Problem Solving Through Programming In C
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Lecture - 51
Interpolation

Today we will first discuss a technique called interpolation. We have seen how a function can be represented in a computer in the form of a table or a 2 2-dimensional array. Now what is interpolation? Let us try to understand.

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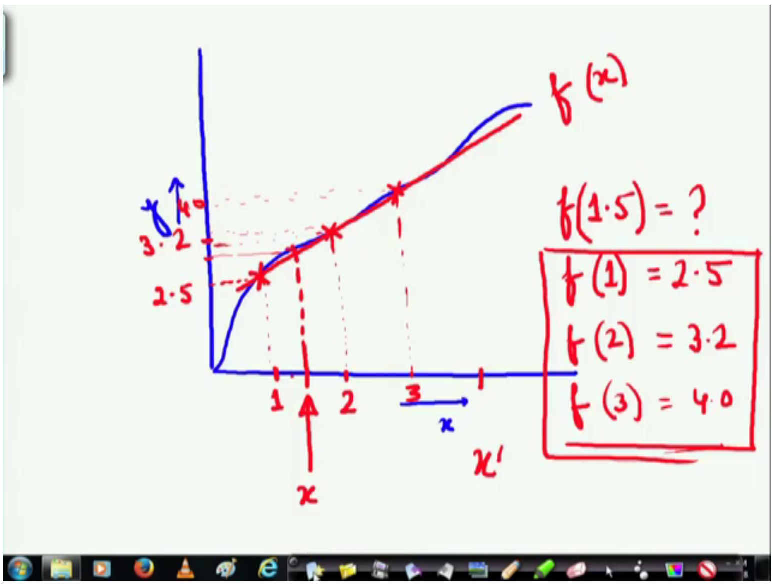

Say, I have got a function, where I know the, I have got a function which is something like this. But I do not know the function before that, all right? If I had known the function, this description of this function, in that case given any x, I could have found out the corresponding y, right.

But suppose the function is not given, instead what is given to us are for some specific excess say, x 1 I have been given this value, all right? This value of y. For x 2 x equal to 2, I have been given this value of y, all right? This particular value of y. Similarly, for 3, I am given a particular value of y, but I do not know what is the value of funk this function for 1.5. So, if I call this to the function f of x, then f of f of 1.5 is not known. Although I know that f of 1 is say something 2.5, I know f of 2 to be may be 3.2, f of 3 maybe something like 4.

But this information are known to me, this part is known to me. Given this known part, can I find out what are the intermediate values for if 1.58, 1.3, 2.5, 2.6, can I find that out? That is the task of interpolation; that means, given some values known, I want to find out the value of the function for some independent value of the dependent variable. So, for a particular x, what is the part? What is the y? That I want to find. Now you see you can look at this blue line and say it. So, simply you draw it like this and you will find the value of y.

But unfortunately, what I have is not the blue line. What I have is only the rate crosses and this line is not there. I do not know that these 3 points whether the curve is something like this, or the curve is could be the curve could be a straight line through this points, right? In that case you see, if it was the case, then the value of x this particular x would be different from this. So, I have to actually see that what is the curve? That best fits all these given points. This problem is known as interpolation.

Now, I can extend it suppose of some a table is given to you, where values for f 1 2 and 3 are given. And so, the range is from one to 3, but you are asked for some particular x prime, which is beyond this range. In that case it is also extra interpolation on the other side; that means, beyond the boundary that is known as extrapolation, all right? Right now, so, basically the concept of extrapolation and interpolation are always the same.

So, here what I try to will try to see is given some values of fx non-values of fx for some particular axis, how do you find out the value of fx for some intermediate value of x, that is the task of interpolation. Now the simplest possible way of interpolating. Now here I show I had shown a curve usually you can also start with a table like this, where some xs are given and some ys are, some x values are given.

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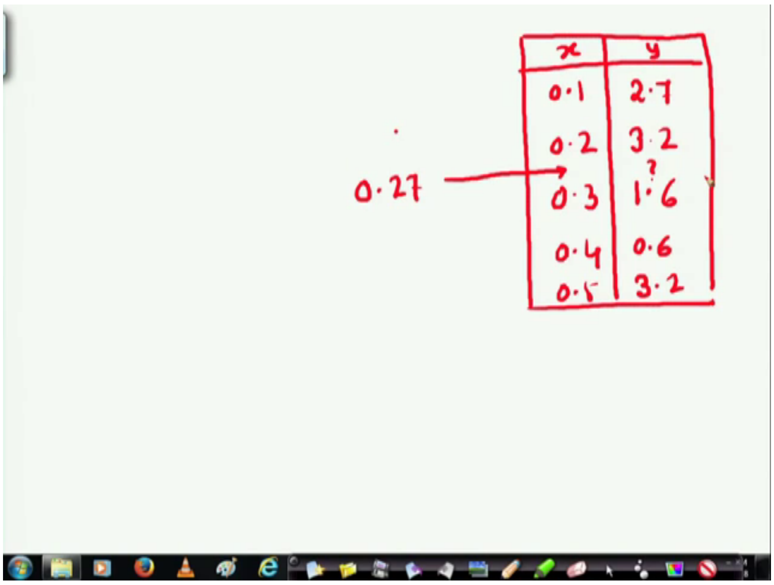

Say 0.1, 0.2, 0.3, 0.4, 0.5 and for each of them we have got some y value 2.7, 3.2, 1.6 come down again, say, 0.6, and again it goes up say 3.2.

Now, if this be the table, then my question is for interpolation, what is the value? What is the value for text which is 0.27? What is the value of this? So, that is the task of interpretation, now, obviously, a simplest possible thing is if I can fit in a line which is known as a linear interpolation.

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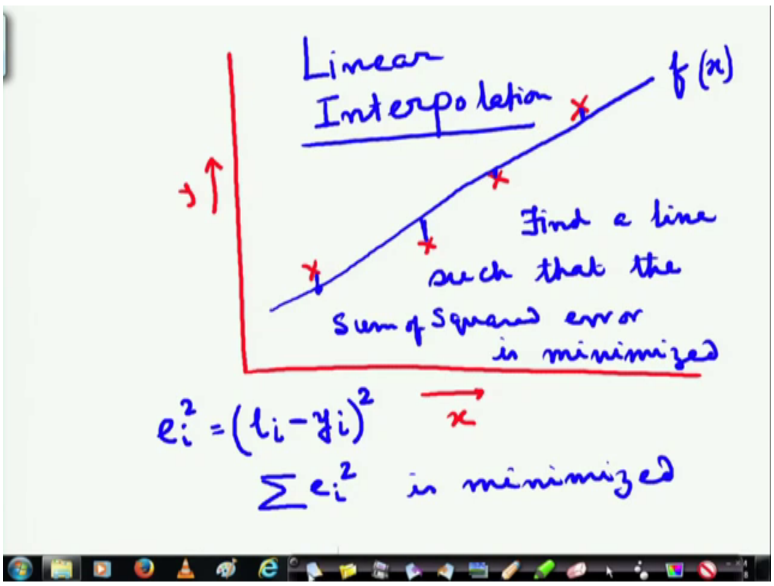

So, I have got some points here, all right? And I try to fit in a line in between them, or let me let me let me do it in a different way not exactly this line, because this line is matching 2 points, but not the others.

So, this is by y, and this is x. So, there could be some points like this, here something here, something here, something here like that equidistant points. Say this, and I try to fit in a line that is somehow I draw line like this. It is a straight line, although my drawing is a little curved, but it is a straight line so, I am drawing the straight line. So, one way is that I try to find out a line such that I minimize the error. So, of course, if I assume that this line is representing my function, then I can see there is some error here, there is some error here, there is some error here, there is some error here.

I could have fitted another line also I could have changed this line a little bit. One of the ways is to find the line that minimises the error, now it depends on how you define the error,. Now here you can see it is a positive error, it is a negative error, it is a positive error, like that it can go on. So, one way is to find out minimise find the line such that the least square , or of let us see less such that the squared sum of squared error is minimised. That can be one way; that means, what is an error error is ei when I take whatever my line is saying, let me call it li, but the line line is telling me the value, let me call it li and whatever is the actual value minus yi.

If I consider that to be the error or the other way now I so, you can see it is a positive or negative when I take the error of all these. So, that could be a sum of error. But here what I am saying is that, I can have the square of this error, and I try to find out a line such that ei square is minimised, ok. So, that I get a very good line. That suppose I get a very good line like this, then I will have I have the equation of that line. Now given 2 points it is very simple, right, let us take the simplest case first.

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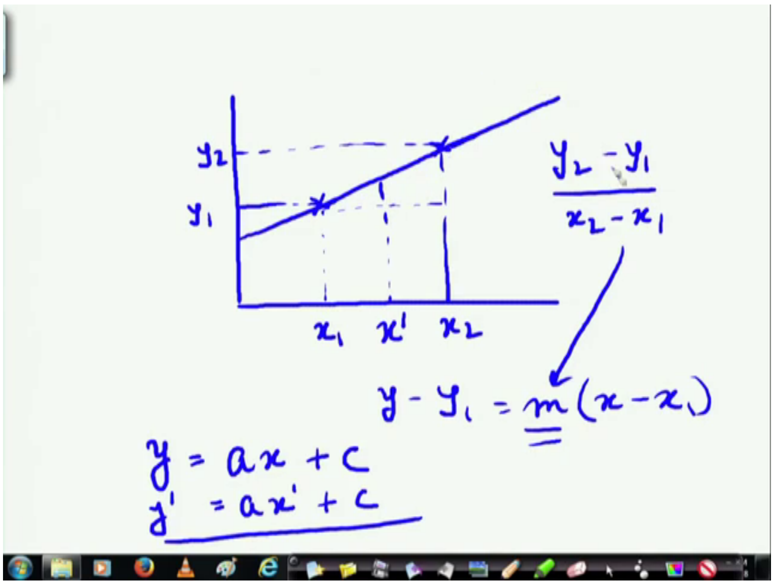

So, if there be only 2 points given, then I can certainly draw a unique line between these 2 points, right? And so, I can find out the equation of these. So, this is x or school level co ordinate geometry x 1 y 1 x 2 y 2. So, you can find out the slope of this line, right, you can find the slope of this line, which will be nothing but y 2 minus y 1 divided by x 2 minus x 1. So, once you get the slope, then you can find out the line. So, the school level equations like y minus y 1 is equal to m into x minus x 1, where m is this m is a slope.

So, you can find the line so, when you find the line, then your problem is solved. Because now given any particular, suppose you get a line of the equation a x plus c, c is this and you get the coefficients like this. Now given any x I give x prime then from that equation I can straight way find the value of y a x prime plus c, right? That straight forward, but the problem is that often it is not only 2 points, but the points are rather distributed as I was showing in the earlier one like this. Where I do not get an exact line; that is cutting through all the points. I have to try it I will try to minimise the error and I will choose such a line.

Now, when I choose such a line; that means, I have chosen an equation. And therefore, again from the linear equation, I can find out the value of y for any given x. So, this is this approach is known as linear interpolation; however, the linear interpolation often as you can see the error will be there. So, people try to find out more accurate solutions. So, we will look at another inter pro so, first of all we have seen what is linear interpolation, the basic idea is very simple we try to fit in the best possible straight line, through all the points. So, that the error is minimized, total error is minimized.

So, one way is to minimise the sum of the squares or there could be some other measures you can take for error, that should be minimised so, you get a line to your satisfaction. So, once you get a line to your satisfaction, you know the equation of the line. Therefore, given any x you can find the corresponding y, ok. Now fitting this line always, the there may be situations where the points are distributed in such a way for example, it can be the points are something like this, all right, is very difficult to find the line over here.

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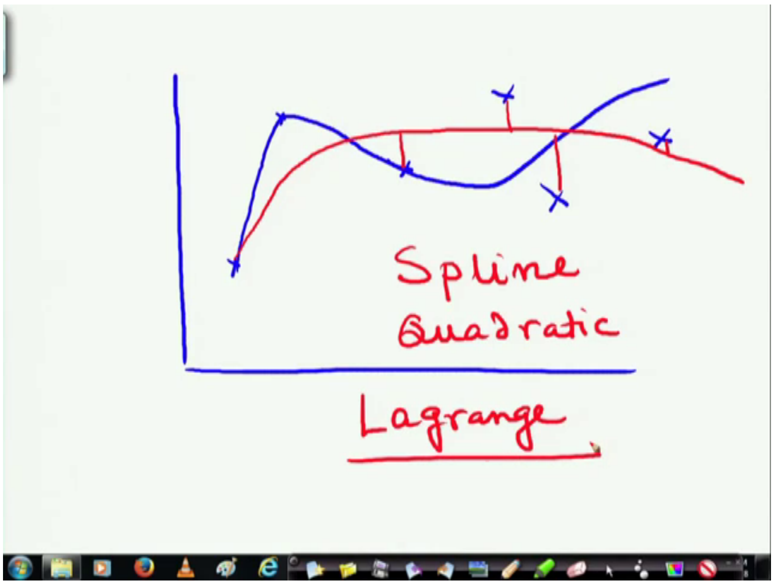

So, often I I would like to find out a curve, or maybe another curve even better, that can be that goes through these points like this, where I have got errors, but I am trying to fit in a curve. So, that is not a line, this is not a linear interpolation, where are further extensions like quadratic interpolation, spline interpolation, cubic spline and all those . So, it is better to know the names, spline interpolation, quadratic interpolation, where I want to fit in a quadratic curve et cetera, et cetera.

But today we will discuss another interpolation technique which is known as LaGrange interpolation, in the name of the mathematician who invented it. So, let us have a look at the langrage interpolation.

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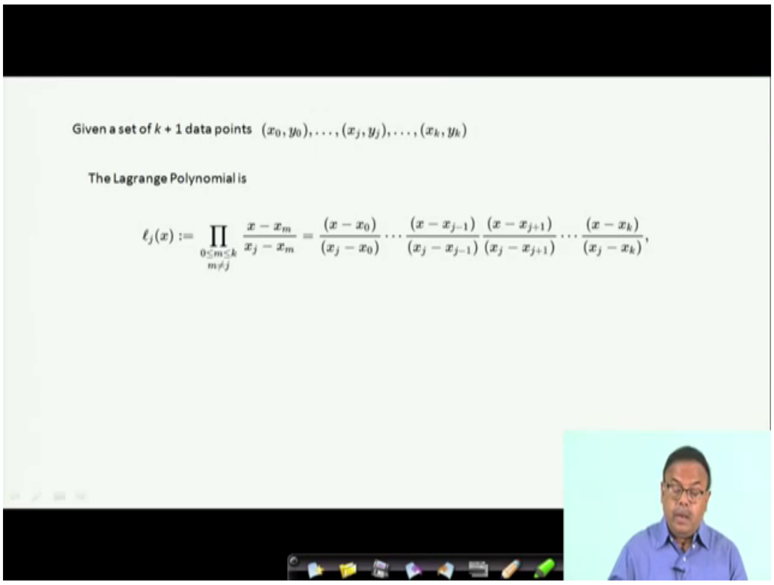

Interpolation as you have understood is given a set of k plus 1 data points, x 0, y 0, x x 1, y 1, x 2, y 2 like that up to x k y k, so many data points are given. The LaGrange polynomial is this that is multiplication of so, I have to find out for a particular x, x minus xn divided by xj minus xm, where m is varying from 0 to k for all these m is varying. So, x 0, x 1, x 2, x 3 for all these I will do that and xj minus xm, where j is for a particular LaGrange interpolation.

So, you see it is for l 1, l 1 x will be, x minus x 0 by x 1 minus 0 x minus x 0 x 1 minus x 0, like that it will go on, all right, so, this is the LaGrange polynomial. So, using this polynomial we can see, let us see how it works, just to give you an example.

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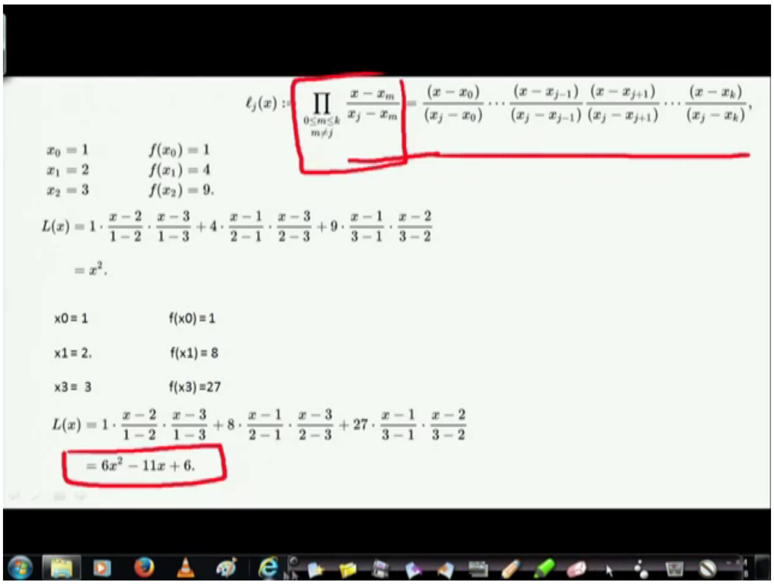

I have reproduced that polynomial, that I show I had shown in the earlier slide here again, you can say this, right? Now suppose I have got the points given look here, the points at a given are for x 0 for x x value 1, fx is 1. For x value 2, fx is 4, 3, this 9 so, I have to fit in a polynomial. So, I my polynomial is capital lx, now each term of this is one small lx. So, this is l 1 x, this is l 2 x, this is l 3 x, like that ok. So, you see the value of x is 1, 1 times x minus x m. M will be something that would be not the same as j.

So, it will be I have got the 3 points so, I will not take this 1, x minus 2 divided by 1 minus 2 times the second term. X minus 3 divided by 1 minus 3 plus 4 now I am taking this one. So, it is 4 and, in that case, I leave out 2, my x j is now 2. So, x minus 1, I will not take x minus 2 here. Because look at this m not equal to j. So, for the other x's, I have selected this row. So, I will only concentrate on the other rows, x minus 1 divided by 2 minus 1 into x minus 3 x minus 3 the other row divided by 2 minus 3. Here I take 9 this value and I am taking this row.

Now, so, now I am taking this row, right? I have taken 9 so, x minus 1 by 3 minus 1, x minus 2 by 3 minus 2. If I solve this I am getting a polynomial x square. Possibly you can see that is a perfect fit with the given data points. X 1, x fx is x square, that is 1, 2 it is x square 4, 3 it is x squared 9. So, it is in this case, there is no it is absolutely a perfect fit . So, this is how I get the polynomial, all right? Let us try once again. For these data value, for x 0 it is 1, x 0 1 the fx is 1, for x equal to 2 it is 8, for x equal to 3 it is 27.

So, obviously, you can see it is no longer x square, all right? It is not also x cube, because here it is cube, here it is cube, it is matching the cube it is actually x cube, right? But let us try to fit in a polynomial here, right. So, what is that polynomial using the LaGrange method, I will first check one. So, I am taking first row taking this value one here, then x minus 2, 1 minus 2, all right? Times x minus 3 3 1 minus 3 plus, next time I take the second row 8 times x minus 1 divided by 2 minus 1, because here it is the value is 2. In that way I for this it will be for the third row it will be again x minus 1 3 minus 1 x minus 2 6 minus 2.

By doing this I am not getting x cube which would be a perfect fit, but I am getting a close enough polynomial 6 x square minus 11 x plus 6, which will be approximating the cube function very closely you can see that. So, I have not it is not that LaGrange polynomial I will always give you the perfect fit, but it gives you a very close fit. So, this is what is known as LaGrange interpolation. So, if I write want to write a program and try to solve it, I left to implement a function, that in I have to write a function that will implement this or this, right?

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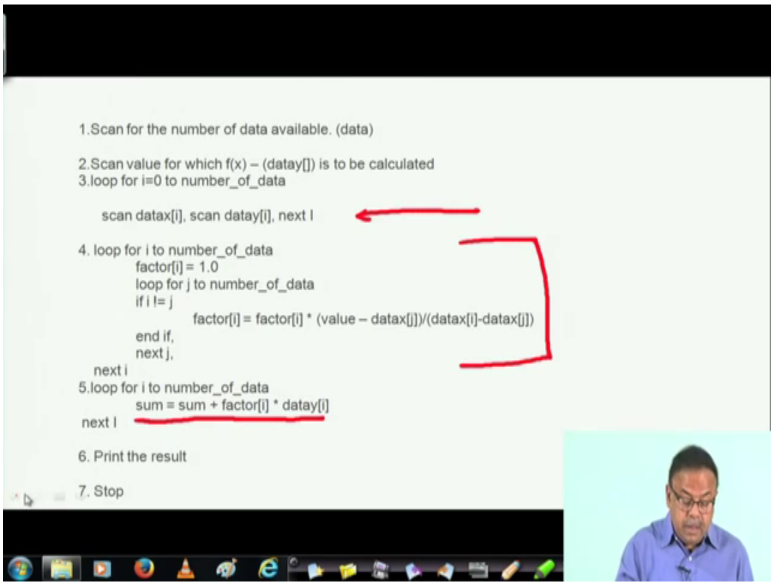

Let us see how we can do that. So, here is an algorithm, scan the number of data available x and y values. Now so, you take the here is the how many data points? This is telling how many data points, you had you are finding the data now here loop for i equal to number of data factor, then here you are computing this. Here you see what we are trying to do? At this point we have just rate the data, and here in a loop we are trying to compute, here we are finding the sum.

But here in the loop, I am finding the factor so, if you go to the earlier the slide, we will find that I have got these are my factors, right? I am finding this factors, each of these factors, I am multiplying them, e this is a factor, this whole thing is a factor, and then I multiply that with the coefficient and add them up. So, I am doing the summation at the end. So, here you see that I am taking the factor then I am taking the, whatever is the value this you can this part you can yourself compute, right?

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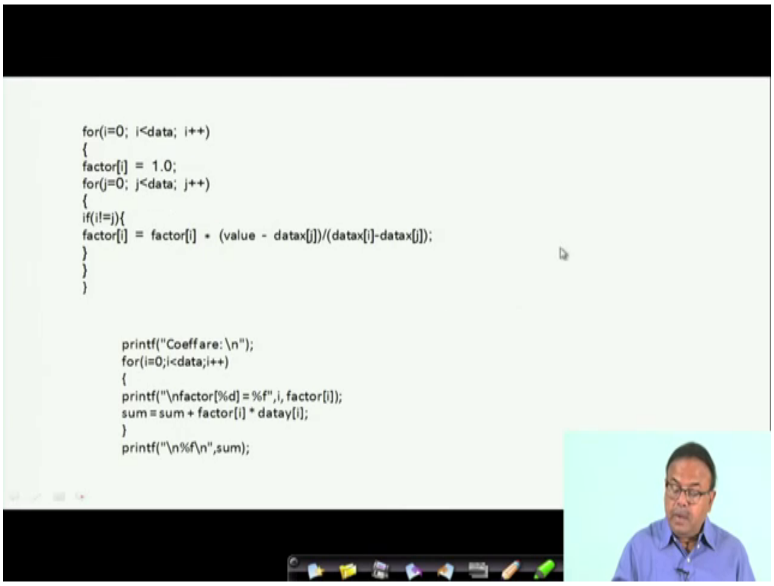

The key part is computation of the factor part, where I am showing this part, you can see that for I 0 to data i plus plus I am initially a factor is one.

Because I am multiplying that, then if i is not equal to j, I will take factor times the value minus the data. So, and then ultimately, I am printing the coefficient. So, you can translate that into in the form of a c function. That is the basic idea of LaGrange interpolation, you can similarly write programs for linear interpolation, and other interpretation techniques. So, the point of this course is not to because I assume that you now know how to do programming, and these are simple loop type of programming.

So, you will be able to do that. So, whenever you want to solve apply some numerical methods, you will have to first look at the technique then try to find out the expression, and you will have to write an algorithm so that you can solve it ok. In the next lecture we will discuss 2 other very interesting techniques for integration and differential equation solver.

Thank you.