Introduction to Soft Computing
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Lecture – 24
Multi-objective optimization problem solving

We are discussing about solving optimization problem and we have discussed one specific problem and used to solve that specific problem the genetic algorithm that we have used. Now, the specific problem in the sense that we use we consider only one objective function to be optimised. So, one objective function subject to a number of constants when a number of design parameters are involved. So, this is a special case.

Now, we are going to discuss more general cases of solving optimisation problem where instead of only one objective function there will be two or more objective function. So, this particular problem is called multi objective optimization problem.

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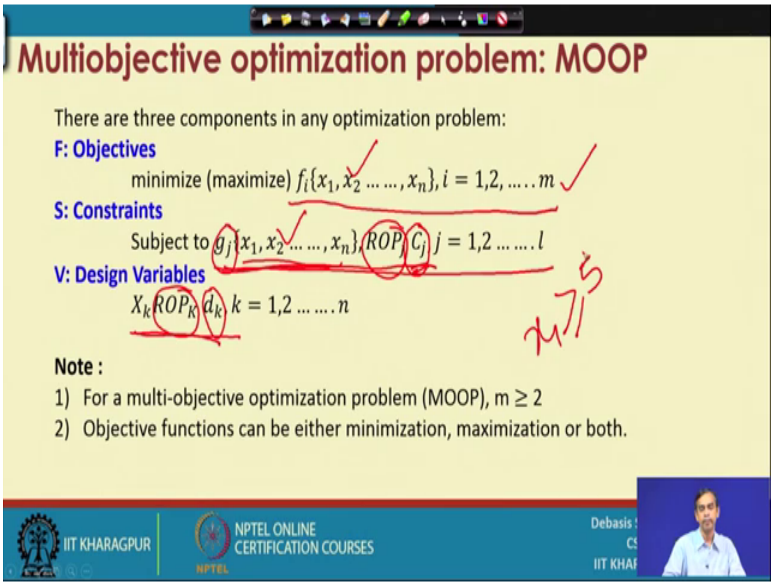

So, in a formal notation the multi objective optimization problem we will may abbreviate as a MOOP, the short form of multi objective optimization problem where we can define in a formal specification which is here. So, like the objective functions related to the single objective optimization we have understood that objectives, constants and design parameters.

So, here we can see the difference between these definition in the context of multi objective optimization problem here the objectives are a number of objectives in this expression we have disused about m objectives and out of this m objectives some objectives are to be minimized or some are to be maximized or all objectives are to be minimized or all objectives are to be maximized. And the remaining parts are the same as in case of single objective function. So, there are constants. So, here we have stated l number of constants are involved and those constants are expressed by the function say g j and you can say that all the constants are expressed in terms of all the design variables. So, here the design variables which are used to define an ith objective functions are the same design variables are used to define the jth constants and these are the design parameters design variables.

Now, here I want to see here. So, all these are the constants are related to some another constants are related to some constant and they are related by this operator, it is called the relational operator; that means, every constant has different relation operator may be say equals to less than or greater than some constant. So, these are the general expression in fact, right and then the design parameters also express in terms of some other relational operators and they are also related with some constant for the kth parameter for example, say if x 1 is a one parameter x is greater than equals to 5. So, this is the one constant.

So, these are the different, these are the statements by which we can express the multi objective optimization problem. And as we have mentioned it is there for the multi objective optimization problem this values of m should be at least 2 and the objective functions are there which are to be optimized either on the subject to the minimization problem or maximization problem or both.

Now, here I want to say one more thing is that in any objective function which is to be minimized can be equivalently converted to a maximization problem and vice versa.

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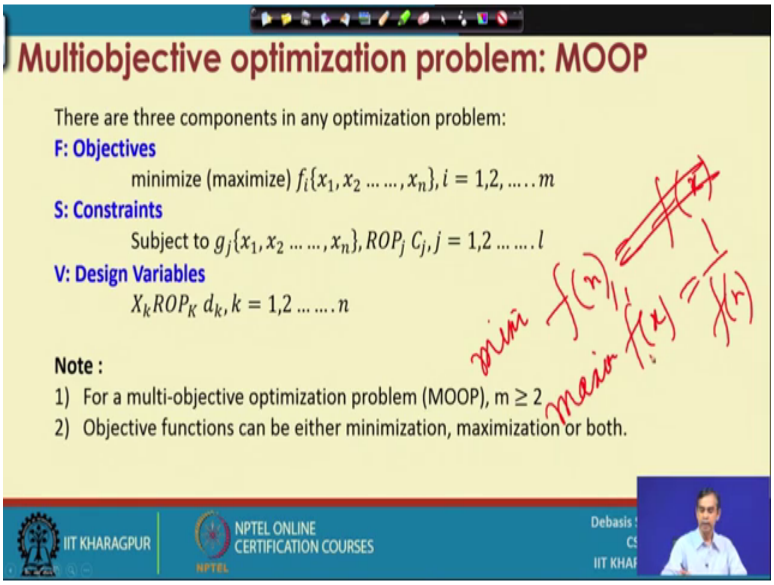

For example, f x this is equals to some function x if this is a problem for minimization then the same objective function, so it suppose minimize then the same function can be solved at maximize f x or we can say f dash x equals to 1 by; so, it is minimize f x when f dash x is basically 1 by f x. So, basically which is the reverse inverse of the function is basically maximization. This is another way, another way also we can write.

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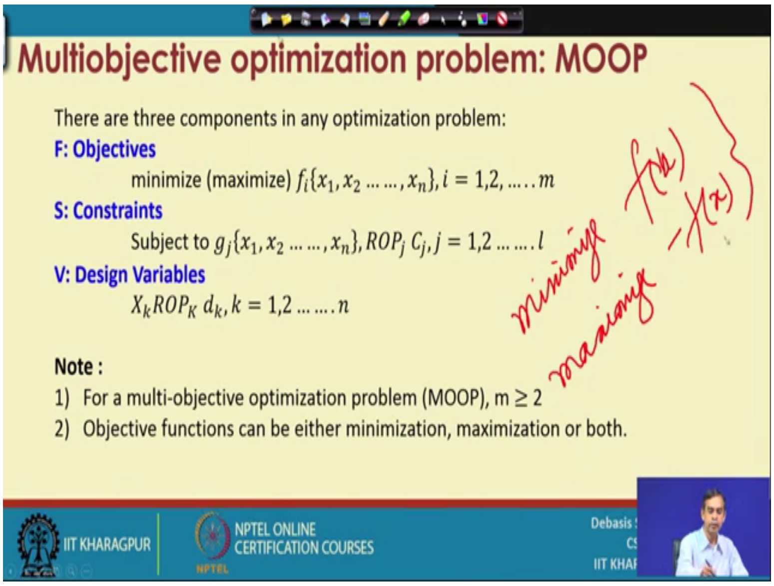

Say one function is minimized f x the equivalent maximization problem can be maximize minus f x. So, both are the same. So, these are the, I mean transformation from minimization to a maximization problem. What I want to say is that if there are some objective function are minimized or maximized then we can express uniformly either belongs to all functions are minimization problem or maximization problem. Now, the statement by which a minimization problem can be converted to maximization and vice versa is called duality problem. So, we can apply the duality problem to transform the objective function from the form of minimization to the form of maximization.

So this is about the problem that we MOOP statement, statement of MOOP problem and having this understanding, will discuss about what exactly the problem it is here, anyway.

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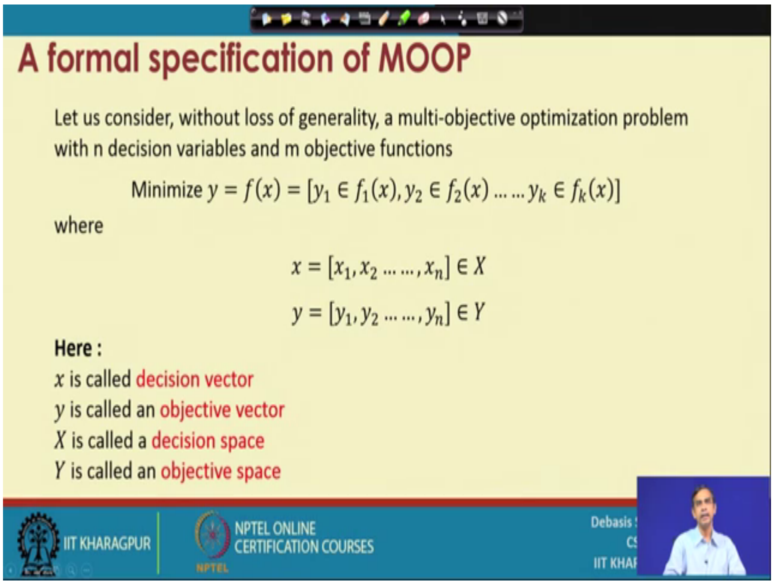

So, in a more mathematical expression we can tell about the MOOP problem where n number of design variables and n number of objective functions are involved and without any loss of generality we assume that all objective function are to be minimized. Then what we can say is that in fact, minimizing all the functions or all the objective function will give some results which is basically minimizing one another what is called the combined function. So, we say that f x is a combination of all the objectives that is there and then these are the output that this combined objective function will return. So, here is y is basically an output and all the design variables solution design parameters are the basically input to the problem.

Now, so we can decide about in the context of this expression. So, these are the x, x is basically set of design variables which is denoted here more specifically and all the values of x which are like this they belongs to some set it is called x. Similarly y it will return the output which objective function. So, all these objective values and we can term this objective functions are the results as an output and all the value of whatever it is this it will be in the domain it is called the y. So, x and y are the 2 domain we can say it is input domain and it is the output domain. So, for some values of the design variables it will produce these are the output and so, this is there.

Now, so here more precisely this x is called the decision vector and all the small y, y 1 and all this thing is called the part of the objective vector. So, this x is decision vector and y which is basically the results obtained from each function constitute a vector and is called the objective vector and this x is called the decision space whereas, y is called the objective space.

So, basically as I know that, so for the objective function solving and optimization problem is using genetic algorithm is basically a searching problem. Now if this is the statement of the problem multi objective problem, now, let us understand what basically to be searched here.

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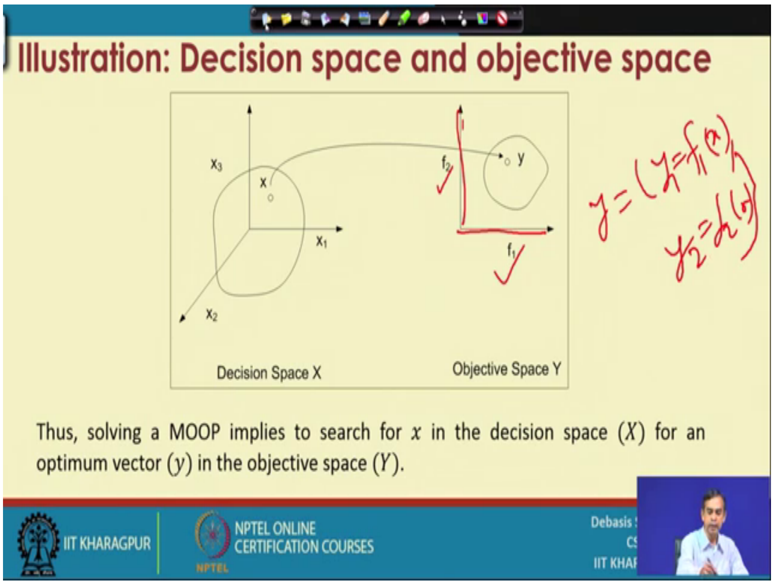

I will explain this things with the only 2 objective function so that I want to give more graphical impression about the ones concept. Here in this example suppose f 1 and f 2 are the 2 objective functions. So, this y is basically is a vector y 1 this is equals to f 1 x and then y 2 equals to f 2 x. So, we symbolically represent this concept y using this expression. So, it is basically idea is that for the different values of f 1 and f 2 the different solutions are possible and that is the solution is called this one. So, this is basically an objective vector. So, y is the objective vector here which has the 2 parts one f 1 part and f 2 part.

Now, again, for input space is concerned that is the decision space. So, this basically represent the decision domain and here we represent a decision vector in terms of 2, in terms of 3 parameters or values that are x 1, x 2 and x 3. So, the same thing can be depicted in the 3 dimensional space and, so for at any instant this point in the decision space represents the instance of a vector having. So, this is the x 1 value, this is the x 2 value and this is the x 3 value. So, it is like this.

So, this way we will be able to express. So, this is the decision space and this is the objective space. Now, for the searching of the solution searching for the solution of an objective problem is concerned is that there is a mapping for every decision vector to an objective vector. So, is mapping is there. So, if we consider another decision vector and then there will be another objective function it is like this. So, there is a mapping from there is a mapping from this phase to this phase and out of this mapping we have to select the best mapping best mapping in the sense that the objective this objective vector will give the optimum value.

So, this problem essentially is basically mapping from decision space to an objective space and that is a searching the procedure or the searching policy should be to find the best map.

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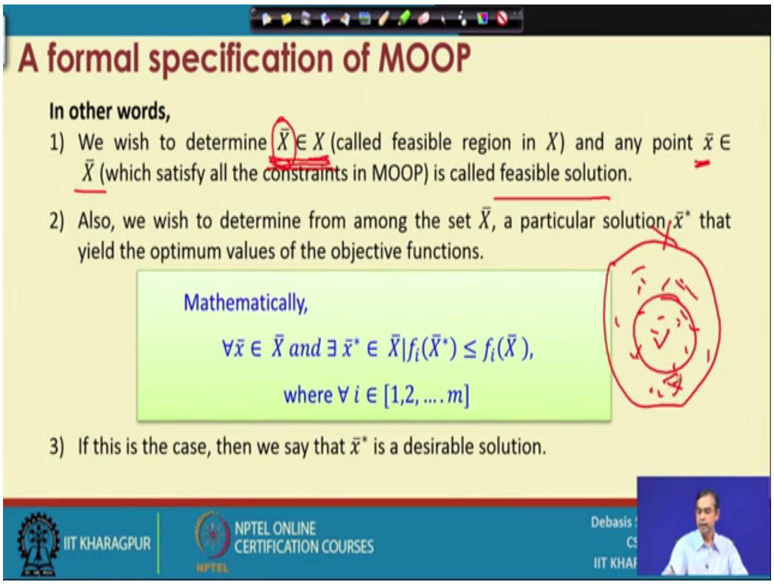

Now, more mathematical again we can define it so that we can discuss the next discussion more in an understanding manner. So, basically the idea is that the X the decision space as we have mentioned earlier is basically the solution region and out of the solution region there is a subset we say that it is X bar. So, X is basically the X is basically the solution space where are X bar is a subset of region because all solutions may not be the feasible solution. So, we have to consider the solution X bar. So, X bar is, X bar is belongs to X. Now, if we consider this X bar and any point say this one X bar into this feasible solution region which basically satisfies all the constant that is there in the MOOP specification then is called a feasible solution.

So, X constitute of many solutions. So, it is basically is the in the domain of decision space all these are the solutions all the possible values, but we have to consider only few things which basically and if this is the X and this is X bar then this basically called the feasible region and the feasible solution. So, it is the idea is that we have to find all the values which are feasible solution first. And out of these feasible solutions we have to select a particular solution and this particular solution we denote as X bar star. So, X bar star satisfy some constant which is mentioned here. So, again we can explain it like this for all X bar in X bar; that means, for all feasible solution in the region of feasible solution and there exist a particular solution which we denoted at star X bar star belongs to this X bar such that such that f i X i bar should be less than or equals to f i X bar if we consider the minimization case, if we consider maximization case then this will be greater than or equals to this one.

That means for the values of these X bar and for the values of X bar, X star bar. So, this objective functions should be always less than any values in the solution space. So, if the solution space contains the m number of solutions. So, here X star bar is called an optimum solution or it is basically a desirable solution. So, mapping therefore, comes into this picture. So, from the set of all design parameters values we basically obtain the solution space, from the solution space we have to decide the feasible solution and out of all the feasible solution we have to select one solution which is called the optimum solution or desirable solution; that means, we have to search the design space for which values X will be a feasible solution and from all the feasible solution we have to search a value which will give us a desirable solution. So, this is the concept that is there in case of multi objective optimization problem solving.

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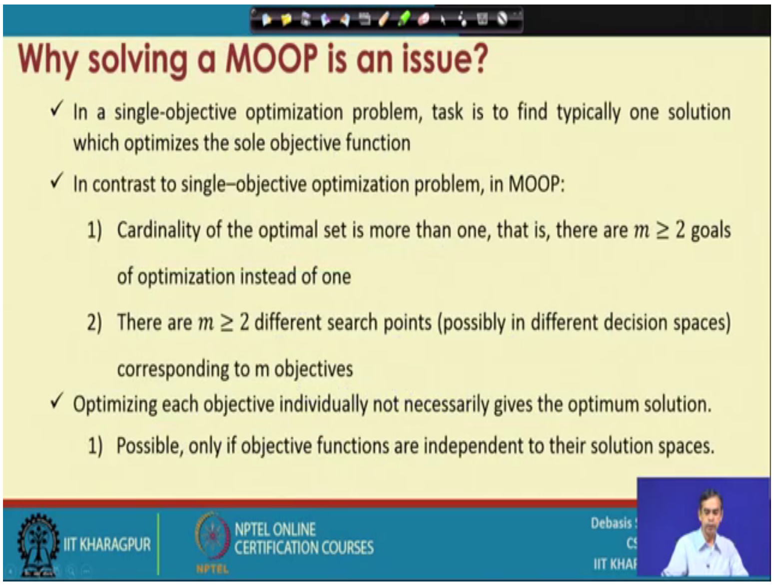

Now, will discuss about few things are involved now the first question that is here. So, if we know the single objective optimization problem solving using genetic algorithm then why we should consider the different what is called the procedure or different techniques or different principles to solve the MOOP problem.

Now, there are definitely many differences between the single objective optimisation problem and multi objective optimisation problem. So, here in this slides I have listed few differences. Now, the first in a single objective optimisation problem that we have learned so far, there the task is to find typically one solution which optimize the only one objective function. And in contrast in case of MOOP problem our objective is to not only a single objective function rather 2 or more objective function to be optimized. And when there are 2 or more objective functions this means that it least to 2 or more search points in contrast to the single objective function where only one search point. So, they are because the different search point related to the different objective function. So, these are the basically constitute and objective vectors or output vectors we can say and then we have to select out of many search points we have to select one search point.

And then optimizing each objective individually it is no issue; however, optimizing the entire what is called the considering all the objective function putting together and then finding a global optimisation is in fact, a non trivial task. Now, we can explain how this become a non trivial task that can, that can be explain with an example.

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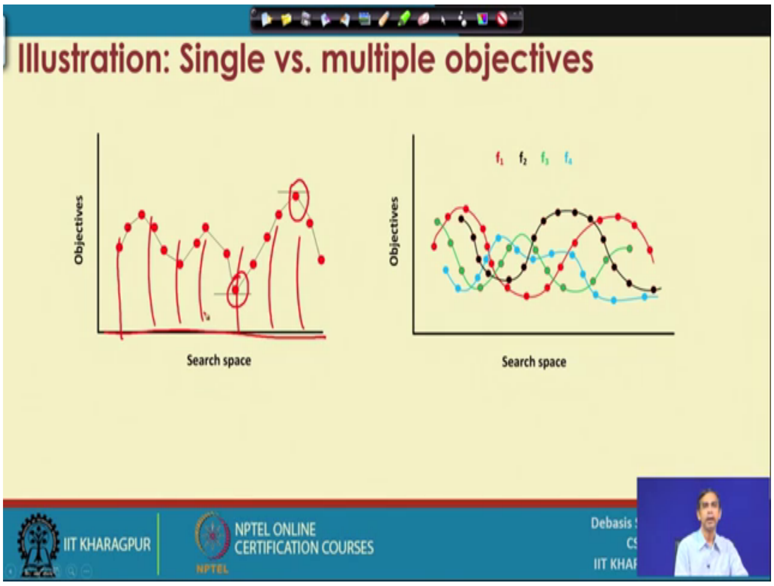

Here if we see the 2 graphs in the left side of the graphs we show the search space for a single objective function and it is as usual, so it is basically you if you have to find a minimum solution then it is the point that can be searched to find it or if it is a maximization problem then this point can be search to find it.

So, the single objective function is concerned is only one objective function this is the search space and for each search space we can find all the solutions and they ultimately find either maximum global maximum or global minimum value. So, this is the simple problem and we have learned about how GA can be applied to search the entire search space.

Now, on the other hand in case of multiple objective function in the right part of the graph if we see this graphic little bit carefully we see that we have plotted the objective function value with respect to the search space. And the different curves, with different colours basically represents a different function like f 1, f 2, f 3 and f 4, and if we see it then it is easily visible it is easily understandable that the different objective function has their different points for which the minimum and maximum occurs. For example, in case of function f 1 if we see the minimum, this is the minimum value. On the other hand if we consider the function f 2 then the minimum value is here. So, this point if this one is the solution space than this one is the solution space there.

Now, again for f 3 if we consider this minimum value maybe it is here. So, solutions space the and similarly for f 4 we can find the solution space here, so this one. Now, what we have conclude from there is that for the different objective function if it is taken into consideration then their solution space will be anywhere in this region; that means, in case of single objective function if only one search point is there, but here the number of search points are there. All search points are related to search point is related to, the optimum value with respect to a particular objective function.

Now, here the question is that out of these search points which point should be taken in order to objective I mean consider the multiple objective function optimisation. So, definitely it is neither this solution or this solution or this solution we have to select out of these one only one solutions in fact. Now, which solution needs to be selected and how these solutions can be explored this is the issue. One issue is that single objective is very fast to do it, but as this number of objective function is it is obviously, the multiple cause to be involved in order to sub optimize this one.

So, the additional genetic algorithm that we have learned is not sufficient to solve this objective function in fact so we have to study a totally different concept to solve the MOOP problems, multiple objective optimisation problems.

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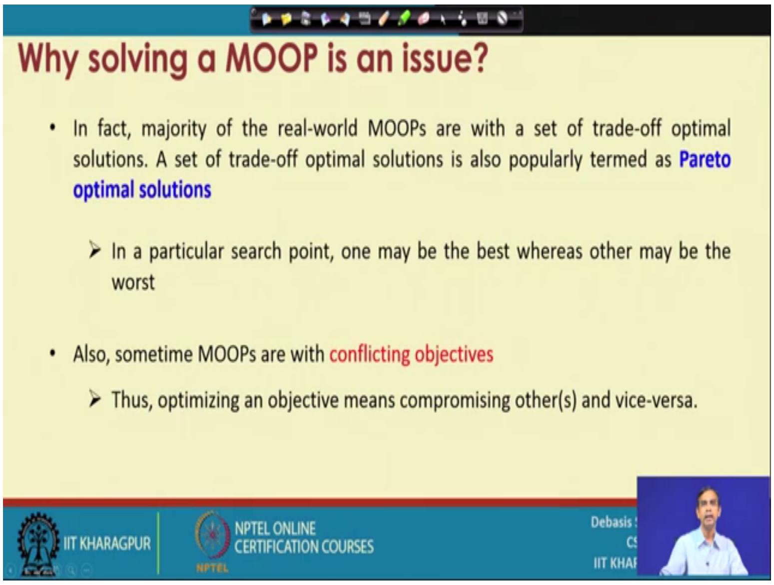

And this is because there are some objectives which are conflicting in nature because if we select this one solution then other with respect to other objective function this may not be solution. I can give an example to understand this concept more clearly.

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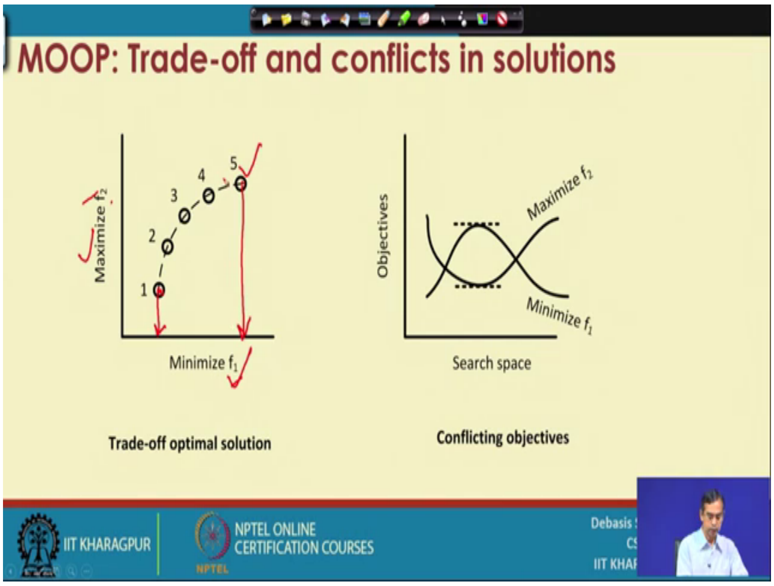

So, here one example I can see. So, these are the again with respect to 2 objective function f 1 and f 2 and how the different solutions are there in order to solve the values it is there. So, there is a 1 solution, 2, 3, 4, 5 these are the solutions.

Now, we can see if we consider this is the solution 1 and it is a minimization problem then definitely this solution is preferable both with respect to f 1 and f 2. Now, so this is a maximization then means that this solution the solution one is preferable with respect to f 1; however, this solution is not preferable with respect to f 2. On the other hand if we consider this solution this is preferable with respect to f 2 because it gives a maximum value for f 2; however, it gives the worst value for f 1; that means so it has one what is called the point is good, but other point is bad similarly this one, and here 2 3 4 either neither f 1 is good nor f 2 is good.

So, this is the concept which called the conflicting objective function. So, objective functions are conflicting f 1 conflicts with f 2 and vice versa. So, the same things which is mentioned here it is shown here. So, these are the fine; if we consider this is the search space then with respect to the search space this is satisfiable, but this is not acceptable. Now, it is like this. So, usually in case of multi objective optimisation problem solving the objectives are conflicting in nature and therefore, finding a unique solution out of these different values of the objective function is a tedious job.

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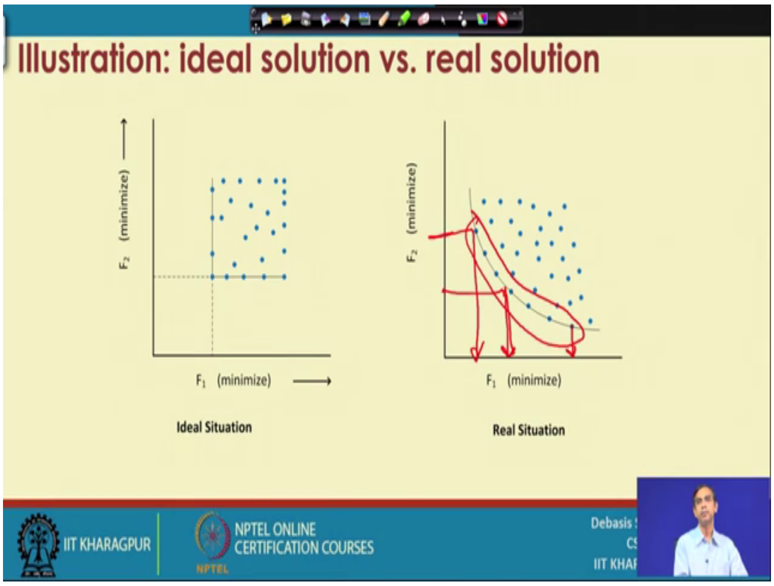

Now, another example, usually there may be some situations if we are very much fortunate enough then we can find a unique solution to solve where both objective functions are satisfying without any conflict.

For example, here, suppose F 1, F 2 both are to be minimized. Now, if it is a minimization problem with respect to all the objectives and these are the solutions suppose. So, these are the solutions and out of which so many solutions we can see one particular solution which is comparable to any other solution in the solution space. Now, this solution satisfies or it is basically desirable solution because it minimises both F 1 and as well as F 2. So, in this case this is a solution which is non conflicting. Now, this is a solution is called ideal solution and the situation where we can get this kind of scenario is called the ideal situation.

So, usually ideal situation is very very far from the real solution. In fact, it is observed that most of the objective functions in case of multiple objective problem solving the solution is like this kind of things are there. So, here basically these are the solution region solution space and out of the solution space we can find some solution it is this kind of solution actually which is basically neither superior to anyone. For example, if we consider this one and then minimization if we both F 1 and F 2 then it is good with respect to F 1, but not both F 2. Similarly, it is not so good with; it may be good with respect to F 1, but not good. Similarly this is one is good with respect to F 2, but not with respect to F 1.

But so, what I want to say is that all the solution that is here in this line there the boundary you can see they are any one solutions which lies on this boundary are neither superior to any one such a solution in multiple objective optimization problem has the special importance. In fact, what is the special importance is that if this is the solutions we can find from the search space then we have to take or we have to select one solution from there for all solutions are acceptable although it is not ideal solution. So, such a solution particularly in the theory of MOOP optimization problem is called the pareto optimal solution means a pareto solution. Means all the solutions are we have to consider in order to decide your own solution. We will discuss about the concept of pareto solution and then pareto optimum solution not in this lecture in other lectures in due time.

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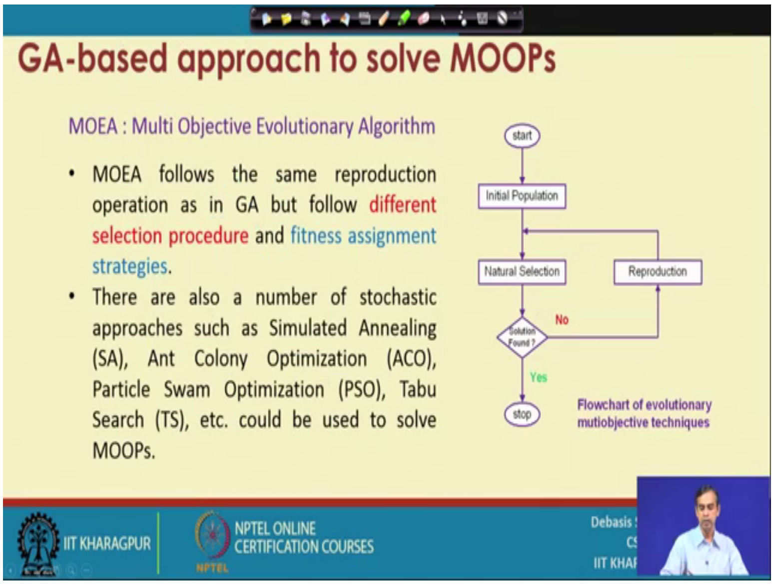

Now, let us see if we want to solve the genetic algorithm basically our objective is basically to apply genetic algorithm to solve multiple objective function, multiple objective optimisation problem. So, idea or the frame work that is used is basically same as the frame work we have considered to solve single objective function that is a one important point and is a, I mean good point to learn it so that the same framework can be applied. However, in the framework we have to do little bit different tactics or the different techniques to be followed, anyway.

So, the idea the basic task that is there in case of genetic algorithm is also followed here namely initial population creation and then selection, and then reproduction, and this is the loop between the selection and then reproduction producing the next generation and so on so on, but the different is there. Different is that whatever the method that we have discussed for the selection that is there in case of single objective optimization is not allowed or not applicable. Mainly the selection operations are to be fine tuned, so for the solving multiple objective optimization problem is concerned using genetic algorithm.

So, solving MOOP problem using genetic algorithm basically the idea is that how the different tac tics or procedure that can be used to using the selection procedure and therefore, the fitness assignment concept there because it is basically to assign the fitness so that we can lead to a better search there. So, these are the thing that we learn it. Now, I am telling you again, let me clear one more thing is that. So, GA is the one approach, parallel to GA there are many other approach in the line also to I mean help us to solve multiple objective optimization problems like simulated annealing, and colony optimisation problem, particle swam optimization problem, tabu search and like this. So, all this thing are the many many theory many techniques many concepts are there, but here will limit our discussion to the genetic algorithm based approach.

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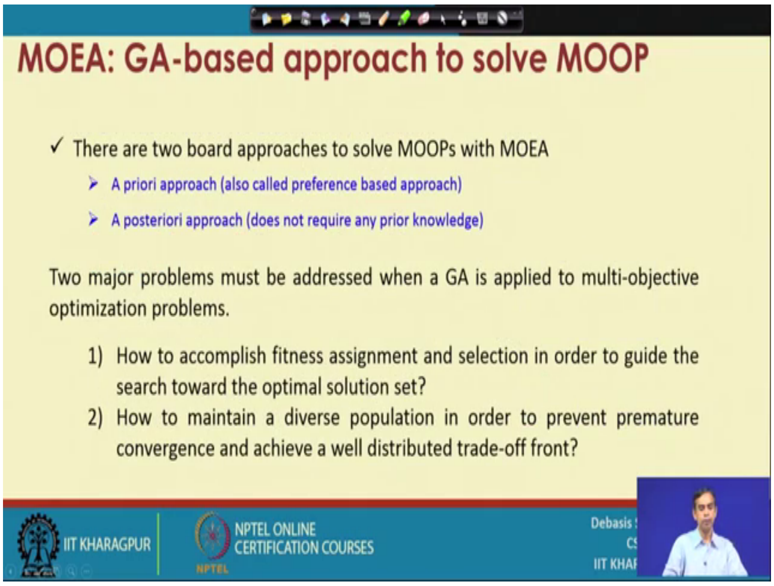

Now, so for the genetic algorithm based approach is concerned there are may be 2 broad techniques, one is called the a priori approach and another is called the posterior approach. Now, will discuss about the 2 techniques in brief and then what is the procedure that is followed there.

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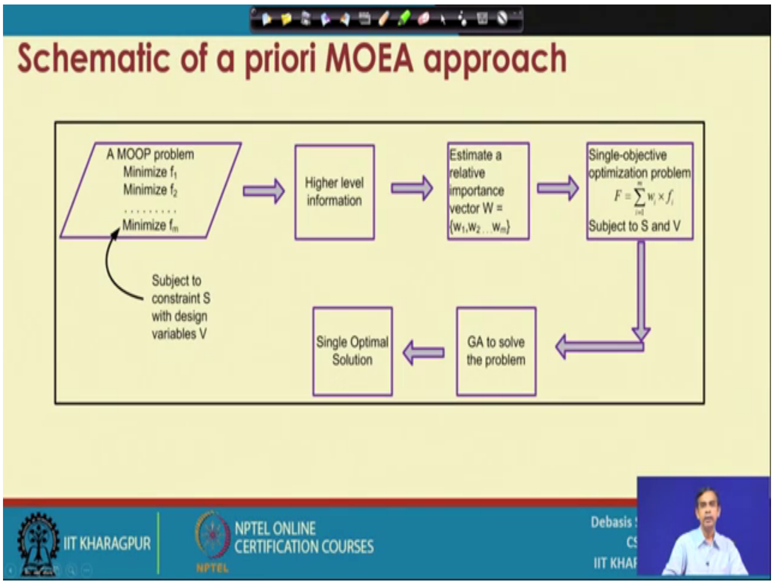

Now, here is basically the flow of the a priori approach, it is easy to understand, so this basically the problem your MOOP problem where m number of objectives are to be optimized. Now, in this approach it is called the a priori approach because it follow certain high level information. Now, what is exactly the high level information I will discuss this lecture within a one minutes or so. Now, we will understand about high level information.

Now, this information basically helps you to understand one weight vector it is there; that means, what is the weightage of the different objective function to be considered in order to decide one solution of our own from the set of solutions provided by each objective function. For example, if w 1 equals to w 2 equals to w m equals to 1, then all objective functions are equally important, but if w 1 is highly important then w m then I can give more weight age to w 1 then w m. So, these are the, this one. Now, in order to have the weight values we have to follow the high level information.

Then in terms of this weights we can express the multiple objective function as a single object function is basically summated weight form w 1 f 1 plus w 2 f 2 plus dot dot w m f m. So, this basically gives the m. So, here basically what is our tricks is that considering the multiple objective optimisation problem and transforming this into a single objective optimisation problem in terms of the weight vectors. Then if it is a single objective function then there is no issue we can apply our traditional genetic algorithm on this and then find the solution. So, this is the prior approach and then we will discuss about another approach it is called the posterior approach.

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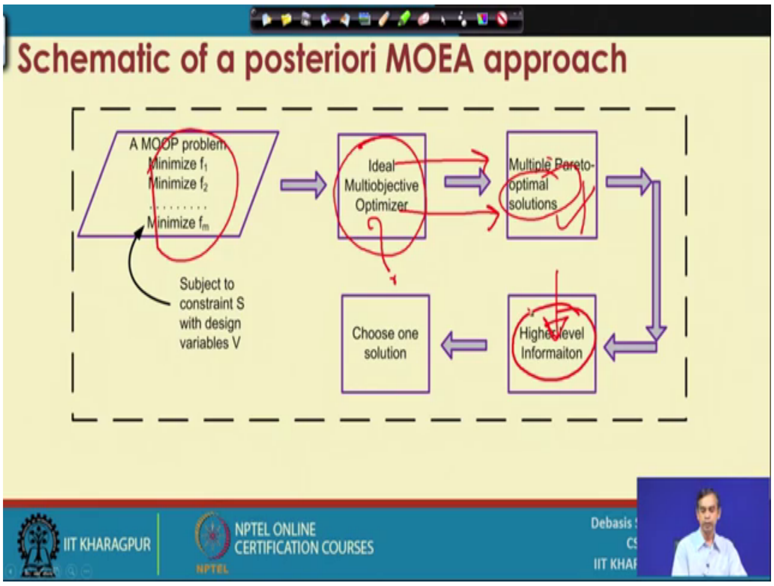

That means we need the information, but at a later stage it is as usual in case of objective function, this is the problem statement, this is basically ideal multi objective optimisation problem, this is a new thing we have to think about it. That means, it will give you the way how the multi objective optimisation problem can be solved.

Now, if we solve it right, for example, one simple idea about ideal multi objective you just solve one objective function at a time. So, this means that it will give you a number of what is called the solutions as we have learned about it. So, this, the solution is called the pareto solutions or pareto optimum solutions. Then we use this and pareto solutions and passes through one high level information because we have to see this will give you a large number of solutions and we have to select only one solution from there, and then the high level information can be used to select a particular solution that is a desirable solution or our required solution.

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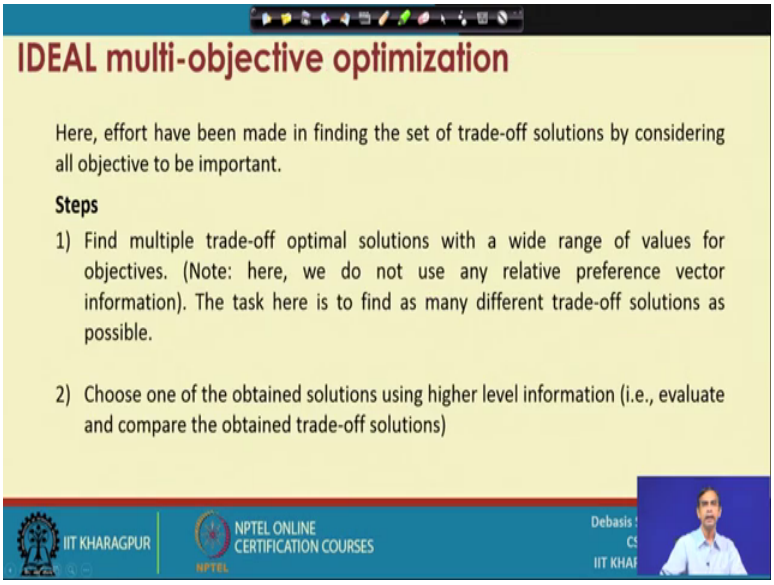

So, these are the 2 approach that is there and now I will discuss about idea about, so is basically high level information.

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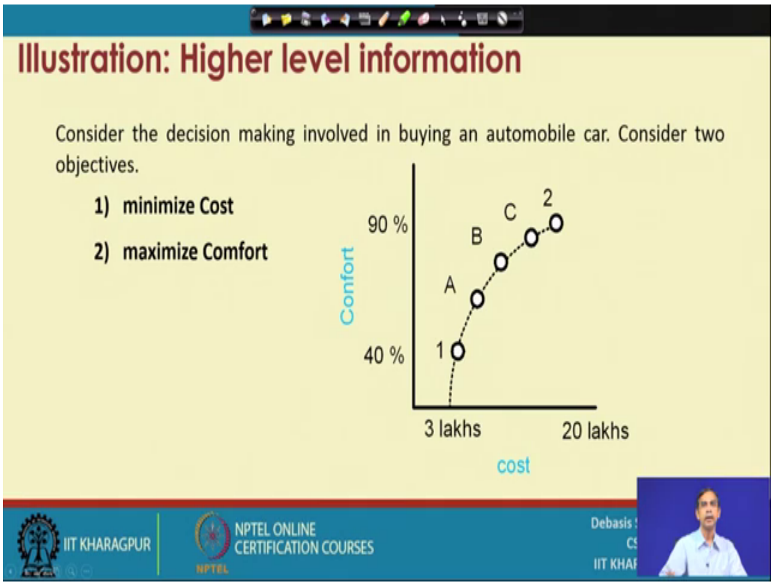

I can discuss the concept of high level information with an example. Say suppose you have to purchase a car and there are 2 objectives of course, the cost and then comfort. That means, you have to purchase a cars with a minimum what is called the cost involved and then maximum comfort possible from the car. Now, if you visit many cars those are there in the car markets then you can find a number of solutions there, for every car have their own cost as well own comfort.

So, here we have given few examples. So, 5 solutions suppose we have serve it and then these are the solution like this one out of which this car is good so for the cost is concerned, but not for the comfort, but this car is so for the comfort it is concerned it is preferable not for, so for that cost is concerned. So, then we have to find out of this the solution is there. Now, we can follow the high level information there.

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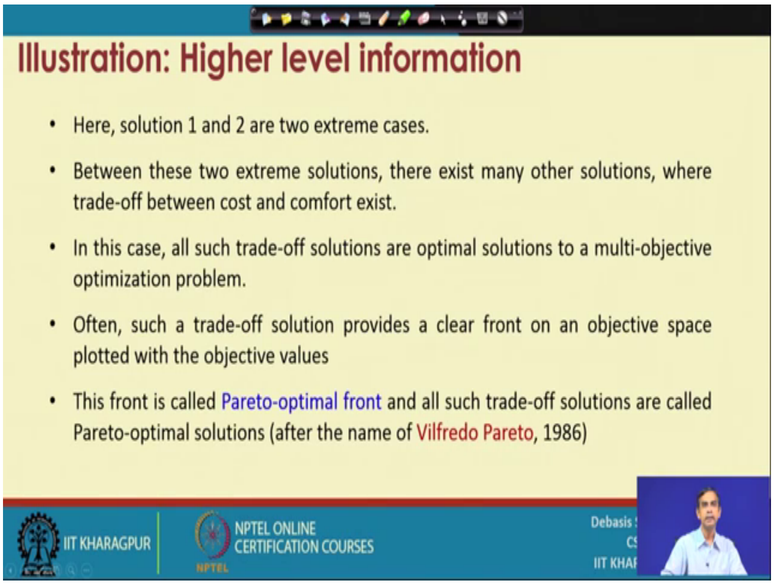

Now, I will discuss about what are high level solution are there.

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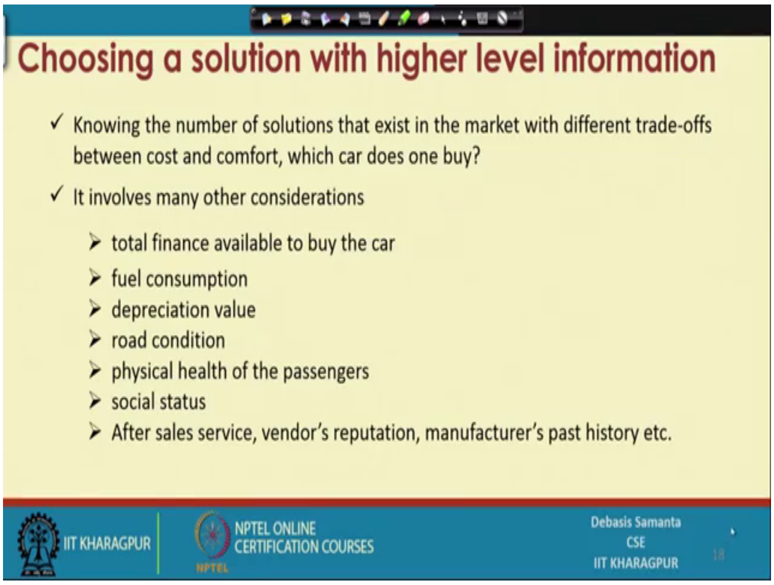

So, they are high level solution right in the context of this car purchase, what are the total finance that you can give to buy the car or what is the fuel consumption each car, depreciation value and in which load condition which car travels better and then physical health of the passengers if a particular car is used and then social status and all these things. So, these are the different high level information if you take into consideration above the solution that you have obtained then it will help you to decide the right solutions or desirable solution.

So, high level solution regarding this thing we learned a lot whenever we discuss more theory about it. So, this is the concept of multiple objective function that we have covered in today's class. In the next class we will discuss many theory and then some treatment and then how to solve this problems in a more pragmatic way, will discuss in the next class.

Thank you very much.