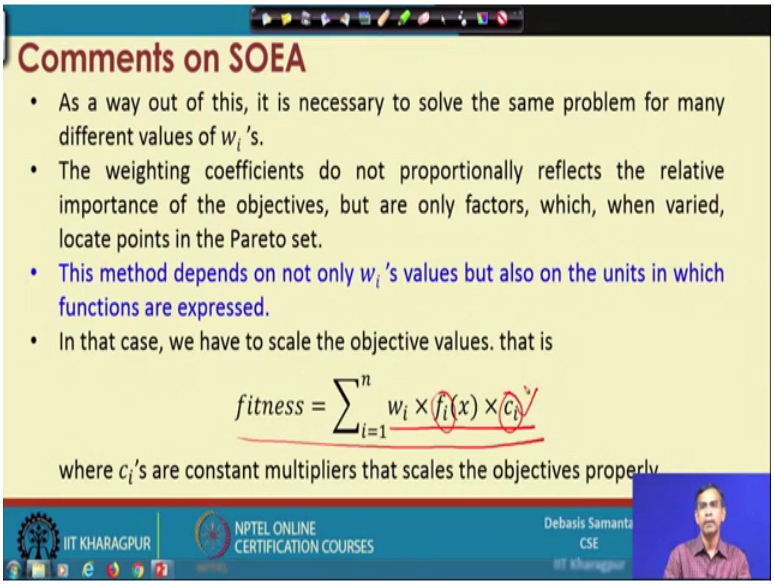

So, the idea it is like this. So, what is the possible remedy to solve this problem it basically to solve the same problem for the different values of weighting coefficients; however, the weighting coefficients as it is infinite sets are possible. So, try out with many possibilities are also not comprehensively visible.

And so, weighting coefficient do not proportionally reflects the relative importance of the objectives that is in case of lexicographic order you know, but are only f factors which one vary it can locates the point on a surface in a different sets. Now the methods, depends on not only the decision of the right values of the weighting coefficients, but also importance on the units in which the different functions are expressed.

Now, so, suppose the 2 objective functions are there one unit is in the millimetres scale and another is in the kilometre scale then definitely weighting formula that can obtain using this formula will be the effective one. So, it required that all the objective functions are to be expressed in the same scale, as a way out this things the idea about that one scaling factors for each objective functions needs to be multiplied in addition to the weighting values weighting coefficients and for say i- th objective f i let c i is a scaling factor it is multiplied by this then the fitness function can gives the proper meaning or the effective meaning.

So, the c i is called it is basically is a scaling factor in order to normalize the values of all objective function or in other words is a scaling factor so, that all objective functions can be expressed in the same scale if it is possible. So, this is the idea about it although it is not an issue it required little bit processing. So, that we can understand the, what are the scaling coefficients for each objective function in the problem. So, this is the idea otherwise the SOEA approach is very effective and useful.

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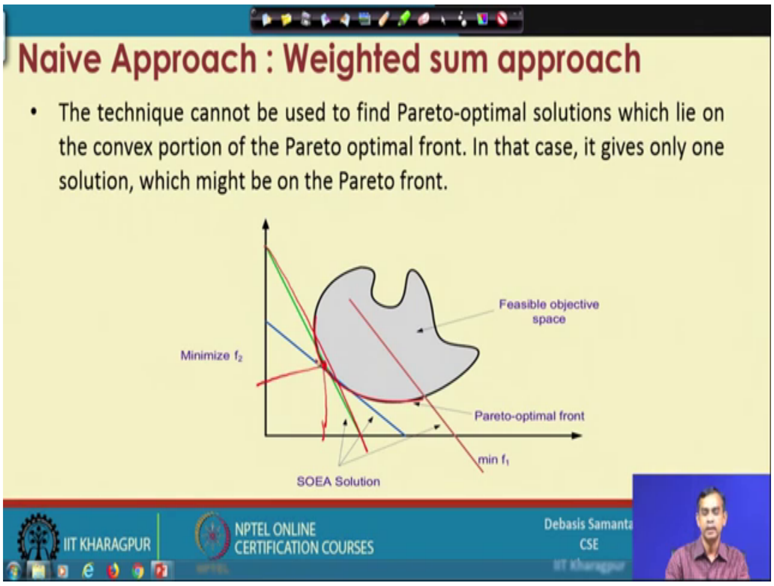

And, so these approach as I told you is a very Naive approach and then ascent approach we can and also it is termed as the weighted sum approach and we have discussed sum pros and cons about it and another advantage another limitations of this approach is that. So, if suppose the Pareto optimal front lies on a straight line it will find on a solution correctly.

Suppose if the Pareto optimal front does not lie on the straight line rather in a convex or in a non-convex region whatever it is here. So, Pareto optimal front it is here or it is here or it is here, whatever it is here, then what will happen. So, in this case it requires only one it returns only one solution because if it a Pareto optimal front if it the Pareto optimal front according to the SOEA approach, but essentially the Pareto optimal front it is then it will find only one solution.

However, this solution is unlikely to get because if you find f 1 and f 2 it is here then it is basically non global solution it is not a solution actually because the solution space is this one. So, it can give some solution which is effectively not a solution rather it does not lie in a solution space. So, it can give only solutions where this line touches this Pareto front only one solution and for which you have to precisely decide what is a f 1 and f 2, then only you able to find it.

So, it is very difficult because it is usually gives a f 1 and f 2 in this region may be which are not necessary lie or within the range all the visible solutions. So, this is why he solution that it will written the SOEA approach not necessarily be a visible objective space rather it is a non-visible solution it can written this is a serious drawback this is a serious drawback of the SOEA approach.

Now, so, this is the concept that we have discussed about the 2 a priori and then non Pareto based approach namely lexicographic ordering and then SOEA approach we have discussed, the another Non-Pareto based approach it is called the vector evaluated genetic algorithm it is also a priori approach we will discuss in the next it is it is not an a priori it is a posterior approach rather we will discuss in the next class. So, it is a posterior approach and then Non-Pareto based approach. In fact so, we will discuss in the next class.

Thank you.