Problem Solving through Programming in C
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Lecture-49
Data Representation

Today, we will be looking at some real applications engineering applications of programming. And we will see how whatever we have learnt till now can be applied to solve some interesting problems like solving equations. One very common thing that we need in any engineering or science is to represent data right, one way of representing data is by in the form of a table right.

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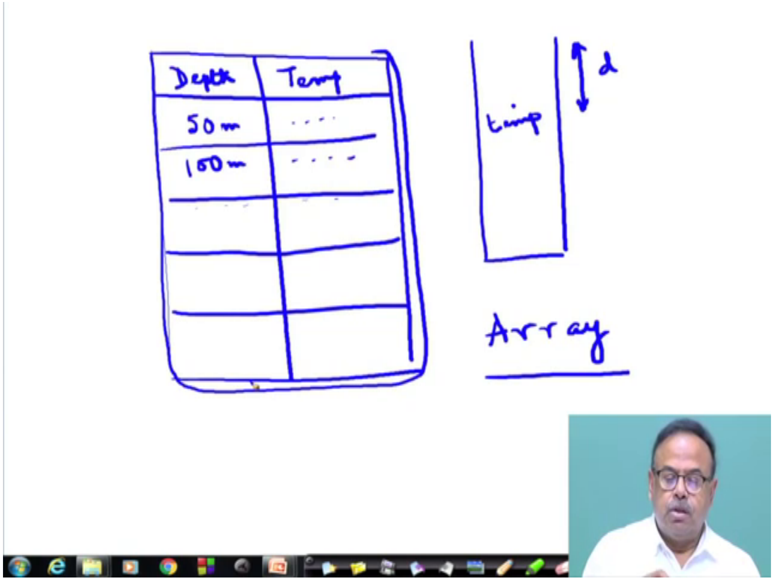

So, I can always represent some data as a table where there can be say on one column it maybe height or depth and maybe as we go down, we go through some point and at different depths, we find we put some sensor and get the temperature.

So, that I can represent in the form of whatever is a depth 50 meters some temperature, 100 meters some temperature etcetera. So, if we need to represent any data in this form we know how to do it? We know that we can as it looks here we can immediately say we will be representing it as array, whether I will be using it using represent them using as 1 array 1 2 dimensional array or two different array will depend on the format of the data the type of the data.

For example if the depth is integer and the temperature is real, then up to the knowledge that we have acquired till now will be representing them in the form of 2 arrays 1 is an integer array another is a separate floating point array alright. So, that is one way one aspect one way of representing data as we have done also in the case of students roll number and marks earlier. Another very interesting thing an important thing is representing graphs.

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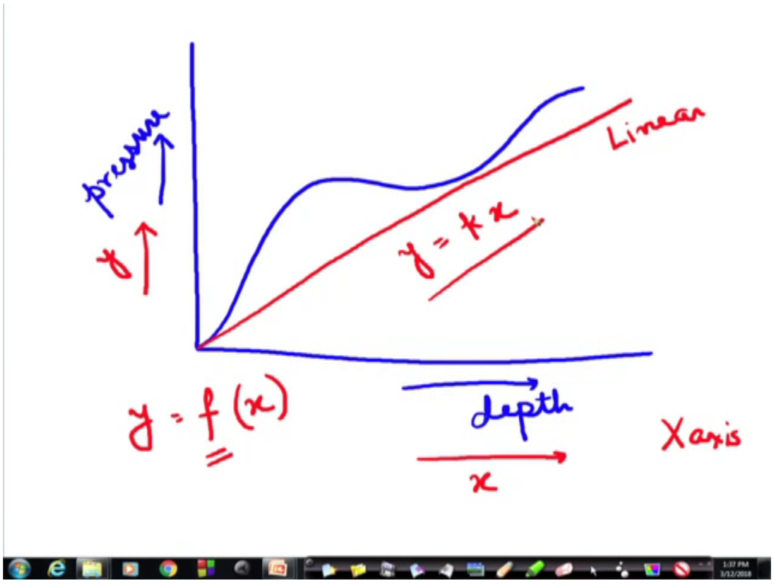

For example, I want to represent a graph like this say maybe this is some this is say again some pressure and this is depth and as this graph shows that it is not increasing in a uniform way with depth right. There could be some graph, which would be just linear this is we call it linear, where along with depth this increases in a linear fashion here it is non-linear ok.

So, the question is how do we represent this sort of graph? Now you know that this graph I can also state if I say this is the X axis and my independent variable is x and the dependent variable is y, then this graph essentially represents a function, which is y is a function of x. Now depending on the nature of variation of the dependent variable with the independent variable, the nature of the function will vary this one will be a linear function right.

For example it can be here y equals to some k x all right, this one is more complex I could have had a quadratic function also this is a linear function. Now the question that we would like to first address is it is. So, very nice to draw a picture on a piece of paper; however, a computer will not be able to just interpret this picture as we do then in that case how would we represent this graph in a computer. So, let us do it again.

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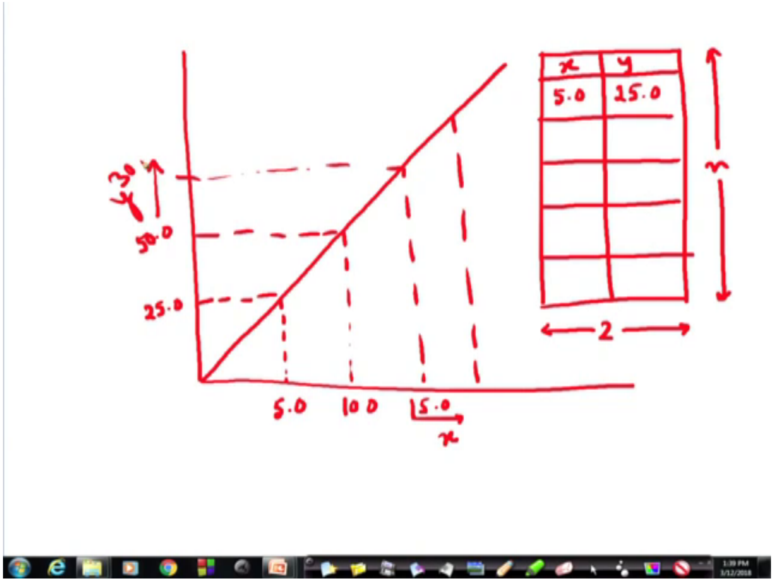

So, I have got a graph let us say a linear graph and I can say that for different values of x what is the corresponding value of y all right now this is a linear 1. So, I can represent them if x and y are the same type I can also represent them in the form of a table; that means, a 2 dimensional array, where 1 side, 1 dimension is the variation of x on 1 column we have got the x and the corresponding y all right.

And what will be the number of rows the number of data points of x that we take will be the size of this array the not the size of this array the number of rows how many n and this one is fixed to be 2 I can suppose that this is 5.0 and this is 25. So, I have 5 here I have 25 here this array is of type float now here it is ten. So, it is 50 it may be 15.

So, this 1 will be 30, because I am talking of a linear curve it need not be linear all the time.

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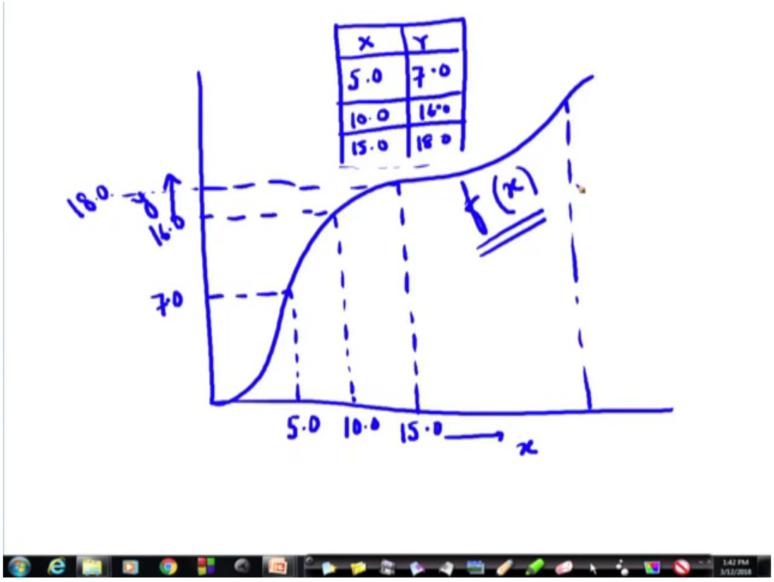

So, I can also have a curve like this, I can also have a curve like this say something like this and here again I have got the y and here I have got the x. So, with a particular value of x say 5 maybe this 1 is initially. Let us say it is it has risen a little sharp so, maybe it is 7 and if I go here. Now, please note that we usually take the samples at equal distances. So, I take it as 10, but here it will not be double it is becoming a little more than that. So, 7 it may be 16 I go here now you see the slope of this curve has reduced right the gradient has reduced.

So, at 15 it will not increase that much it will be 18 say this is 18, because it is flattening out still flattening out and here there is a sharp change etcetera right. So, this one also I can represent in the form of a table like say I have a 2 dimensional table, where I have got 5 this is my x and this is my y. So, 5 corresponding to that 7 first row that is the first point I could have taken 0 0 also that would be another one.

So, 10 corresponding to that 16 then 15 corresponding to that 18 so, in that way it could go on; that means, whatever is this function f x a function can be represented as an array 2 dimensional array so, that all as a table. So, this we have to remember either as 1 array or as 2 arrays, because if this was integer this was float then I would have needed 2 different areas right it can be. So, here is a function now let us look at another aspect sometimes some data.

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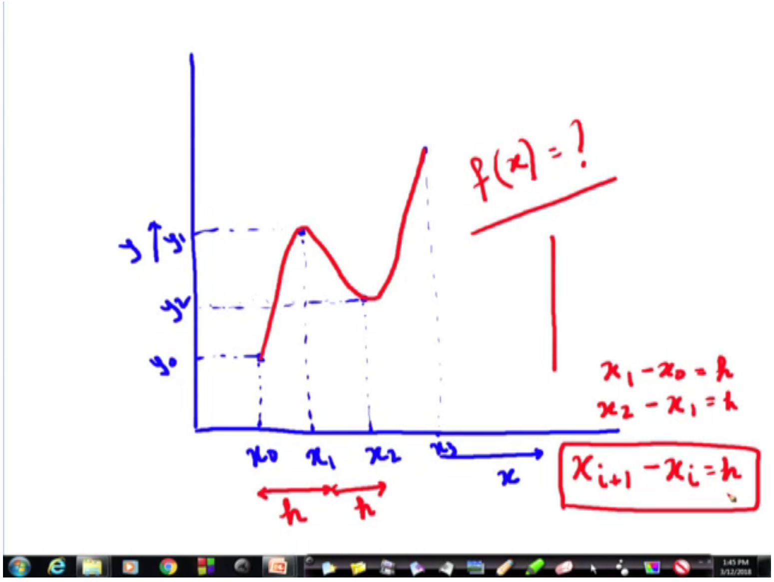

I do not have the function, but I have got some data right. So, here x and here is y and I have got some data points for x equal to this is the y point alright. So, if I call it x 0 this is y 0, next one could be here that x 1 it is y 1 here maybe it has come down sorry x 2 it is y 2 this will be y 2. So, I do not have a function right now, but I have got different data points and I have to find out the function. So, that is also another very interesting thing finding out the function that can represent this distribution of data points might be here again there is x 3 and it is again gone up here.

So, it is not a straight line if I want to fit a curve to meet that what how that curve look likes the curve will be something like this right. And I have to find out what function is this curve what is the function I do not know as it that is another challenging point all right.

So, first of all graphs or data points graphs can be represented as table and the distribution of data points can also be represented as table, because here in this scenario I do not have a function, but certainly I have got the x and y values known to me. Therefore, I can represent that as a table another point to note here is that usually this independent variable the sampling points that I am taking our usual equidistant.

So, I can say x 1 minus x 0 is some value h x 2 minus x 1 will be the same h usually we represent that in order to see for an incremental fixed incremental increase of the independent variable, how much does the dependent variable vary ok. So, I can say in general x i plus 1 minus x i is equal to h right.

So, this is typically how we write now we have got a number of problems to solve using data.

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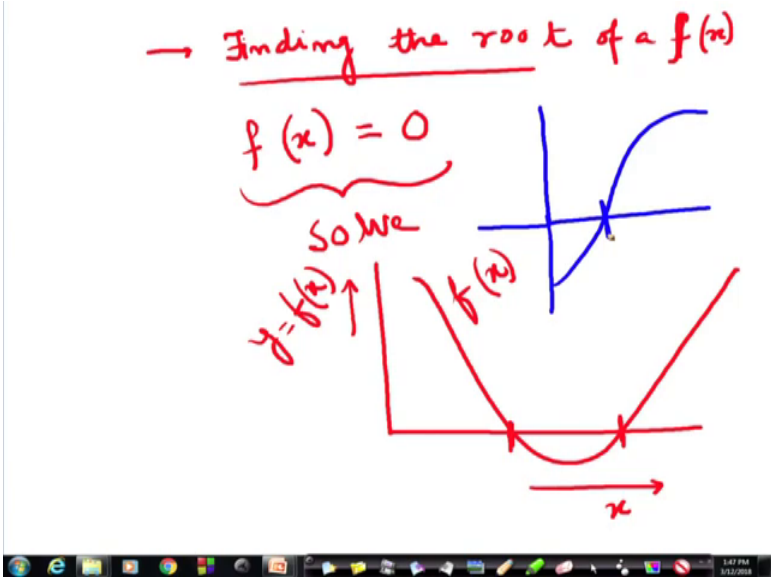

So, for example, let us start with one problem finding the root of a function finding the root of a function some function fx. Now finding the root means essentially it is solving the equation. So, if I have got a function fx, then my equation is fx equals 0 and I want to solve this right and I want to find out that root at least 1 root.

Now what do I mean by the root the root is that if there is a function like this say there is a function like this. So, for what value of x f x equal to 0 is the equation. So, this 1 is fx y or fx. So, we are to find out for what value of x for which value of x f x is 0. So, that is the root now this particular function if this be an fx, then it has got 2 distinct roots right another function could be something like this could be just like this where I have got 1 root this is the point where f x equal to 0 all right.

So, one of the major problems is finding out the root of a function or we will say in general the root of a polynomial why are we saying a polynomial, because you know that any function say fx is equal to 3 x squared plus 2 x plus 3 is a polynomial of degree 2 right or this is f 1 x.

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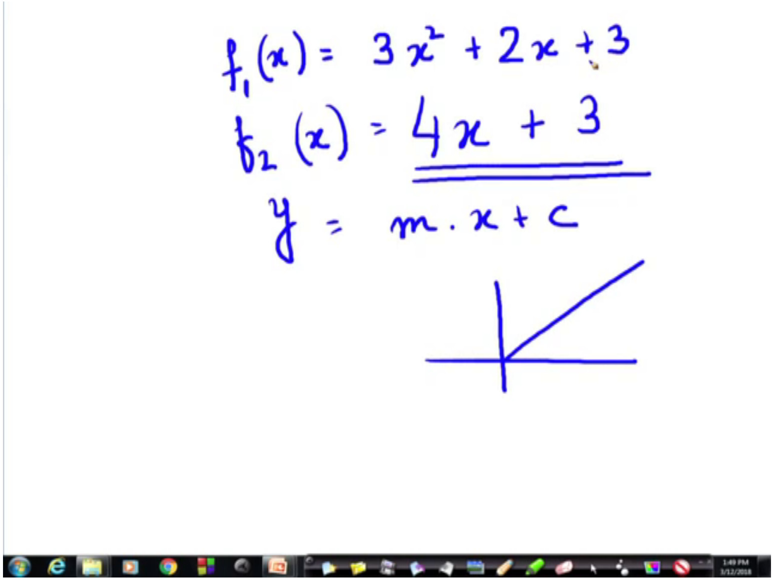

I could have another f 2 x, as 4 x this is a polynomial of the 4 x plus 3 maybe all right. Now anybody who remembers cool coordinate geometry this is a linear equation, because it is a polynomial of degree 1; that means, if we say this to be y then this is y equals m x plus c. So, m this 4 is nothing, but the slope of this straight line.

So, there will be some slope of this straight line this is linear all right. This one is not linear this quadratic ok. So, I can have the different functions can be written as a polynomial and when the polynomial is equated to 0 it becomes an equation and we want to solve that equation. Problem number 2 is interpolating a function I will describe what interpolation is a little later ok,

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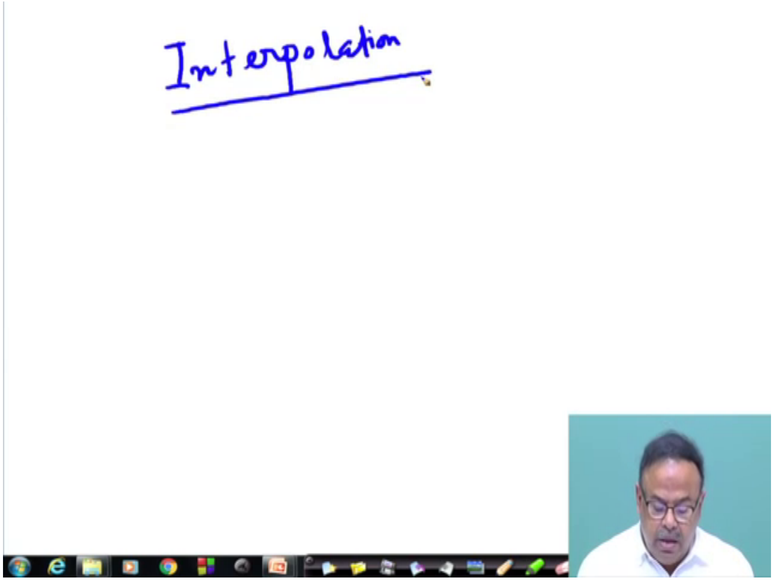

But, before that before that let us try to see a very simple way of finding the root of a function; say I have got a function x and y alright and a function is something like this.

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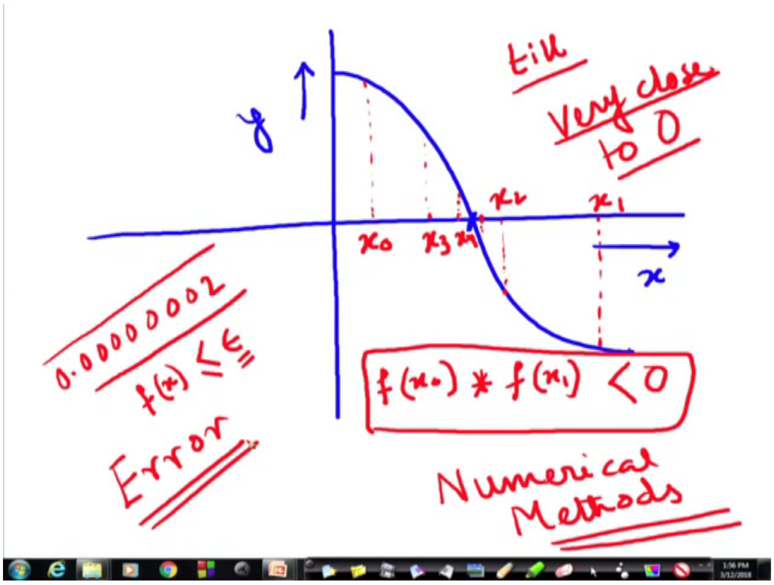

Now, I want to find out this point what is the value of x this value ok, for which this y is 0. Now the method that we will be talking about is known as the bisection method this is a very very common and interesting approach you have seen in binary search also we have partitioned the array into 2 halves and then went to 1 half very similar to that let us see how the bisection method tries to find out the root. First we start finding 2 arbitrary points say x 0 and x 1, such that fx 0 and fx 1 are of different signs. So, here you can see this is plus this is minus right, f x 1 is the y value corresponding to x 1. So, if the y value of x 0 and x 1 are opposite then it is immediately understood that the root must lie somewhere in between ok.

So, we will then try to find out the midpoint of x 0 and x 1 suppose the midpoint of x 0 and x 1 is x 2. So, if x 2 fx 2 and f x 0 are out of opposite sign, then I will again find the then the root must be I have I have reduced my space right. So, it is between x 2 and x 0. So, I again find out the value the midpoint of x 2 ending 0 suppose that midpoint is here, suppose this is x 3 and these 2 fx 3 and fx 2 are of opposite signs. Therefore, I will find the mid midpoint of these 2. So, suppose the midpoint of these 2 is this x 4 here.

Now still this 1 and this 1 out of opposite sides therefore, I will try to find out the midpoint of these 2. So, I will come somewhere here all right in that way we approach till what till we find that the value of y is very close to 0 the value of y is very close to 0, very close to 0. Why am I saying very close to 0 not exactly 0 there are reasons for that the reason say, reasons we are coming to that a computer works with finite representation of numbers.

So, we may not get exact 0, but suppose I get this and then I can assume that to be 0 because it is very small and often we call it that that the value of fx at that point is less than equal to some very small value epsilon that will decide a prior ok.

So; however so, you have seen this approach now 1 thing that you can quickly think of that how would I know that these 2 are of opposite signs fx 1 and fx 0 are of opposite signs, if I take the product of this if I take the product of this then; obviously, if they are opposite signs the product will be negative that is less than 0.

So, at every point we check whether they are less than 0. If there is no root suppose it goes like this then I will not find any point where they are of opposite signs ok. So, that is the basic idea of bisection method. Now, when I carry it out through a computer such attempts to solve such problems known as programming numerical methods.

And we will start with a some representative relatively easy 1 examples of those some numerical methods one of them is finding the root of a function, root of a polynomial root of a polynomial ok. Now, while doing these numerical methods we always encounter errors and our algorithm will be better, if the error is less. Now what do I mean by error say, I am computing 4 by 3 all right. What is the result is 1 point correct result most accurate absolutely correct result do you know that no 1.3 3 3 3 goes on.

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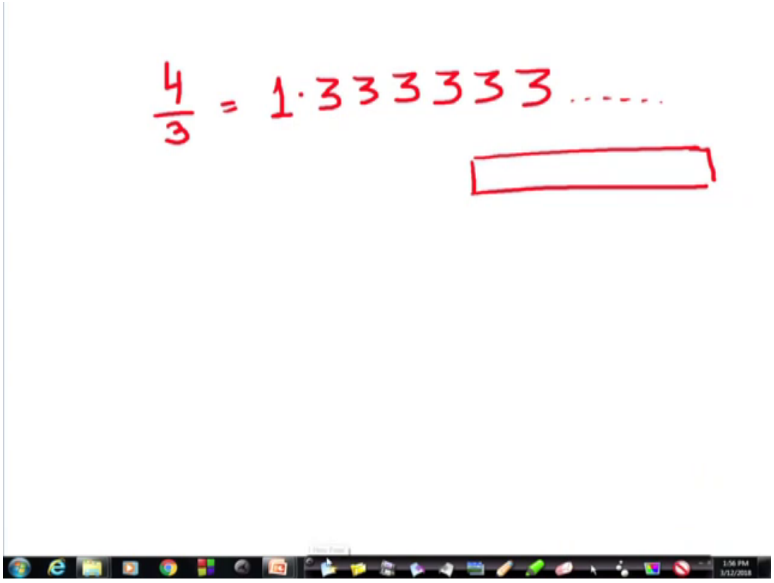

Now, how much my computer you know our computers have got some storage locations of some 8 by 8 bits or 16 bits like that.

So, depending on that I have got a finite capacity to store the data therefore, I may represent this as says 1.3 3 3 3, I just start with say 6 6 digits all right, 6 7 8; I can store 8 digits maximum 6 after decimal 1 decimal and this so, 6 places.

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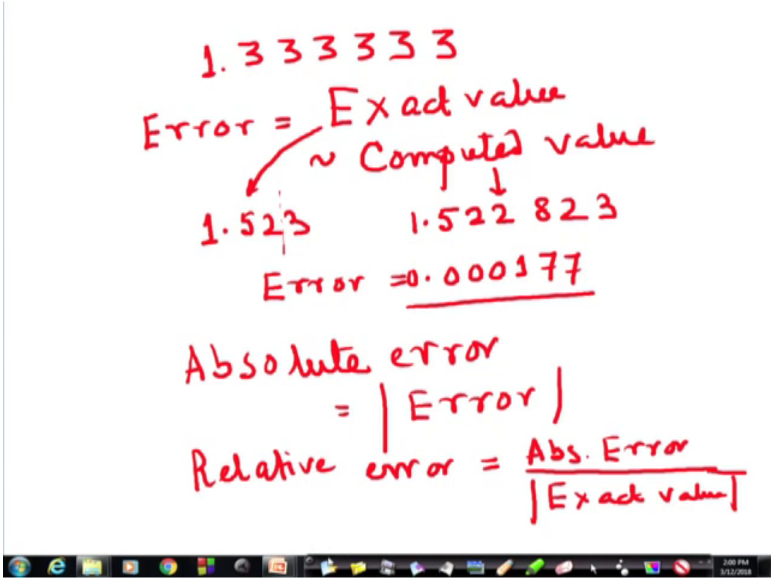

Therefore, I am actual thing was much more. So, I am encountering I am actually committing some error. So, what is the error in computation is the exact value, the difference of the exact value, by this m and the computed value, it can be positive or negative all right. Suppose the result is actually 1.5 2 3 that is the exact value suppose.

And during my computation I got 1.5 2 2 8 2 3 then the error is the difference between these 2 and the error is therefore, if I subtract this it will be 0.0 0.0 0 0 1 7 7 that is the error now the smaller this error is more accurate my result is I think it is very clear. Now there are 2 types of errors also round off error for example, here I could have rounded it off as 5 1.5 2 2 5 1.5 2 3.

So, actual suppose the actual thing was this and I rounded it to 1.5 2 by approximating if I go by 2 bits, that is one type of error, other type of error is truncation error this was there I have just dropped this 1.5 2, because I could not store more.

So, there are two types of errors right. So, truncation error and round off error. Another term that you need to know is the absolute error absolute error is nothing, but the absolute value of the error ok. And relative error is absolute error divided by exact value the absolute of the exact value alright absolutely here is the errors absolute part and by the exact value and percentage error will be relative error times 1000 relative error times 1000.

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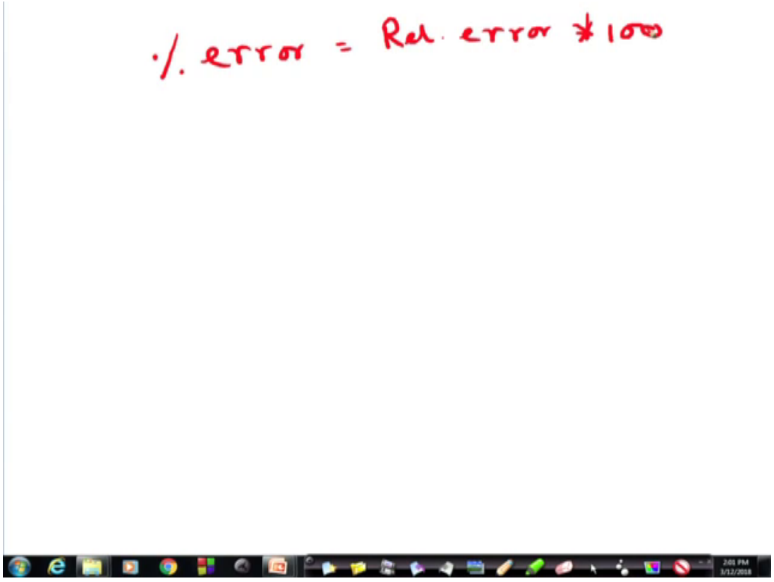

So, we can say that percentage error is relative error in to sorry into 100 what I am saying.

So, now just look at this have a quick look at these definitions absolute error means the absolute value of the error relative error is absolute error divided by the exact value and then we come to the percentage error, which is relative error times 100 ok.

Now, another important thing that we have to consider is accumulated error, but that will consider later next in the next lecture we will move. So, whatever we do the way we do the computation we must be very careful about the algorithm be such that it is it does not accommodate too much error.

However, discussing about error analysis in general is beyond the scope of this course right now. So, we will in the next lecture start with the bisection method as I have explained and then move to some other methods.