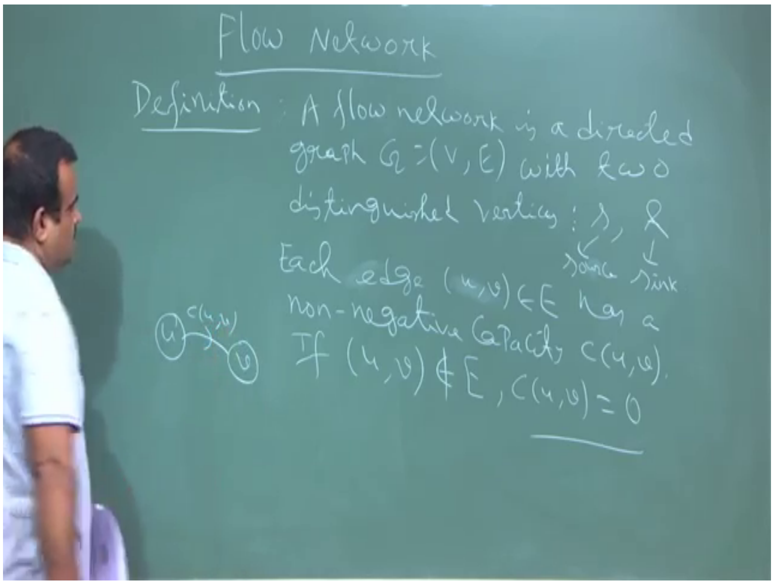
An Introduction to Algorithms

Prof. Sourav Mukhopadhyay
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture – 54
Network Flow

So we talk about network flow or flow network. So, what is the problem? Problem is we are given a graph.

(Refer Slide Time: 00:28)

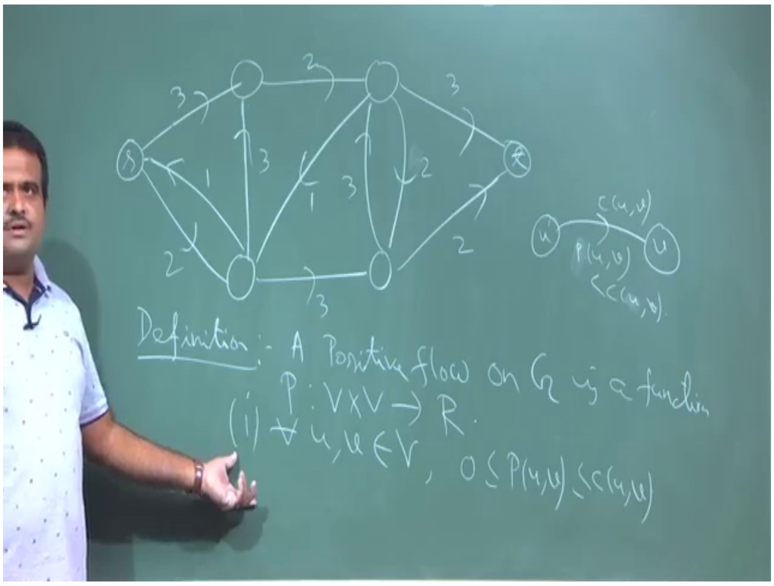


So, this Is the definition. So, this is called if flow network is a directed graph or diagraph directed graph G v comma e with 2 vertex distinct vertex which is called one is source other one is called sink with 2 distinct vertex distinguish vertex vertices s and t this is called source and this is called sink and each edge there is a weight associated with each edeg and that is called capacity.

So, each edge u v has a nonnegative capacity. So, weight some non negative non negative capacity that is c u comma v. And between if there is 2 vertex where there is no h then we different the capacity to be 0. So, if u comma v is not to edge then we defined as c u v this is the convention there is no edge. So, there is no question of capacity.

So, this is the this is called flow network or flow graph. So, so basically we will take some. So, basically each node. So, if you have u v. So, this is a directed graph. So, if there is a edge from u to v then there is a capacity the value a integer non negative value. So, will take an example. So, will take such an example of such a flow network. So, it is basically a directed graph. So, let us draw a directed graph.

(Refer Slide Time: 03:24)



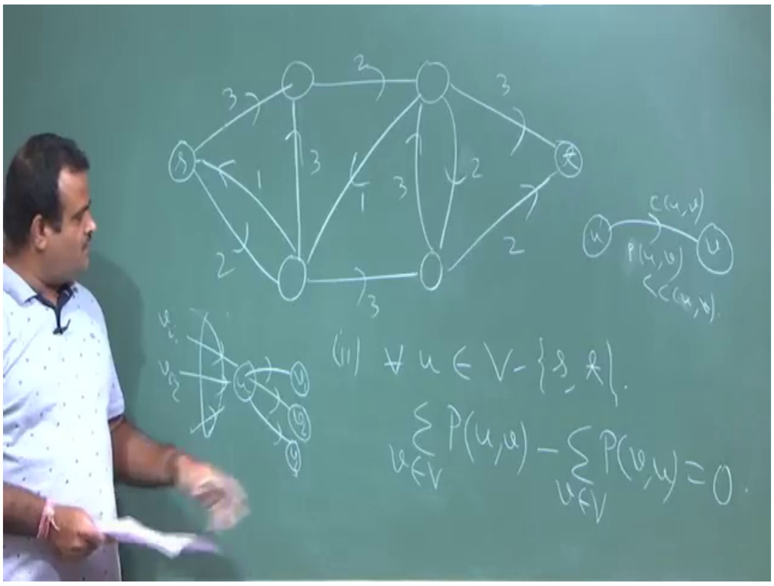
So, this is the source and we have a edges these at the direction this is the directed graph these are the vertices these are the edges, we have a then we have here then we have a edge over here we have a edge over here ok.

So, this is the sink. This is the source that is the sink. And this is the directed graph now we have to put the capacity. So, suppose the capacities are say 3 these are all non negative integer 2 1 3 1 2 3 3 2 3 2 suppose these are the 2 3 2 suppose these are the capacity. So now defined a network positive flow. So, this is another definition. So, a positive flow a positive flow on G on this network G is a function p which is basically v cross v to r, satisfying the conditions 3 conditions.

First condition is called capacity constant. So, we want to we want to flow some current. So, say this is a this is the current these are the capacity of each where now we want to flow some current. Now obviously, if the capacity is 3 then we cannot flow the current more than 3. So, that is the first condition. So, this is called capacity constant. So, for all u v belongs to v such that So, this flow this positive flow was p positive and it must be less than c u v.

So, if we have 2 vertex in v, if this is u v now this is c u v, now you want to have a p u v, we want to defined a p u v positive flow. So, such that So, if the capacity So, such that it should be less than the capacity because we cannot send more current then the capacity. So, that is the idea. So, this is the capacity constant. So, you cannot say more than the capacity. So now, the second condition is second condition is called flow conservation.

(Refer Slide Time: 07:07)



So, it is basically number 2 it is basically taking telling if you take any vertex other than sink and source.

So, flow conservation should be 0. So that means, we are not storing any current we just passing the current. So, in other words suppose this is u, this is a node which is not source and not sink. Now there are some edges coming over here and there are some edges going out to there. So, there are So, this is coming flow. This is the current coming to u and this is the. So, there are say more edge this is the current going from u total. So, this difference should be 0. So that means, whatever current it is taking it should distribute.

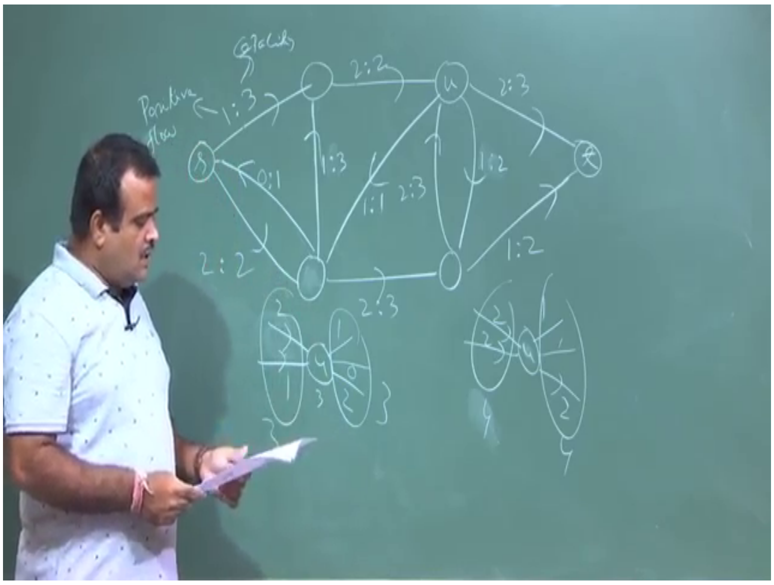
So, it is basically it is not holding any current. So, it is just a distributed sort or media. I mean it is it is just a middle man it just So, this we have to write in a mathematical form. So, this is basically telling for all u which is not in not a source or sink summation of p of u comma v. So, this is basically v is belongs to. So, this is the this is basically the current which is coming from. So, u yeah, So this is basically current which are going from u to v this is v 1 v 2 v 3 like this.

So, these are the all vertices which are which is just basically out going vertices this sum. Then minus the sum of a capacity v comma u this is. So that means, this telling this is the all the vertices v i 1 v I 2 which is basically. So, this is this is basically sum of all this. So, whatever current we are having you have passing that. So, these sum should be this difference would be 0. So, flow conservation. So, we are not any middle nodes other than source and sink are not conserving any current I mean it is just passing all the currents.

So, this is sort of kirchhoffs current law. So, whatever current is coming it is just going out. So, in that holding any current in the middle point. So, this is the this is the this is the definition of positive flow if the there are 2 condition, one is the flow the p of u v should not be more than the capacity and in the middle not we should not have any conservation of the current. Now let us take an example of the positive flow on this network. So, this is a given network flow network where capacities are given and this is the source and that is the sink.

So now we want to have a positive flow on this network, which satisfy this 2 condition. So, let us try to have that. So, this is basically capacity.

(Refer Slide Time: 11:01)



So, you can Have something like 1, 1 flow on this then 2 is the capacity we can have at most 2 flow this is the 0 then this is 3. So, we can have 1. So, this is the basically positive flow, this is basically positive flow and this is the capacity. So, our flow must be capacity of that direction. So, our flow must be less than that capacity.

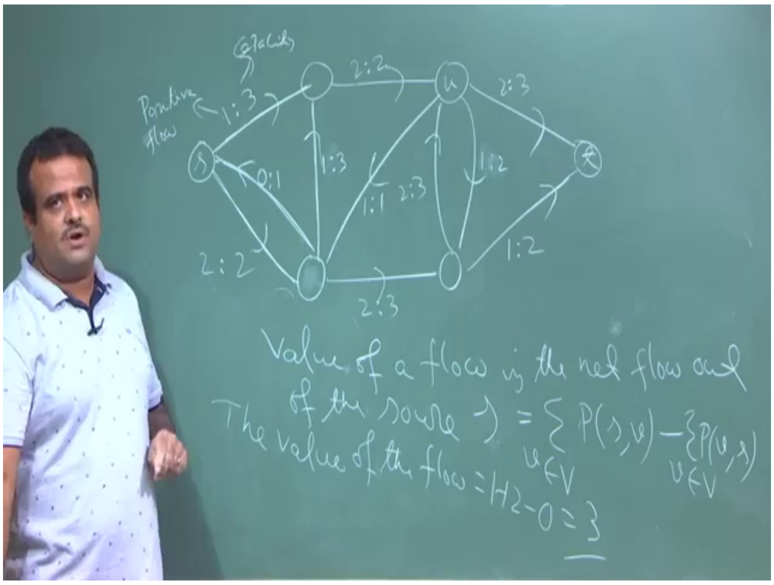
This is sort of current passing on this wire. Now let us give some. So, on the other edges. So, this is say 2 this is say 1, this is say also 2 this is say 1, and this is 2 and this is 1. So, this is satisfying first condition. Everywhere if you check every node suppose this is u, now if this is u and this is v. So, every node the capacity the flow positive flow is less than the capacity. So, that first condition is satisfying.

Then the second condition is if you have to take. So, the consumption current consumption. So, that consumption flow consumption must be 0 for any intermediate node which is not s and t suppose for example, this node. So, this node must what is the flow consumption? So, this node this is the flow coming. So, 2 2 is coming flow and then 2 and then this is another flow is coming 3. So, basically. So, this is u So, what are the flow coming. So, 2 this is 1. So, total flow is consume s o 3.

Now, any other node is incoming node a incoming vertex h node. And the going is basically this one and this one and this one. So, this is 0 this is 1 this is 2. So, this sum is 3 this sum is 3. So, no current consumption. So, kirchhoffs law satisfy that is second condition is satisfied. So, this we can verify form any other node suppose this node, for this node suppose this is our u for this node what you have how many are incoming? So, this is u incoming is this 2 this 2, and another 2 this 1 and no more. So, 4 total 4 and what is the outgoing? Outgoing is this one 1 and this one 1 and this one 2. So, 4 this is 4 this is 4.

So, incoming and outgoing flow positive I mean flow is said. So, this is said we can verify for other vertices also. So, so this is the this is a positive flow, this is a positive flow now we want to defined the value of this flow, what is the value of a flow?

(Refer Slide Time: 14:51)



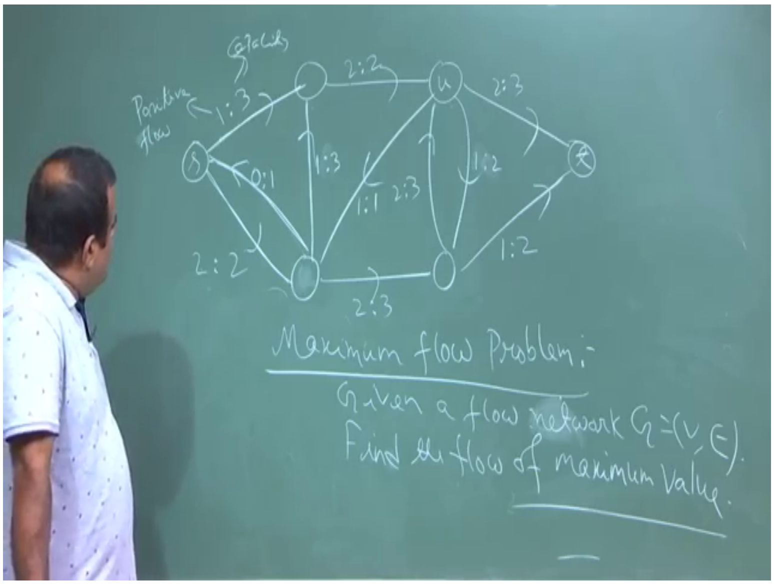
So, Value of a flow value of a positive flow is the net flow which is outgoing from s. So, this value is basically is the net flow out of the source is, So that means So, what is the net flow out of the source So that means, this is basically all the outgoing vertices outgoing edges from s. So, this is basically p of s comma v minus if there are any incoming edge from to s minus summation of p of v comma s. So, this is the net flow.

So, p of s comma v minus p of. So now, for example, here. So, what is the net flow? What is the value of the flow? So, value of the flow is. So, sorry not net flow value of the flow. So, these are the flow going out this one and this one. And this is the flow coming in, but this flow value is 0 and this is 1 this is 2 3. So, value of this flow is 3. So, the value of this flow is basically 1 plus 2 minus 0 3. So, this is the value of the flow. Now is this is the maximum flow? And this is the value of the flow now this flow. So, what is the flow at t? What is the value of the flow at t?

So, here So, this is 2 this is 3. So, we have flow at t is 1. So, whatever current we have pass here that is we are consume here. So that means, and in between they are basically the intermediate node they are just passing the current not holding any current in the in their. So, which is basically the. So now, now we want to defined what is the max flow. Is this the maximum current we can flow in this network? That is the question. Is the 3 is the maximum flow maximum positive flow we can pass into this network?

So, that is called max flow problem. So, that problem is referred as max flow problem.

(Refer Slide Time: 18:01)



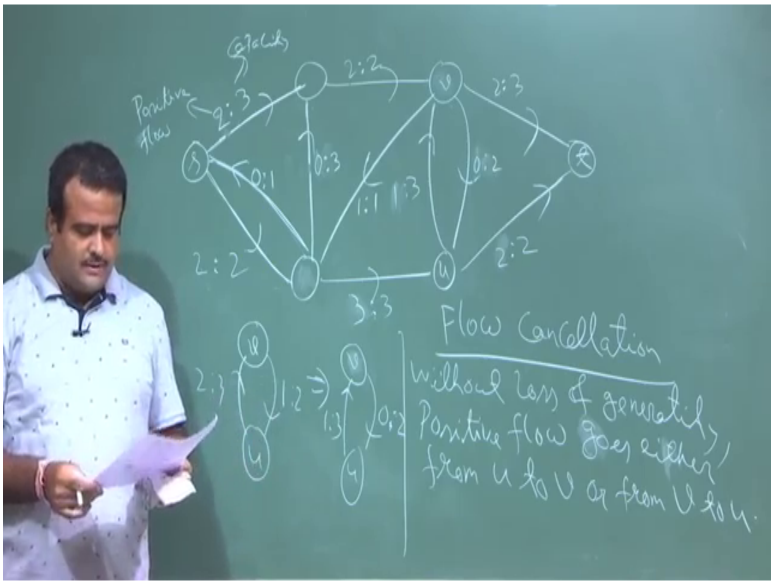
So, we just defined that maximum flow problem. So, we have given a flow network G. So, this is the definition given a flow network this graph G v comma e and the capacity vector. Find the flow of maximum value. So, flow of maximum value. So, that is called max flow, this problem is called max flow. So, we have given this for example, we have given this graph and we have a flow this is the flow p. So now, is this a max flow. Can you do more than this? Can you pass more current than this that is the question.

So, can you try to pass more current this, I mean more value on the flow. So, let us see. So, this is the flow value of this network is I mean positive flow is 3. So, can you can you pass it 4 like let us see. So, here we cannot pass more than 2, but here we can pass 2 or 3. So, let us try with 2. So, if you pass 2 over here, then this is this (Refer Time: 19:50) problem because this is consuming, how much? 2 3 2 3, now it is now it is passing. So, if you just make it 0. So, you make it 0 and then So, this is basically now we can pass it 3.

So, this consumption is basically 2 and here 3 yeah. So, 2 and here 3 we can pass this 3 because capacity is allowing us to pass 3. And then what we do? So, we have one we have one over here. So, you can instead of 1 we can pass 2 2 we can pass because capacity is 2 that is it. So, this is the way we just pass this current this is. So, this is the way we can pass 4. Now is this the maximum can we pass 5? Because this is 3 this is 2. So, this capacity is outgoing capacity is 5.

So now the question is can you pass 5? So, that we have to check. So, that may not be possible. So, we can check that you can check that that is not possible, I think 4 is the maximum flow value. So, the max. So, 4 is the maximum value, we will we will have a, So we will see how we can get that.

(Refer Slide Time: 21:29)



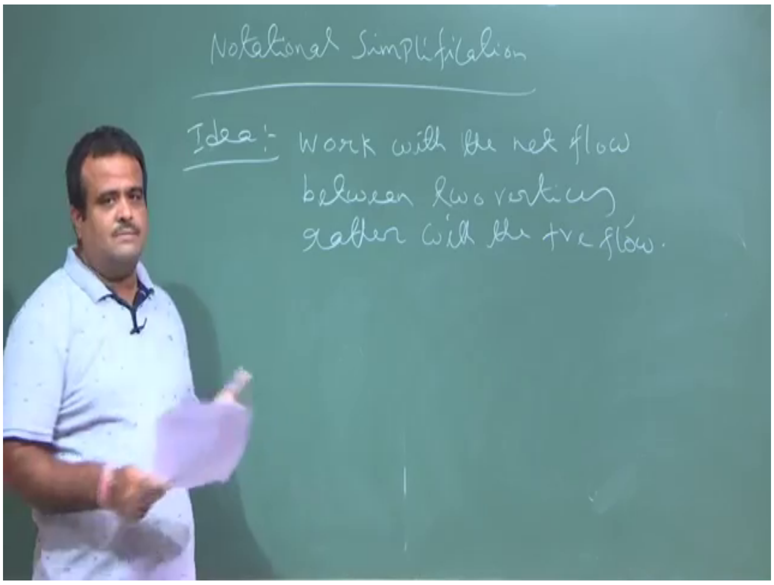
So, the value of Value of the maximum flow is 4. We cannot get 5 we I suggest you to verify that. So now, how we can get this maximum flow? So, for that we need to do some theorem and some more definitions.

So, first step we are going to do what is called flow cancellation, flow cancellation. So, what is that? Without loss of generating without loss of generate a the positive flow goes either from u to v or v to u, positive flow goes either from u to v or from v to u. So that means, if you have u v like this. So, suppose yeah, So suppose this is this is the example this is u and say this is v. So, here both the direction positive flow are going because from u to v we have a positive flow 2 and from v to u you have a positive flow 1.

So, this we want to do the some cancelation. So, what we do? We make it 0 we make it 1. So that means So, net flow this is sort of net flow. So, net flow is remain same it is basically 1. Because 2 was going one was coming. So, there is So, we can make it 0. So, positive flow without loss of generative we can assume the positive flow is going only to the one direction. Because if you have a flow in 2 direction we can cancel one I mean which is the lower one. So, we can make it 0 and we can reduce the flow in other one. So, that is what we did.

So, this what we did. So, this was 2 this was 1. So, we convert into this v u. So, we make this 1 and we make it 0. So, this is called referred as flow cancellation. So, you have canceling flow; that means, we are allowing the flow positive flow to from only one direction not for the both direction. So, net flow from u to v is in case 1, because earlier also net flow is one now also net flow is 1. So now, we want to do some notation simplification of this. So, we want to defined the net flow. So, we want to defined the net flow.

(Refer Slide Time: 25:53)



So this So, this is one notational simplification. So, in set of positive flow we just we are just now interested in the net flow. So, net flow is idea is. So, the idea is because positive flow we have both the direction. Now without loss of generative we see that we can cancel the one direction flow. And then that direction on direction is 0 another. So, that is the net flow from u to v. And from v to u that minus of that. So, that is the idea. So, idea is work with work with the net flow between 2 vertices rather than the positive flow. Rather with the positive flow, because we can cancel the one direction flow because net flow ultimately we are looking at how many flow we are passing.

So, that way this net flow with this is some sort of simplifying we are (Refer Time: 27:20) things. So, we have a positive flow we will check this 2 definition are same. So, we have an defined it in the next class we will defined this what is the net flow. And then we will see the how this 2 definitions are basically equivalence, if you have given the positive flow, we can have a net flow if you have a net flow we can defined the positive flow so that we will discuss in the next class.

Thank you.