Introduction to Soft Computing
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Lecture – 02
Introduction to Fuzzy Logic

We will start our lecture these are beginning of the topics are the fuzzy logic. So, fuzzy logic is an essential component for the soft computing. So, today we will learn about basic concept of fuzzy logic and to understand the fuzzy system we should familiar our self with different terminologies. So, we will explain the different terminology related to the fuzzy logic.

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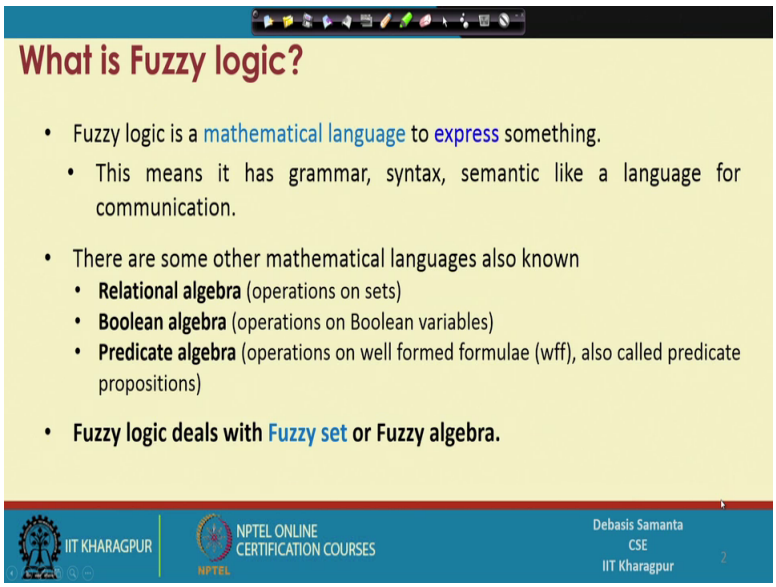

So, the first is what is fuzzy logic? In fact, fuzzy logic is a language we can say more precisely is a mathematical language like any language you know. So, this language is also used to express something which is meaningful to others. So, it is a language this means that it has grammar it has it own syntax the meaning like a language for communication like English.

Now, like fuzzy logic there are many mathematical languages we know. So, one language is called relational algebra which is based on the operations on set. So, this is also called relational logic Boolean logic is basically based on the operations on Boolean variables it is Boolean algebra it is also called and predicate logic or it is called the

predicate algebra. Which is basically operations based on the well-formed formulae or proposition also called the predicate propositions, it is very interesting that fuzzy logic like relation logic Boolean logic and predicate logic it also deals with some elements; the elements on which this fuzzy logic depends is called fuzzy set and it is also alternative we call the fuzzy algebra and another interesting fact is that the fuzzy logic essentially combined the different algebras like relational algebra Boolean algebra and predicate algebra together.

So, it is basically a mixture of the different mathematical languages to define another new language that is the fuzzy logic.

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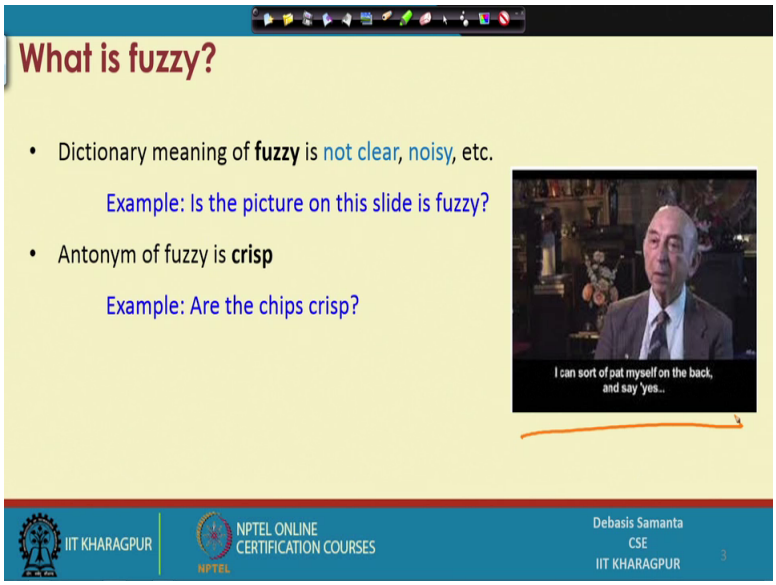

Now, so the word fuzzy may not be new to us. So, if we search dictionary the meaning of fuzzy is not clear or it is called noisy it is like that. Now as an example so we see one figure here we see one figure here this is the figure now. So, sometimes we see that is the picture on this slide is clear. So, we can say yeah the picture on this slide is fuzzy; that means not clear whatever the wording it is clear we may say that it is not clear or there are many noise in the image. So, the image is noisy image is fuzzy.

In other words we can understand the meaning of the fuzzy if we see it is antonym the antonym of fuzzy is crisp. Now crisp in the sense that if we say there is a there are 2 regions and if we say the boundary if the boundary is not clear then we can say the 2 regions are separated fuzzily, on the other hand if there is a strong boundary by which we

can easily distinguish 2 regions clearly then we can say that the boundary is crisp. So, these way we can understand the fuzzy versus crisp we learn many thing about this fuzzy versus crisp in our next slides next discussion.

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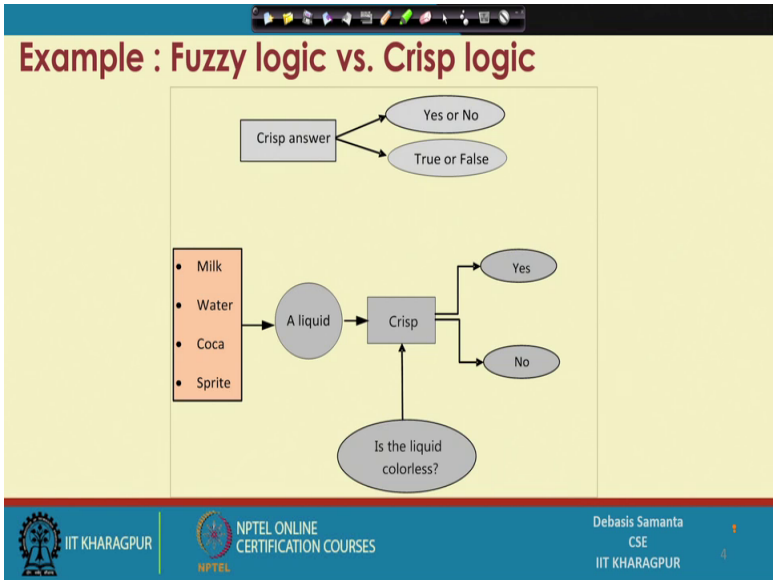

So, we have I will give you understanding about the meaning of fuzzy our next discussion exactly with some examples. So, how the 2 logic may be say logic with fuzzy sets and logic with crisp set. So, I say here set anyway. So, I will discuss about what exactly the crisp set it is anyway. So, fuzzy logic versus crisp logic now say if we ask some questions and then answer to that question if has the clear meaning then we can say that answer is having crisp answer.

So, crisp answer usually expressed in the form of either yes or no true or false like this as an example. Suppose the question is that we have to identify a liquid now any liquid like milk, water, coca, sprite is given and then if we ask the question that is the liquid colorless now you have to give the answer in terms of only 2 things, yes or no then it is called the crisp answer. So, this way we can understand exactly the crisp crisp system.

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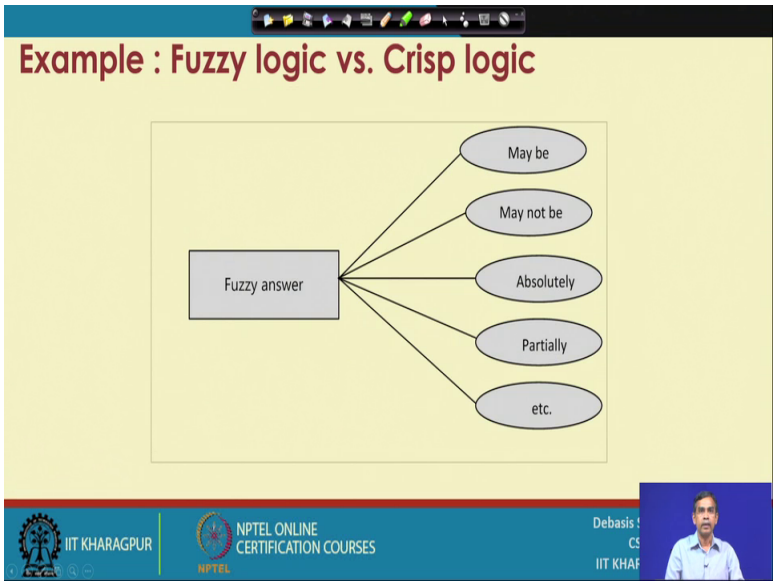

And alternative to crisp system the fuzzy answer let us see how the fuzzy answers can be.

So, if we ask one questions and the answer can be of many instead only 2 solid answers. So, the answer may be may be may not be absolutely partially etcetera. So, there are many many form may values for the same answer.

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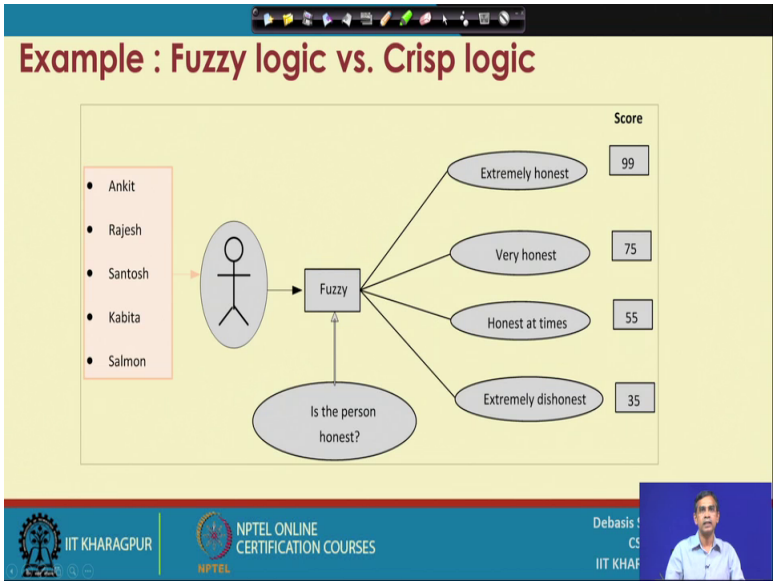

So, this is basically the concept of fuzzy answer I can illustrate the concept with an example say fuzzy system is like the question is the person honest and a person is given as an input let they are the Ankit or Rajesh, Santhosh, Kabita, Salmon like.

So, if we ask the question is the person honest say Ankit. So, their answer may be extremely honest very honest honest time to time or extremely dishonest. Now so for a for a question unlike the crisp question, for the fuzzy questions, or for a question the fuzzy answer, on like the crisp answer may have different what is called the answers. Now so, this different answers if it is there for the same question then which is the correct answer actually because all answers seems to be acceptable or rejectable now which answer is there.

Now, we can give a score to each answer. So, here I have given a score here for each answer for example, extremely honest 99 very honest 75 honest at times 55 extremely dishonest 35. So, this means that if it is a 2 valued answer like say crisp answer then only 2 and then score will be on 100 and another is 0 whatever it is there, but here the different values of the score these means that the answer which is a very honest it is also the correct, but correct with a validity score it is called the 75.

Now; obviously, question that arise that how we know what is the score actually. So, we will discuss about that how the score for a for an answer can be calculated and that can be tagged into that answer to signifying that how answer is significant or how answer is acceptable. So, for the question is concerned anyway so the idea is that these are the answer is called the fuzzy answer for a given question unlike the crisp answer.

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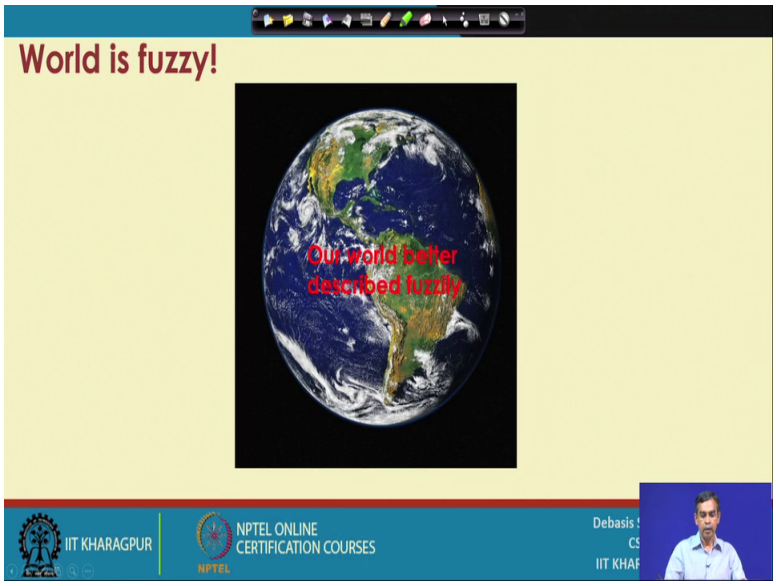

Now, in fact our world can be better described fuzzily this is because if I say what is the temperature today. So, you can get an answer like very hot some people can say that comfortable, then extreme or very cold it is like this. So, this means that for the same question the answer can be different if the same question is for a too many people, but everybody can give the answer according to their own estimation whatever it is there, but the answer is like that like the temperature another what will be the weather today. So, if I ask to predict it to some expert person then he will give the answer fuzzily; that means, yeah weather is sunny today may be sunny may not be sunny may be cloudy or it is like that.

So, the answer can be for same questions of different form and different form has their own value and then we have to take all the values. In fact, and then process it. So, that the answer is acceptable to us.

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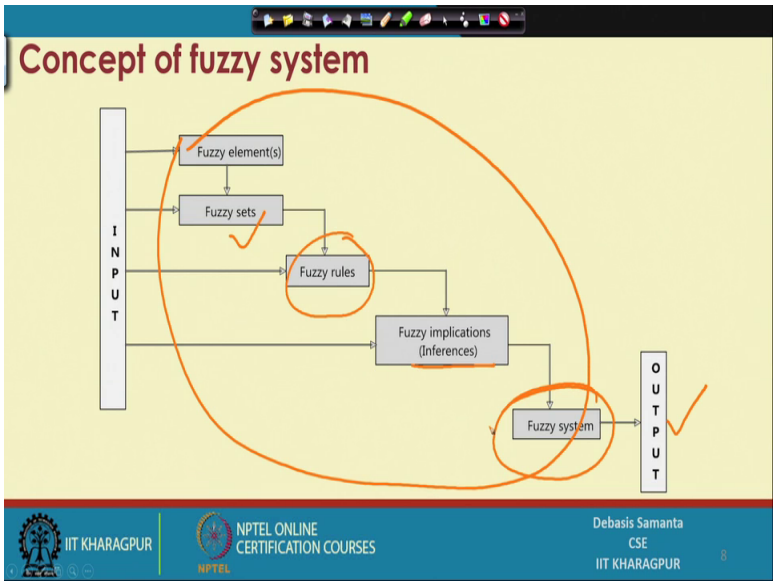

Now, so the best idea it is that the system that we are using it is basically better can be described fuzzily or it is basically the system or we can say that everything is in the form of a fuzzy and then we can take the fuzzy manners or the fuzzy way to describe any system rather.

So, if we describe a system in the in the way of in the way the fuzzy decides then this system is called the fuzzy system. Now typically the fuzzy system has many ingredients or elements so; obviously, the input and output is the part of any system that we have already discuss in the first lecture itself. So, if this is the entire system then these are input and then output, now need not to say that input is; obviously, in the form of a crisp because we usually give input to the system in the form of a crisp value.

Similarly, the output also should be in the form of a crisp value. So, in this system there are 2 boundaries one input and output input and output are in the form of a crisp value; however, input can be transformed into some fuzzy form and then the fuzzy system can come into play. And so in this type of fuzzy system there are many constituents many elements are there.

So, the first element is called the fuzzy elements and taking one or more fuzzy elements we can discuss about the fuzzy set and then many fuzzy sets can be connected with the set of another element is called the fuzzy rules and finally, a set of fuzzy rules can govern

us to decide is called the fuzzy implication or it is called the inferences and these whole the things constitute what is called our fuzzy system.

In other words to understand the fuzzy system it is our task to understand what exactly a fuzzy element it is and then what is a fuzzy set and then using the fuzzy set how the fuzzy rules can be obtained and then how the inferences can be described in the form of a fuzzy rules and that all these things, if we learn it then we will be in a position to discuss about the fuzzy system. So, in our subsequent lectures we will basically discuss about all these elements one by one today will discuss about the fuzzy elements first.

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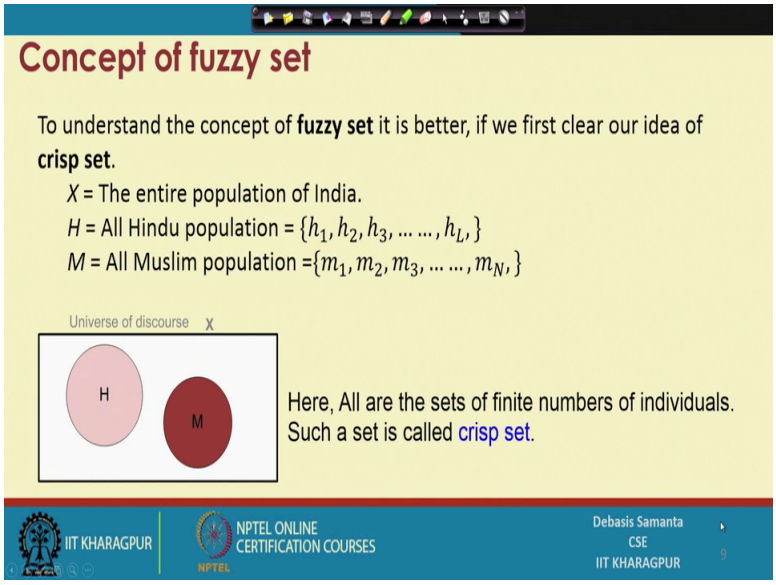

Now, let us see exactly what is the fuzzy elements. So, fuzzy elements basically essentially it a fuzzy set. So, we can better describe a fuzzy set in the form of a crisp set actually. So, we know exactly the concept of set. So, this the traditional set that we know it is. In fact, is a crisp set for an example say X denotes a crisp set and it denotes the entire population of India, then you can say what are the elements yourself, myself, or the elements belongs to the set X.

Now, I can derive one set again from this set X or some other means suppose it is the H H is the another set it denotes all Hindu population. So, any element that is means any person or any individuals belongs to this set is the set composition itself. For example, here h 1, h 2, h 3 all these things are the elements to this set they are basically individual who basically satisfy some characteristics being Hindu population. Like Hindu

population we can define another set say all Muslim population for example, these are the set of all Muslim individuals.

So, these are the example of crisp set and we know any crisp set can be better describe in the form of a graphs or it is a ven diagram. So, we have we have shown one ven diagram here for this and X whole the things are basically shown here and we can see that there are the 2 boundaries the 2 boundary essentially difference or basically define solidly the 2 regions 1 a regions belongs to H and another region belongs to M and these 2 regions a basically belongs to another bigger region.

So, this bigger region is basically called universe of discourse in this case it is X. So, all the regions whatever it is their has a solid boundary and that is why they called the crisp set.

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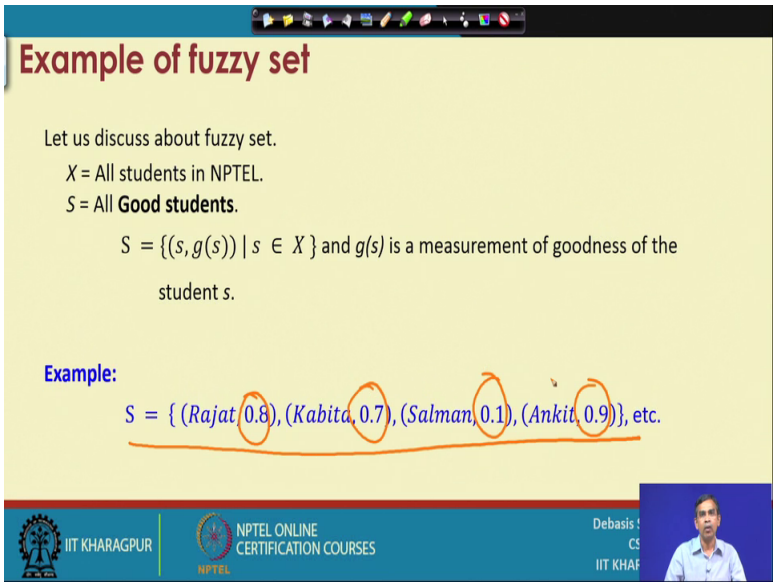

Now, so like crisp set the fuzzy set is also almost similar, but little bit difference is their; difference so for the presentation is concerned. For an example; so suppose X X denotes a set and let the set be all students in NPTEL. So, this is the universal of discourse in this case.

Now, I let us define one set belong to this X let this set be S and will define this set s as all good students. Now let us see how the same thing can be defined using a fuzzy manner. So, we define the set S as a 2 things in each elements one is s the elements itself

and g s some measurement of S itself where s is any element belong to X and g s is a measurement now this measurement. In fact, we can say measuring the goodness of a student.

Now, for example, if I want to measure the or evaluate a student. So, how I can evaluate I can take some exams and I can take the marks obtained by the students in that exam. So, g s can be same type of measurement like. So, it is a goodness measure it is or rather it is called the measurement that that the S belongs to the set S. So, for example, here again we can see. So, suppose there are few students which are Rajat, Kabita, Salman, Ankit like and their measurement is expressed here for a Rajat is having the score 0.8 Kabita having 0.7 Salman 0.1 and Ankit a 0.9.

So, this set signifies that all students who belongs to this set like Rajat, Kabita, Salman they are the good student, but goodness is defined by means of measure. In other word Salman if he is a good student then Ankit is also good student, but Salman being a good student his score is 0.1 and Ankit his score is 0.9. So, the difference between the 2 is basically how they have their own membership values; that mean, 0.1 0.9 whatever it is there, but all them belongs to the good student. In fact, all though Salman may scoreless or Ankit may score highest here all of them are the good students belongs to the good students actually.

Now, here another point you can note that the measurement value that we have mentioned here is basically in between 0 2 1. Actually it is the concept that is followed in fuzzy logic all the measurement value g s like. So, value should have in 0 2 1 both inclusive. So, any value in between 0 and one are the basically taken as the membership value for this 1.

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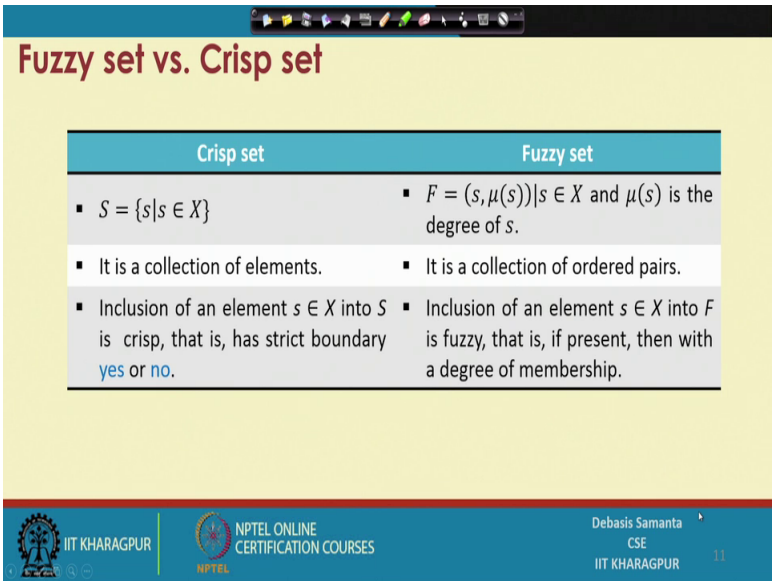

Now, so we have a little bit understanding about the fuzzy sets and let us see what are the difference, the salient differences between the crisp set and the fuzzy set. So, the differences between the 2 sets are defined in the form of a table. So, if we define a crisp set it is basically is a collection of elements; that means one part only. So, S where as if it is a fuzzy set then it is a collection of ordered pair it is called the first part is the element itself and second part is the measurement of that element itself.

So, sometimes this measurement mu S in fuzzy theory it is called the degree of S or it is also called membership value of S. So, what you have understood is that a crisp set is a collection of elements or as a fuzzy set is a collection of ordered pairs so 2 things together from one element in the fuzzy set.

Now, inclusion of an element any element say S into the set a S capital S is crisps that is it has strict boundary yes or no, if that element belongs to the set yes or no we can easily justify that one; however, inclusion of an element S into F the which is a fuzzy set is present then with a degree of membership. In other words a same element say X can belongs to 2 fuzzy set F and g, but with different membership values for example, if F denotes the good student and g denotes the bad students then same element say S, can belongs to the good student as well as bad student, but with different membership value. For example, S appears in F with membership value 0.7 whereas; the same element belongs to the set g with membership value say 0.3. So, it is like these.

So, same elements may appear into the 2 sets with different membership values whereas, same element may not appear into 2 crisp set, it is either in one set or another. So, there may be ok.

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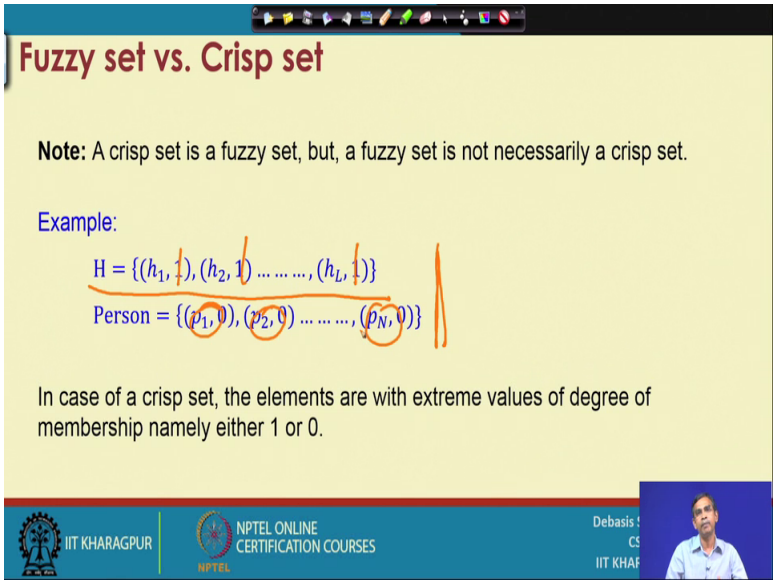

So, we have understood about few definition about the fuzzy set versus crisp set, now we will discuss about 1 point you can note is as, I already told you the membership values a degree of membership value we can say alternatively like this degree of values degree of membership values or membership values that can be their if an element belongs to fuzzy set with any value 0 to 1, inclusive and then any value in between 0 to 1 inclusive.

On the other hand the same element actually if it is a crisp set can also express in the form of a fuzzy form with the membership value 1 and 0 only. For example, here is the set H. So, if that element presents their then I can say the degree of membership one if the element does not belongs to that set then we can say the degree of membership is 0. So, basically this is a fuzzy set essentially with the membership value 0 and 1, now with this understanding if we do not write these one these one and these one then we can say that h is a crisp set which elements are h 1, h 2, h L.

On the other hand if it is the membership value 0; that means, this element does not belong to this set. So, in this case the first one becomes a null set. So, basically 0 and 1 being the 2 extreme values can be expressed to define a crisp set in the form of a fuzzy set. So, this way we can say that a crisp set is a fuzzy set because anyway crisp set can be

converted in the fuzzy set easily, but a fuzzy set cannot be expressed always in the form of a crisp set because there membership value not necessarily always 0 and one in between 0 and 1.

So, this is the one conclusion that we can it from our discussion that the crisp set is a fuzzy set, but a fuzzy set is not necessarily a crisp set.

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So, we have understanding about the fuzzy set versus crisp set or we can say crisp logic versus fuzzy logic little bit now let see one important point here. So, for the fuzzy set decision is concerned the membership value and there is a question that how the membership value each elements can be decided and who can decide this membership values for each elements which belongs to the fuzzy set I can give an example.

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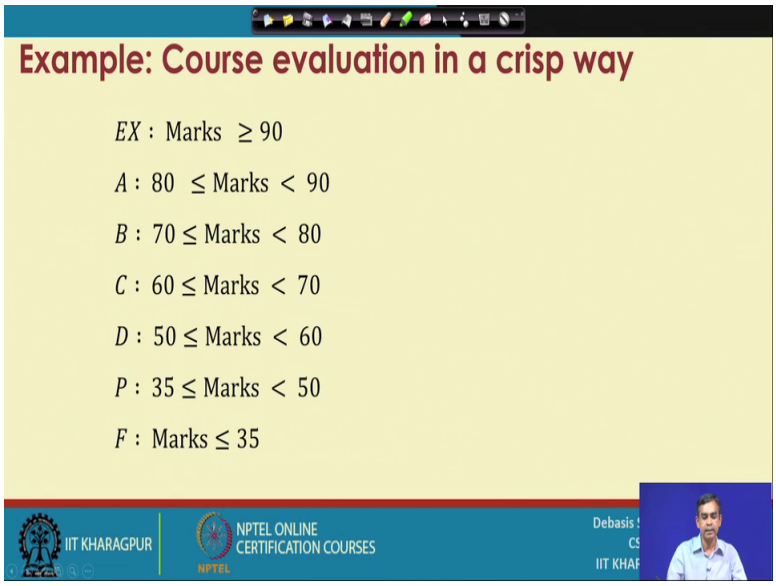

Say suppose all cities in India or more precisely say suppose there 6 cities in India like Bangalore, Bombay, Hyderabad, Kharagpur, Madras, Delhi right and I want to define one set let the name of the set is city of comfort. Now I have decided some value let the values for each side belongs to the set like Bangalore is 0.9 5 and so on. Now so the idea is that how this comfort of the Bangalore city 0.9 5 can be decided, there are certain population boat or something like population opinion or any way a feedback whatever you can consider by this feedback, if we normalize those feed back into the value this one then it will give us to a fuzzy values.

So, this way we can have the fuzzy membership and regarding the membership value we will discussing details in due time, now there is another example which I would like to mention here. So, that we can understand the concept of crisps versus fuzzy the idea it is here say we know exactly how to grade the marks obtain by a students in a subjects.

So, basically this is the grading formula, now these are the grading that we can we can see that there is a strict boundary between one marks to another. So, A marks will be either belongs to the grade A or it is AX or B, but cannot be a same marks belongs to the 2 different grade for example, one mark which is there in this it can belong to this one, that mean the marks can be EX, marks can be A, marks can be B, if it is marks in EX it is definitely with certain membership value is a 0.2 if it is belong to b then maybe it is 0.3 if it is belongs to a then maybe it is 0.9.

So, there is a there is a concept and this is basically the example of crisps formulation for the marks now the same thing if we do it in a fuzzy formulation it will look like this.

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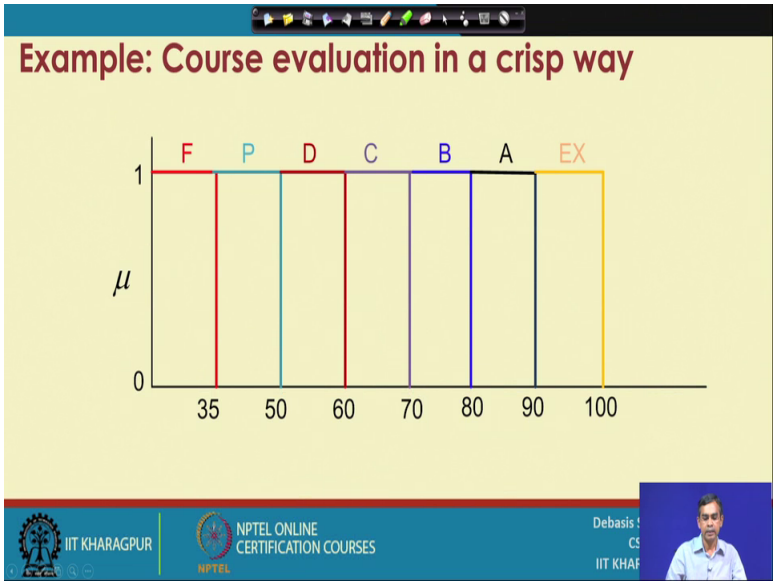

So, this is basically the graphical display of the crisp formulation and the fuzzy formulation we can see it is the fuzzy formulation.

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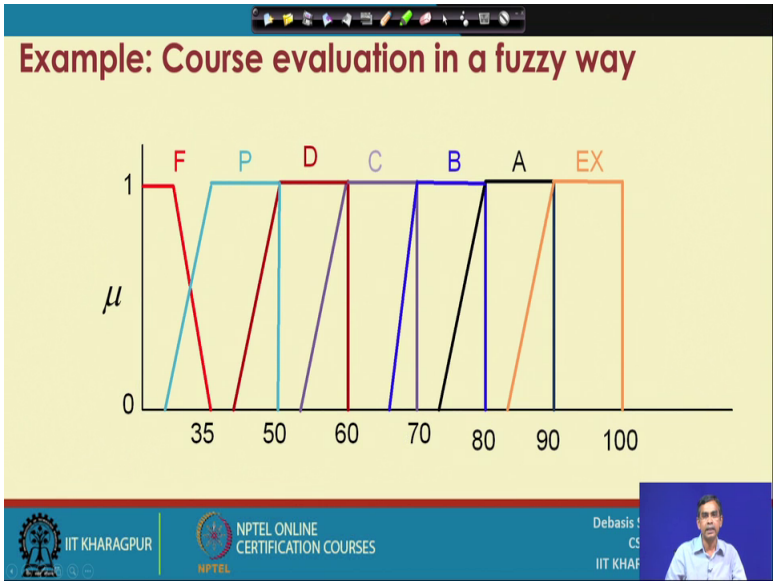

So, here we can note that any marks for example, any marks say it is these one this is the marks. So, this marks is basically these basically denote the D grade and these basically denotes the P grade. So, this marks both belongs to the P grade and both belongs to the D

grade. If it is if we draw like this so if it is a D grade then this is the membership value and if it the P grade then this is the membership value.

So, the same marks belong to the 2 sets P or D with the different membership values.

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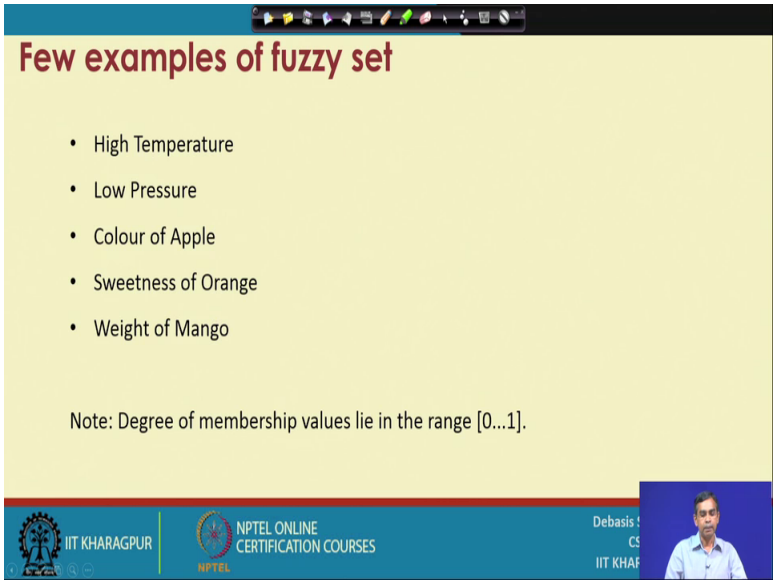

Some examples further can be used for example, temperature is high. So, we can discuss it with is in a fuzzy form low pressure color of apple sweetness of orange weight of mango and so on so on. So, these are the few examples which basically we know. So, these are the input and then they can be discuss in a fuzzy form also.

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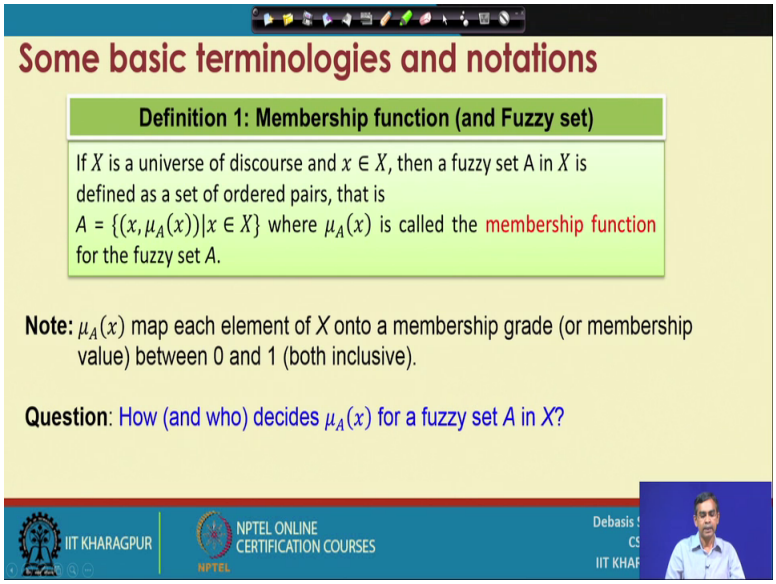

Now, for the definition we will start with few terminologies. So, the first that a membership function and so it can be defined like this if X is A universe of discourse and if any element x, which is belongs to this X, then a fuzzy set A which is defined in X is defined as a set of ordered pairs, as I told you ordered pairs x and mu x. So, this is the concept of fuzzy sets and definition of basically membership function and these fuzzy set.

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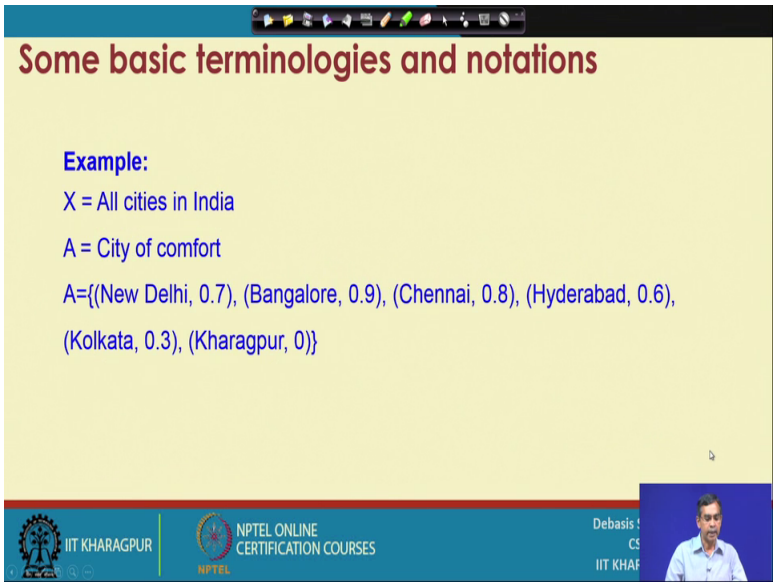

So, here as an example that how fuzzy set can be X is the all cities in India and A is A fuzzy set of comfort and then this fuzzy set can be discussed using this form.

Now, membership functions may have any value they are with either discrete membership values here I can show I show one example here where the all.

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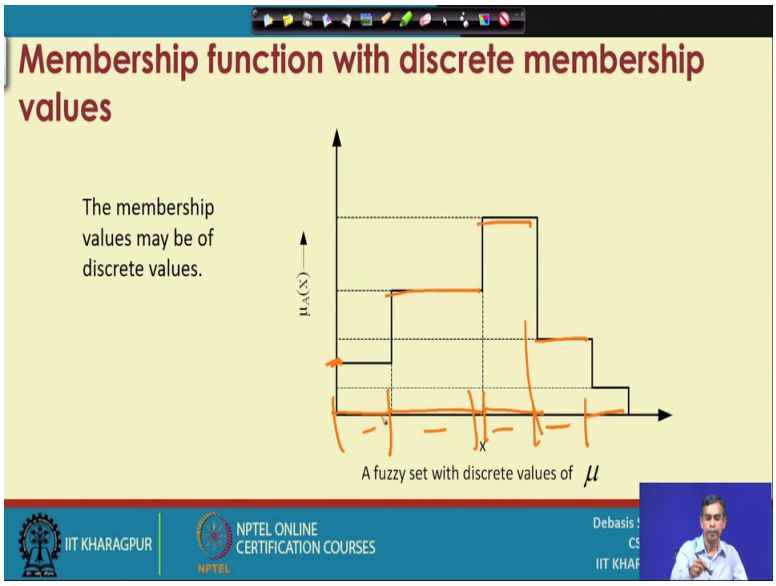

So, these are the elements in between has the membership value this one in these element the membership value is this one here these one so these one. So, the different elements so different element have the different membership value and it is called the discrete what is called the values of the membership function.

So, the membership values can be discrete the membership values can be also continuous domain.

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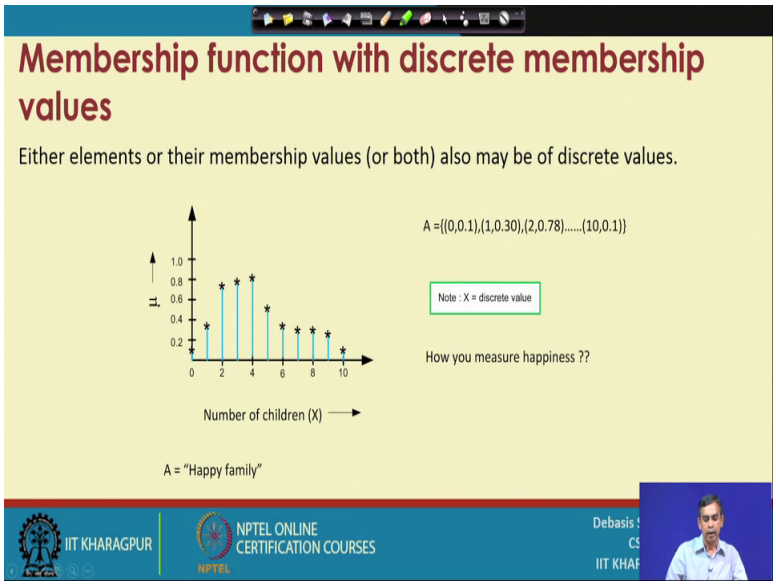

And the element also can be a discrete here in this example all the element that belongs to these are defined in terms of discrete, quantities, the membership values also may be discrete or continuous.

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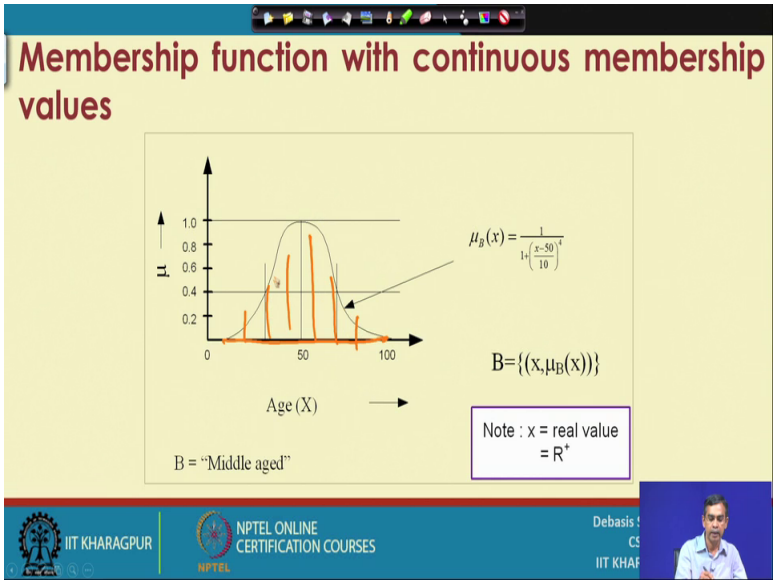

So, what I want to say is that the element can be either discrete value or continuous value likewise any membership values for element can be of discrete values or it can be of continuous values. So, this is an example which basically shows how the membership values is continuous for example, in this region. So, membership values for any element in a continuous domain can be described by means of this curve. So, it like this whatever it is.

So, membership value can be a discrete value element can be discrete value the membership the value can be continuous the elements also can be continuous.

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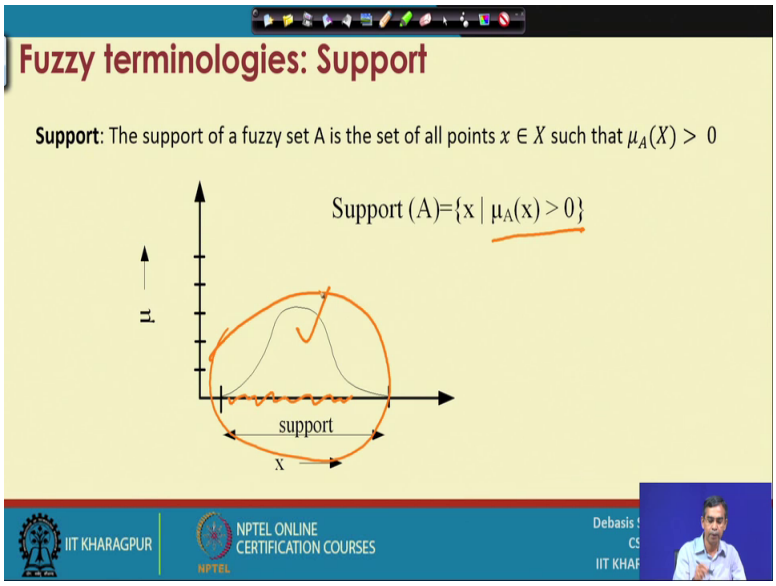

Now, there are a few more things or their terminologies are there I will quickly cover this terminologies within one minute. So, there we can understand about it. So, the first terminology is called the support the support of an element is of a fuzzy set denotes that whose membership value is greater than x.

So, all these elements are basically the support which is belong to define this fuzzy set whose membership function is like this. So, what you can say that a fuzzy set. In fact, can be disclaim by means of a graph.

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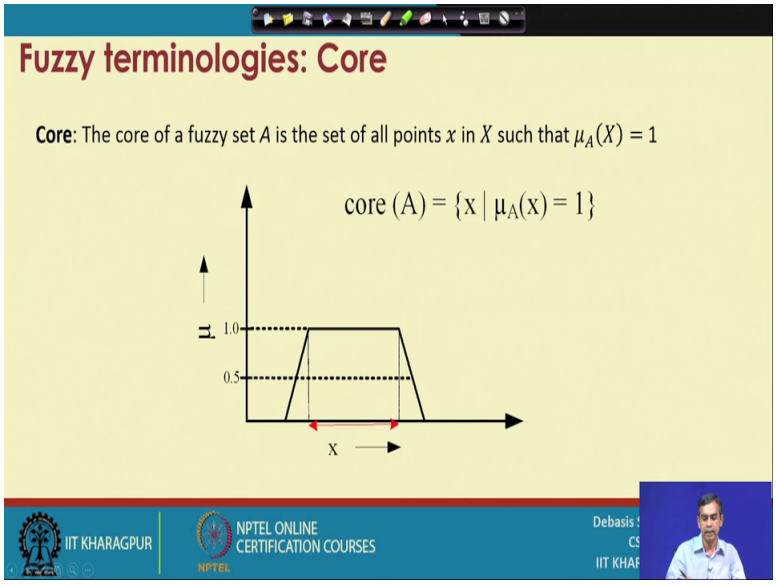

With regarding these things will discuss in detail later on now here core A core A is basically all elements a which are having membership values is equal to 1. Now here the core A all these elements having the membership values 1. So, these basically denotes the core A and we can understand that core A basically essentially a fuzzy sets.

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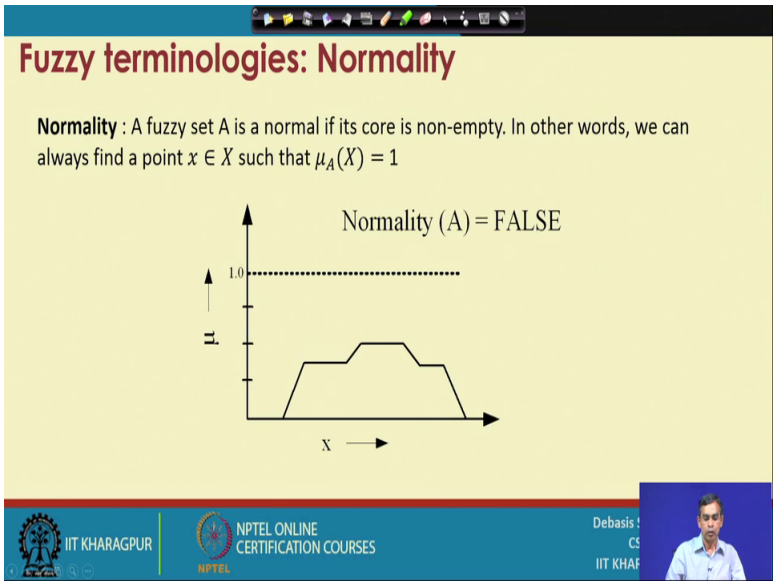

So, normality now a fuzzy set can be a turn by the normal it is basically a Boolean value either 0 or 1 or the crisp value if it contains at least one element which core value is non-empty; that means, it has at least one element whose membership value is one. And if it does not contain any element whose membership value is not equal to one then it is not a normal. So, normality is false.

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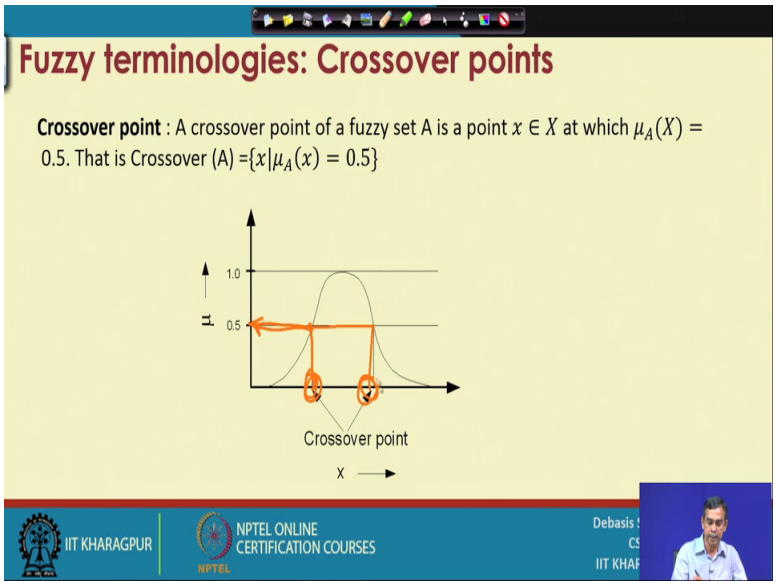

Now, crossover point so there are the elements whose membership value exactly 0.5 is called the crossover point. For example, in this graph we can say this is the 2 elements it has the membership value 0.5 these also has the membership value 0.5. So, these element and these element whose belongs to the set x is basically the crossover point in this case few more terminologies we will discuss as the time is short so will discuss in the next lectures.

Thank you.