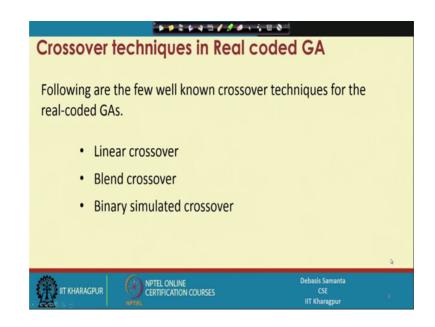
Introduction to Soft Computing Prof. Debasis Samanta Department of Computer Science & Engineering Indian Institute of Science, Kharagpur

Lecture – 21 GA Operator: Crossover (Contd.)

In this lecture, we shall try to learn the crossover technique; the crossover technique applicable to the real coded GA.

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Now, there are 3 broad techniques which are followed to perform the crossover operation in case of real coded GAs. The 3 techniques are linear crossover, blend crossover, binary simulated crossover. So, these cross over techniques are based on the different policies in fact, so we learn the different policies that is here. Let us, first learn about linear cross over technique for the real coated GA.

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Linear crossover in Real-coded GAs	
• This scheme uses some linear functions of the parent chromosomes to produce the new children.	
Example:	
Suppose P_1 and P_2 are the two parameter's values in two parents, then the corresponding offspring values in chromosomes can be obtained as	
$C_i = \alpha_i P_1 + \beta_i P_2$	
where $i = 1, 2 \cdots n$ (number of children).	
$lpha_i$ and eta_i are some constants decided by the user.	
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So, in this technique, we use some linear function that is why the name is called linear crossover; linear function of the parent chromosomes to produce the new children. So, we can discuss the technique better with an example; suppose, P1 and P2 are the 2 parameter values in 2 parents, basically they are the 2, any 2 genes values belongs to 2 different parents P1 and P2. Then, the corresponding offspring values say it is C i can be obtained using this formula; the formula says, that C i is equals to alpha i P1 plus beta i P2. Here, P1 is the gene values in parent P1 and P2 is the gene values for the parent P2. So, it is like this, if this is the chromosome belongs to 1 parent P1 and these are another chromosome belongs to another parent P2.

So, for any gene value this 1 and this 1, so it is basically P1 and P2. So, any 2 value gene values this 1 and this 1, we denote this as a P1 and we denote this as a P2. So, having this structure, so that means, we want to calculate the gene values for the ith chromosome, which belongs to the ith children; that means, here 1 or more children's can be produced. So, here we are considering how from the 2 parents P1 and P2 values the n number of children's can be produced.

Now here, so the production is basically based on 2 parameters, these 2 parameters are called alpha and beta, for the ith children will use alpha i and beta i. So, these are the values alpha i and beta i, if you want to produce n number of children will be decided by

the user. So, it is give the responsibility decide the values of alpha and betas in order to produce the children.

So, this is the formula that is there, this formula is basically for how this ith children's gene value can be calculated.

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Linear crossover: An example	
Example : Suppose $P_1 = 15.65$ and $P_2 = 18.83$ and $\alpha_1 = \beta_1 = 0.5$ $\alpha_2 = 1.5$ and $\beta_2 = -0.5$ $\alpha_3 = -0.5$ and $\beta_3 = 1.5$ Answer : $C_1 = 0.5 \times (P_1 + P_2) = 17.24$ $C_2 = 1.5 \times P_1 - 0.5 \times P_2 = 14.06$ $C_3 = -0.5 \times P_1 + 1.5 \times P_2 = 20.24$	
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So, let us have an example. Say suppose, this is the 1 gene value belongs to the parent P1 and another gene value belongs to parent P2 is 18.83 and we are considering several cases or different values of alphas and betas. So, first case 1, let us take this is the value of alpha and beta, in this case we take the 2 equal values which is 0.5 and then another case, where we take another alpha and beta value and then 3rd instance we can consider this one.

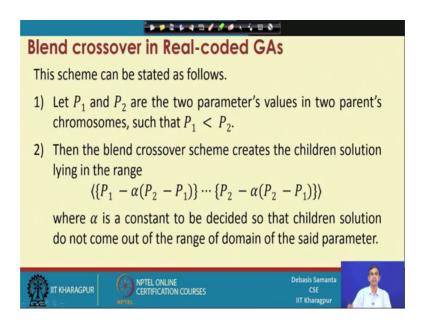
So, if we decide 3 set of values for alphas and betas, then we will be able to produce 3 children's. So, how the 3 children's can be produced in terms of this alpha beta values and given the gene values for the P1 and P2 is shown here. So, this is the first case, where the alpha and beta equals to 1. So, it gives this is the gene value and for the 2nd case the alpha equals 1.5 and beta equals to minus 0.5. So, it gives these values and the 3rd case alpha is minus 0.5 and beta is 1.5, it will produce these value.

Now, so you have learned about how the linear function. So, these are the basically these are the linear function that is followed here. So, these are the linear functions and in terms of linear function we are able to calculate the children solution.

Now, here the different values of alpha, beta has their own significance; have their own significances for example, here if we take alpha beta like this, then we will see that the children's value will be, so this is the parent value P1 and this is a parent value P2. So, if we take the alpha, beta is like this, then the children will be within this. For example, in this case the children value is this one.

Now, if we take alpha beta like this 1, alpha is heavy then beta 2, then the children will be beyond the P1. Similarly, if we take this 1 alpha and beta is this 1, this case then it will this one. So, depending on the different values of alpha and beta, so the children solution will be either here or here or here or within any regions, right within any point in the 3 regions can be considered. So, that is the importance of the different values of alpha and beta. That means, if you have a good knowledge about the different what is called the values, a range of values, then you will be able to generate the different children's accordingly.

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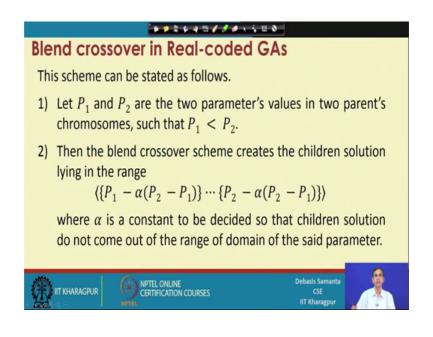


So, this is the idea about linear crossover, it is a very simple, but it has many advantage as well as limitation. The first is that, it is very simple to calculate because it use a linear function and calculation of a linear function is straight forward and that is why this technique is very fast so for the computation is concerned and as we change the different values of alphas and betas, we will be able to generate large set of offspring from only 2 parents, so this is the 1 advantage.

So, we can generate as many as solutions from the 2 parents and so this basically results population exploration and then, controls are possible to choose a wide range of change variations as I said in the last example, some values within the values P1 and P2, some values beyond P2, some values beyond P1. So, all these things can be possible, if we choose the values of alpha beta properly.

Now, here these are the advantage of course, the simplicity is the most important advantage in this case; however, it has certain limitation as well. The first limitation is basically the programmer should decide the values of alpha and betas and that is really headache for the programmer and in case of the programmer experience, then deciding right values for alpha and beta is really tedious job for the inexperienced user and more serious limitation is that if you do not choose the values properly, then the solution that will produce may leads to either premature convergence or stuck into a local optimum. So, there is a chance that this solution may not I mean with this cross over technique, the solution may not be optimum always, so this is the advantage and limitation for the linear crossover technique. Now, another technique basically tried to address all those limitations, this technique is called the blend crossover; blend crossover in real coded GA.

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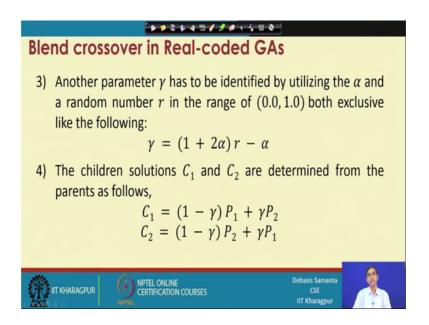


Now, we can explain this strategy like this, again will consider 2 gene values from parent P1 and P2, let they are represented as P1 and P2 and for simplicity will assume that P1 is less than P2.

Then, the blend crossover scheme it basically objective of the screen to create the children solution within the range, one is, this is the lower range and this is upper range within the range P1 minus alpha into P2 minus P1 and these are lower range and this is the upper range P2 minus alpha into P2 minus P1. Here, the alpha value is basically decided by the programmer; that means how much you want to have the region wider or narrower, so that alpha value will decide.

So, it is basically is a constant and this constants should be decided by the programmer before using this operation. Once the alpha value is known to us, then we will be able to follow this technique.

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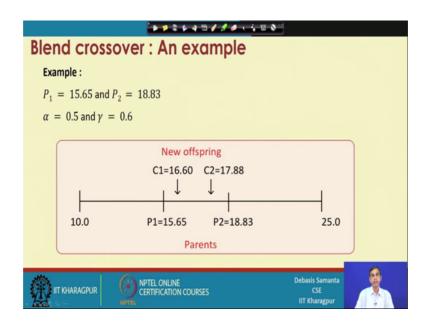


Now, this technique is basically calculates another parameters; now we denote this parameter as a gamma and this parameter is denoted in terms of another random number r. So, alpha is already known, r a random number is generated a random, these random number r should be in the range 0.0 to 1.0 and then based on this r and alpha is already known to us, will be able to decide the value of gamma.

So, basically gamma is, in this case a random value inside because r, as it is r is a random number and alpha is a constant, so gamma again it is a random number. Now, having this gamma random number, any 2 children's C1 and C2 can be calculated taking the confidence of the gene values of P1 and P2, which is shown here, for example, so 1 minus gamma into P1 plus gamma into P2 and the another solution is 1 minus gamma into P2 and this is a P1. So, changing the values of P1 and P2 like this, we will be able to calculate this one.

Now here, the unlike in case of linear crossover technique, we can generate a large number of solutions C1, C2, C3 if we take the different random values in fact. So, taking the different random values, we will be able to have the different gamma values and the different values will produce the different solutions according to this blend crossover technique. So, this is the scheme in fact, in case of blend crossover technique.

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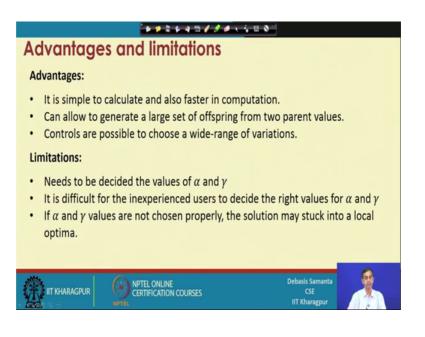


Now, let us illustrate this technique with an example, here is a simple example, let the gene value which is belongs to the parent P1 is this 1 and another gene value belongs to parent P2 is this 1, we consider alpha is 0.5 and gamma based on some random number, which is not mentioned here say that gamma at the moment is obtained as 0.6.

Now, with this we will be able to calculate the C1, that is 1 minus gamma into P1 plus gamma into P2 minus P1 and this is basically the value that can be obtained for the 1 solution and another solution.

If we take some another random number, which will give an another gamma, then another solution can be this sides or another solution can be this sides can be obtained. So, this way we will be able to generate a large number of solutions like a linear crossover, but only in terms of a probabilistic way random number.

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Now, so this is the plane cross over again it has the limitation, it is also simple because it also follow linear 1 equations for the calculation and like linear crossover it is also known to be a faster 1 technique and like linear crossover it also produce a large set of offspring from any 2 parent values and controls are possible because here the wide range of variations can be possible, if we choose enough random number as we wish. So, this is the limitation, like linear crossover technique it is also the, is a good point of this technique is that it is simple and then fast.

However, it has the limitation, the first limitation is alpha, but again you can note that alpha can be chosen with certain calculation, that how much the range that you want to have? So, that alpha can be obtained little bit by a prior calculation and then so alpha calculation is not a big issue. Now, so alpha the calculation can be done by some estimation, then gamma also can be done with the help of alpha values which are obtained and then just generating a random number.

So here, alpha and gamma can be decided, but it can be decided little bit calculated manner as it is not possible in linear crossover. So, this is the 1 difference between linear crossover and blend crossover. Again, for the inexperience user, deciding alpha is little bit difficult although it is not as such difficult as the linear crossover technique. So, it is little bit simpler for the user to decide the alpha value in fact and obviously, the alpha and gamma are the 2 deciding values in order to decide the right chromosome values for

the children. So, if we do not decide alpha and gamma values properly, then it may leads to premature convergence as well as stuck to a local optimum solution. So, this is the blend crossover technique and we can understand that it has little bit it, it is comparable you compare to the linear crossover technique.

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Simulated binary crossover in Real-coded GAs	
 This scheme is based on the probability distribution of generated children solution from the given parents. 	
• A spread factor α is used to represent the spread of the children solutions with respect to that of the parents, as given below. $\alpha = \frac{C_1 - C_2}{P_1 - P_2}$ Here, P_1 and P_2 are represent the parent points and C_1 and C_2	
are two children solutions.	
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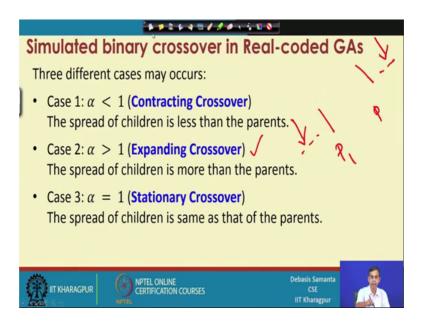
Now, we will discuss another technique, this is basically another statistical technique also called and it is called the binary simulated crossover technique and so in the binary crossover simulated techniques, the idea is more what is called a statistical in nature, is it gives I mean variation; more variation compared to the linear and blend crossover. However, this technique is little bit compressionally expensive and you will discuss it about.

So, this scheme, the simulated binary crossover is based on the concept of probability distribution and then, they basically follow certain probability distribution function to generate the children solution, that is the 1 advantage and it has been observed that if we use the probability distribution function rather than the simple random number as we have discussed in blend crossover, it basically produce the better result and can avoid or can address the premature convergence and then start to the local optima. So, this technique is preferable in the sense that it gives better solution, then the solution that can be obtained following the linear crossover technique and blend crossover technique.

Now, the basic idea in this technique is basically, it consider 1 factor it is called the spread factor. So, it is spread factor and denoted as alpha, the spread factor can be calculated by this formula, we have to assume any 2 C1 and C2, that means, how much variations in the children's solution that you want to have and these a P1 and P2 are the input value that you are having, then knowing or anticipating these are the C1 and C2 value, then you can calculate alpha. So, alpha calculation is basically under your control that how much division between the children solution that you want to have and decide, that will decide the values of alpha. So, alpha is a little bit can be calculated, so in that case user does not have to be an experienced user in fact.

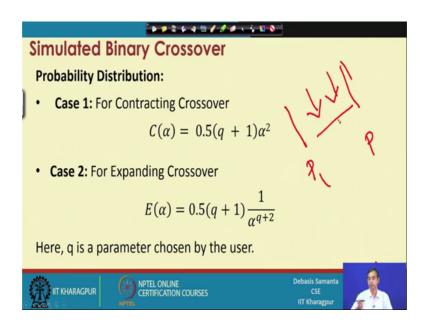
So, once the alpha, the spread factor is calculated, then we will be able to calculate the solution; children solutions which can be, which has the 3 different situation. So, will discuss about the 3 cases.

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So, 3 different cases means if alpha is less than 1, then the simulated binary crossover is called contracting crossover, so in fact, in case the spread of children is less than the parent.

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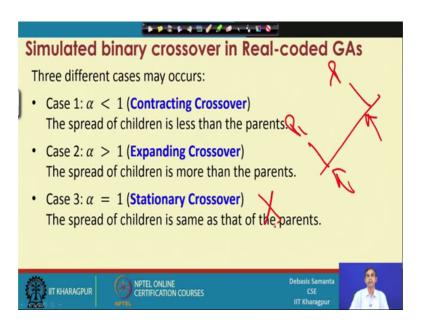


So, is basically if this is the parent P1 and parent P2, then C1 and C2 will be within this one. So, it is called the contract that means within the parent P1 and P2.

On the other hand case 2, if, so case 1 is basically alpha when it is less than 1, it is called the contracting crossover. In this case, the P1 and P2 are the 2 parent values, then the C1 and C2 can be obtained anywhere in between the parent and P1 and P2.

Now, the second case is case 2, in this case alpha is greater than 1 and in this case it is called expanding cross over. So, expanding crossover means if it is the P1 and P2 then C1 will be calculated beyond P1 and P2 here or there.

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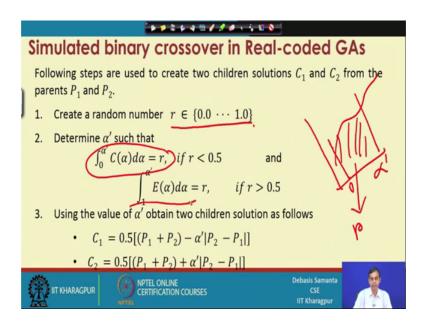


So, these are the expanding and the 3rd case is if alpha equals to 1 and eventually alpha equals 1 means P1, P2 and then C1 and C2. So, this is not a useful fact because it will not produce any variation. So, usually this technique is not considered or the alpha equals to 1 is not they acceptable value. So, will consider only this is alpha using this 2 cases that is alpha is less than 1 and alpha is greater than 1, that is either contracting cross over or we have to use a simulated binary crossover as a expanding crossover.

Now, let us see, how the different crossover can be realised? Now, as I told you simulated binary crossover basically follow a probability distribution function and so here actually the probability distribution function you can choose any probability distribution function, but it is recommended to follow the 2 specific probability distribution functions they are basically called C alpha and E alpha.

C alpha the probability distribution function is usually followed for contracting crossover where as the E alpha, the another probability distribution used for the expanding crossover. Now, so the probability distribution function that it is followed in case of contacting crossover is shown here. So, this is a function description for a given values of alpha; alpha is already known to us and they it consider q is a constant, the constant can be decided by the user based on the users experience. Similarly, in terms of q if the alpha is known then this is basically the another probability distribution function this one. Now, here these are the 2 recommended probability distribution function, other probability distribution function like Gaussian distribution or some other distribution function also can be followed. Anyway, we will consider these are the 2 recommend probability distribution function to calculate the children solution following the contracting crossover and then expanding crossover, let us see how these can be done.

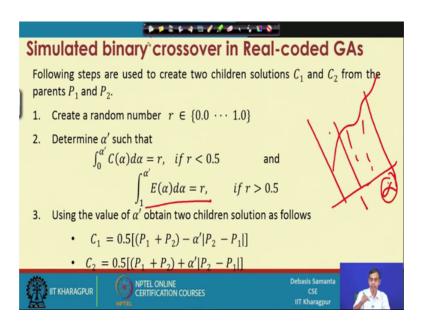
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Now, we will consider the contracting crossover first. So, here basically the 3 steps are to be followed. So, the first step is that, we have to generate a random number r, that is the random number r we, this is a first step; generate random number r in between 0 and 1.0, then we have to determine alpha dash, how the value of alpha dash can be determined? Alpha dash is a value that can be determined using this function.

So, it is basically area, so if this is the probability distribution function and then area covered by these value into 0 and alpha dash, so that this area is basically equals to r. So, if r is less than 0.5 and in case of if r, the random number that we have generated here, if it is greater than 0.5, then will calculate this one.

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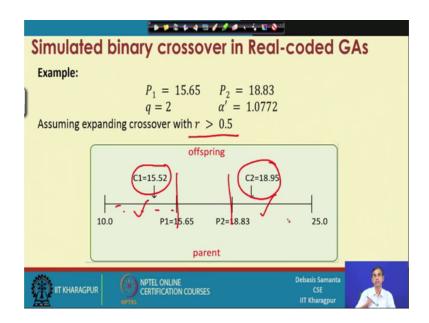
So, it is basically in this case the distribution function it is like this, if this is the distribution function and then it is basically say 1 and then it is basically alpha dash, then we will calculate this alpha dash value following this expression, if this one.

Now here, so basically the idea is that the r less than 0.5 the chance that 50 percent will be belongs to the contracting crossover and then r greater than 0.5 that means, another 50 percent will have the chance to have the expanding; what is called the expanding crossover. So, both way, both expanding and then contracting cross over can be followed to calculate the 2 values alpha dash according to this distribution function. 1 alpha dash is known to us, then any 2 children can be calculated using this formula. So, this is the formula recommended by the developer of simulated binary crossover, this is the formula you have to just follow it.

So, the formula says that is 0.5 and then value of P1 plus 2 minus alpha dash and then it is a what is called the absolute difference between the parent values. Similarly, taking the plus sign, the another will be obtained. So, this way the 2 solution C1, C2 can be obtained based on the contracting as well as expanding depending r different values, so this way this they will give. Now, here actually this technique is good, good because we do not have to take any parameters that is required both in linear crossover as well as blend crossover except only the calculation or competition. The competition because this operation or this operation is basically is a comparisonally expensive operation, but if

you able to do it right then all this things are pretty simple and then it is useful and more effective; effective in the sense that it gives better result compare to the linear crossover and then binary crossover technique.

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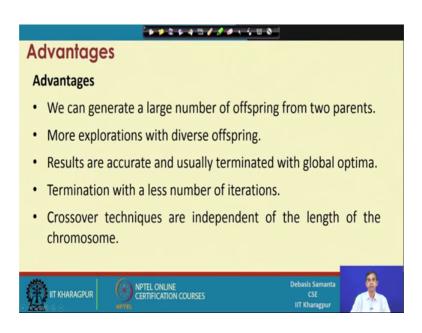


Now, finally, I would like to give an illustration of the simulated binary cross over technique. So, let us see these are the 2 parent gene values P1 and P2 and we assume q equals to 2 is a defined constant that can be varied if you want to have the different results like, it is basically the q value is decided by trial and error, that means, if you take very large values of q whether it is quickly converged or if it is a small value then it can converge with a better solution and all this things. So, here little bit experience of the user is required, usually the user can gather experience by means of trial and error method, that means, they have to run the same program for several cases with different values and for a certain values it will be better result it is taken as that value as the standard value.

Anyway, so suppose q equals 2 known to you and then alpha dash can be calculated based on the random number generation and then the probability distribution function that we have discussed and having these value for example, in this case, so r is 0.5 which gives alpha dash according to the expanding function, then the 2 values can be obtained, now 1 is C1 using the formula that we have already discussed and as I told you r is

greater than 0.5, so it is expanding, that means, this is the P1 and P2, it will calculate the chromosome any 1 region within this region.

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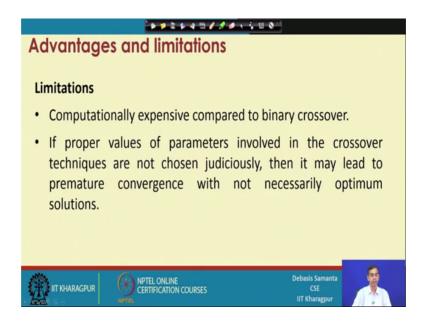


So, this is the idea about the simulated binary crossover techniques that is there and now, simulated binary crossover has a number of advantages compared to the previous the 2 techniques that we have discussed and as in case of linear crossover and blend crossover in this technique also, we will be able to generate large number offspring from 2 parents. So, in that case what we have to do is that we have to generate as many as random number as many we want to have the children's and it in fact allows more exploration with diverse values of offspring, which is comparable to the both linear as well as the binary the blend crossover technique.

Here the results, usually gives more accurate results compare to the linear and blend crossover techniques and usually it gives the global optima, whereas other 2 techniques usually stuck into the local optima and it basically terminates with a less number of iterations because number of iterations that is required to run the GA is it is in fact, observe that more in case of linear than blend crossover. So, in that sense it is also cost effective, although it is the costly operation in case of crossover, but so far the GA iteration is concerned it require less, so that means, effectively it is a faster GA algorithm than the crossover technique if we follow linear and blend cross over technique.

And here, actually crossover techniques are independent of the length of the chromosome, whatever be the values of the chromosome, that means, the parent values has many number of gene absolutely no problem, we will be able to run effectively using the same techniques. So, it is fast in that case because the same alpha dash that can be used to calculate the different gene values for the chromosomes, if we take the different this one. So, in the same line we will be able to follow, no need to discuss, no need to consider the different gene values and the different alpha dash and then different random number generation, it is not require. So, in 1 set the same alpha dash can be used to calculate the different gene values for the offspring for the different parent values or different parent values of the different genes.

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So, this is the advantage of this one; however, it is suffering for another limitation as well as computationally expensive compared to the binary crossover technique. I am now comparing not the linear crossover or blend crossover, but the binary cross over which we followed in case of the binary coded GA. So, if we see, all the binary cross over techniques are very fast and straight forward and pretty simple also, whereas this similar type binary crossover is little bit compresionaly expensive.

Now, again there is a decision regarding the probability distribution function, if you do not choose the proper probability distribution function or if you do not choose the q values, that is required in case of I mean in probability distribution function decision, then you may leads to a erroneous results and premature convergence. So, user needs to be little bit careful about choosing the right values of probability distribution; right probability functions for contracting as well as expanding function and also the correct values the q that is to be used in the probability distribution function.

So, this is the technique that we have discussed so far the simulated binary cross over is concerned and so we have so far discussed about the 2 different GA technique, 1 is the binary coded GA, another is real coded GA and there are several crossover operations we have learned in binary coded GA and then, just now we have learned about the cross over techniques in the real coded GA, there is another GA coding scheme which we will discuss, this is the order GA and we will discuss the crossover techniques that is required for the order GA in the next class.

Thank you.