Introduction to Soft Computing
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Lecture – 25
Multi- objective optimization problem solving (Cont.)

So, we are discussing about solving optimisation problem. In the last few lectures, we have learned about how to solve single objective optimisation problem. In the last lecture, we were discussing about solving multi objective optimisation problem which are more applicable in our real life applications.

Today, we will continue the same discussion today mainly we will discuss about some properties, some characteristics which is very much essential to solve multi objective optimisation problem. And after we learn the different characteristics then we will discuss the different approaches to solve multi objective optimization problem.

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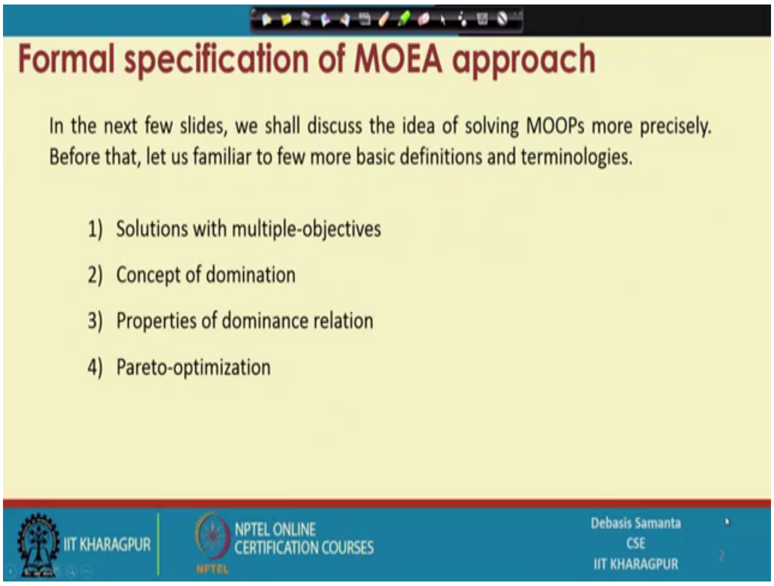

Now, so we have planned our discussion in subsequent lectures like. First, how the solutions with multiple-objectives are characterized and then the solution possesses some important concept it is called the concept of domination we will discuss it. And then will try to learn about the relation that is the properties that a dominance relation holds. And finally, we learn about Pareto-optimization techniques or use a Pareto-optimization concept. So, all these things we will done one by one.

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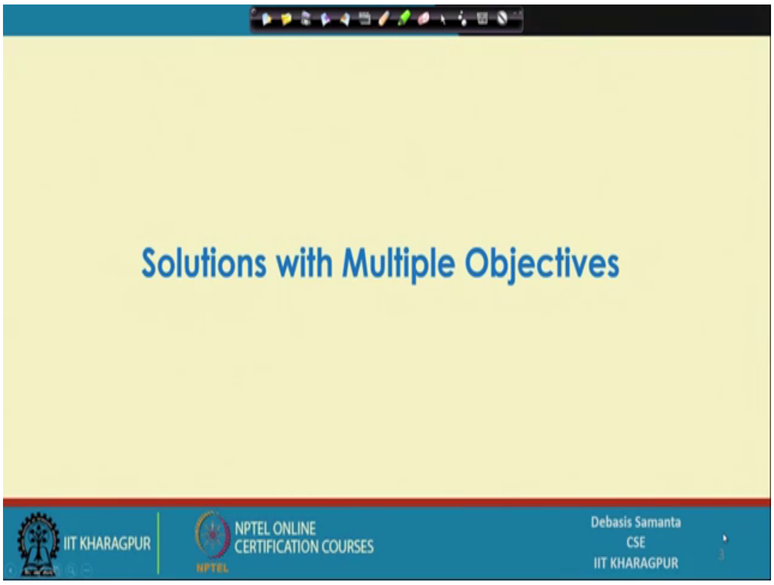

So, let us first start with solutions with multiple-objectives, and how they can be characterized.

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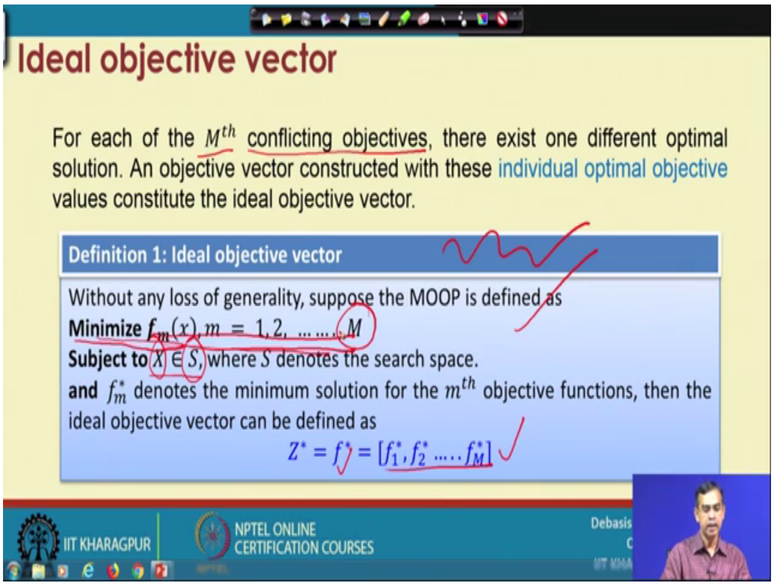

Now, let us consider for our discussion say that there are M number of objective functions. And these objective functions are conflicting objectives. Conflicting objective means if you try to minimise both say f 1 and f 2 then when you try to minimise f 1 f 2 not necessarily to be minimised or vice versa that means, there is a trade off. If we want to minimize both then there may not be any solution. So, we have to have some conflicting objectives if it is there then essentially there exist many optimum solution.

Now, if it is possible to exist one different solution which is basically satisfying both the objectives simultaneously, then we can say that such a solution is called the ideal solution. So, solution which has the ideal solution also called solution ideal solution and then the objective functions which basically holds good like this it is called the ideal objective vector.

Now, so here the same thing is discussed in a more precise or more formal method formal way. So, say here basically for the simplicity, we assume that all the functions all the objectives are to be minimized. So, it is basically it is generalization, it can be there are may be some objective function to be minimize some to be optimized, but we know with the virtue of principle of duality problem, all objective function can be converted to only one type. So, let us consider there are M number of total objective functions and out of which each objective function are to be minimized.

And then they are basically subject to X S, where S denotes the search space that mean we have to find a solution X in the search space S which basically satisfy all the criteria simultaneously. That means, here if f star is the optimum function, then it is optimum with respect to all that mean f 1, f 2 and f m then this solution is called the ideal solutions. And all the objective vectors, these all the objective vectors are called objective vectors.

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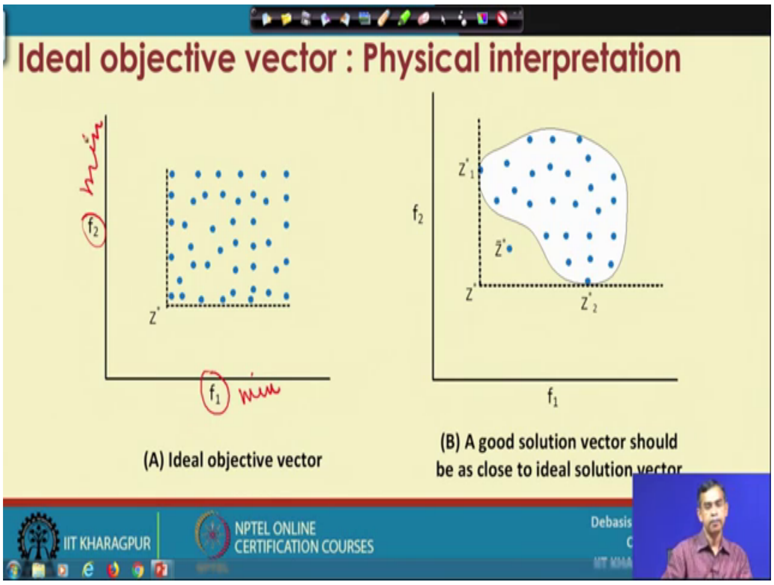

Now, the same thing can be discussed with a visual description and illustration right. Now, here suppose f 1 and f 2 are the two objective function. And we assume that f 1 is to be minimized and f 2 is also to be minimized for simplicity. Now, so, in this is the search space.

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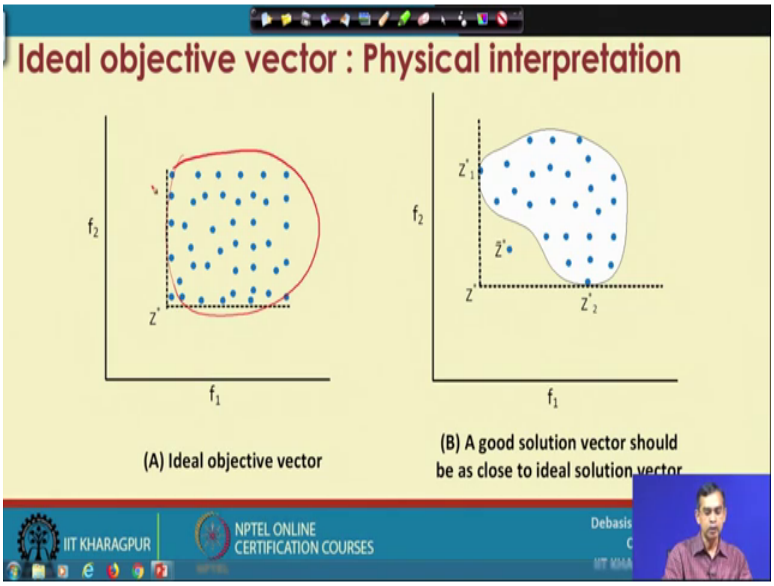

This is the search space for the entire what is called the searching of towards the optimum values. Now, out of this search space, we can see there is one solution.

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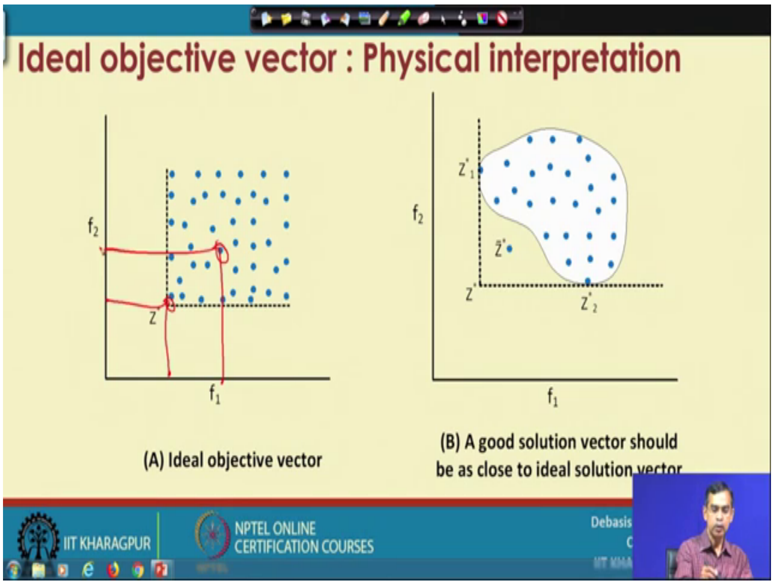

This one which is basically satisfying all the objective functions simultaneously. So, if this is the solution then it is f 1 is minimum and f 2 is minimum. Now, if you consider any other solution we can say that this is not the solution which simultaneously satisfying this one. For example, if it is minimum f 1, f 1 is also not minimum, then f 2 is also not minimum, so like this. So, the solution point which basically signifies this one, it is basically corresponding to the ideal objective solution. And if there is an objective function which exist there then it is called the ideal objective vector. So, this is the concept it is there. Now, so this solution as we have discussed about it is called the ideal objective solution.

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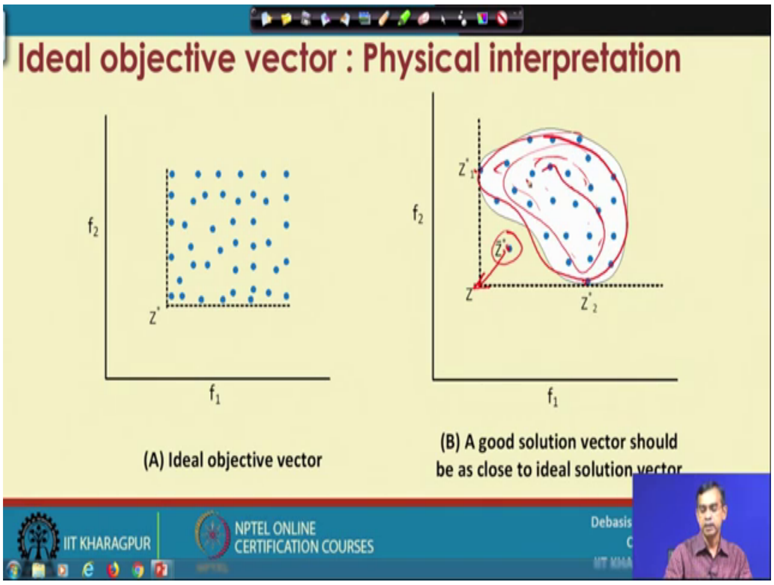

Now, in this in this figure, if we see it again, we can see that this is a one solution; this is the one solution. This solution may be minimum with respect to f 2, but the solution is not minimum with respect to a f 1. Similarly, this solution is minimum with respect to f 1, but not with respect to f 2. However, if we can find one solution like this which is obviously not exist here then we can say this is the one solution which is minimum with respect to both f 1 and f 2.

Now, so, this is the case if we have, then say suppose this is the one search space that means, we find many solutions in this region. Then definitely this is the one solution this is the trade off solution; that means, if it is good with respect to f 2, but not good with respect to f 1 and vice versa. Now, if there is any other solution which is very close to this ideal solution, then we can say this solution is more preferable than any other solution in the search space.

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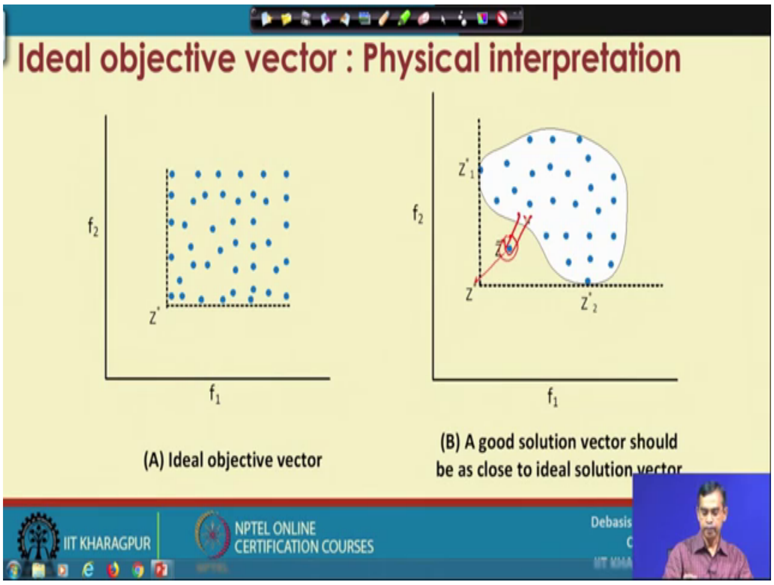

So, while we are searching if we find one solution which is not necessarily and ideal solution, but very close to the ideal solution right then this solution can be considered as a solution of our objectives. So, this is a desirable solution. So, a good solution, so vector should be as close as to ideal solution vector. So, this is the interpretation that we can have from this concept ideal solution ideal objective solution.

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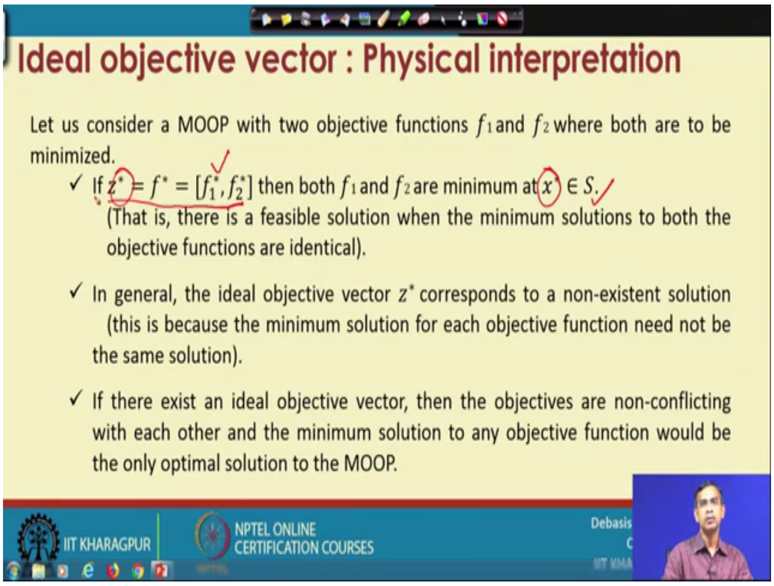

So, here now we can generalise little bit precise about say suppose there is a multi objective optimization functions with two objectives f 1 and f 2 where suppose both are to be minimized, both are to be minimized. Now, so if there is a solution we say that the solution is f 1 star f 2 star then both f 1 and f 2 are minimum at x star that is the solution space belongs to the search space. So,. So, in general the ideal objective vector. So, this is basically the ideal objective vector.

So, z star we can say it is an objective vector which is ideal right, right in fact, the ideal objective vector corresponding to a non existence solution that is why we called it is an ideal. Because many multi objective optimization problem are conflicting objective it is it is very rear it is in fact, impossible to see one objective function which is one objective vector which is minimum with respect to both the objectives. And if there exist an ideal solution then the objectives are non conflicting with each other, and then minimum a solution to any objective function would be any optimum solution to the problem. So,. So,. So, this is the concept of ideal. So, ideal objective vector is non existing one solution we can say.

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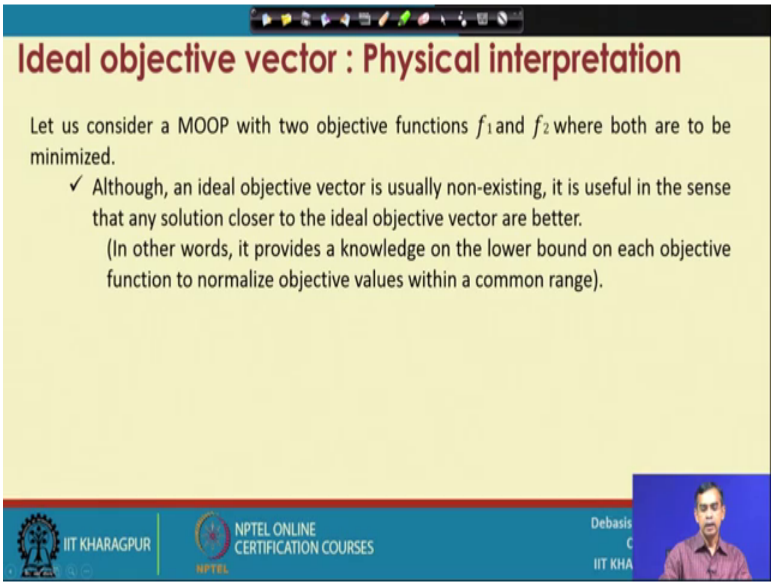

Now, so, fine now so, we have learned that ideal objective solu[tion]- ideal solution is usually non existing, but it is useful in the sense that any solution closer to the ideal objective vectors are preferable. So, this is the usefulness of this concept of ideal objective vector.

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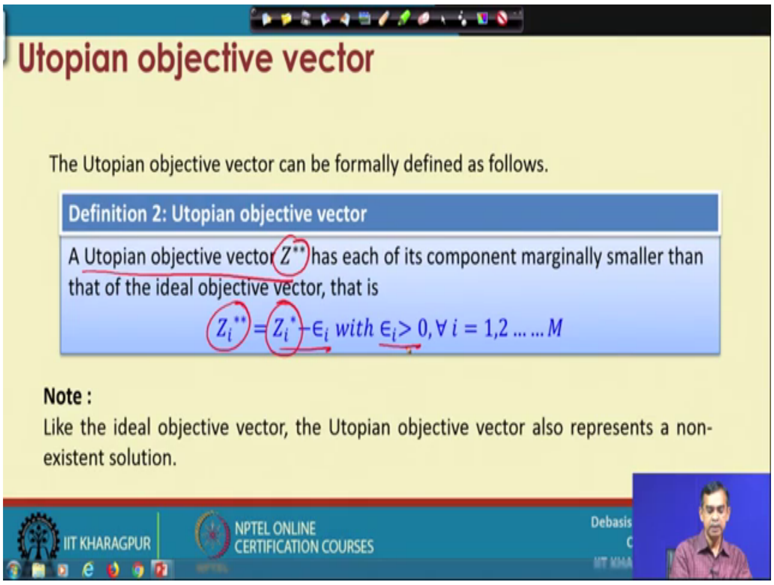

Now, we will discuss about another solution such a solution is called the utopian solution. Utopian means it is it is it is a is a fictitious, it is never possible like an ideal also, but it is an another extent. Now, let us define how the utopian solution can be. A utopian objective vector, so we denote it at z start star has each of its component marginally smaller than that of the ideal objective vector. So, if z 1 star is the ideal objective vector then j star will be far better than ideal objective vector. So, it is basically with epsilon i greater than 0, so that means, z star z 1 double star is far better than z 1 star then such a solution is called the utopian solution. And then solution vector if it is possible then it is also called the utopian objective vector.

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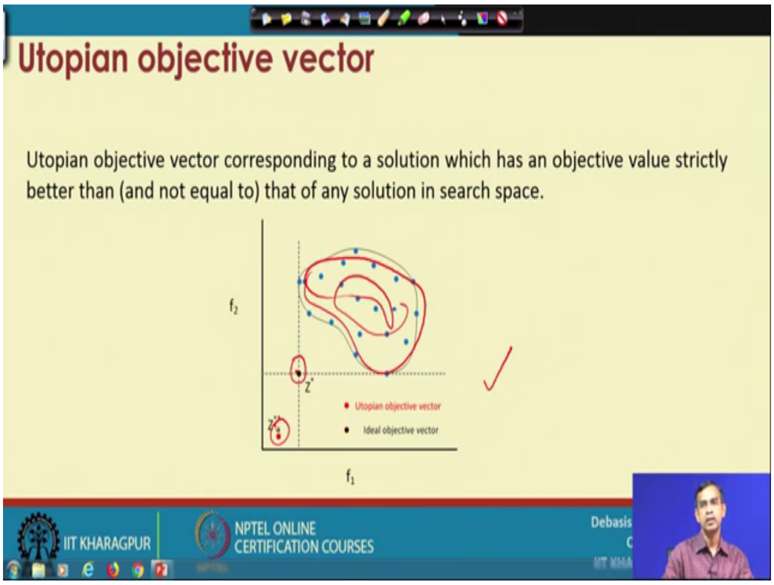

Now let us illustrate the concept with a with a with an illustration. Now, here this figure is planned to illustrate the concept of utopian objective vector. Now, now this is the search space this is the search space. And we have learned that this is the one solution which is called the ideal solution. And then utopian solution is far better than the ideal solution mean this is the one solution is an utopian solution. And the vector for which satisfy the solution is called the utopian objective vector. Now, so we have learned about the ideal solutions and thereby the utopian objective vector.

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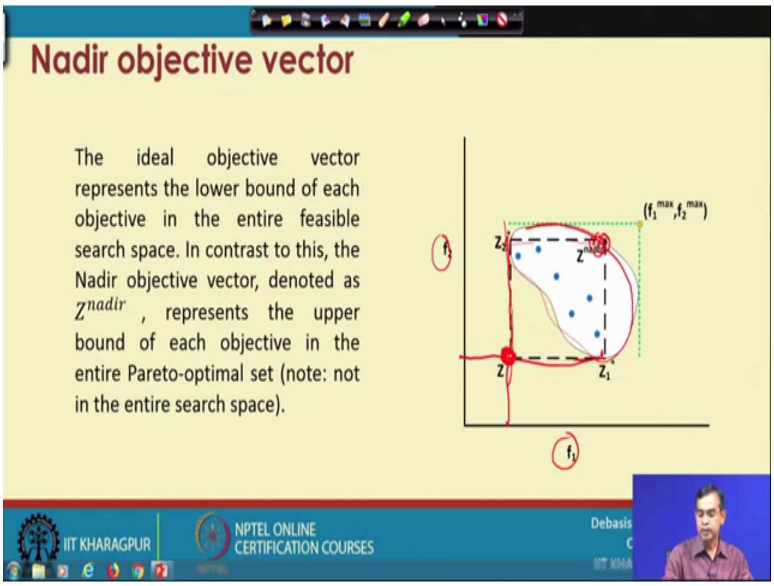

Now, as we have learned about it like say like ideal solution is non existing solution for conflicting objectives, similarly the utopian solution is also a non existing solution when the objectives are conflicting of type. So, this the two solutions types, one ideal solution and utopian solutions. Now, we learn about another solution it is called the Nadir solution.

Now, let us see what exactly the Nadir solution is ok, we can explain this concept by means of a example of an example. Say this is the solution search space all solutions possible. And both f 1 and f 2 are to be minimized for the sake of generality. And then so this is basically in our case it is the ideal solution. And then there is one z star solution which is basically minimum with respect to f 1, similarly there is another solution z 1 star which is minimum with respect to f 2.

Now, so for this z 2 star, we can see the f 1 is maximum ; and for the z 1 star we can see the for the z 1 star, f 1 is maximum value; and for the z 2 star also f 1 is maximum. So, this is the one solution if we say which is on the boundary or basically close to the solution then there is a extreme solution such an extreme solution is called the Nadir solution. Now, so, so nadir solution is like this. So, it is basically the extreme values with respect to the optimization objectives.

And then and there is another solution suppose this one and this is the another solution is this one these are the basically again another extreme solutions in the solution space. Now, extreme solution with respect to we say it is f 2 and this is an extreme solution with respect to f 1. And then there is another solution which is like this having this satisfaction then this solution is just like a utopian solution in case of ideal objective, it is basically the another utopian solution with respect to z nadir. Now, so this basically gives an idea about how the scope or range of the solutions maybe there. Now, so, this is the idea about the nadir objective vector.

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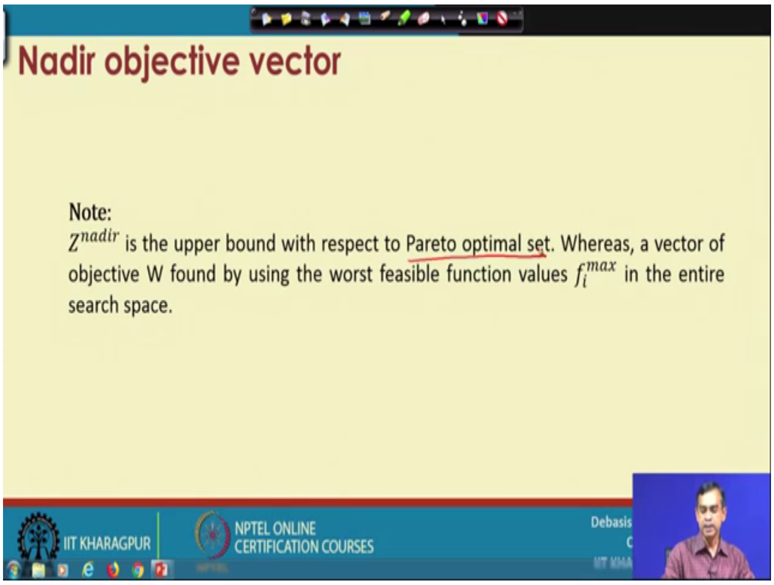

And, so, in a simple word, the nadir objective solutions is the upper bound with respect to the set of all optimum solutions. We will turn all the solution as the Pareto optimum solution. We will turn them as a Pareto optimum solution. So, it is basically the upper bound in that sense now. So, having this is a concept we will just see exactly what is the usefulness of these solutions ok. We have learned about the usefulness of ideal objective vector. Now, similarly the there is an application of the nadir objective solution for a given this one.

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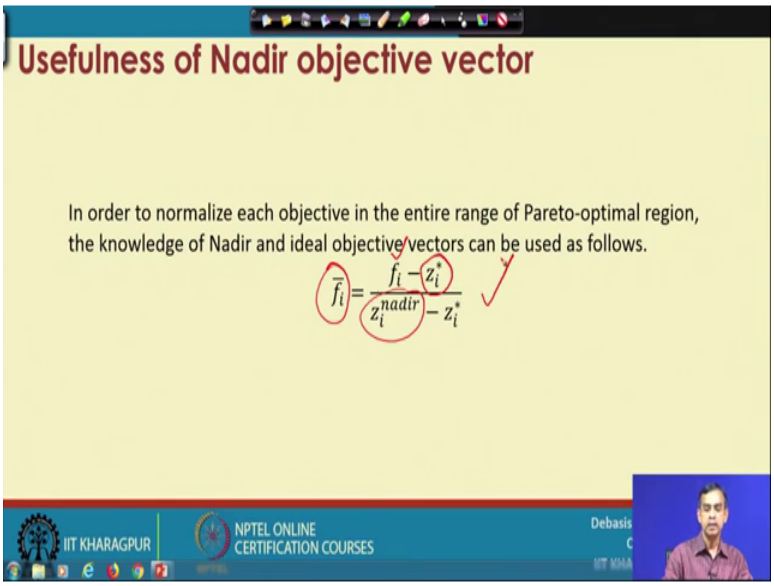

So that that can be explained by this form. So, this solution the nadir objective solution usually used to scale the objective vectors the scaling of objective vectors we have learnt while we are discussing about scaling in case of single objective genetic algorithm. Now, so this is the one idea here. If z i star is the is the ideal objective solution and f 1 is the any objective vector and the z i nadir is corresponding to the nadir objective solution then the scaled objective function can be denoted by this. And it is satisfied this formula. So, this is the one what is called the idea where the ideal objective solution and the nadir solution is used. So, it is basically used to scaling the objective vectors.

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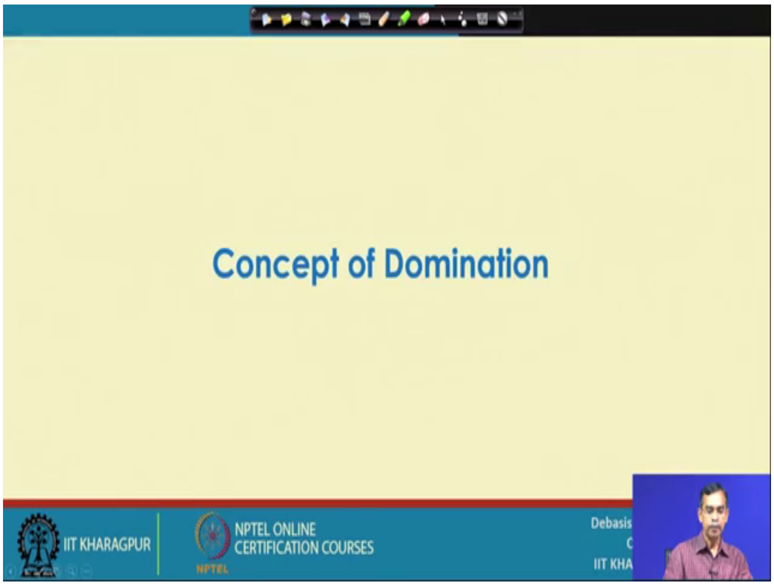

So, we have learned about the different solutions that may have there in with respect to two objective functions we have discussed, but that idea can be extended with respect to multiple objective factors as well as. Now, we will discuss about another important concept, it is called the concept of domination.

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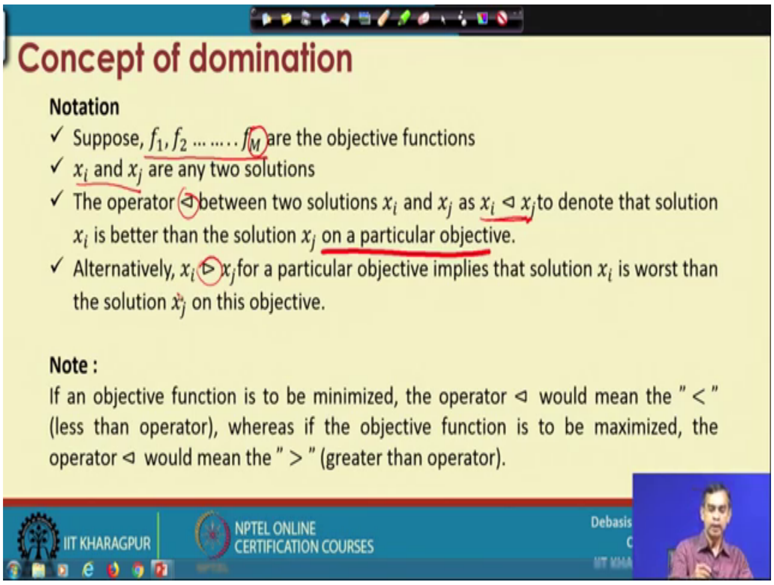

Now, concept of domination as I told you so there are some objective functions which are conflicting ok. Then let us suppose the M so there are M number of objective functions and there exist any two solutions say x i and x j are any two solutions in the search space. Now, we can define an symbol it is basically an operator between two solution x i and x j to denotes that x i x i dominates x j to denote that solution x i is better than the solution x j for a particular objective on a particular objective with respect to one particular objective. Similarly, if x i dominates x j or x j dominates x i this kind of symbol can be used.

Now, here our objective or the discussion that we are going to discuss about that if two solutions are given to you, then how you can decide that the one solution dominates other solution or no dominate each other. So, dominate in the sense that one solution is better than the other solution. So, this concept is called the concept of domination.

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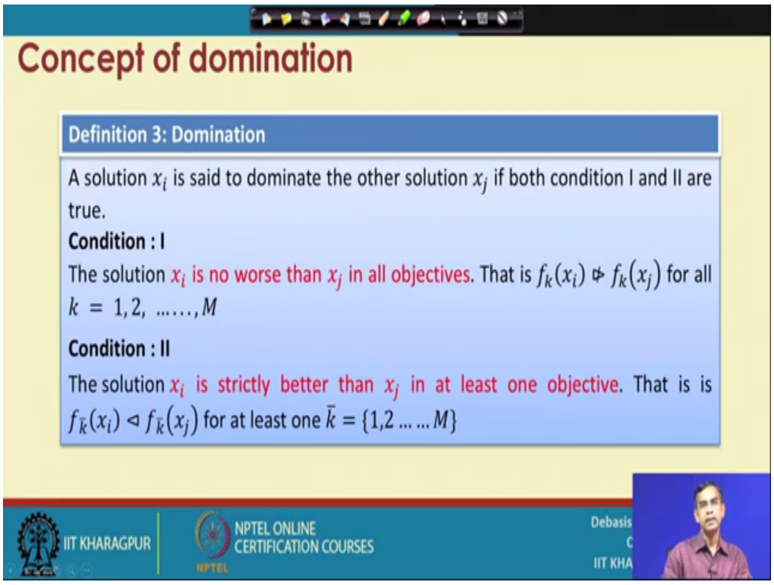

And it basically so there is a precise definition for the domination. Supposed two solution x i and x are there, then we can say that that solution x i dominates another solution x j if they satisfy two conditions. So, both the conditions are to be satisfied, then only we can say that x i dominates the solution x j. In other words in a simple words x i is a better solution then x a.

Now, so, the condition first. The first condition is that x i is no worse than x j in all objective that mean there are objective function for all objective function k, where k equals to 1 to capital M and then that solution does not dominates any other solution. So, it is basically the first condition ; that means, we will illustrate an example anyway. So, this is very important one statement that x i is no worse no worse then x j in all objectives in all objectives. So, these are the things to be noted.

And the second condition is that x i is strictly better than ; that means, it is better than always in at least one objective. So, it is basically again this point to be noted is strictly better than and at least one objectives. So, if these two conditions are satisfied between x i and x j then we can say that x i is x i dominates the solution x j.

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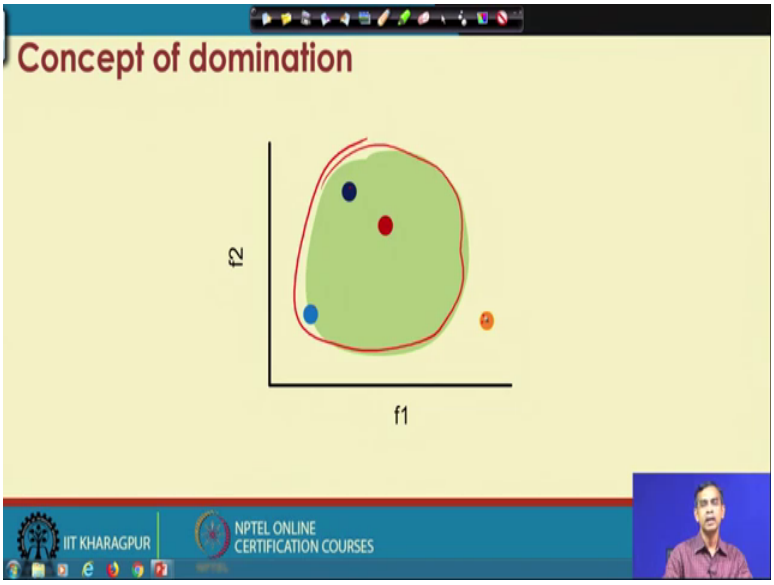

Now, let us say let us see the example so that you can understand about it. Now, in this example f 1 and f 2 are the two objectives. And so these are the solution space. This is also another solution which is not in the search space or whatever it is there. Now, out of this solution this and these we have to check that which solution dominates other solutions. Then definitely our objective is to select those solution which dominates all other solution if there exist one solution which dominates all other solution that will be the our desirable solution. Now, let us see what are the different situation may occur, so that we can understand which solution dominates other or how the dominant property is can be this one can be satisfied.

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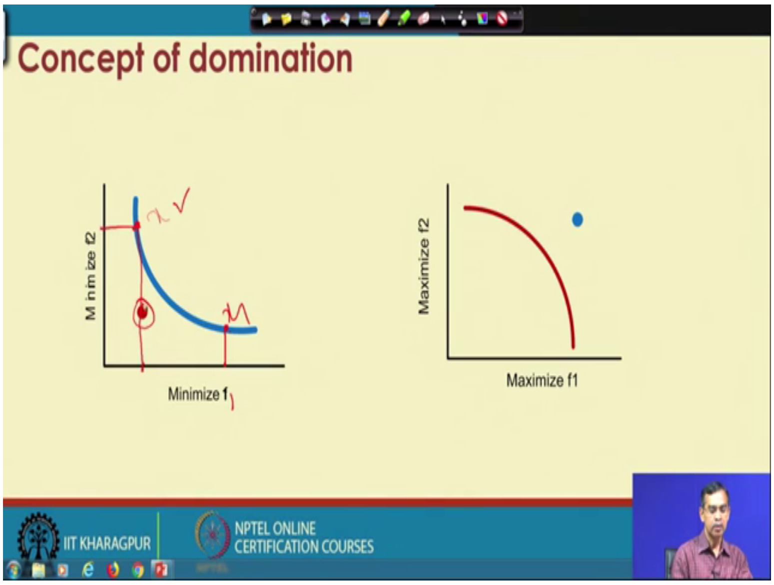

Now, this is the diagram that I have planted it, so that you can understand it. And I am discussing about with respect to the different what is called the different objectives type. For example, here this is minimize f 1, this is minimize f 2, this maximize f 2, and like this.

Now, anyway so if we see in this slides we can understand that this is the one solution, and this is the one solution right. Now, this solution and this solution if we see so this solution so for the f 1 is concerned right f 1 is concerned it is worst or it is not good as the this solution f 2 is concerned. Because this solution f 2 has the better with respect to f 1. On the other hand, this solution is also good with respect to f 2, but not with respect to f 1. However, this is the one solution, which is good with respect to both the solution as well as this solution.

So, we can say that this is the one solution which dominates this solution. On the other hand, this solution or this solution, so if this is x 1 and this is x 2 then we can say that neither x 1 dominate x 2 or x 2 dominates x 1. Because the condition one and condition two are not satisfied for the solution x 1 and x 2. However, condition one and condition two both the conditions are satisfied for this solution. Now, so, in this context what you can say is that this solution dominates these are the solution whereas x 1 and x 2 does not dominate each other.

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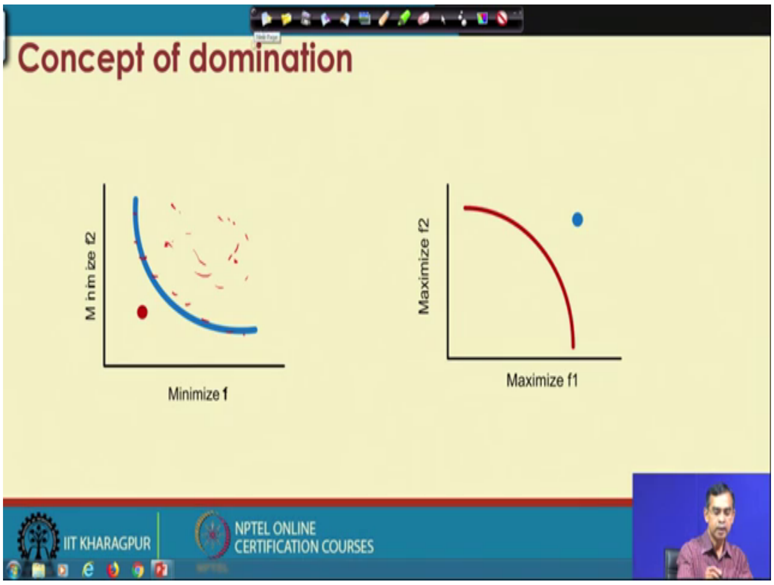

Now, similarly all the solution which lies on this region they are basically non dominating, non dominating no one dominates other. But any other solution or any other solution here also we can say this solutions is better than this solution in some respect. So, we can say that all solutions dominates all other solutions here. So, this concept can be can be understood from if we verify the two properties.

Similarly, in this example the same as this one, but is a maximize f 1 and f 2 we can extend the same idea for this two solution. And then we can check that these are the solutions which do not dominates one solution. And this is the one solution which basically it is dominates any more solution in this one. So, this is basically a dominating solution and these are the non dominating solution fine.

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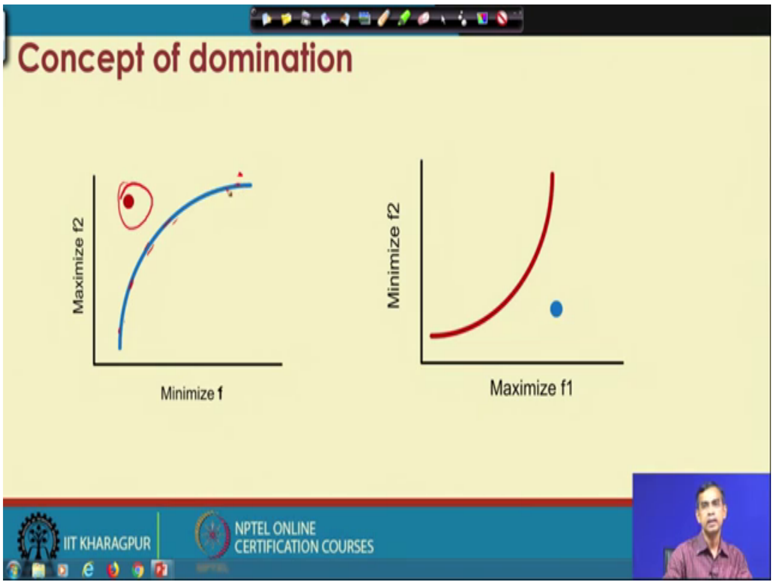

So, the same thing can be extended for the other minimize and this maximize, and this is maximize and this minimize. And these are the solution is basically these are the solution we can these are the solution is basically dominates any other solutions here, but all the solutions is lies on this region are non dominating, the same thing it is there. Now, so, this is the concept of domination if you want to understand more precisely about this concept, then I can plan one example.

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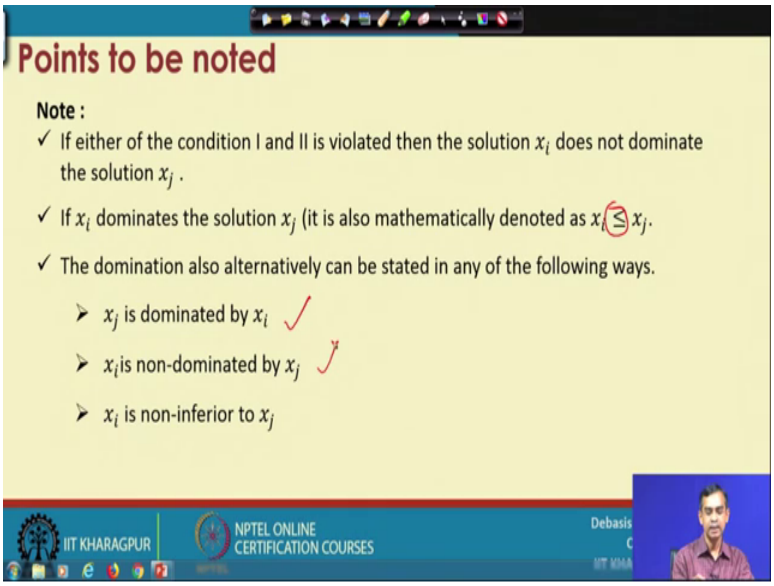

Ah ok. So, let us illustrate the concept of domination because it is very important and then it is we have to we will explain the three cases, so that you can understand the concept it is there. So, case one, and suppose f 1 f 2 and f 3 are the three solutions are the three objective vectors. And there are two solution one and solution two.

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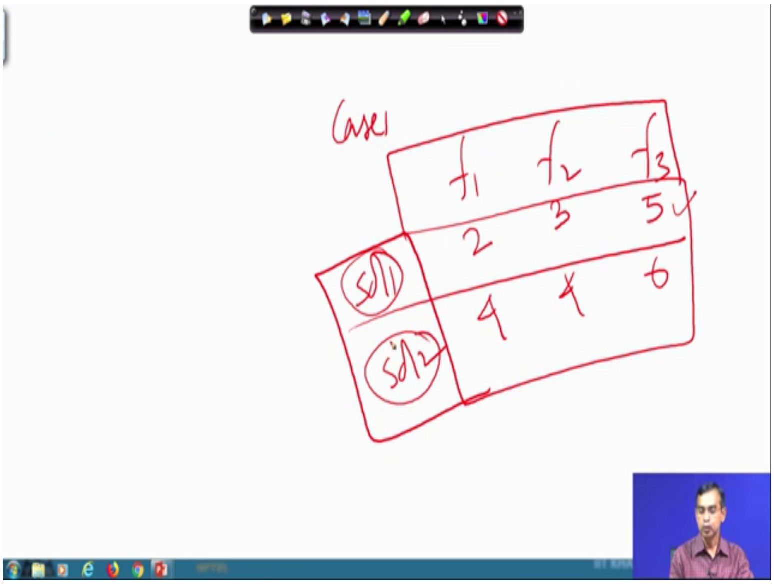

Now, say solution one is suppose both are to all these objective vectors are to be minimized. And say solution one has the results say 2, 3 and 4; and then solution two has the result 4, 4, 6. Now, in this case ok, so f 1, f 2 and f 3 are all to be minimized. Now, if we consider so then this solution one and solution two, if we compare then solution one condition. So, condition one so far this is a it is no worse than any other objective function.

So, the condition one is satisfied for the solution one and then also with respect to at least one this solution has the satisfied right so that means, in this case both 4, 4, 6, and here 2, 3, 5 with respect to all this solutions satisfy this one. Then we can say that this solution one dominates the solution two.

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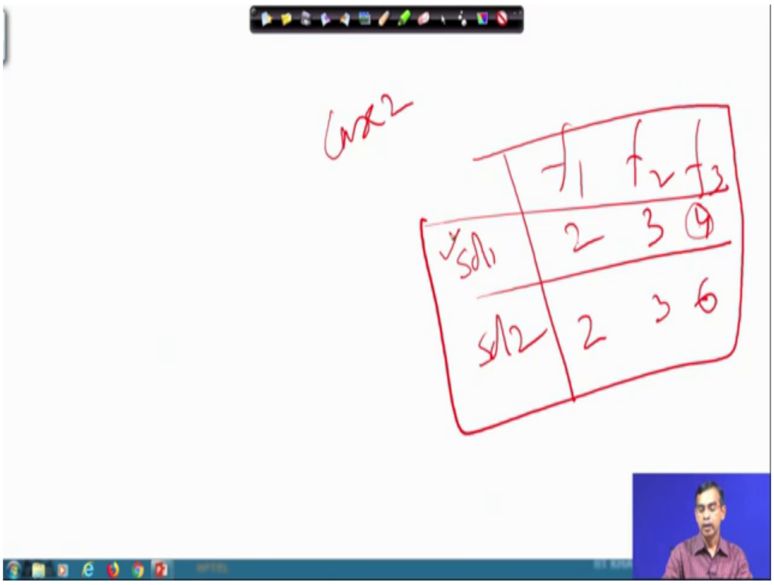

Now, another example so it this case two same thing earlier so, again f 1 and f 2 and f 3 are the three objectives or which are to be minimized. And there are two solutions solution one. And suppose solutions one is 2, 3 4; and solution two is 2, 3 and 6. Now, in this case, so solution one is no worse than any solution there in solution two. And it solution two and three, and solution for f 1 with respect to f 1 and f 2 both the solutions are same, but at least with respect to f 3 this solution is better than this solution. Then we can say again here solution one and solution two solution one dominates solution two.

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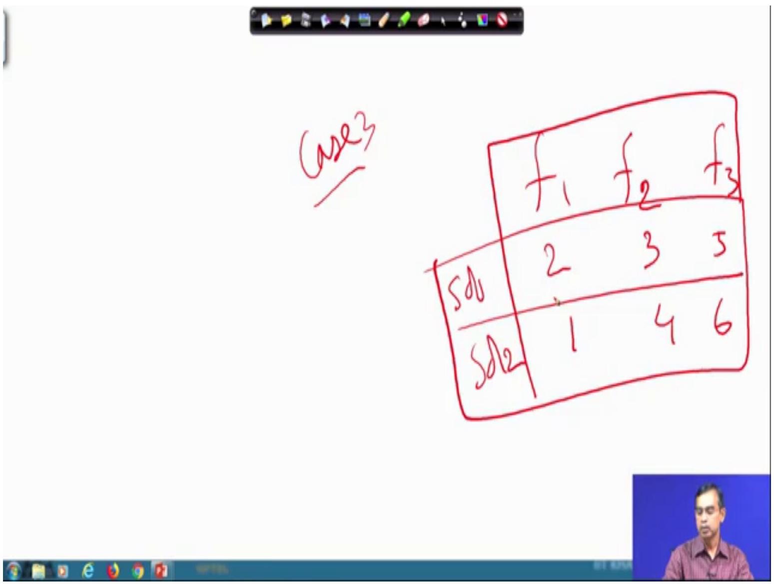

Now, let us see another case three. Again we will consider f 1, and f 2 and f 3 both are to be minimized. And there are two solution one and solution two. And let the solution 1, 2 3, 5, and 1, 4, 6 this kind of this kind of solutions are there now here we can see ah. So, solution two is better with respect to a f 1. However, it is no it is worse with respect to both f 2 and f 3. So, this means that neither solution one and solution two dominates that means, solution one neither dominates solution two or solution two does not dominate solution one. So, this is the example that can be helpful to understand the concept of dominance there.

Now, so we have learned about the dominant the concept of domination. Sometimes many language that can be used to represent or notation also. So, here if x i dominates the solution x j, then we can write this kind of symbol also used for (Refer Time: 29:18). And also it can be termed as x i is dominated x j is dominated by x 1 or x i is non-dominated by x j, or x i is non-inferior to x j this kind of solutions are there any way. So, meaning whatever the way we can express it the concept is like this one.

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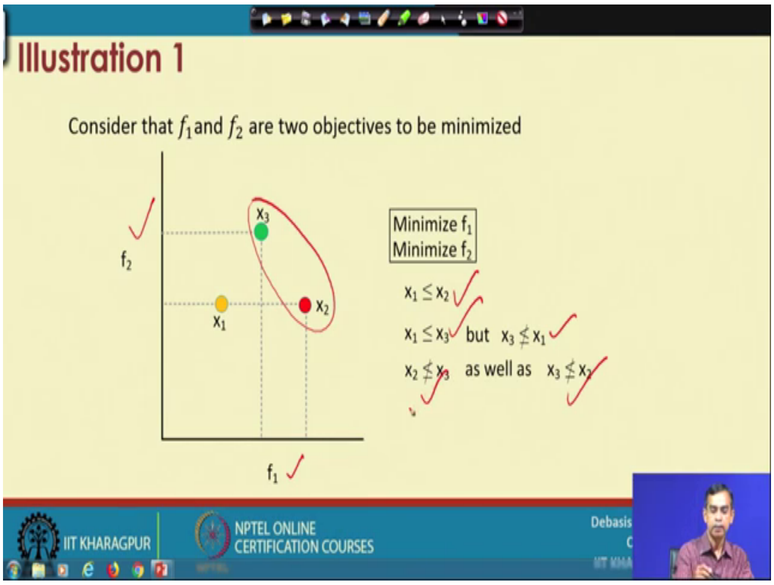

Now, we can illustrate the same concept two more slides are there as you have discussed already, you can verify yourself that. Here x 1 and x 2 x 1 dominates x 2. Bow, similarly x 1 and x 2 if we consider then x 1 dominates x 3 but if we consider x 2 and x three then neither dominates this one. So, here we consider f 1 are to be minimized and f to are to be minimised that you can verify yourself. So, this is the thing that we have written here and these are the things you can verify yourself.

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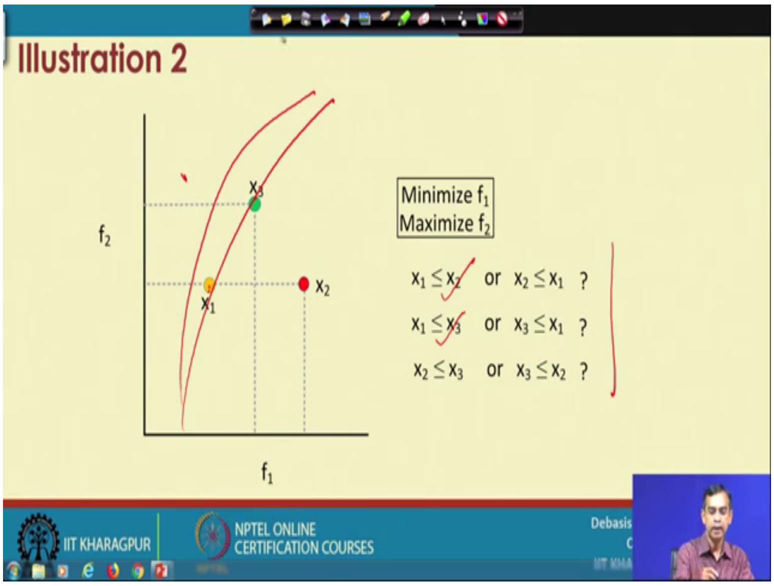

Now, second illustration here adjust this is minimize f 1 and maximize f 2. So, here also the ok, so it is basically minimize and maximize that means, the objective that solution the solution it is basically it is like this or whatever it is there. So, if we consider x 1 and x 2 because it is a minimize and maximize the solution region will be look like this one. So, all the solution which is on this region they are basically non-dominating each other.

However, any solution if you consider this one it is preferable and optimise this one, so this way we can verify that x 1 and x 2 x 1 dominates x 2 in this case, and then x 1 also dominate x 3 right and. So, so, but x 1 and if you consider x 2 and x 3, they are not dominating each other so that you can verify and then understand about it.

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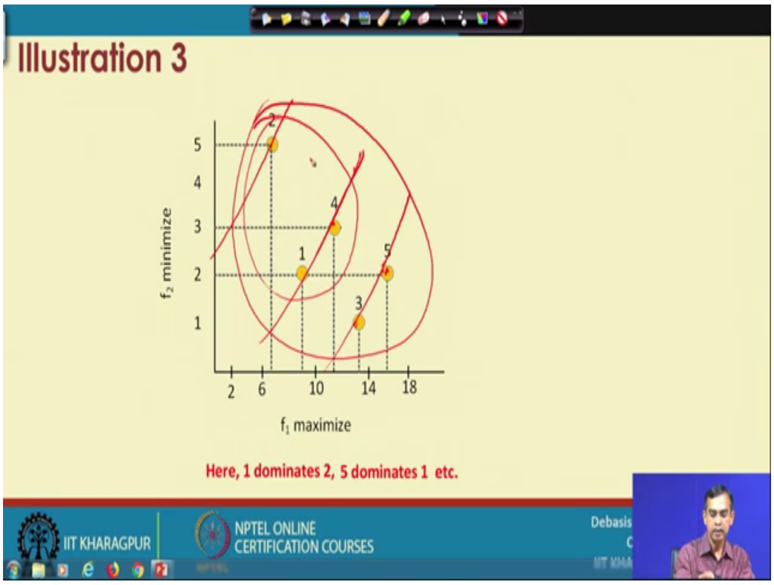

And similarly this is a another case where the f 1 is maximize, and f 2 is minimize. So, the idea it is that it is like this. So, you can say that is there. So, any solution not dominates this one, but here if we consider this solution all right or these are the solutions which dominates all other solutions. So, it is like this. So, all solutions dominates any solution which is on this range on this line or so similarly.

So, we can say that if this is the search space then all the solutions are dominated by all the solutions, but here no solution dominated by this one and then the concept is like this anyway. So, it require little bit thorough checking and then you can understand that which solutions dominates which of the solutions ok.

So, this is the concept of domination. And then we will discuss about important properties this domination solution holds good that will be covered in the next lecture.

Thank you.