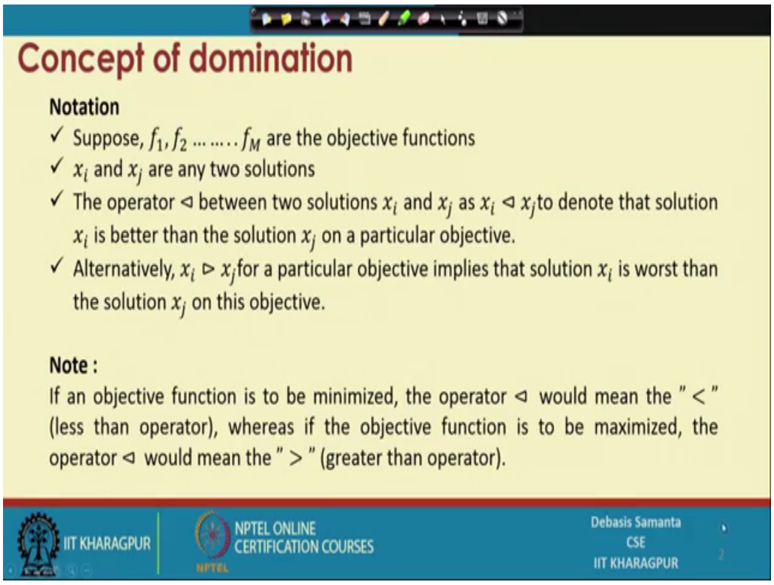
Introduction to Soft Computing
Prof. Debasis Samanta
Department of Computer Science & Engineering
Indian Institute of Technology, Kharagpur

Lecture – 26
Concept of Domination

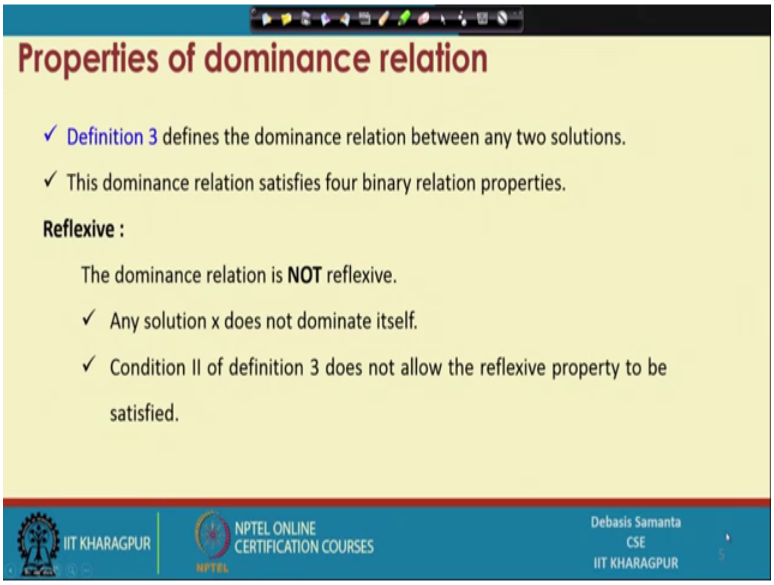
In the last lecture we are learning about two things the concept of the solutions. So, for the multi objective vectors are concerned, and then we discussed about concept of domination. In this lecture we will try to learn about the properties there the concept of domination holds and then we will discuss about the pareto optimal font which is a very important concept so far the multi objective uh solution is concerned.

(Refer Slide Time: 01:03)



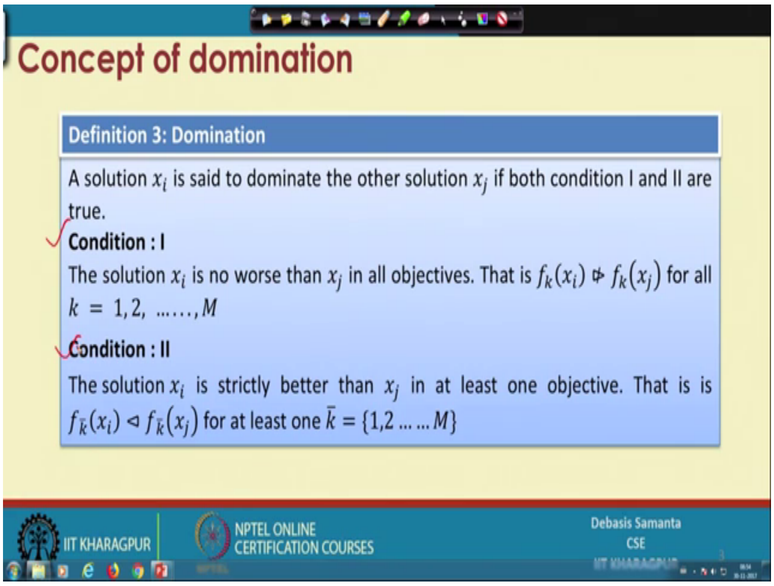
So, now, this is in continuation with the previous discussion we just introduced the concept of domination whether two solutions x i and x j with respect to aim number of objective vectors are dominating or not. Now, today we will discuss about the properties that this dominations relation holds. That means, if they are easy to solutions x i and x a where x i dominates x j then what kind of relations that it can holds.

(Refer Slide Time: 01:39)



Now, we can start with the concept it is here. So, definition 3, the definition 3 we have the definition see that we have discussed in the last lecture we say that to solutions are said to be the or two solutions x i and x j, if it is there then x i said to dominate x j if two conditions condition 1 and condition 2 which is stated here, satisfied here.

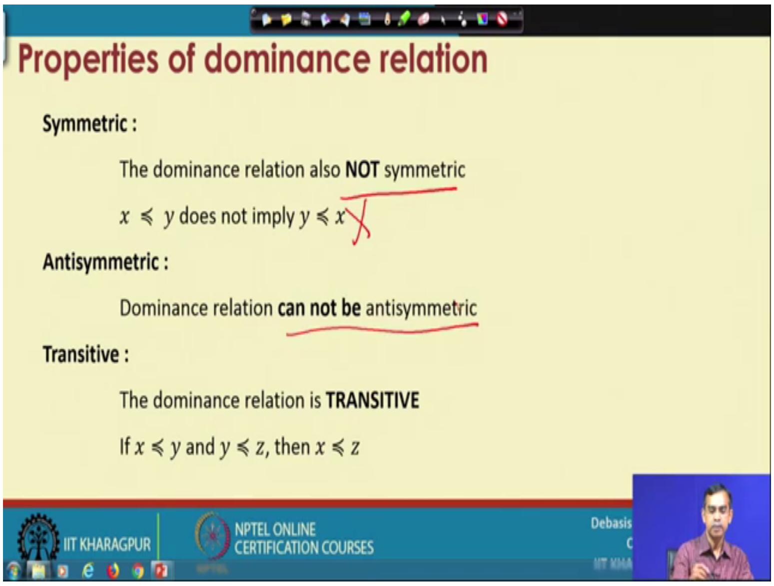
(Refer Slide Time: 01:44)



Now, let us see if there exist two solutions and then what relations that they holds good. Now, so, it required basically it is basically the relation between the two solutions. So, it is the binary relations. We can say and we know the binary relation concept it is there and that relation can reflexive can be symmetric or can be transitive, and we can say that the solutions x i and x j uh is the dominance relation is not reflexive that mean a any solution x does not dominate itself. So, this is a one important concept that it is not reflexive. Condition because, so condition ok.

So, so far the condition one is concerned x 1, x i and x i, the first condition holds good, but the second condition that it should be strictly better than with respect to at least one which is not holds good there. So, that is why it does not satisfy the reflexive property. So, that is why this relation is not a reflexive relation.

(Refer Slide Time: 03:24)

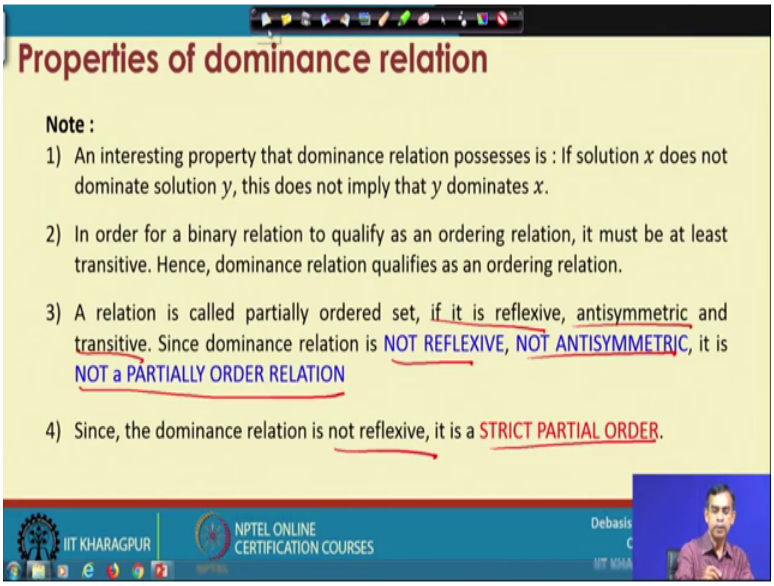


Similarly, this solution is also not symmetric relation, not symmetric relation, so not symmetric relation. So, means that x, if x, if x dominates y it does not imply that y dominates x. So, it is not a symmetric relation.

Now, it also can be anti symmetric; that means, a dominance relations if x y does not satisfy y and then y can be also can be also dominate this x also that is why it is called the can not be anti symmetric. However, it satisfy one property it is called the transitive property that mean if x and y two solution. So, that x dominates y and similarly there is a solution z such that y dominates z then we can say that x dominates z. So, it is called the transitive property.

So, what we have understood is that, so dominance relations is not reflexive, not symmetric, not anti symmetric, however it is transitive.

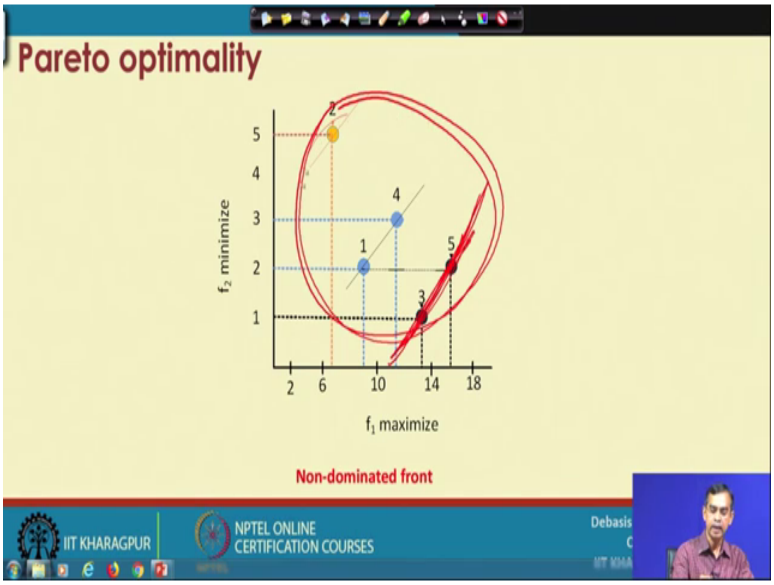
(Refer Slide Time: 04:49)



Now, so, based on this concept a relation a binary relations are a binary can be termed as a partially ordered set, partially ordered set if it is reflexive, it is anti symmetric and transitive. So, in this case as it is not reflexive, not anti symmetric then it is not a partially ordered set. So, that is why it is called is not a partially ordered relations. However, seen it is not reflexive and then it basically strict strictly partial order. So, the concept the domination relation is therefore, not a partially ordered relation it is a strict partial order relation.

So, this is a property that the domination relation satisfied.

(Refer Slide Time: 05:49)



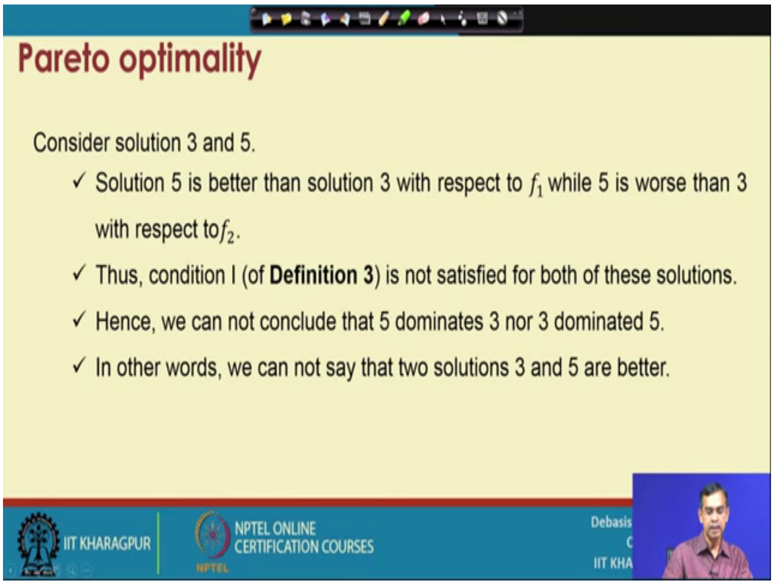
Now, we will discuss about pareto optimal solutions, the concept of pareto optimal solution. Now, before going to discuss this things we will continue our discussion of the concept of domination where we have discussed about the two objective PPTs here then how they can dominate each other or this kind of things here. Now, in this figure we can say this 3 and 5 when a font is maximise and f 2 is minimise we can say that 3 and 5 neither dominate each other; that means, this solution 3 does not dominate solution 5 or solution 5 does not solution 3.

However, if we consider solution one solution then we can say that this solution one or the solution 3 dominates 1 or solution 5 dominates 1 or solution 5 dominates both 1 and 4 solution 3, this one. Now, here all the solutions which rise on this line is basically not dominated by any other solution; however, they dominates all other solutions these and this one all the solutions. So, these are the solutions dominates all other solutions in this region, and as there is no solution here we can say that this solution is not dominated by any other solution. Now, the solution set which lies on this line like is called the optimal solution set or non-dominated solution and these are the front it is called the non-dominated solution front.

So, this is the concept it is there. Now, solution, so these are the optimal solution with respect to our searching of multi objective solution and ok, so this is a entire surface then all the solutions are desirable solution because they are at least better with respect to at least one objective vector if it is not there.

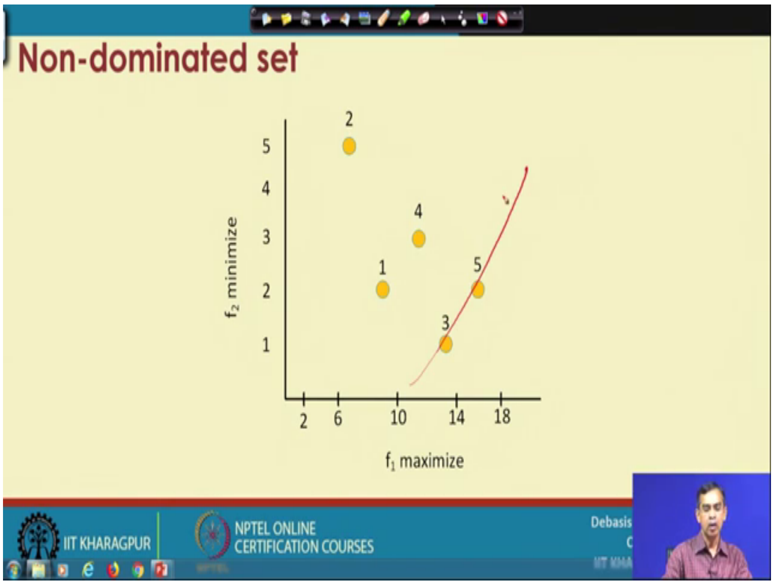
Now, so this is a concept it is here and we will see exactly the pareto optimal solution if all the solutions that is this is a entire surface that is possible or all visible solution those are there and out of these all visible solutions if this is a solution that can be on the non-dominated front then we can say that this is the pareto optimal front. This front is also called pareto optimal front. So, the condition is there the solution will be termed as pareto optimal only the entire surface is covered here. So, this is the concept of pareto optimality and then we will discussed about the pareto optimal front.

(Refer Slide Time: 08:28)



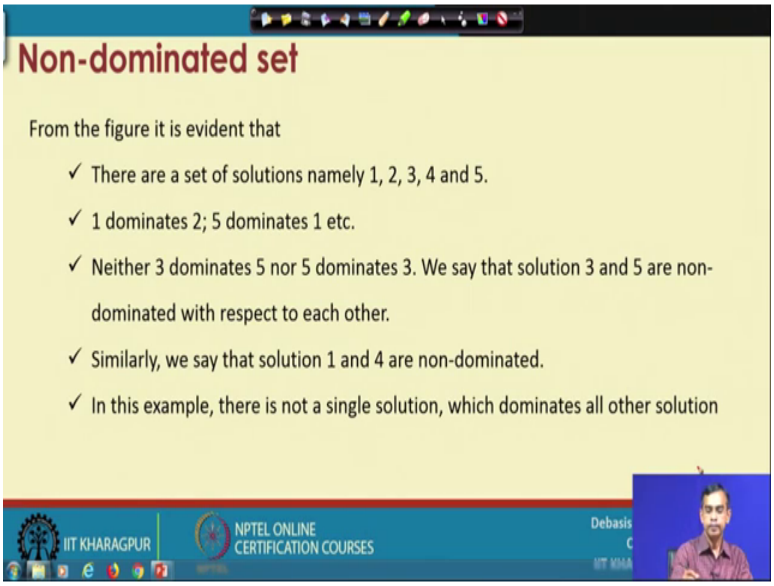
So, so that idea that we have discussed it is basically the same concept.

(Refer Slide Time: 08:36)



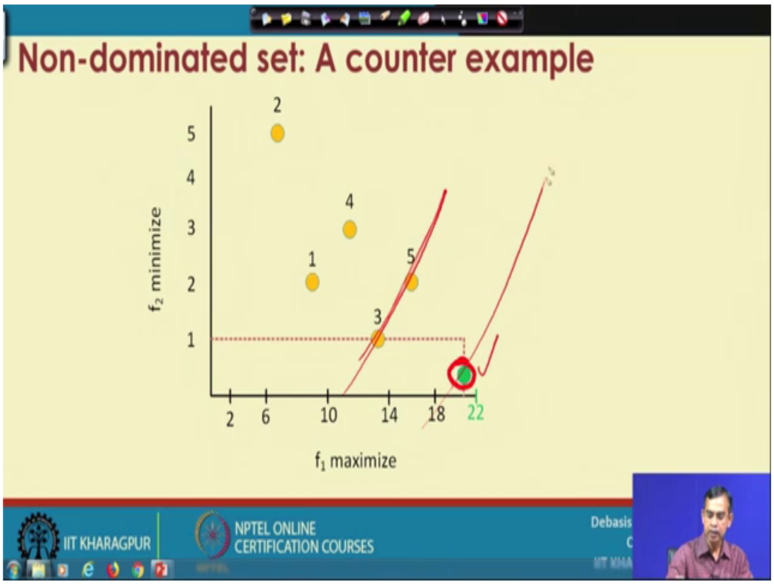
And so, again, so this is the front that front can be termed as a non-dominated set front that we have already discussed.

(Refer Slide Time: 08:50)



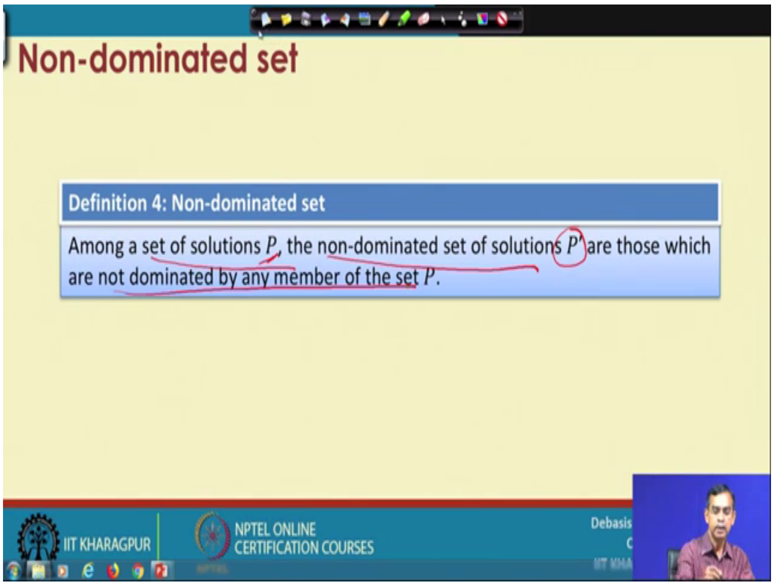
And. So, we if we check it all these you can verify with respect to the previous slides we can verify there are a number of solutions the solutions are 1 2 3 4 5, 1 dominates 2, 5 dominates 1 like this kind of concept is there and then we can see that there are some solutions 3 and 5 which are non-dominated solution ok. So, that you can check it and you can find that these are the conditions all conditions hold good.

(Refer Slide Time: 09:27)



Now, so, non-dominated as I told you, if this is the solution is a non-dominated and if we can find any other solution then they are no more non-dominated solution. So, this solution is become the dominated. So, this is basically another dominated front actually ok. So, this is a example when they are not the dominated like this if there exist some solution is here.

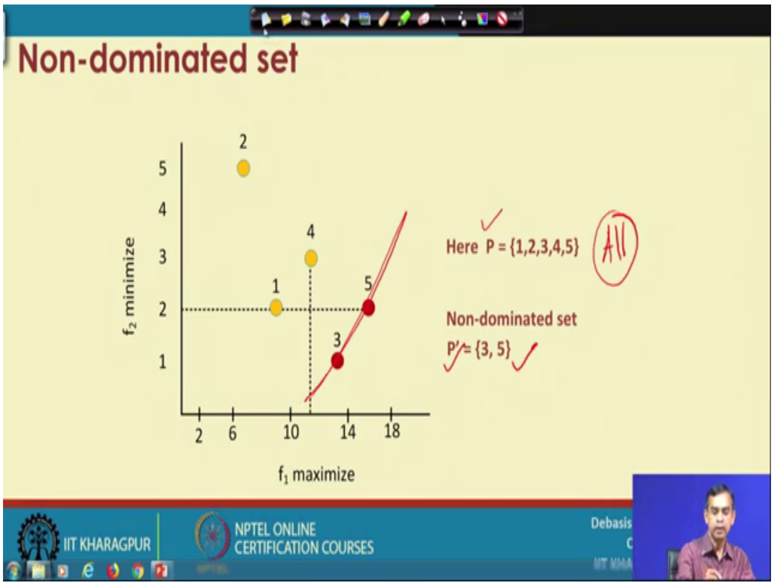
(Refer Slide Time: 09:52)



So, now, we can precisely mention about when a solution will be termed as a non-dominated solution or all solutions because they are not a single solution which can be there, there may be multiple solutions also, all solutions which are non-dominated they are called the non-dominated set.

So, it is like this idea, set of solution all solutions P then non-dominated set of solution there is a subset P dash of P are those which are not dominated by any member of the set P. So, it is a concept of non-dominated set. And the concept of non-dominated set is very important in the concept of optimization of objective function.

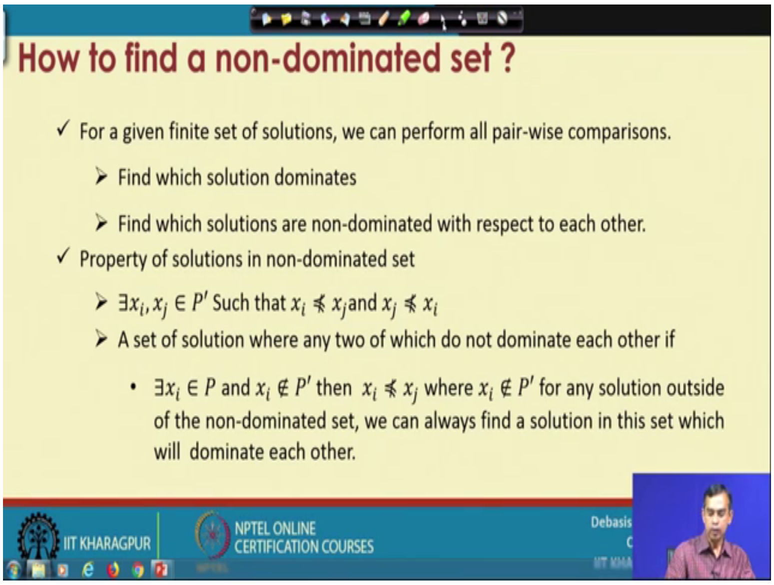
(Refer Slide Time: 10:37)



Now, again we can elaborate it, so the same idea that we have discussed about. So, non dominate in this case is 3 and 5; that means, P dash and P is the all solutions in the solution space.

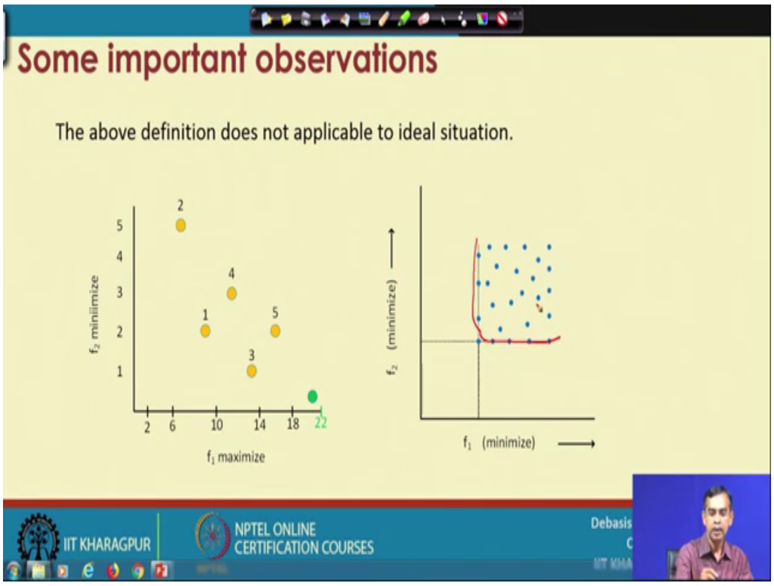
Now, if it all solutions this all solution in the all solution, then this is the optimal front and particularly this optimal front is called a pareto optimal front.

(Refer Slide Time: 11:12)



Now, we will discuss few cases are there so that we can learn about how to find a non-dominated set ok. So, basically we have to apply the dominant solution for all solution with respect to any other solutions and if we can find the solution which is not dominated by any other solutions, but ok. So, then it is the non-dominated front like this.

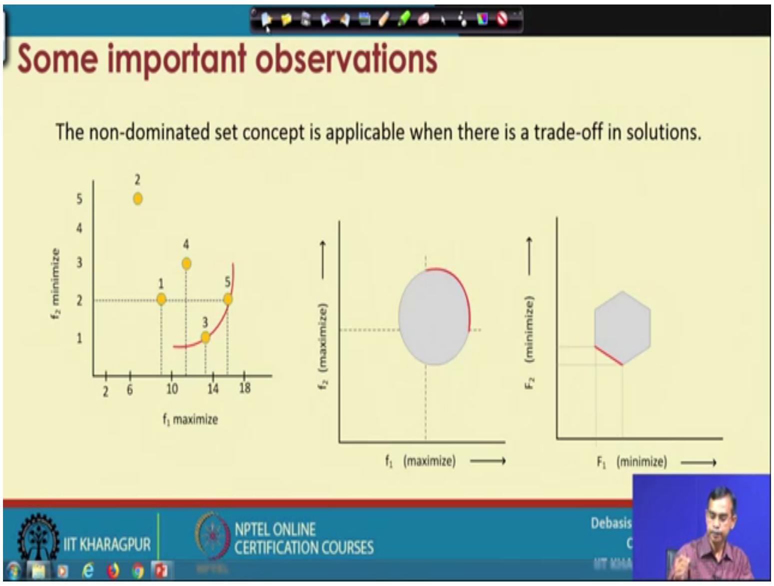
(Refer Slide Time: 11:44)



Now, ok, so [vocalised-noise] this is a idea that the solution can be obtained by this concept of dominant solution and then we can find it. Now, so, again we can note that if there is an ideal, there is an ideal of solution, so ideal solution does not satisfy these kind of concept actually. So, it is not applicable to, so there is no front actually.

So, for example, if this is a solution space both f 1 is minimize and f 2 minimize then, then in fact, this is not the front actually rather the ideal objective front only one solution. So, this is not the front actually. So, this is the only solution that is there. So, for the ideal solution is concerned.

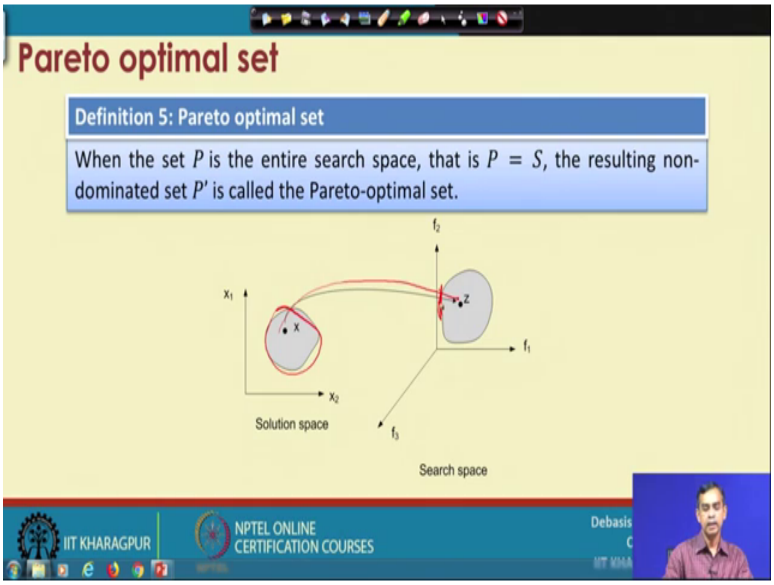
(Refer Slide Time: 12:41)



Now, we can generalize this concept with reference to many examples if it is there. Now, if you can say that if it is maximized if it is minimized as we have learned about this is the pareto optimal front. Now, if we consider another case f 2 is maximise and f 1 is maximise again then the this is the front and this is the entire solution out of this entire solution this is the front; that means, all the solutions which lies on this region they are called the pareto optimal solution.

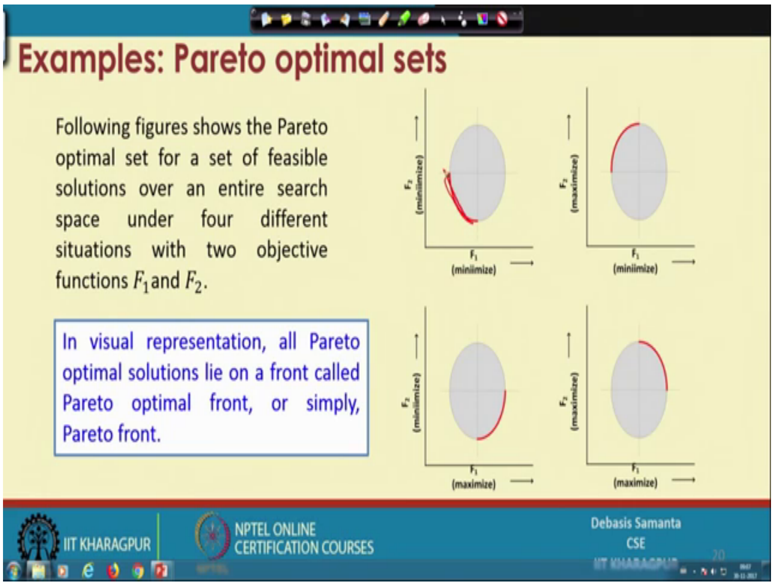
A another solution if f 1 is minimised, and f 2 is minimised then these are the solutions are the pareto optimal solution. Then this front is called as pareto optimal front because its satisfy this concept of domination.

(Refer Slide Time: 13:46)



Likewise you can extend few more example ok. So, we have learned about the pareto optimal set. So, if the, so pareto optimal set this is the entire solutions space, out of this entire solution set which are on the pareto optimal front then they are the pareto optimal solution. This is the concept there.

(Refer Slide Time: 14:14)

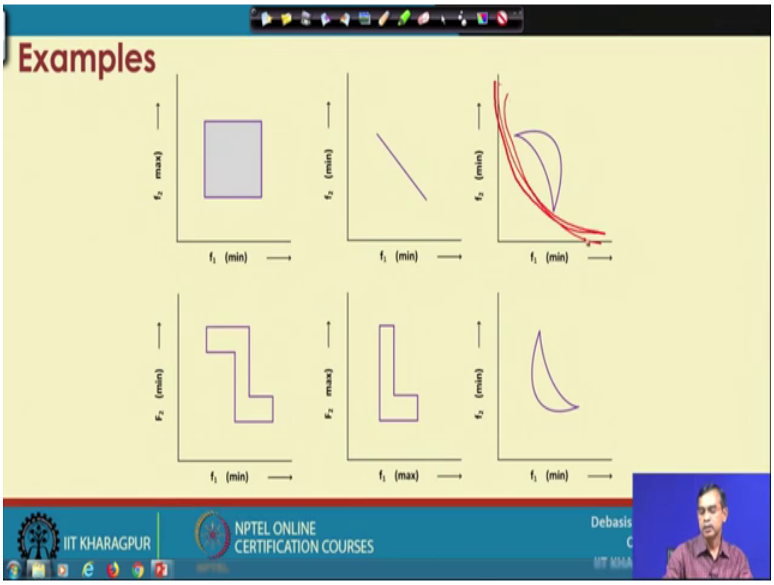


Now, few examples in order to identify the different concept of the different situations where the pareto optimal solutions because some time we have to decide only the; so it is the pictorial description of the different situation by the pareto optimal solution can be thought of. Now, here, depending on f 1 is minimise and f 2 is minimise, so this is the pareto optimal front. Similarly if it is maximise and if it is minimise then this a pareto optimal front and if it minimise and it maximise this is the pareto optimal front and it is maximise and maximise this is the pareto optimal front and these are the entire solutions space or the surface, the entire surface that is there.

So, these -are concept here. But we should not worried about the different situations there it is just only matter of understanding the important thing is that any objective function whether it is minimise or maximise or whatever it is there they can be converted into one form, either all maximise or all minimise then our idea will be very simply. If we all minimise then we can say that these are the concept. So, optimal solution we can easily identify a particular front which basically the pareto optimal front. So, the idea is that all the solution is given to you and as I told you the pareto optimal front is our desirable solutions. So, if we can identify the pareto optimal front then we can take all solutions and these are the trade of solutions.

So, all pareto optimal solutions which lies on a front is called the pareto optimal front sometimes it is simply called as the pareto front. Now, this is the idea that we have elasted for the two objective function is very difficult to visualize in case of in dimensional, if there n objective vector is there, but it is the concept a mathematically the same concept can be applied whether the two objectives or more than two objectives are there.

(Refer Slide Time: 16:21)

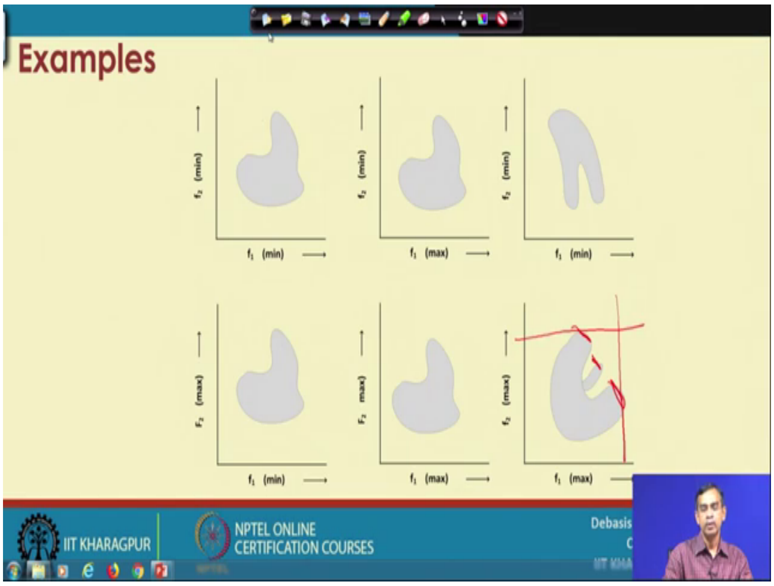


So, this is the concept about it and we would conclude this concept with the few examples here. Now, the first is that the f min and max. So, it is a basically maximising this one and minimising this one. Now, you can say that which is the pareto optimal front here in this case. So, in this case because if it is maximise and it is minimise we can re call in the one slides in the last lecture we have different the cases there we can say that this is the pareto optimal front in this case.

Now, second example here both f 1 and f 2 are minimise. So, this is the pareto optimal front in this case. Now, here is a one another the typical curve it is here. So, f 1 is minimise f 2 is minimise. So, this is the entire surface then this is the one front this front is basically pareto optimal front; no, no; so is ok. So, is basically ok. So, if it is minimisation, so now can you tell me which is the pareto optimal front. So, basically it is basically so far the minimisation is concerned this is a function which basically the able the basically the pareto front look like the. So, here in this case this one and this one are the two solutions which are lies on the pareto optimal front.

Now, here if it is f 2 is minimum and f 1 is minimum both the minimisation and this is the front then then we can say that these are the pareto optimal front in this case. Now, similarly for the max if it is here. So, then these and these are the pareto optimal front in this case. And here also min and min and this is the pareto optimal front in this case. So, you have, so given the different what is called as geometry of the solution space we will be able to find what are the different front there and which front is essentially the pareto optimal front that is a important thing, that you should learn it and you should know it.

(Refer Slide Time: 18:41)



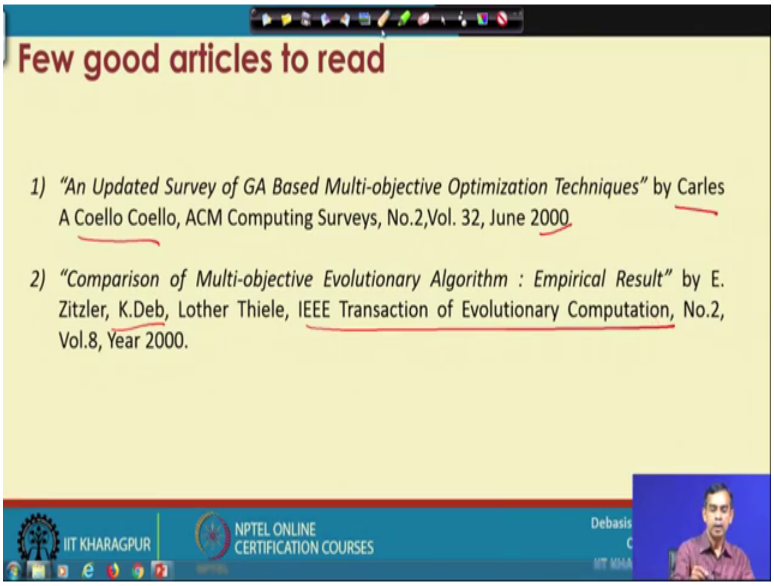
After visiting few examples I would like to give few more examples actually it happens in many real life solutions are there ok. Now, I left is an exercise for you. So, you can check it and then verify it. Now, here a f 1 is minimisation and f 2 is then you can find which is a pareto optimal front. So, it is basically it is like this and I thus given an example for your hint. So, this region is the pareto optimal front in this case. Now, likewise the same idea can be explained if it is maximise and it is minimise then this front is the pareto optimal front in this case.

Now, here again minimise. So, it is basically minimise and this is minimise. So, this and this, so these are the pareto front and this is a pareto front in this case. Now, again it is minimise and it is maximise, so it is minimise and maximise uh. So, this concept it is like this. So, minimise and maximise; so it is ok. So, it is maximise means this one and minimise this one. So, this front is basically the pareto optimal front in this case. Now, is a maximum, is a maximize.

So, it is basically maximize and this is the maximize, so this is pareto a front in this case. So, the pareto front like here and for the maximum this is basically maximum and this is maximum. So, this front is basically a pareto front in this case. So, pareto front not necessarily be a continuous front actually it may be discrete front as we have illustrated with few examples here.

So, we have learned about the different solutions and then we discussed about the ideal solutions. We discussed about the utopian solutions and their application. And then we have discussed about the concept of domination and then the relation that the domination can satisfy, and then we have discussed about the pareto optimal front which is important to understand about different solving the multi objective optimization problem in this case.

(Refer Slide Time: 21:08)



Now, so, in order to understand this concept better I would suggest to follow few articles because it needs lot of patience and more studies to understand the concept. So, the fast there is a survey paper it is an updated survey of GA Based Multi-objective Optimization Techniques by Carles A Coello and Coello this is (Refer Time: 21:32). And this is published by ACM Computing Surveys in 2000. So, this is very good article to read.

And there is one very nice paper which is written by K Deb, Kalyanmoy Deb. He has many contribution in the field of multi objective optimization solving and here basically comparison of multi objective evolutionary algorithm some results by Zitzler, Deb and Thiele. It is publishing IEEE transaction of evolutionary computation. Now, this is a one transactions published by IEEE is very famous and many articles related to our discussion can be obtained from here. So, these are the paper that you can follow to understand the concept.

So, with this discussion I would like to stop it here. And we will discuss about the approaches, the different approaches to solve multi objective optimization problem that we will start in the next lecture.

Thank you.