Embedded Systems Design
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Lecture - 15
Quantization Noise, SNR and D/A Convert

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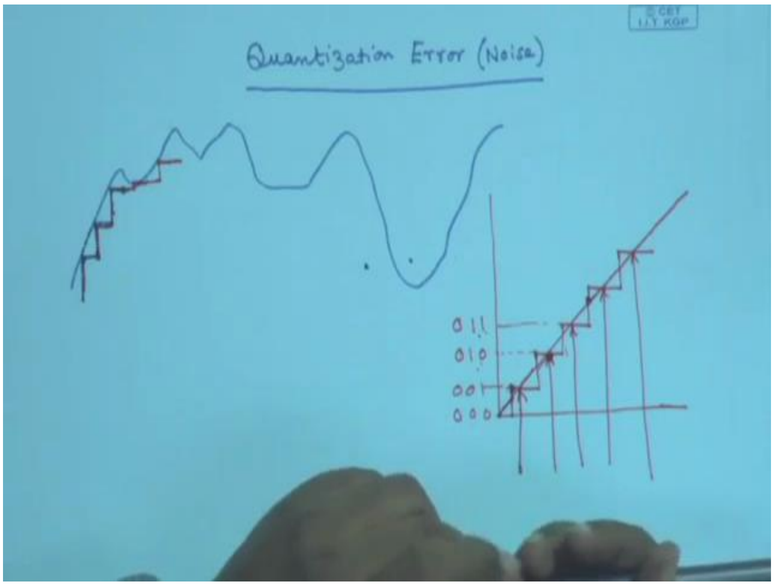

So, next topic that we will be discussing is quantization error, which is also known as quantization noise. What does it mean? We have given a particular signal, and we are quantizing it by sample and hold circuit, and A to D conversion all those things. So, I am taking samples. So, this is my sample, this you can understand this. Let me use a red color, I do not know whether that will be visible. This is my sampled value right, depending on the times when I sample.

Now, in between this point, there are several values which I am not getting access to, I am missing that. So, I am losing some sort of information. So, that error is the quantization error. This error occurred, because I have quantized. Now, those of you who are scared about electronics a little bit, I mean this sort of error is very important to consider, because in computer science we always try to gather information, and we process information. Now, this signals cannot, may not be electronic signals also in general the different events happening the stock market price change whether the time domain may be longer and we are not actually sampling or observing at every continuous point of time. We are sampling it all the time.

So, whatever we learnt earlier, of sample and hold and all those things, we do that all the time. We take some data and we assume that data to be true till I take the next data, but in between whatever happens that will give rise to some error. So, let me draw another diagram here, suppose this is a linear signal of a linear signal, and I have quantized it at different points of time and I have given some bead values to this.

What are my sampling points? My sampling point is: this is a sampling point, this is a sampling point, this is a sampling point, assume that all these are equidistant; my drawing may not tell you that. So, in that way it goes on this is another sampling point. Now, I have sampled here and I have assumed that actually I have sampled here, I am sorry and I am assuming say I have sampled here let us take this one forget about this part there is some error in this time. So, if I sampled it here and I assume that it is all through it is holding that particular value and accordingly I put in some values like 0 0 0 for this, for this value 0 0 1, this one may be 0 1 0 like that I go on putting the values, etc.

So, these are the realities to me, but the actual realities were lying here. So, whatever I missed is a quantization error or quantization noise.

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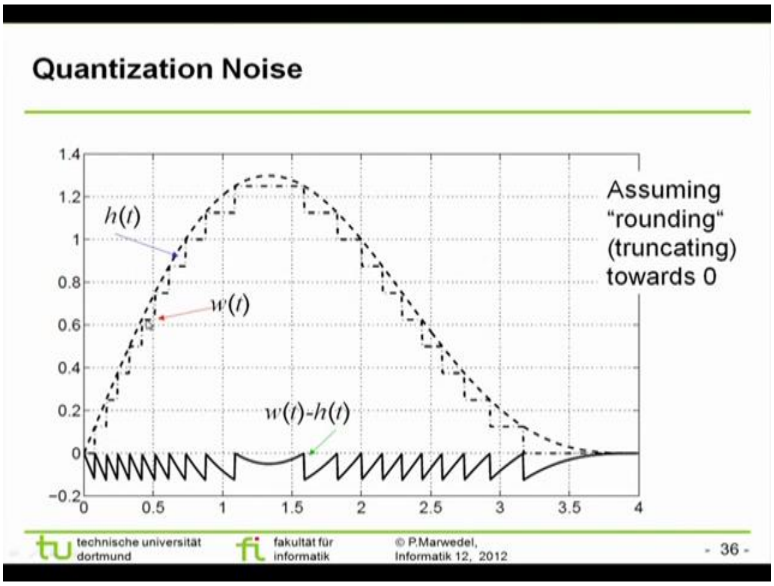

So, if we look at this slide, you can see that this is the signal; a very nice uniform sinusoid like signal h t and I have sampled it at different points of time. So, here you see, look at my mouse here that, I have sampled here and I assume that the value is constant all through and therefore, depending on what I sample and what the actual value is the difference is my quantization error. So, I can say that w t minus h t the actual signal is my quantization error, some pattern like this. This is also known as quantization noise.

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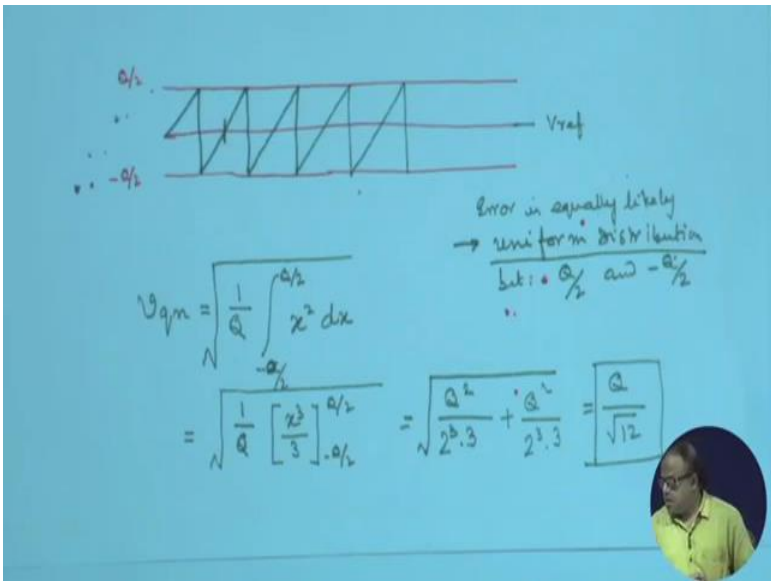

Now, see in this case, let me draw it once again; if I take a signal of this form there is one level that is; plus Q by 2 and here minus Q by 2 and here is a 0 level and the signal is like this, this is the cross over points, and the signal goes on like this. Now, here is the V reference voltage, and the signal is (Refer Time: 07:58) like this. Now, any value of error in between this, the swing is Q. So, on one side this Q by half plus Q by 2 and minus Q by 2. So, since the error is equally likely in the distribution range of error is equally likely any error.

So, we have got a uniform distribution right of error. Now, between Q by 2 and minus Q by 2, therefore, the error can be quantized with an RMS value, RMS value you remember; root mean square value, RMS value. Let us call that v q n quantization noise it will be what; since, it is a uniform distribution I can need not memorize, but you know that it will be 1 by Q we will come to some very interesting result out of this minus Q by Q 2 plus Q by 2 X square D X that is the normal formula. So, that will come to 1 by Q if you do this integration X cube by 3 with Q by 2 minus Q by 2, is it visible. So, it will be, if you solve it Q square by 2 to the power into 3 plus Q square, this negative will go out will be 2 to the power into 3. So, that is coming to Q by square root of 12. This is the quantization error.

Why this is important? It is important because, there is a term called signal to noise ratio. Whenever we are doing this quantization, because of the quantization, why do we do the quantization? We do the quantization because I have to leave in the digital world and I cannot call a monitor though I am not monitoring the continuous signal, so depending on the infrastructure that I have I am doing some things for my convenience and in the process I am introducing some errors.

Now, the errors should be such, that it actually does not affect the original signal, the interpretation of the original signal to a large extent. Giving a simple example; if somebody is singing and the signal is coming towards the system, and the system because of its own idiosyncrasy, its own peculiarity is introducing some noise and if the noise kills the signal then; obviously, you will not be able to hear the music. So, the signal to noise ratio is a very important figure.

So, now, we have to, we know the signal, but we have to deal with the noise also. So, because of quantization, in our scenario because of this digitization we have introduced some quantum this noise.

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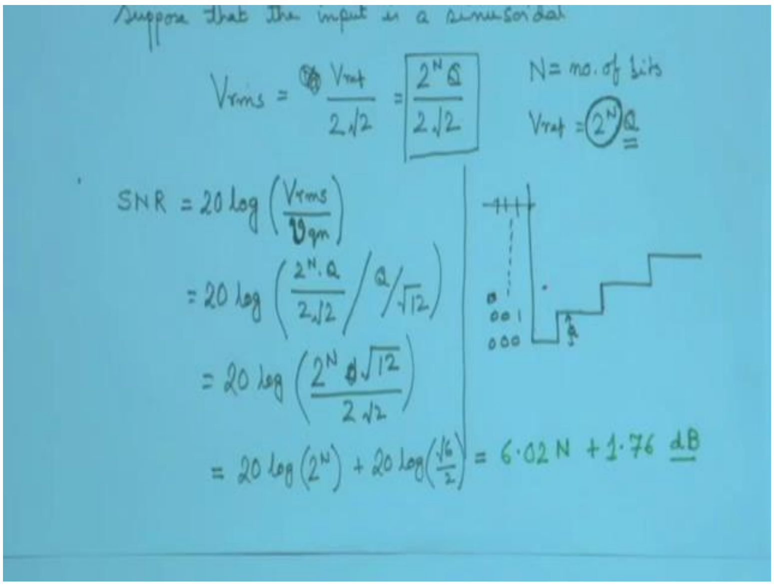

So now, if I think of an input sinusoidal signal suppose, sinusoidal signal, in that case the RMS value of that will be v reference by 2 root 2, now, this I can represent as 2 raised N by 2 root 2. Now, the question is, this part is clear; RMS value is defined as this. The reference peak, the range by 2 roots 2, for any signal we know that.

Now, where from do I get this? What is this N? N is a, since I am digitizing it, N is the number of bits, that are available for our discretization. Number of bits you using, which I am digitizing it, and where from the this V ref is equal to 2 raised N Q come; because V ref is divided into with N bits, I am dividing the V ref into two raised N steps. And what is this Q? Q is the quantum of the steps. So, it is clear, I am digitizing it like this.

Now, this is the quantum of the steps, depending on the frequency at which I sample, and how many such steps n bits I can have, So, with n bits say; I can have this encoded as 0 0 0 0 0 0 0 1 0 like that I can have 8 levels here. So, that 2 to the power N will be therefore, 8 with 3 bits, if I have 3 bits then this will be 8 and for 8 each of them has got this much Q. So, the total voltage range, the total voltage and this is V ref where I have got 1 1 1 then that is the V ref, the total voltage range is being divided into so many quantum. So, therefore, it is coming. So, this equation is understood now that V ref is to raise N Q.

Now, the signal to noise ratio, here I am, the signal to noise ratio; in short SNR is 20 log V RMS that is this one by the, how did I denote? That I denote it the quantum noise v q n I had found out the v q n, there is a quantum noise the RMS value of the quantum noise because it is a uniform distribution right. So, this is the value. So, this will be therefore, 20 log, what should be 2 to the power N Q by 2 root 2 divided by Q, Q divided by root 12. So, this will come to 20 log 2 to the power N root over 12 divided by 2 root 2. This can be further simplified to 20, log 2 to the power N plus 20 log root over 6 by 2.

Now, this value is a very interesting value. This value is coming to 6.02 n plus 1.76 dB, dB is a decibel. So, the signal to noise ratio is expressed in the form of dB. So, that is how we get the signal to noise ratio. Now, summing up from the earlier lecture, we are getting the signal, we are filtering it, to cut off the high frequency components. Now, why is it that I am so concerned about the sampling rate? You can say, why you are filtering up, filtering out sir, you can sample at a very high frequency; as I increase the sampling frequency the number of bits will increase, the register size will increase.

So, there is an engineering optimization at every step. So, we are sampling at a particular frequency, then doing the analog to digital conversion, either using flash A to D converter or successive approximation A to D *c*onverter depending on my requirements, speed and resolution. And while I do that, do all these things A to D conversion. I am introducing some noise starting from the process of sampling and I must be sure that signal to noise ratio is not hampered, affected adversely because of that. So, signal to noise ratio should be high, the signal should be high. And how do we calculate this? This is the way because each of these steps is leading to some quantization, and quantization errors are coming in and therefore, we are doing it.

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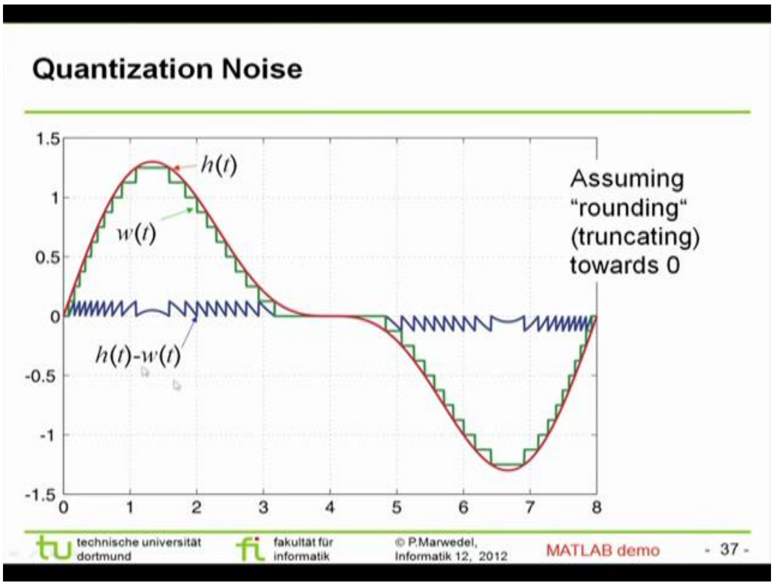

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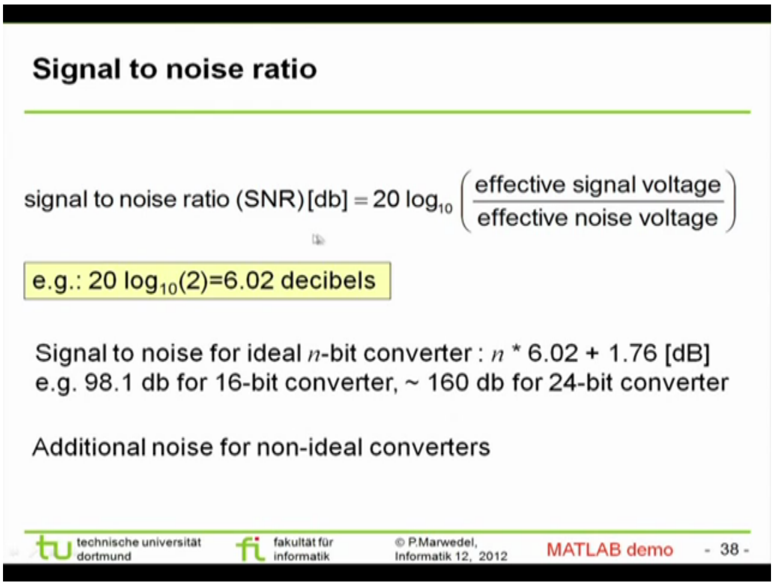

Now, let us see. So, here is the quantization noise; we can see it is a purely coming out from a MATLAB demo, that this is a quantization noise. The signal to noise ratio is essentially effective signal voltage by effective noise voltage. What is effective? Effective is the RMS val. For example, 20 log 2 is 6.02 decibel that is how we compute. It log to the base 10 decibel right. So, signal to noise ratio for ideal n bit converter is we have already computed that, that is we have found out 6.02 N plus 1.76 dB and that is what is, for example, if I have got a 16 bit converter what it would be converted?

Now, one thing is we had talked about speed, we have talked about the hardware complexity, well also a third dimension is now coming into picture that is; the quantization. Therefore, for a 16 bit converter we will have the signal to noise ratio 98.1 dB based on this equation that we have just now derived. 160 around, 160 dB for 24 bit converter and if I have non ideal converter there will be other additional noise is coming up. So, that is the input part. Now, we will come to the output part, and we will discuss about is it clear I have been this.

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No not in a linear fashion, it is a uniform distribution, because everything is equally probable here, any of these are equally probable.

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No not always, it is just a probability. We are assuming that any of these errors can take place. So, it is a uniform probability of all of them. That is why it is uniform distribution. Next, we will come to again; I think a little bit of, not too much hardware left, but little bit of hardware will come here that is, now, when we come to the other side on the output side. So, we have got an embedded system, on input side we are getting the signals we do A to D conversion and now the output side when we get the digital things and we have to activate the actuators we need the digital to analog converters.

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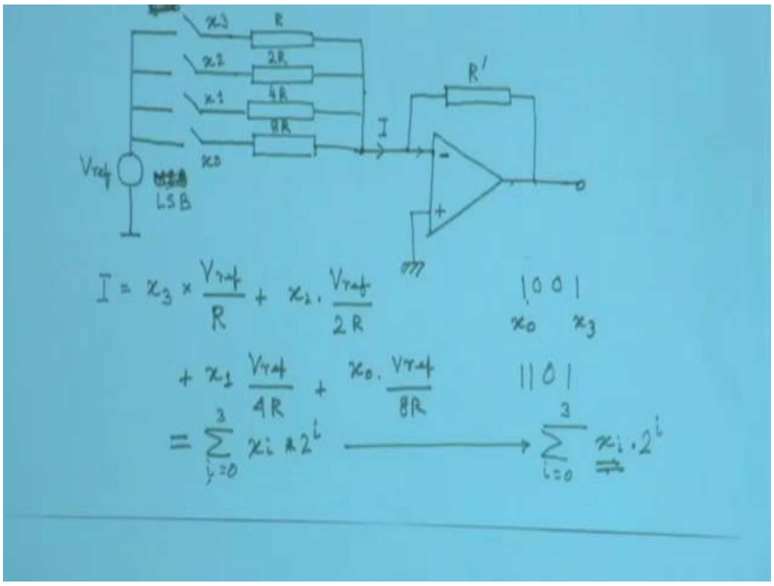

And again our good old friend the operational amplifier will come in handy in doing that.

So, our digital to analog converter very simply I am putting it. Here, that I have got a reference voltage here and I have got a number of switches here. If the bit is 1 then the switch will be closed, if the bit is 0 then this switch will be open and there are resistances at with each of these inputs, and these resistances are weighted. So, this is 2 r, this is 4 r and this is 8 r, just like 1 2 4 8; however, positional weightage of the binary system. Now, this, all these will be connected to the inverting input of an operational amplifier, and we know that the inverting input and with the non- inverting input 0 if I fit something in the inverting input that will do the task of amplification, provided I put a nice, proper, some other resistance r prime as a feedback.

Now, we can understand that, there are 4 bits here let us call it this to be the X 0 bit X 1 bit X 2 bit and X 3 bit. Now, if I take the pattern for example, 1 0 0 1 a digital pattern;

that means, this switch is closed and this switch is closed. So, current will flow through this. So, I will have the voltage R. So, this is actually the l S P. So, let me write down here so that you do not feel confused, and there is the M S B side. So, this is X 0, this is X 3. There is normally, we write it in other way.

So, if these two are closed 1 and 1, I will have current flowing through this, with the voltage R and a voltage 8 R. So, it will come here. So, for this I will have a particular voltage at the output of the O P A M P and if I, this bit is changed to say 1 1 0 1 then current will flow through this direction as well as these two directions and these are where ending. So, all of them will come and this is coming in all these currents are flowing over here and we will get this amplification.

So, according to Kirchhoff’s; I mean again school level knowledge Kirchhoff's junction rule; we can quickly write down that the current that will be flowing here. If I call that I is nothing but, X 3 times I am sorry, please hold back to this our notation was right it is actually this current is actually is being driven by the current and. So, it is being driven by the current. So, this is the MSB and this is the LSB. the convention is fine, why? Here X 3 by the V into the V reference by R that will be the current. So, for this the weight it is more, we actually dividing with a resistance. So, the current is going through this right plus X 2 into V reference divided by 2 R plus X 1 into v reference by R by 4 R plus X 0 into V reference by 8 R.

So, that is actually coming to sigma X I times 2 I, I from 0 to 3. I write it in a bigger way, sigma of X I times 2 raised I, I assigned 0 to 3. That is how the actual voltage will be determined, the current that is coming and that current will be amplified by this. So, the output voltage will be proportional to the number represented by this X I. So, depending on this X I, X I is a pattern of 0 and ones depending on that this current and the consequently the voltage, that will come here at the output will vary. So, that is how digital to analog conversion is done.

So, the output generated, we look at the typical output that is generated by this do not have that with me here, the output generated from this. So, ultimately what are we having here, ultimately, then we are now in a position to conclude this by drawing this diagram.

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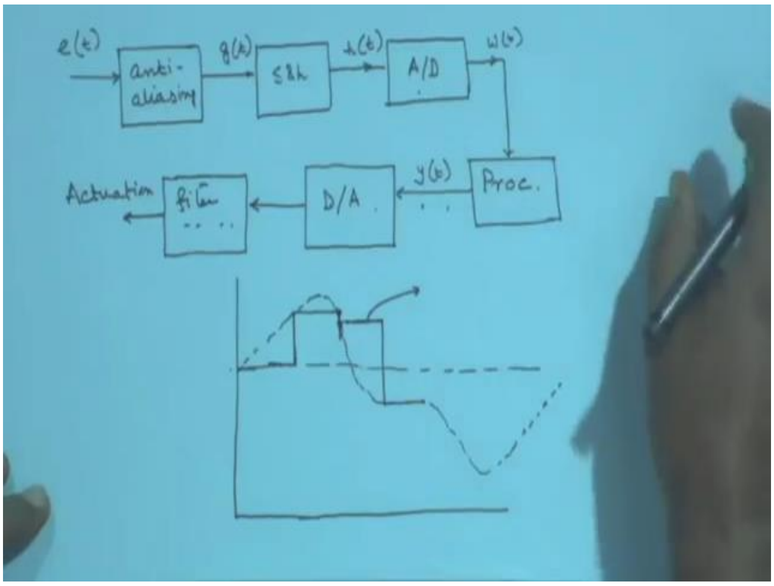

The signal coming up, let us call it whatever, let us call the original signal to E T and that is coming to anti aliasing filter and then, let us call that signal to B E G T, there is a filtered signal, that is coming to a sample and hold. And from there we are getting the H T time domain signal, which is coming to the A to D, and from the A to D we are getting another signal that is, after doing that we can give it a different name like say; W T and that is coming to the processing, whatever processing we do using a processor and then the process signal.

So, W T and we are getting an output signal Y T for example, and that one I am putting it to digital to analog converter. And then we can now, we are getting something analog then if you like. You can put a filter if you like, with that filter and other things whatever you like filtered, etcetera for the actuation. So, typically, these are whole circuit and we have learnt about each of these now.

So, typically if I have a signal, like say this form, like this and if I have 0 somewhere here, my 0 point is here it is going positive or negative after that after sampling I mean we are, if we assume a 0 order hold; that means, we are not delaying the signal in any way in that case I will get some digital signals like this, at each of these points after sampling. And from this say here, then I sample this point I get different signals and from this digital signal, which is this one W T or Y T I get from there, I can reconstruct back this signal. So, that is the whole thing.

Now, one thing that we have not discussed, briefly, let me touch upon that; that is once we have these filters, once we have this analog output, I can as you as you already know depending on my system that I will be using. I can directly feed the digital signal to the actuator depending on my actuation. We will see some digital actuations data or we may like to do the digital to analog and then from there we will feed it to the actuator there can be different types of actuators, which we will discuss in the next class some at some actuators and some digital actuators also, and we will be doing a couple of examples which processes.