An Introduction to Algorithms

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Lecture - 25
Binary Search Tree (BST) Sort

So we talk about binary search tree sorting I mean how binary search tree can help us to have a sorting algorithm. So, before that let us talk about what is the binary, recap the binary search tree, binary search tree or in sort it is call BST.

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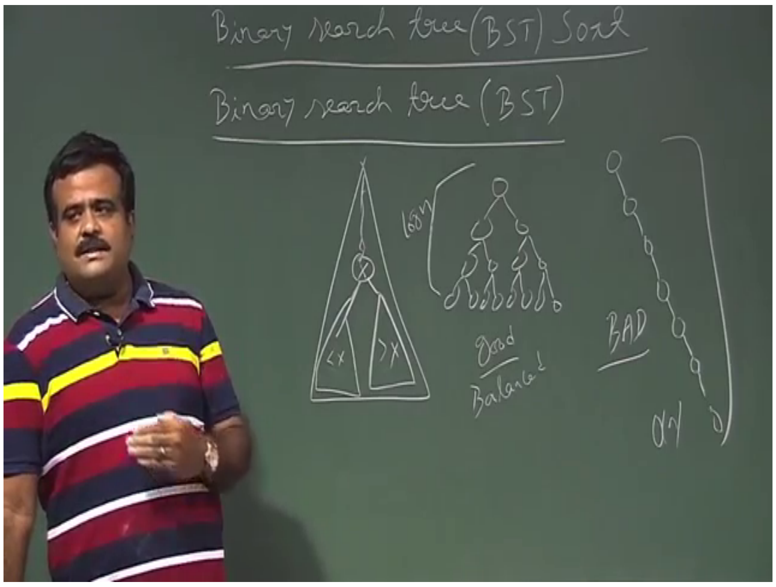

So, it is basically a tree and it has some property when it is search property. What is that? It is suppose we take, so this is a tree suppose you take any node over here in this tree this is key value is X. Now we considered the left subtree rooted at this right subtree rooted at this. Now binary search tree means all the key value over here must be less than X all the value must be greater than X. So, this is the property. So, if they distinct then it is must be strictly this. So, this is the property of binary search.

So, a binary tree which is having this property is called binary search tree. So, if the all elements in the left side of the tree is less than X and all the element in the right side of the tree must be greater than X. So, there are some good tree and bad tree. So, what are the good tree? So, suppose we have a tree like this, this is a good binary tree why because it is balanced this one is good tree, good in the sense it is balanced height is log

n. So, if there are n nodes height is log n. So, this is a balanced tree. And what is the bad tree? Bad binary tree like this if we have this. So, say height is n ok. So, this is a bad tree because it is not balance tree.

So, why balance tree is good because we can do some search or we can do some query in a height time and if height is log n then it is a log n time. So, that is why if it is good to have a balance tree. So, today now we will talk about yeah. So, BST sort binary search tree sort how we can use the binary tree to have a sorting algorithm. So, let us just talk write this as BST sort. So, we have given n numbers we can sort them by using the binary search tree.

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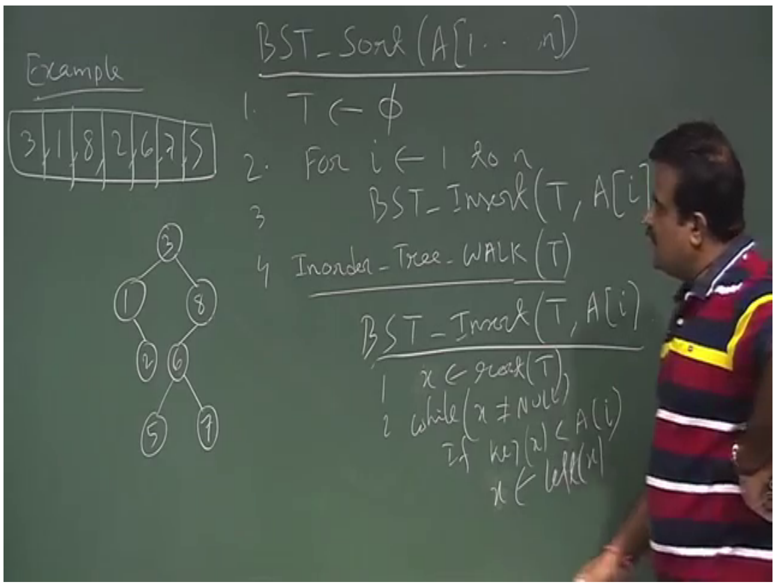

```
BST_Sort(A[1 .. n])
1. T ← φ
2. For i ← 1 to n
3.     BST_Insert(T, A[i])
4  Inorder_Tree_WALK(T)

BST_Insert(T, A[i])
1  x ← root(T)
2  while(x ≠ NULL)
       If key(x) < A(i)
           x ← left(x)
```

So, this is BST sort we have given n numbers or we have given a array of size n. So, what we do? First our t is empty we are initialising a t tree binary search tree empty and then we will from the tree. So, we just for i is equal to one to n we insert this node into the tree.

So, this is BST insert basically we insert this A i into the tree and then we will do the inorder traversal of the tree. So, this is this is after this we form the tree, form the BST. So, we will talk about how the BST insert will work we form the BST and then so to from BST basically we start with the root until we reach to a nil or the leaf node, if tree is initially empty then the first node will be the root and in the subsequence step. So, if we start with the root if the element is less than that we go to the left side of the root left subtree otherwise we go to the right subtree and this way you continue until we reach to a nil or leaf node once we reach to a nil we insert that node there. So, this is the basically BST insert.

And then we do the inorder tree walk inorder tree walk of this tree inorder tree walk means we first print the this is basically the way to print a tree or a traversal, tree traversal. So, we first print the leaf subtree then the root then the right subtree. So, that way we just visit the all the element of the tree. So, this is one. So, there are inorder, pre order, post order.

So, this is the way. So, inorder, if you do the inorder traversal then we will get the sorted one. So, let us take an example - suppose we have some number say 3 1 8 2 6 7 5 suppose this is our a arraying this is our given numbers, there are how many numbers 1 2 3 4 5 6 7, there are 7 numbers is given. So, now, how to form the tree? So, initially tree is empty now we insert this 3, 3 will be the root and then we then we insert slowly, so 1. So, this we have to insert 1. So, we start with the root 1 is less than 3. So, we will go to the left part of the tree left part is empty. So, you will insert there 8, 8 is greater than 3, then 2 we start with the root 2 is less than 3.

So, we will go to the left part of the tree then again 2 is greater than one. So, we will put it here and then 6, 6 is greater than 3, but less than 8. So, we will put it here then 7, 7 is greater than 3, but less than 8 again greater than. So, we will put it here then 5 5 is greater than 8 say greater than 3, but less than 8, but again less than this. So, this is the structure of the tree. So, basically this is the BST insert.

So, BST insert means, if you just write the BST insert code, BST insert, so we first. So, we first go we first start with the root. So, we take this X as a root, root of the tree initially root is empty root is null. So, while we are not reaching to a null while X is not null. So, we do this if the key of key of X; that means, key key means here the values if the key of x. So, we are going to insert some number A i, if key of X is less than A i then we call the this is the recursive call. Then we go to the, then X is this is while loop then X is left of X else X will be the right of X, if key is greater than this. Ao that means, we start with the root if the root is empty that is the initial condition t is empty then we insert that that is why 3 is inserted has a root and after that we start with we say 8.

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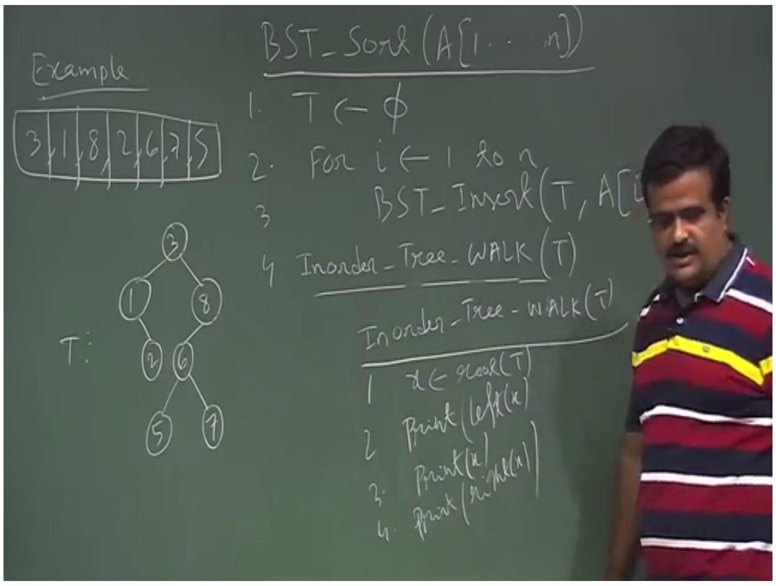

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2. For i ← 1 to n
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4  Inorder_Tree_WALK(T)

Inorder_Tree_WALK(T)
1   x ← root(T)
2   Print(left(x))
3.  Print(x)
4.  Print(right(x))
```

So, we know 8 is greater than this. So, we call this in this then again 8 is seating here. So, like this. So, this is the BST insert now what is the inorder tree walk inorder tree walk basically. So, inorder tree walk. So, inorder tree walk is basically we first, so X is the root, root of t. So, we first print the we first traverse the left part of the left of a X then we print the root and then we print the right of X print means we again call the inorder tree walk. So, we do until we go until it search no child then we will print it print right of X print means this is basically inorder tree walk again.

So, for example, here we start with we want to call. So, this is our tree after this is insert BST tree insert. So, we start with the root we first call this inorder tree walk of this and then we print 3 and then we call the we print the right part of the tree this is the right part. So, again for the left part so this is the tree now, now we again call the inorder. So, this has no left part, this is one then we print the right part.

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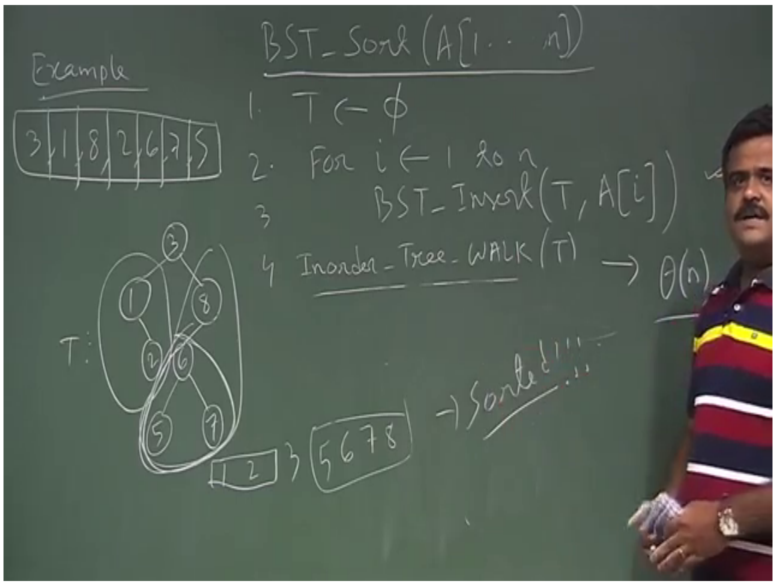

And similarly for this tree, so 8 is the root for this subtree now we call the inorder tree walk, it will 8 will be. So, there is no right subtree, so 8 will be printed here and this as to be print the left subtree. So, left subtree is this one. So, again for this we call this is the root this is the right sub right and this is the left. So, 5 6 7 sorted. So, inorder tree walk if we if we do the inorder tree of on this BST we are getting the sorted array. So, this called BST sort. So, now, we have to analysis this how must time it is taking. So, how much time it is taking how much time it is taking to do this inorder tree walk it is order of n because basically we are just seeing the node only once we are just print we are just travelling the tree. So, we are visiting the nodes only once. So, there are n nodes. So, it is basically order of n we are just printing the nodes. So, order of n.

Now the question is how much time it will take to build the tree build the BST this is the build the BST. So, how much time it will take, so time to build the BST, so that is the question. So, that time will depend on this the total time for the BST sort. So, how much time it will take to build a binary search tree for a given n input for a given n arraying. So, how much time it will take to build a BST? So, we will come back to this example again.

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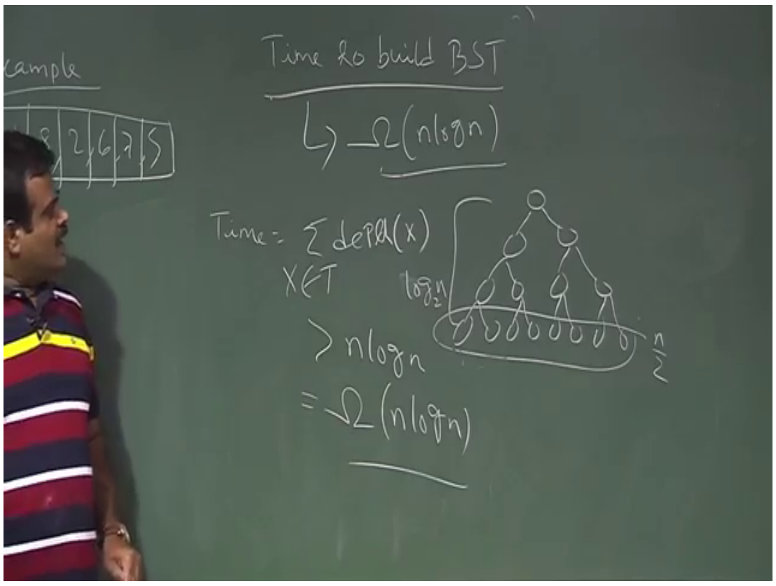
Time to build BST
$\hookrightarrow \Omega(n\log n)$
Time = $\sum_{x \in T} \mathrm{depth}(x)$
$> n\log n$
$= \Omega(n\log n)$
$\log_2 n$
$\frac{n}{2}$

So, time to build BST binary search tree. So, any answer I mean. So, any lower bound upper bound. So, any upper bound how much time it will take. So, how to build a BST? So, basically time to build a BST is basically summation of the depth of X, while X in the tree because. So, we are every time are we are comparing and we are reaching to a position and that position is the depth of that node. So, depth means that many times we compared to insert this node. So, this the time complexity to build the BST basically time of the depth of X where X is in the tree.

So, this will be basically big omega of n log n this is the lower bound we cannot do better than big omega of n log n why because when is the best case for this when the we have a balance tree if the tree it dependence on the nature of the tree. If the tree is balanced if the tree is balance suppose there are n node this is the balance tree if the tree is balance now how much time it will take to insert how much time it will take to build such a tree. So, how many node are there here in the leafs? So there are basically n by 2 nodes there are total n nodes and this is the height, height is basically n height is basically log n less 2. Now this is the depth of this node this depth is log n. So, for each of this leaf node we have to compare root then these then these with these then only it came here. So, that is the depth of this node, depth of this node is log n.

So, at least for each of this node we have to compare log n time. So, n by 2 is log n by 2 n is the comparison for all these node and even we have these node. So, that is why height is height must. So, time is must be greater than n log n. So, this is the depth, this must be greater than n log n because there are n by 2 nodes. So, at list for this node we have to we have to the depth is log n for this node. So, depth is log n means they have to compare with this node this node. So, how many comparison at list log n comparison each of this leaf at to compare with the log n many nodes. So, that is why we inserted in that height level. So, that is why this time is basically big omega of n log n this is the lower bound. We cannot do better than this. And any upper bound whether this is any big O. So, this we have to think. So, this is the best case.

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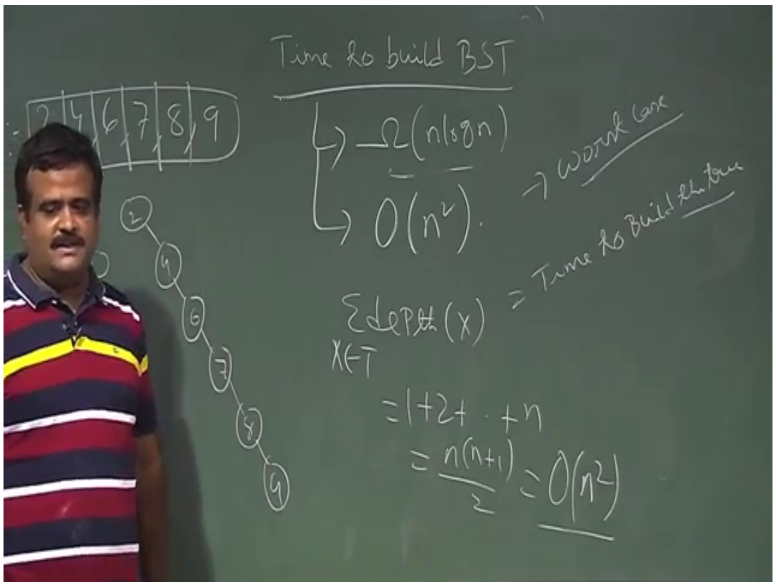

Now can we say this is n square big O of n square. So, when is the worst case of this scenario? Worst case is if the tree is bad tree like if the tree is like this if there are n nodes then height. So, then what is the summation of depth of X? X is in tree, so this basically arithmetic series, so this is n into n plus 1 by 2. So, this is order of n square. So, if the tree is bad tree so that means, if our numbers search is ascending order or descending order suppose we have given numbers are like this say 2 4 6 7 8 9 suppose this is our input. Suppose this is our input sorted, the our array is sorted already then what is the tree, tree will be like this. So, 2 then 4 6 7 8 9 see. So, this is basically to from this tree we need order of n square because everybody has to compare with the that previous all the element.

Even if it is reward sorted also this will be like this if it is reward sorted the tree will be looks like this. So, this is the worst case. So, worst case is order of n square to build the tree. So, this is the time complexity for time, time to build the tree. So, this is the worst case so that means so this will the time for BST sort. So, this will be the time for BST sort because inorder traversal will take linear time and this will take the measure time will take by here to build BST to build the binary search tree and that itself is a its taking the time of order of n square.

So, the time complexity for BST sort in the worst case it is order n square and in the best case it is order of n log n.

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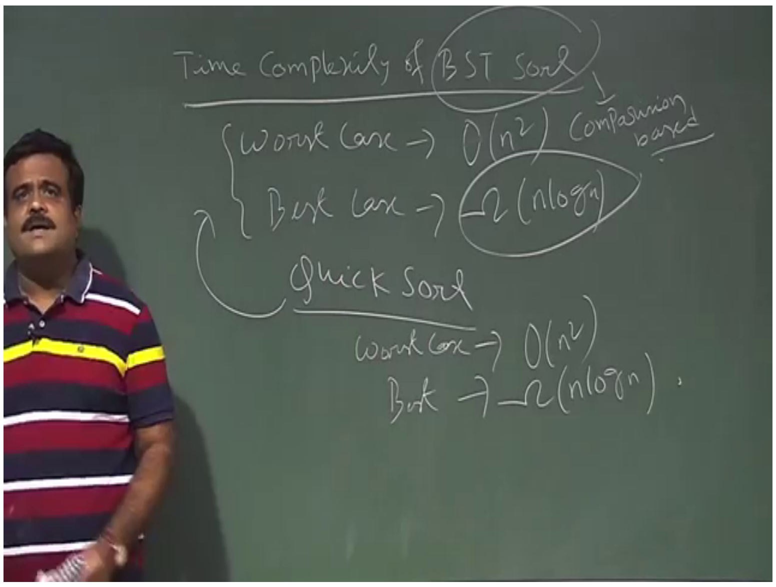
Time Complexity of BST Sort
Comparision based
Worst Case → $O(n^2)$
Best Case → $\Omega(n \log n)$
Quick Sort
Worst Case → $O(n^2)$
Best → $\Omega(n \log n)$

So, the time for runtime or time complexity of BST sort. So, worst case it is order of n square and the best case it is n log n. So, BST sort, this is the sorting algorithm which is taking worst case what about n square and best case n log n. So, is it remain us some sorting algorithm we know who performance is like this, like worst case is n square and the best case is n log n sorry n log n. So, we know any sorting algorithm we have studied comparison best sort because here this is also the BST sort is also comparison best sort because this is also a comparison best sort and that is why lower bound has to more than n log n I mean that is I mean we cannot do better than n log n we have seen by the this is entry method module that we have seen any comparison best sort worst case time complexity is more than cannot be faster than the n log n.

So, this is also comparison best sort because we are comparing the element and then we are forming tree and then we are inorder traversal. So, any comparison best sort we know having this time complexity yes quick sort. So, quick sort is having same type of time complexity worst case quick sort worst case is order of n square and the best case is order of n log n. So, quick sort is having the same time complexity has BST sort. So, that is the, that is the our next point to see whether is there any relationship between BST sort and the quick sort why they are giving us the same time complexity. So, that we want to see that what is the relationship between BST sort and the quick sort. To see that let us take the example again and we will see that they were having same number of comparison we are doing in quick sort and the BST sort. So, let us take that same example A array, 8 sorry 3 1 8 2 6 7 5 this is our given input and in the BST sort we form the tree 3 then 1 then 8 then here 2 here 6 and then 5 7.

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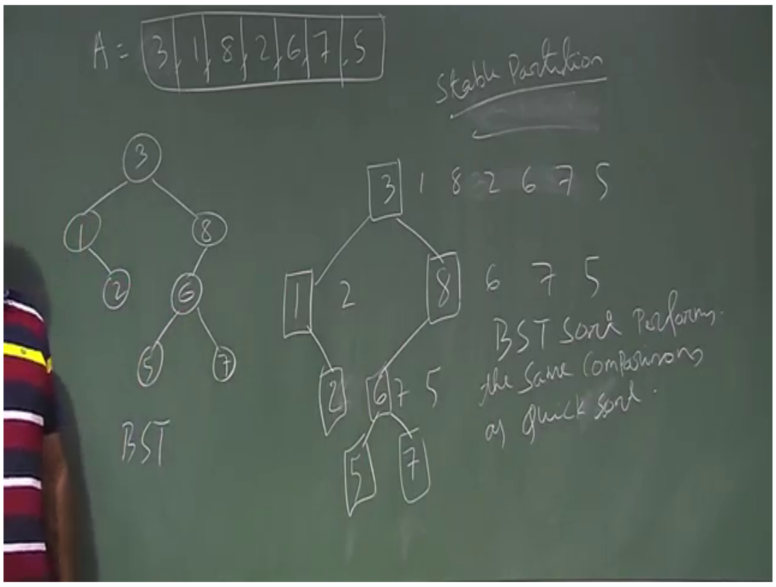

So, this is the BST binary search tree and then we do the inorder traversal to have this thing. So, have the sorted one to have the sorted one array. So, this is BST sort. Now let us talk about quick sort performance on this input. So, in the quick sort what we are doing we are choosing a pivot element suppose we choose and we choose the first element as a pivot if we choose the first element as a pivot. So, what will be look likes 3 is a pivot. So, what pivot will do, pivot will partition this array into 2 sub array all the element less than X must be left sub array all the element greater than X possible right sub array. So, that way it will do like this. So, 3 this is the array 3 1 8. So, let us just 3 1 8 2 6 7 5. So, this is the given input now we choose this as a pivot element.

So, it will partition this into two part all the element will be less than X must be in the left sub array all the element greater than X must be in the right sub array. So, who are the element? So, 1 2 must be seating here and 8 6 7 5. So, this is quick sort partition and here we are assuming one thing this partition must be stable partition because we are assuming this ordering will not changing. So, we know this is 1 2 will come in left sub array, but it is not guaranteed that 2 will come. So, which ordering, but here we are using a partition which is call stable partition and that is what very top to get the code of stable partition. So, stable partition means we are assuming the ordering of this input ordering same has this in the array. So, there are 8 6 7 5 there in the same order.

Now again for this we are taking this as a pivot now all the elements. So, 2 will be seating here and we are taking this as a pivot. So, who are element are greater than all the element are greater than less than 8. So, 6 7 5 then 2 then nobody is there then we call 6 is here, so this is basically 5 and 7. So, this is the recursive call of the partition. Now if you just form the tree like this, like this, so like this if you look this 2 tree as same, so that means, we are doing the same number of comparison. So, in the partition recursive call of partition the number of comparison we are doing is same as to form the BST because to form the BST this 6. So, when you insert 6, 6 as to compare with 3 6 as to compare with 8 here also.

So, to come here 6 as to compare with 3 because this will partitioning into 2 part again 6 as to compare with this 8. So, the same number of comparison we are doing for the both the cases. So, this is the idea. So, BST sort. So, this is the observation BST sort basically to form the BST performs the same number of may be different order, but we are doing same number of comparison as in quick sort same comparison, comparison as quick sort. So, that is the reason there are giving the same time complexity, the time complexity is coming out to be same because of this fact that we are basically doing the same number of comparison in both the cases to form the BST and for the quick sort.

Here we are assuming this table the partition is stable partition because to and that is also not top to achieve to have this 2. So, the same tree we are having this. So, this is the observation and that is the reason it is giving us the same time complexity for the BST sort and the quick sort.

So, in the next class we will talk about randomized version of the BST sort and there we will see the height of the randomly build BST. So, that will expected height we will see to be balanced log n. So, that will cover in the next class.

Thank you.