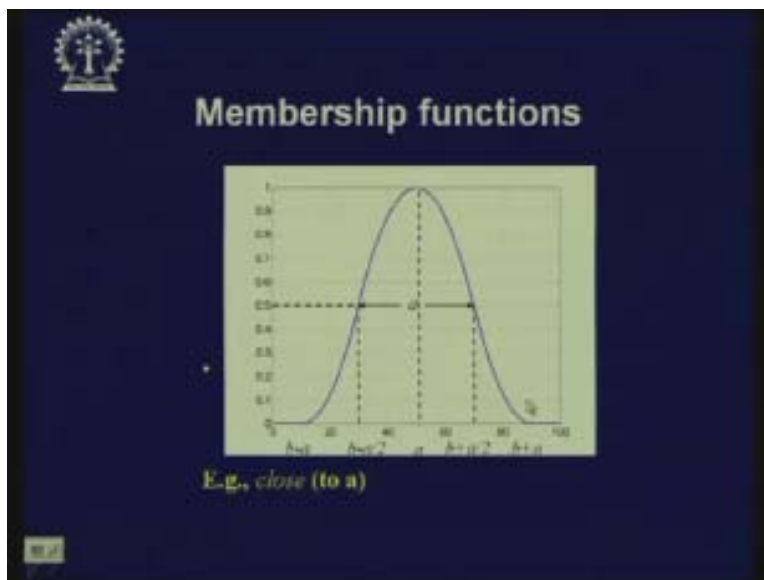


Artificial Intelligence
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Indian Institute of Technology, Kharagpur
Lecture - 31
Fuzzy Reasoning - II

In this lecture we will continue our discussion about fuzzy sets and fuzzy reasoning. In the last lecture we concluded with an example membership function like this which is a bell shaped curve representing some number like close to a. Now, as was mentioned earlier this can be represented in the computer as a table.

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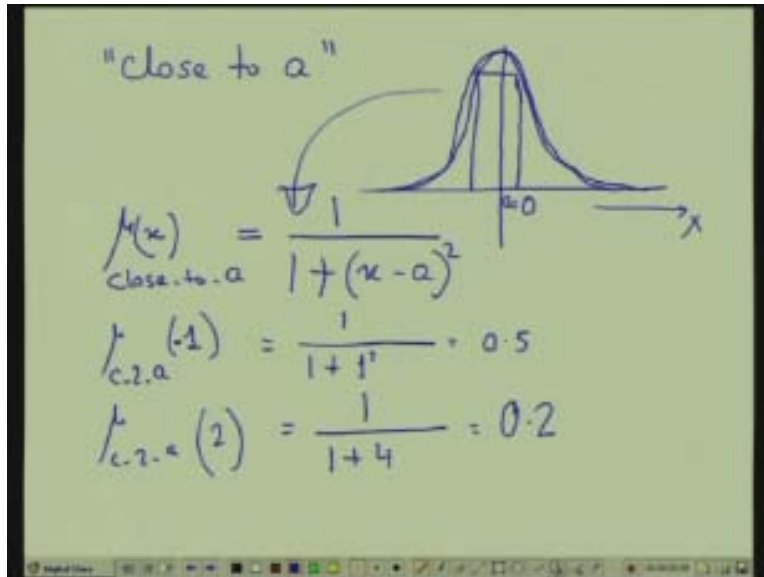


For each of these values, for 20 we stored the membership value 0.1, for 30 we stored the membership value 0.5, for 40 we stored the membership value 0.9, for 50 we stored the membership value 1 etc. There is also another way of dealing with such curves and storing them in the computer as a function. For example, if I ask you to represent a fuzzy set close to a, the curve will be something like this where this number is a, and any number which is close in this z1 will have high membership values. So if I want to store a curve which is of this shape I can either put it as table or I can also find out some function.

For example, if I write that the μ_{close} of any number x in the set close to a to be represented as $1 - \frac{1}{1 + (x - a)^2}$ then also we will be able to represent this sort of a curve just to verify this. Suppose this a is 0, in that case μ_{close} of -2-a(1) the membership of 1 in that set will be $1 - \frac{1}{1 + 1^2}$ that is 0.5, the square part will take care of minus 1 also because minus 1 will also be 0.5. We can see that of curve is asymmetric nature. Similarly, for c-2-a(2) is equal to $1 - \frac{1}{1 + (2)^2}$ that is 4 that is $1 - \frac{1}{5}$ that is 0.2. So quite sharply it is falling down. So, instead of storing all the

possible values and their membership function as a table we can decide on the nature of the curve and find an equivalent function which can be computed in the computer.

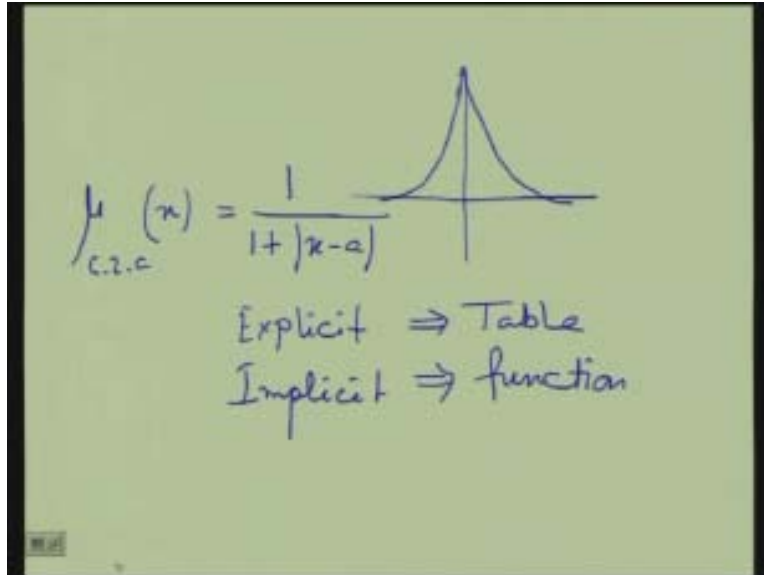
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Now if I really want to have a much more sharper curve like instead of the bell like curve I like to have some curve like this or even sharper sort of curve you can try in that case at $\mu_{\text{close to } a}(x)$ is equal to $1 / (1 + |x - a|)$. If you try this you will find that the nature of the curve is becoming quite sharp. So we can have either an explicit presentation of a fuzzy set which is in the form of a table, data structure or an implicit presentation which is in the form of a function. These are the two possibilities of representing a fuzzy set in a computer.

We usually prefer to have piecewise linear like triangular or this is another sigmoid but instead of being a very continuous curve it is a piecewise linear curve. This is often preferred because it is easier to represent and also calculation is easier it saves the computation.

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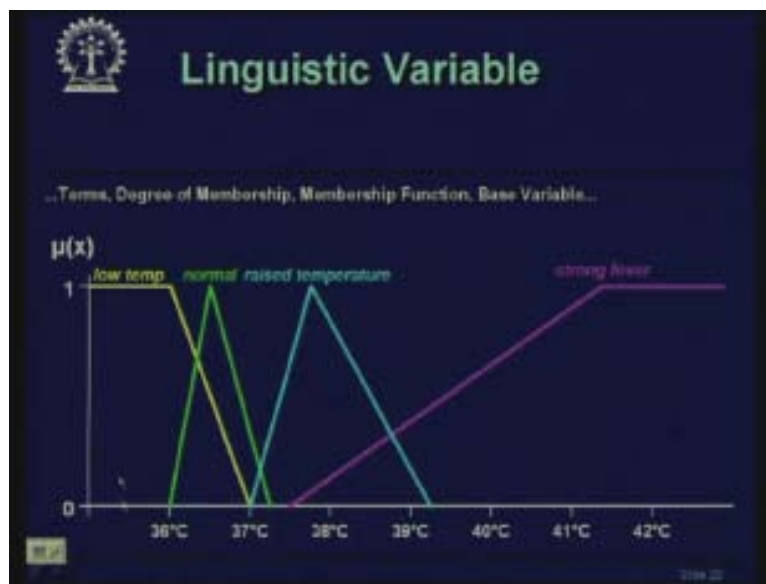
The slide features a logo in the top left corner. The title "Linguistic Variable" is centered at the top. Below the title, there are two examples: "Ravi is tall (adjective)" with a sub-note "- set of tall people is fuzzy", and "Europeans are mostly (adverb) rich (adjective)". A definition follows: "Definition – A linguistic variable is the predicate of a sentence and typically is an adjective (often qualified by an adverb).". At the bottom, a note states: "A linguistic variable to be amenable to fuzzy logic, must have an underlying numerical quantity."

Next we come to the notion of linguistic variable. What are linguistic variables? For example, we use the sentences Ravi is tall. Here tall is an adjective and set of all in English is an adjective. Set of all tall people is fuzzy and statements like Europeans are mostly rich, now this tall is a linguistic variable, rich is a linguistic variable. Now in this sentence we are using adverb which is qualifying this word rich, Europeans are mostly rich. Now how to deal with mostly? That is, as we have some x and square of x similar to that mostly adverb can be assumed to be an operator over this adjective. So we can define a linguistic variable as a predicate of a sentence and is typically an adjective which is often qualified an adverb. This is an observation; A linguistic variable in order to be

amenable to fuzzy logic representation must be related to some crisp set on which we can define the fuzzy set.

For example, when we say rich then we may have a crisp set of possible incomes and we can define the set rich on that crisp set. Unless we have got some underline numerical definition corresponding to crisp set we will not be able to define a fuzzy set over that. A fuzzy set is always define over a crisp set and said to be the subset of that crisp set a fuzzy subset as it is said. Linguistic variables can be of different types usually they are terms and associated with every term we have got a degree of membership and that degree of membership can be computed through a membership function and some phase variables. Here are some interesting curves.

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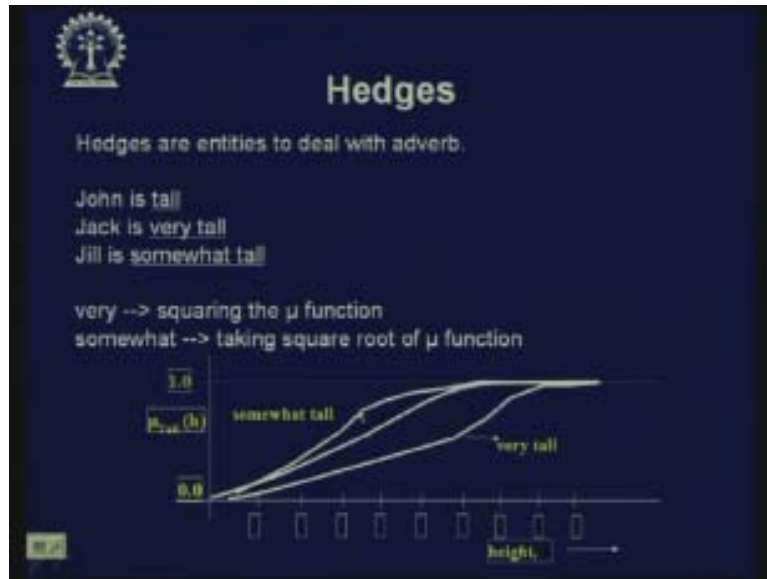


Here is the underlined numerical crisp set of the different temperatures. And on this side is a membership function of the fuzzy set that we defined over this crisp set. So we can define low temperature as in this way which we can again translate in the form of a table or some function if possible. This is piecewise linear so I can always have up to 36 or less than equal to 36 should be 1 otherwise it will be a straight line equation with a given radiant up to 37. This is a normal temperature that means from 36 to 37 so somewhere in between 36.5 is a normal temperature raised temperature we start from here and strong fever starts from here goes up like this. Therefore over the same crisp set we can define different fuzzy sets based on different linguistic variables. So these are the different terms. Corresponding to each term we will have a different fuzzy set and they will be different membership functions which define the degree of membership of the different values of the crisp set.

Now if you have 38 degree centigrade as a particular value then obviously we will get membership around 0.8 which we can say is pretty much raised and again the same value is intersecting in a slightly strong fever here is our corresponding comment on this

membership. Therefore from a crisp value we are coming to fuzzy value so I am getting a membership function and corresponding to the membership function again I can have a corresponding linguistic state.

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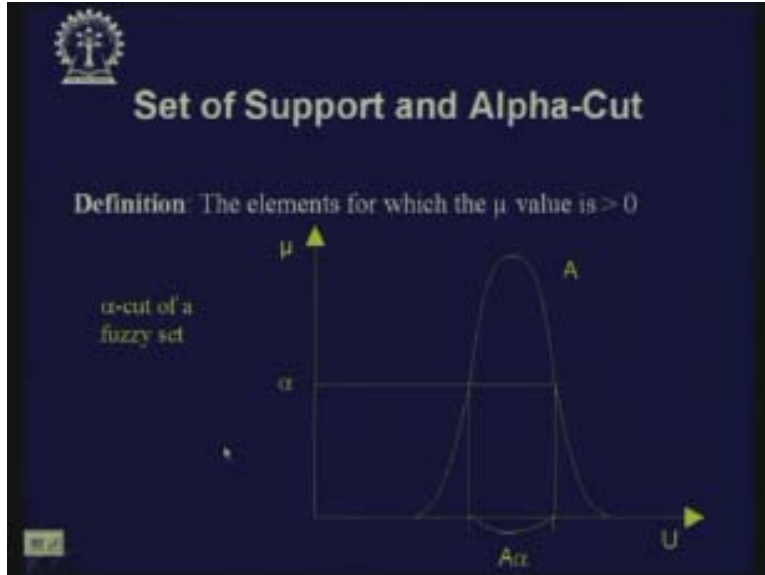


There is another term that needs to be introduced called hedges. Hedges are entities which deal with adverb. For example, John is tall so we can define a fuzzy set tall, this is the height and this is the fuzzy set tall so on this we can say Jack is very tall, very tall is further qualification over this and so that curve will be something like this. And somewhat tall is somehow a sort of relaxation and will look like this. Now at this a point I would like to mention about two operators.

One is the concentration operator and the other is the dilation operator. Suppose for some value of x I have got a membership in tall as 0.5 and the fuzzy set tall will be looking as something like this. Now for a particular height maybe 5 ft 6 inches I have got a membership value something like 0.5. Obviously for this height the membership in a set very tall should be less. So intuitively we can think of curve that the set very tall will be something like this, this will be very tall.

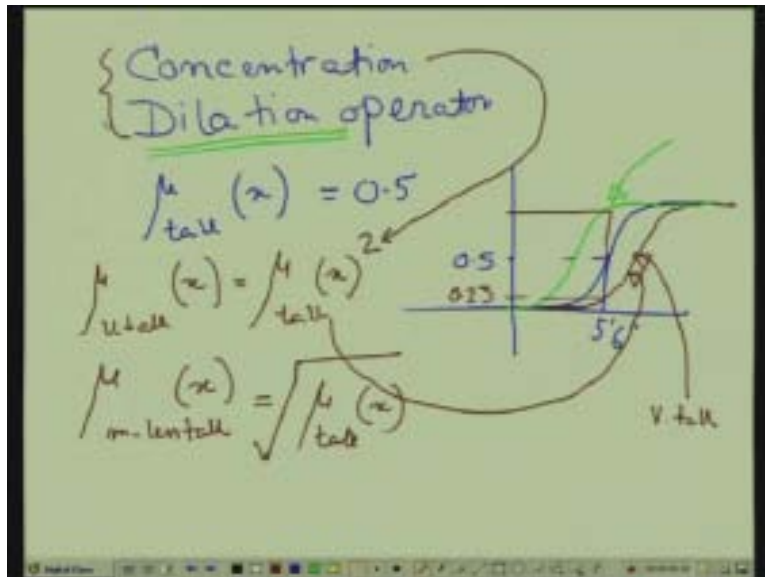
Now, according to **za** it is very convenient to have the membership of very tall obtain very of x obtain from this as new of tall x whole square. So this operation square is known as concentration operation and that is reflected in this curve. on the other hand if we look at the same height and we want to define a set more or less tall then obviously the membership of 5 .6 should increase so that curve will be somehow like this. So this is a dilated curve and this is known as dilation operation. And this one we can obtain as square root of new tall x so this is one way, this square root raised to the power half is a standard dilation operation and squaring is a concentration operation.

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Therefore by this concentration operation this curve will automatically be a membership of this 5.6 in very tall will be 0.25 whereas more or less tall will be quite high. So these are again two very important operations in fuzzy set or fuzzy logic and this is used to compute hedges. The boy is running, the boy is tall, the boy is very tall, so how do I handle very tall when I know the definition of the fuzzy set tall?

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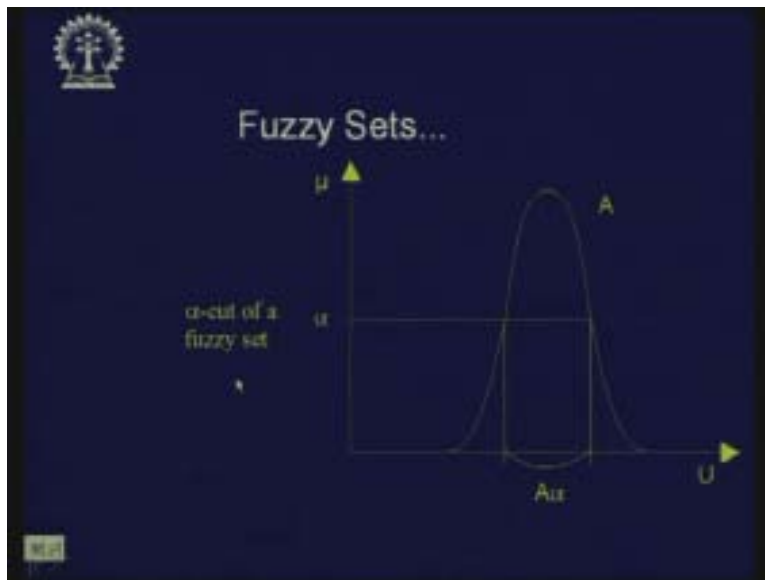


That is how the hedges are represented. These are also called linguistic hedges. Now there are two more important concepts before we move to fuzzy operators. One is set of support. The set of support for a fuzzy set is defined as the set it is a crisp set, a crisp set

consisting of elements whose membership values in the corresponding fuzzy set is greater than 0. So all the elements say height in tall, so all the heights in the set tall, all the heights which have got a membership value greater than 0 in the fuzzy set tall will be called the set of support for the fuzzy set tall. Another thing is alpha cut, it is threshold.

That is, we put up some threshold alpha say 0.3 and all the elements whose membership are less than alpha will be taken out of the set. So, if you look over here we have put in some alpha here. All the elements whose membership value is less than alpha will go out of this alpha cut of the fuzzy set. Therefore the values which will have all these elements in this zone and those values having membership value greater than equal to alpha will only be in the alpha cut of the fuzzy set a. So please note that the set of support and alpha cut which are derived from the fuzzy set a will be two different crisp sets.

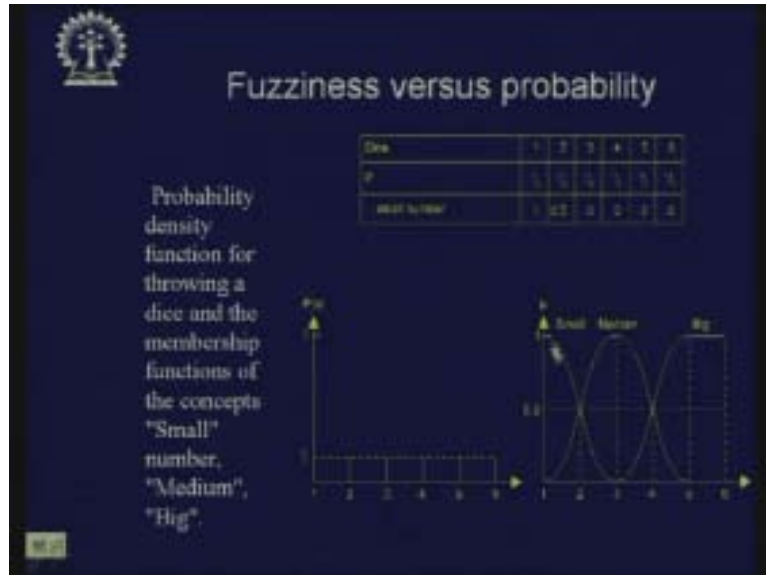
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So this is again the alpha cut. Now let us look at probability and fuzziness. So when we throw a die then the die can have 6 phases 1 2 3 4 5 6 and the probability of each phase will be 1 by 6 1 by 6 etc. But if we say, from this dice if I define a fuzzy set called small number then 1 will be a small number certainly so that is 1, 2 will have a membership 0.5 in this set small number, 3 and all those things will be 0.

So this will be a definition of small, similarly this will be a definition of medium and this will be a definition of big number etc. On the other hand, probability-wise all of them are the same. Therefore we must understand the difference between fuzzy number and a probability value, they are conceptually different. Now our ultimate objective is to go towards fuzzy inferencing. But in order to deal with fuzzy sets just as we do in conventional sets we need to carry out some set operations. So here we are introducing some fuzzy logic operations just like AND, OR, NOT as we did in the case of a normal set and for normal logic. Let us see what they are for fuzzy sets.

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For example, equality: Two fuzzy sets a and b are straight to be equal if for all x their memberships are the same. The membership of all x in the set a is the same as it is in the set b. Now look at the difference between this. In the case of a normal set we said that two sets that a and b are equal if all the elements in this set are equal to this. But in this case all the elements are same but this is a fuzzy set and this is another fuzzy set then unless the membership of this is not the same as the membership of this element and similarly for all the elements, just the belongingness will not do but the membership values should also be the same then only we can see that the theory works.

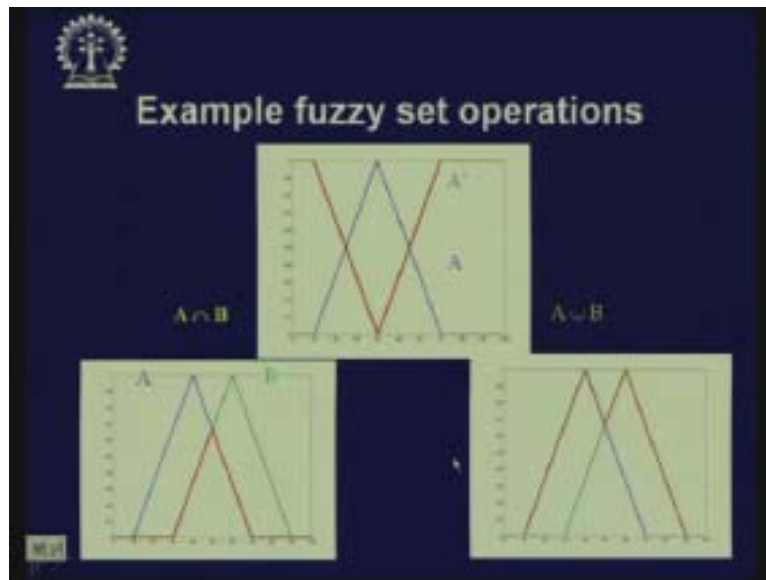
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The other thing is complement. In normal set and fuzzy set it is the same the complement is here. The complement of fuzzy set a is denoted as a' or a complement, membership of a complement x is 1 minus membership of x in a and that is for all x. Containment that is a is the sub set of b a is contained in b is true in case if all the membership values for all the xs' in a or less than equal to membership value of the same element in b. This is the way we are dealing with the membership values.

Here are two important operators union and intersection. The union of two fuzzy sets a and b is defined as a new fuzzy set where the membership value of all the elements here will be the max of their membership elements in the individual sets. And similarly in intersection the membership value in the intersection set will be the minimum of their membership in the individual sets.

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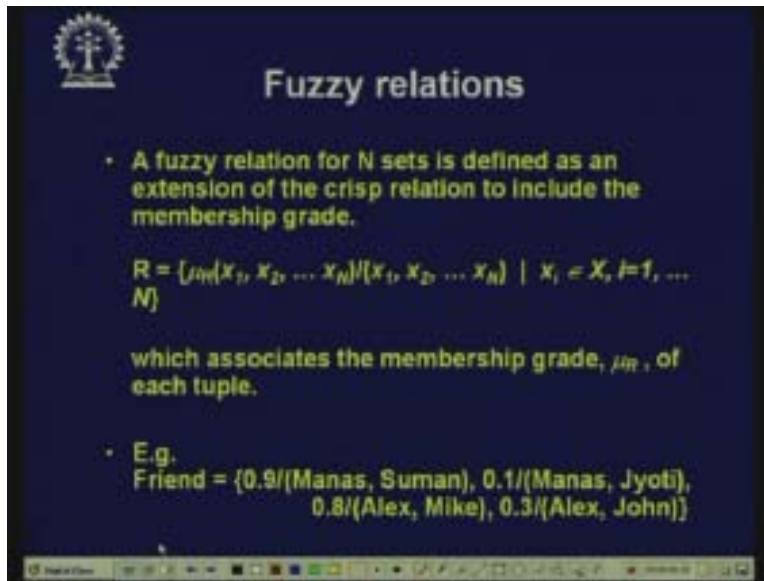


Here is an example; We are looking at a fuzzy set a this is a fuzzy set a, this blue line is a fuzzy set a and then the compliment of fuzzy set a compliment is defined by this red line, wherever its membership is, suppose it is membership of 20 or 0.4 then the membership of compliment for 20 of will be 1 minus 0.4 to 0.6 so this is the compliment set. Here we are looking at the intersection, this blue line is the fuzzy set a and this green line is a fuzzy set b, then the intersection of that is for all the xs' which are common to both a and b and whose membership value maybe this 40 the 40's membership value in a is something like 0.8 and 40's membership in b is something like 0.4 so in the intersection forties membership will be the minimum of this 0.2 to 0.4 so it should be 0.4 50, its membership in a is 0.6 5, its membership in b is also .6 5 so in the intersection set membership of fifty will be the minimum of this so the minimum of 0.6 5 and 0.6 5 0.6 5 so in this way with this red line we will define the intersection set.

The union set on the other hand will look like this. This was a and the green was b and the union is red line where I have taken max of the membership values. And 40's

membership in b was 0.4 and in a it was something like point 8 so I have taken that 0.8 so this is the union in this set. As we know we can modify the meaning of fuzzy set uses hedges like very more or less slightly. And typically very f is nothing but f square like tall, is very tall and more or less is the square root of that. Next we come to a very important notion that is a fuzzy relation.

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What is a relation?

We know, if we have got a crisp set a in which we have got Tom and John and in the set b we have got marks like 80 and 70 then we can define a set a into b which will be something like all the possible tuples Tom that is one element from this and one element from this, John 80, Tom 70 and John 70 so these are the possibilities. Then we define a relation r which is a subset of a into b. Hence may be all the things are not there depending on what r is, Tom 80 and John 70.

Now I give a name to this relation marks obtained. So, marks obtained is a relation, Tom got 80 and John got 70. That is what we know about what relation is all about in the typical Computer Science, Discrete Mathematics sense the Discrete Structures. This is the definition of a relation. Similarly, I could have defined, if there was set c like Mary and Susan I could have defined a relation likes between a into c so Tom likes Mary and John likes Mary could be a set, could be relation.

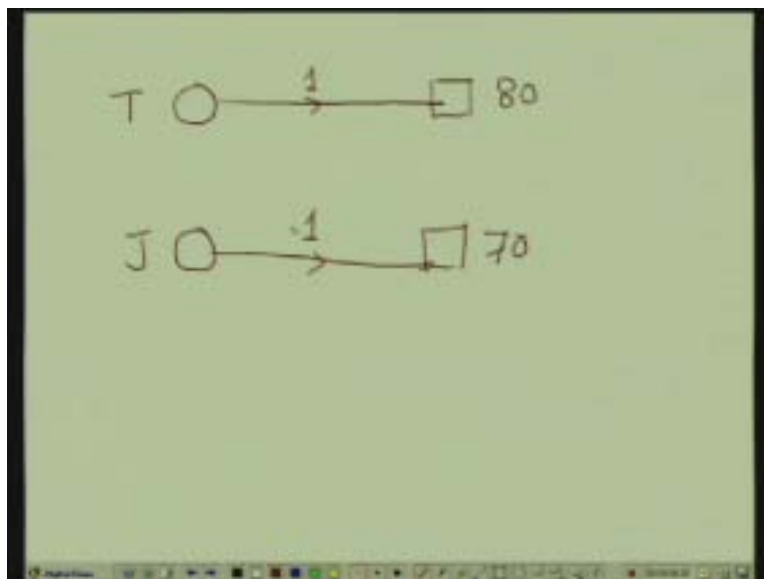
Now here you see that, in a relation either this tuple belongs to this relation or it does not belong to this relation. From this a into b either it belongs to this or it does not belong to this. But in the case of a fuzzy relation this belongingness will again be fuzzy, there will be degree or grade or belongings. So it will belong very strongly or it will belong very weakly. Now this relation Tom John etc I can further represent it in a form of a graph where I have got Tom and John and here I have got those marks say 80 and 70 and my relation was defined as the graph that is Tom got 80 and John got 70.

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$$\begin{aligned} A &= \{ \text{Tom, John} \} \\ B &= \{ 80, 70 \} \quad C = \{ \text{Mary, Susan} \} \\ A \times B &= \{ \langle \text{Tom}, 80 \rangle, \langle \text{J}, 80 \rangle, \\ &\quad \langle \text{Tom}, 70 \rangle, \langle \text{J}, 70 \rangle \} \\ R &\subseteq A \times B \\ \underline{R} &= \{ \langle \text{Tom}, 80 \rangle, \langle \text{John}, 70 \rangle \} \\ \underline{\text{Marks-obtained}} &= \{ \underline{\langle \text{T}, 80 \rangle}, \underline{\langle \text{J}, 70 \rangle} \} \end{aligned}$$

Now the strength of these links are one. I can also have another relation say Tom, John another relation Mary and Susan and a relation may be something like this Tom likes both Mary and Susan and John likes Mary. So this graph can be used to represent the relation.

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Now the strength of relation of these links which is denoting that relationship from Tom to Mary are all having a strength of one in the case of a crisp relation. But in the case of a fuzzy relation there is a degree of association or degree of belonging. So we define it as r the fuzzy relation will be a grade of membership of all these elements x_1, x_2 etc. It is like

x_1 's membership in this relation, x_2 's membership in this relation etc. So this relation is actually associating the membership grade μ for each tuple.

For example, friendship, so Manas and Suman are friends; they belong to the friendship relationship membership of 0.9. Manas and Jyoti are friends but not that strong so that is 0.1, Alex and Mike 0.8, Alex and John .3 so here along with those hedges we had shown we are also putting some grade of membership. Next operator we are looking at is the typical AND OR NOT. We know that NOT of a is denoted by compliment of a is 1 minus $\mu_A(x)$, a and b which is the intersection essentially is when we have got min we have got two memberships that x has got only for the element which is there in both. Of course the membership that is minimum in a and b I take its membership in a and is membership b and take the minimum, a or b will be the maximum. So, if I just take a crisp set that a and b can only have values 0 and 1 which is the truth table and if I apply min a b for the case of 1 1 is 1 and everything else is 0 this is the truth table for conventional a and b.

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Fuzzy logical operations

- AND, OR, NOT, etc.
- NOT A = A' = $1 - \mu_A(x)$
- A AND B = $A \cap B = \min(\mu_A(x), \mu_B(x))$
- A OR B = $A \cup B = \max(\mu_A(x), \mu_B(x))$

From the following truth tables it is seen that fuzzy logic is a superset of Boolean logic.

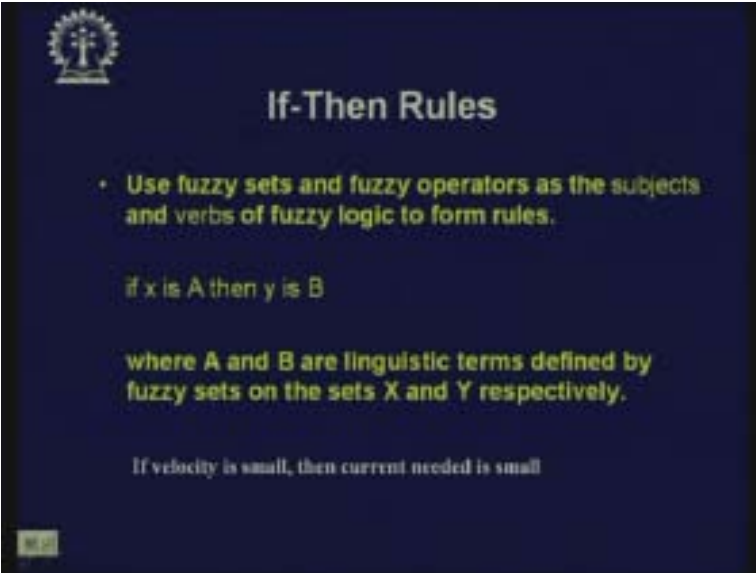
A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

A	not A
0	1
1	0

But if I apply min a b then also a and b will be 0, min of 0 and 1 will be 0, min of 1 and 0 will be 0, min of 1 and 1 will be 1. Similarly for r the max of 0 and 0 will be 0, 0 and 1 will be 1, 1 and 0 will be 1, max of 1 and 1 will be 1. Therefore in this way you can see that the fuzzy logic is actually covering our conventional logic. So fuzzy logic can be considered to be a super set of the Boolean logic.

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If-Then Rules

- Use fuzzy sets and fuzzy operators as the subjects and verbs of fuzzy logic to form rules.

if x is A then y is B

where A and B are linguistic terms defined by fuzzy sets on the sets X and Y respectively.

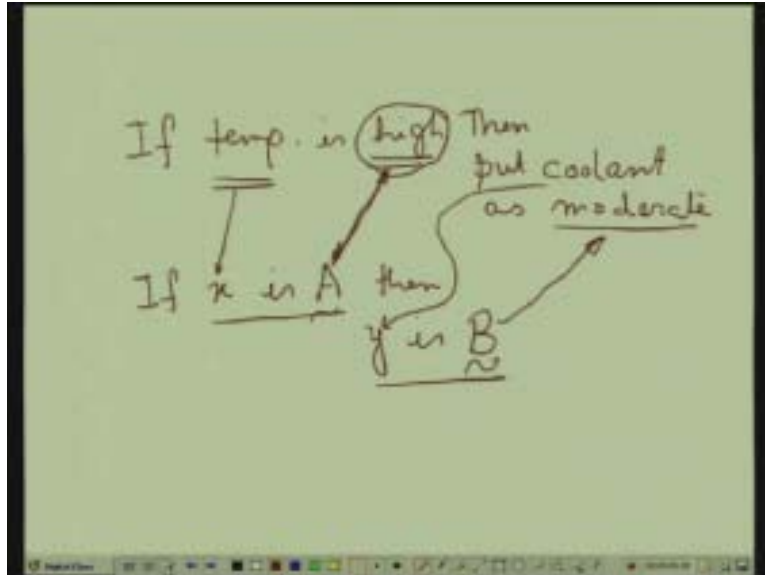
If velocity is small, then current needed is small

Next in order to do some inferencing what we need to do is, we have to look at if then type of rules. We use fuzzy sets and fuzzy operators as the subjects and verbs of fuzzy logic to form the rules. Now consider this rule; if velocity is small then current needed is small. Now obviously this is a rule where the statements are different than what we had in the case of conventional rule because here we have got all these fuzzy terms here. So such rules can be written as if x is a then y is b.

If the boy is tall then the boy is strong. If the boy is tall then A is fuzzy set tall. So A is a linguistic term defined by a fuzzy set tall, then Y is strong, B a linguistic term which is a fuzzy set strong defined over some crisp set Y. So we can have a couple of more examples; if temperature is high then put coolant as moderate. Here if I write this as, if x is a now x is this linguistic term which is taking a value from a fuzzy set A. And that fuzzy set A is the fuzzy set high, then y is B, y is the variable coolant and B is again the fuzzy set moderate. So this is a fuzzy set and this is another fuzzy set. So fuzzy rules can be generalized in this way if x is a fuzzy set then y is B.

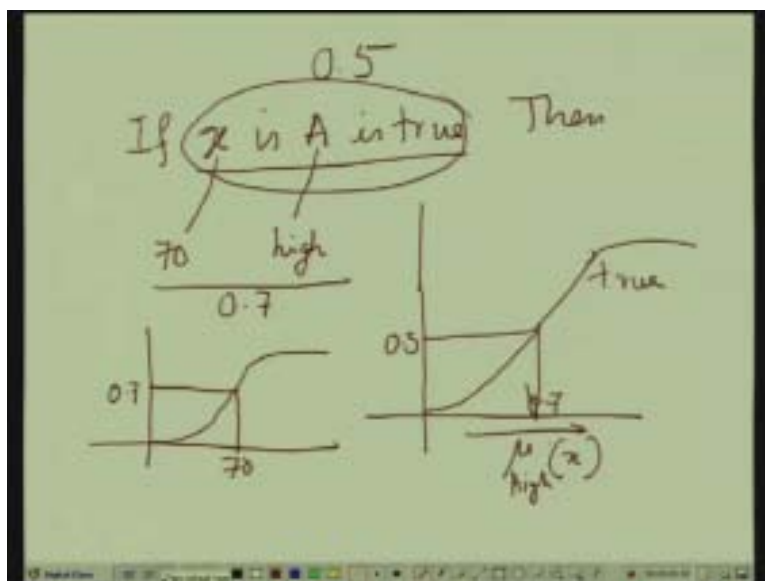
Now the question is, if I have a rule like this if x is A where A is a fuzzy set then y is B this is another fuzzy set. Now, I have to evaluate in order to fire this rule in the form of forward changing I have to first evaluate this antecedent and earlier we found that this antecedent usually is evaluated to true or false. Now, if temperature is high now I have got a fuzzy set high and suppose temperature is 70 degree and in the fuzzy set high the membership of 70 degree is mew of 70 in high is 0.7 so this will evaluate to not 1 or 0 but it will evaluate to 0.7. Therefore unlike the conventional rule base where the antecedent was evaluating to true it is now also having a quantity of truth.

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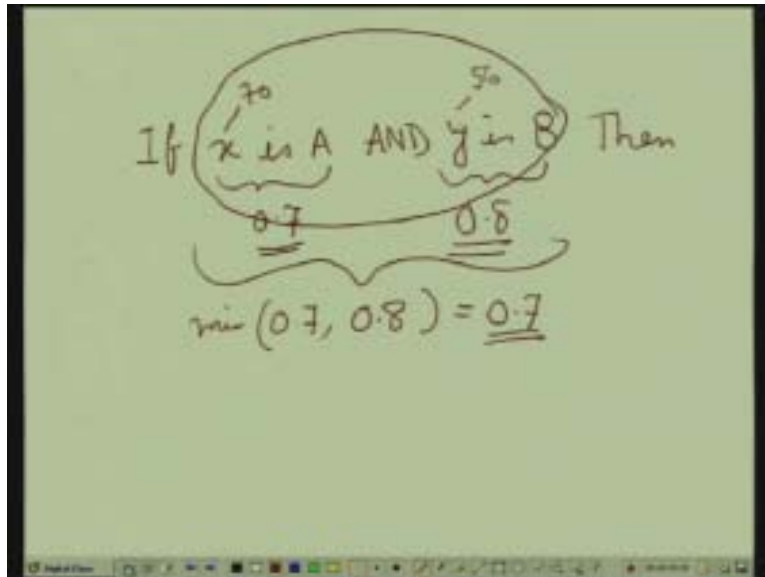
I can even make this antecedent a little more complex. If x is A is true now what will be the value of this antecedent? So x is again temperature 70 degree and A is high so I evaluate this in some fuzzy set high temperatures as something like this where I got 70 degree and in that I found its membership to be 0.7 so this one comes to 0.7. Now this is true and here is the new value of x in high and there we got .7 and this is my definition for the value true, this is the fuzzy set true as something like this. So 0.7 will have a membership say 0.5 here this is a two level thing. So this entire thing will now evaluate to 0.5. This is how the antecedents are evaluated.

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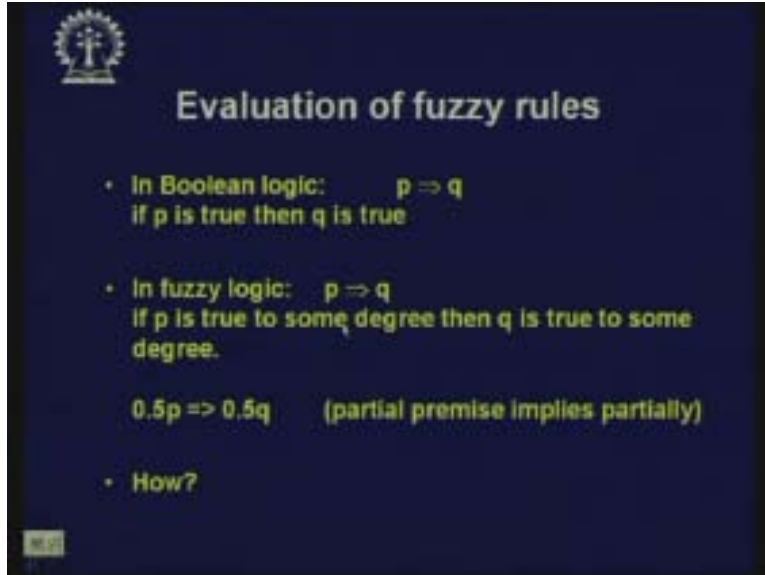
Again we can have some rules like; if x is A and just a conjunction y is B, if the temperature is high and the pressure is low and as in the earlier case it was 70 this was high so this value evaluated with 0.7 and this was some value of 56 PSI and in the set low pressure is some how evaluated with 0.8 then what will be the evaluation? What will be the truth of this entire antecedent? We actually take the min of these two so min of these two 0.7 and 0.8 we will take so it will become 0.7 so we evaluate this **conjunct antecedent** with a strength 0.7.

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Evaluation of fuzzy rules: In Boolean logic we know that p implies q if p is true then q is true. But in fuzzy logic if p is true to some degree as we have seen x is A true with the degree 0.7 then q is true to some degree, it is not just as q is true absolutely. Like here simply if p is true with 0.5 then q will be assumed to be true with 0.5.

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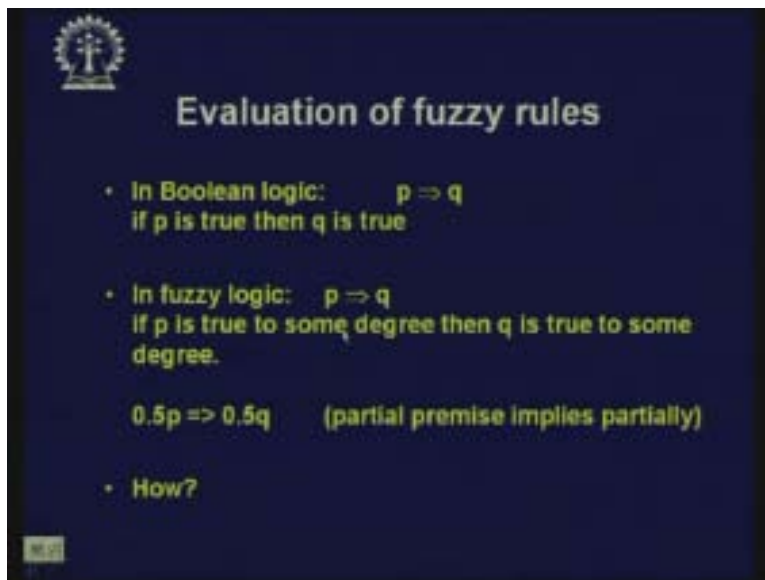
Evaluation of fuzzy rules

- In Boolean logic: $p \Rightarrow q$
if p is true then q is true
- In fuzzy logic: $p \Rightarrow q$
if p is true to some degree then q is true to some degree.

$0.5p \Rightarrow 0.5q$ (partial premise implies partially)

- How?

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Evaluation of fuzzy rules

- In Boolean logic: $p \Rightarrow q$
if p is true then q is true
- In fuzzy logic: $p \Rightarrow q$
if p is true to some degree then q is true to some degree.

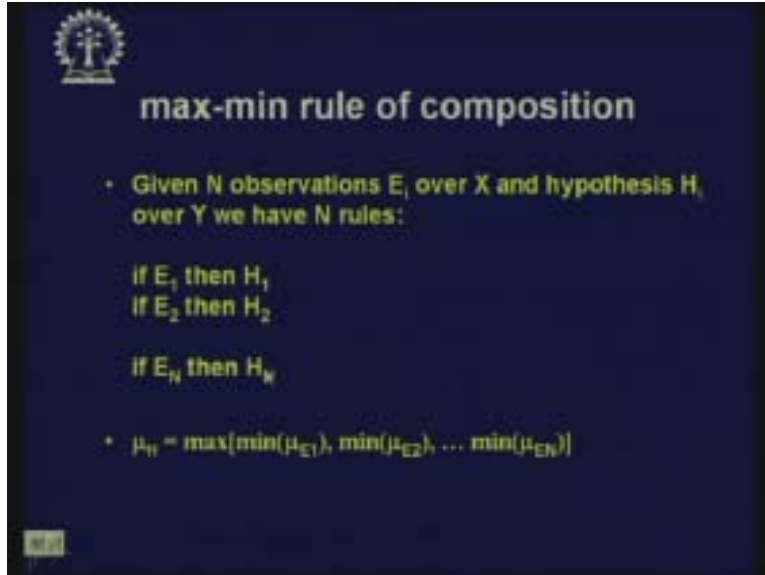
$0.5p \Rightarrow 0.5q$ (partial premise implies partially)

- How?

Max-min rule of composition:

Suppose I have got multiple rules, there are n observations E_1, E_2, E_3 up to E_N over x and we have got n different hypothesis for them. Now I want to evaluate the max, I want to evaluate the strength of the hypothesis and here this can be a conjunction as we said x is A and y is B so what we will do is we take the min of that so we will get a min value of E_1 , I will get min value of E_2 , I will get min value of E_N . Then out of those I will take the max of those.

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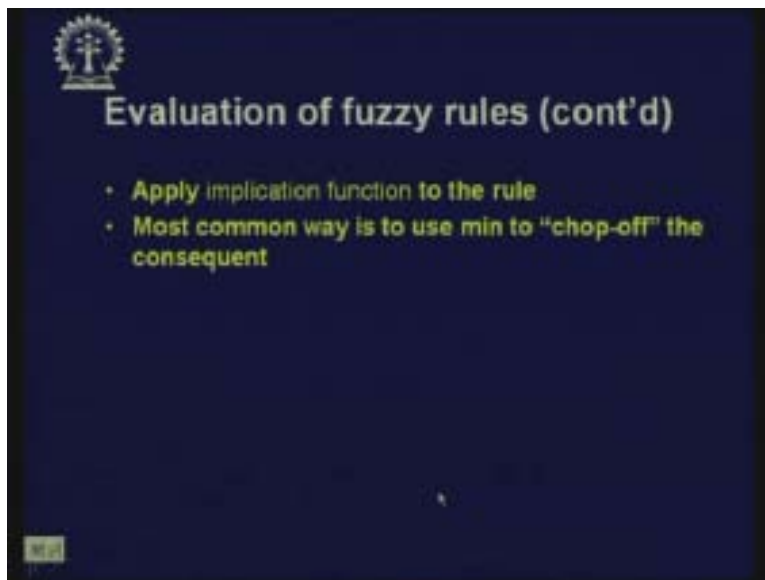


max-min rule of composition

- Given N observations E_i over X and hypothesis H_i over Y we have N rules:
if E_1 then H_1
if E_2 then H_2

if E_N then H_N
- $\mu_H = \max[\min(\mu_{E_1}), \min(\mu_{E_2}), \dots, \min(\mu_{E_N})]$

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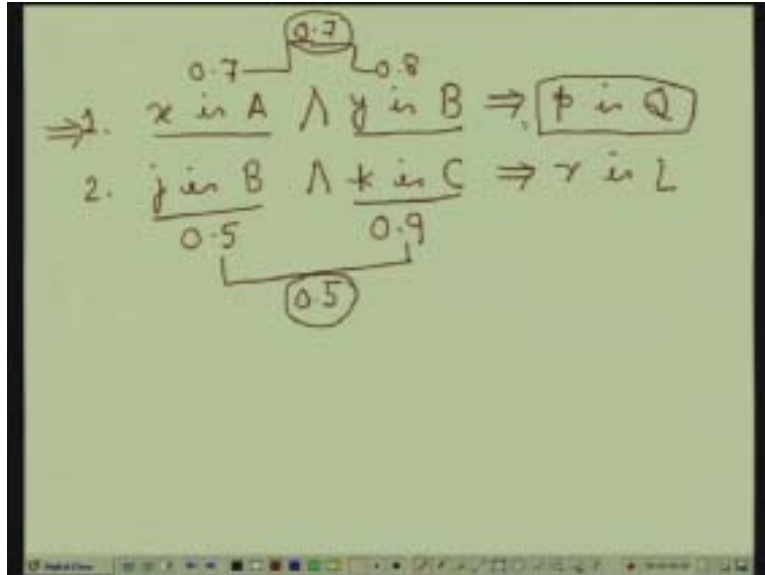


Evaluation of fuzzy rules (cont'd)

- Apply implication function to the rule
- Most common way is to use min to "chop-off" the consequent

Again the same example; x is A is the if part and y is B then maybe something like b is q is one rule. There is another rule say j is B and k is C implies r is may be q or l . Now x is A is evaluated with 0.7 and y is B is evaluated with 0.8. So we took the min of these two and this antecedent is evaluated to 0.7. Suppose this part evaluated with 0.5 and this part evaluated with 0.9 then I will take the min of this so it will be 0.5. Now, out of this 0.5 and 0.7 I will take the max of these two so I will select this rule and I will fire this concept, so this is one approach of doing that.

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This is what is told here; the max of all mins. Evaluation of fuzzy rules: The most common way is to chop-off the consequent.

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Multiple rules

- We aggregate the outputs into a single fuzzy set which combines their decisions.
- The input to aggregation is the list of truncated fuzzy sets and the output is a single fuzzy set for each variable.
- Aggregation rules: max, sum, etc.
- As long as it is commutative then the order of rule exec is irrelevant.

Just now we saw the example where we aggregate the outputs into a single fuzzy set which combines a decision. Now let us come to fuzzy inferencing problem. There are some things we need to do for this. The first thing is fuzzification.

What is meant by fuzzification? We are given some number, why do we need to fuzzify?

We need to fuzzify because our inference rules are not crisp rules as we have shown in the conventional rule based expert systems. In this case our rules are themselves fuzzy. If the velocity is small then the current is quite small. Therefore when we get some value maybe the velocity is 5 meters per second now 5 meters per second is it really a small velocity?

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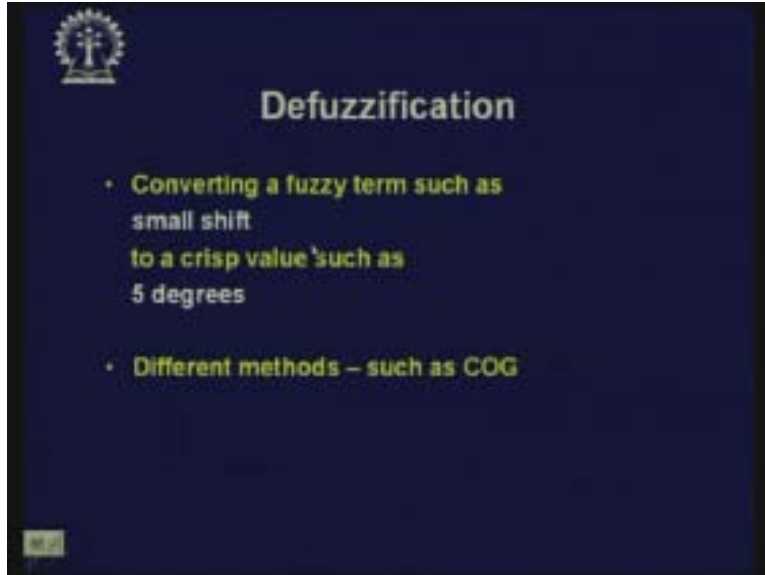


What is its membership in the fuzzy set small is what we have to find out. And this process is known as fuzzification. So we get some crisp value converting a crisp value such as height for example 5 ft 6 inches to a membership value of fuzzy set such as medium or may be tall. There are different ways of fuzzification. Some are experimental and some are subjective. And fuzzified values serve as the input to the fuzzy rules. So fuzzification is converting a crisp value to a fuzzy membership. The other thing is defuzzification.

What we mean by defuzzification?

It is converting a fuzzy term such as small shift. Suppose some rule is like this; if the temperature is moderate then turn the coolant control and give that coolant control a small shift. Now the rule is stating small shift. Now for the robot arm or some automated machine we will have to give a small shift and then actually command using some crisp term say 5 degree, 10 degree whatever it is. And this process of converting this small shift to a crisp value is known as defuzzification. There are different methods of defuzzification such as centre of gravity method.

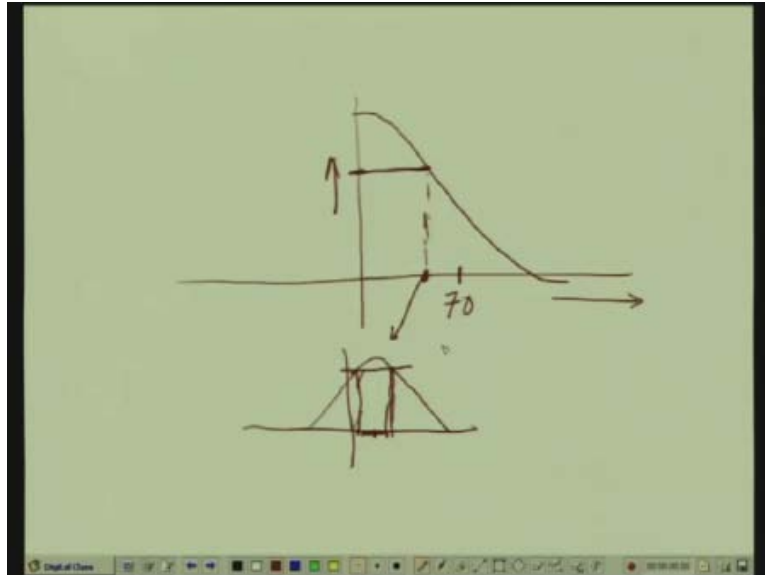
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For example, I have got some fuzzy set like this any arbitrary fuzzy set where this is the variable and here is the membership. Now I get some crisp membership value here. Now this membership value will have to be translated to some crisp value which is known as the process of defuzzification.

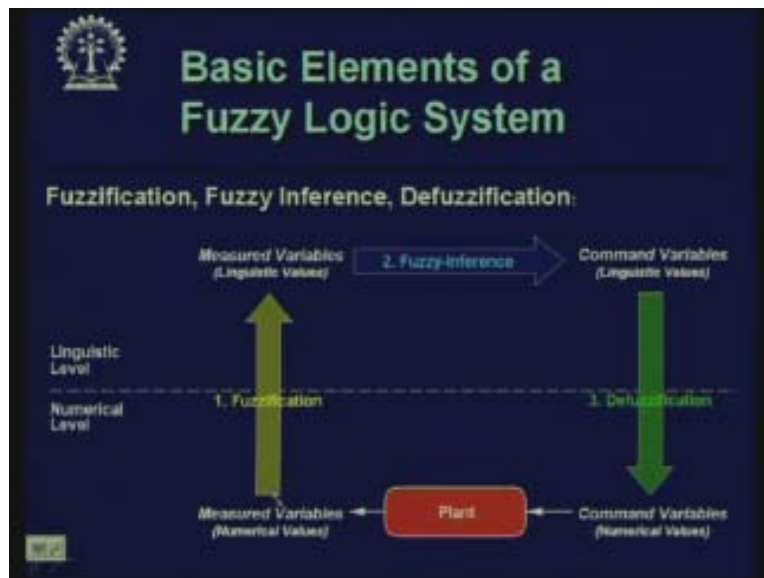
Now, suppose this is a curve and a curve is of this form as something like this and a particular membership value is actually cutting this curve in two points here and here so I am getting this so which one will I take? There are different ways, I can take the mid point of this or I can take the center of gravity method that means for each of these points I take membership and I multiply this value with their membership and then divide by these values so that I get the average of this. So there are different methods like max, center of gravity method etc. There are a number of good books to refer like [.....] 49:56 and Falger and the book by Timothy Ross.

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Now we will show an example where we will be applying this fuzzy inferencing. Let me first explain the problem here. The problem is something like this; we are going to control something.

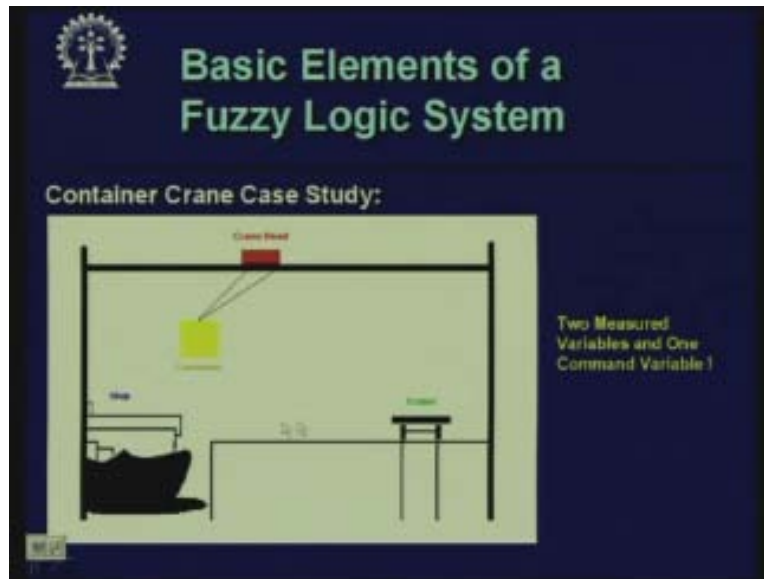
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We are given some measured value from a plant. I get some numerical value but my inferencing method is fuzzy. Therefore first thing that I have to do is fuzzify this and get the linguistic variable. Then using my fuzzy rules I will get some output in terms of linguistics, in terms of give a small shift or something like that.

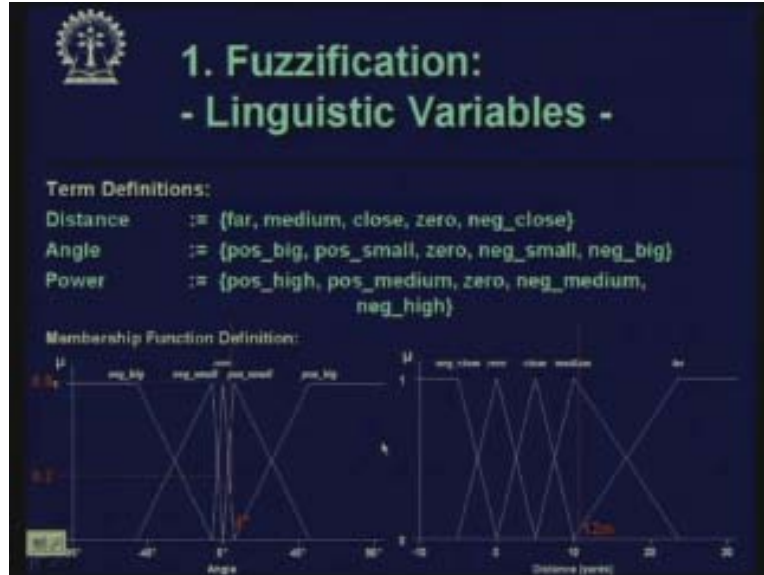
Then I will have to defuzzify that and I will get a numerical value which I will again feedback to this control. So here you see I am working at two levels numerical and linguistic. Now here is an interesting example. There is a crane moving along this axis and there is a container hanging from here. And the control must leave this container at a proper point of times so that it falls on the ship which is this or it takes it from the ship and falls on the trailer. Now it is very difficult to model such systems using normal state phase approach.

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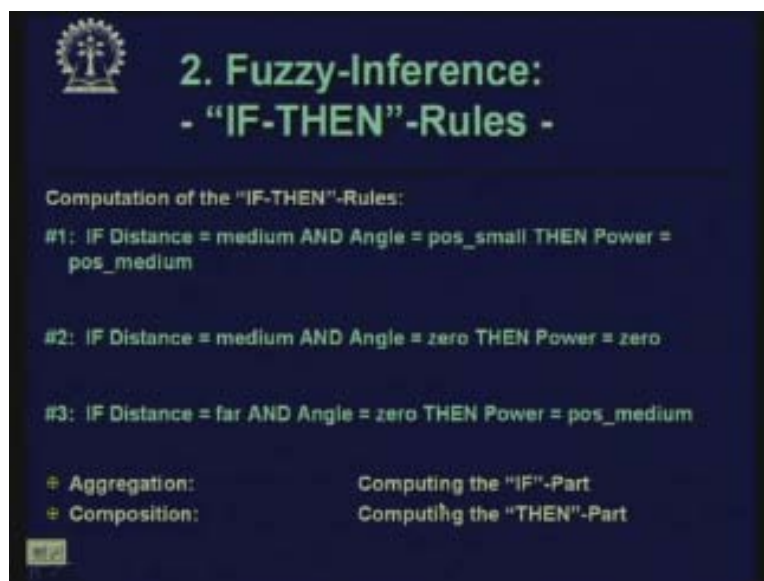
Here, there are two measured variable and one control variable. So what we do is, we have got angle and distance as two issues, now the question is, at which angle I should leave it or from which distance I should leave it. Now I have to give the container grain a particular power and depending on the power I give we the grain will move with different speed and accordingly the angle and distant vary so I have to control that power which is my objective. So I read the numerical values the angle and distance and then fuzzify it. So I get some linguistic values and then get the power and then defuzzify it and again actually put the real numerical power to the controller crane. That is the problem we are trying to solve. So the first thing that I have to do is to define a set of fuzzy sets like distance, is far, medium, close, zero, negative close etc. Angle again may be positive big, positive small, negative small, negative big etc. power can be positive high, positive medium, zero, negative medium, negative high etc. Now these are very big so I have to actually define their membership functions.

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Therefore I can define the negative big here on an angle, the negative big is this fuzzy set, negative small is this fuzzy set, positive small is this fuzzy set in this way I can define all of them. And given any particular value maybe 4 degree of angle I can find out the membership that its membership in 0 is 0.2 and its membership in positive small is 0.8. So, in that way I can find out their memberships. So this is fuzzification. I get an angle and get these membership values.

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This is how I do the fuzzification first. After doing the fuzzification we will apply the rule. Suppose there are couple of rules; if distance is medium and angle is positive small then power is positive medium. If distance is medium and angle is 0 then power is 0. If distance is far and angle 0 then power is positive medium. So we need to do the two things. First we need to do the aggregation. That means first we have to do the computation of the 'if part' and composition is the computation of the then part. The computation of the 'if part' is what we have already discussed.

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**2. Fuzzy-Inference:
- Aggregation -**

Fuzzy Logic Delivers
a Continuous Extension:

- ⊕ AND: $\mu_{A+B} = \min\{\mu_A; \mu_B\}$
- ⊕ OR: $\mu_{A+B} = \max\{\mu_A; \mu_B\}$
- ⊕ NOT: $\mu_{\bar{A}} = 1 - \mu_A$

Aggregation of the "IF"-Part:

- #1: $\min\{0.9, 0.8\} = 0.8$
- #2: $\min\{0.9, 0.2\} = 0.2$
- #3: $\min\{0.1, 0.2\} = 0.1$

In the first rule we get the distance medium was 0.8 and the angle positive small was 0.9, the angle had a membership 0.9 here and the distance was 0.8 in medium so this thing will be the min so it is 0.8. And for the second rule suppose in the same the two antecedents get 0.2 and for the third rule it is giving 0.1. So for each of these rules we do the aggregation, we compute with the min function the truth of their antecedents then I will select to the composition so all these rules will give you power to be positive medium powered to be 0 powered to be positive medium.

Then when I compose them positive high is not coming with any membership, positive medium is 0.8 because it was the max of 0.8 and 0.1 where from this 0.8 and 0.1 come? Again looking back, positive medium was 0.8 the first rule was saying positive medium with 0.8 and the last rule was saying positive medium with 0.1. So I have got these so I have taken the max. Therefore out of this I have taken max so I have got 0.8.

Now with 0.8 what is the positive medium power I have to give? But how would I give that 0.8? So 0.8 is somewhere here, 0.8 membership is positive medium so 0.8 is coming somewhere here so I will again compute the crisp value corresponding to this using some method like center of the maximum and I will get 6 0.4Kw and that is the power I will have to give. So this is the approach we adopt in the case of fuzzy inferencing.

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2. Fuzzy-Inference:
- "IF-THEN"-Rules -

Computation of the "IF-THEN"-Rules:

#1: IF Distance = medium AND Angle = pos_small THEN Power = pos_medium

#2: IF Distance = medium AND Angle = zero THEN Power = zero

#3: IF Distance = far AND Angle = zero THEN Power = pos_medium

⊕ Aggregation: Computing the "IF"-Part
⊕ Composition: Computing the "THEN"-Part

Given a set of fuzzy rules suppose you have a set of fuzzy rules where the antecedents are fuzzy and the consequents are fuzzy so you have got some value, then the first thing you have to do is you have to fuzzify. And in order to fuzzify you have to define the fuzzy sets before hand. And those fuzzy sets can be stored as tables or functions. So fuzzification means you will get some membership value. So, for every rule that is applicable you will compute the truth of their antecedents.

How do you compute the truth of their antecedents?

If the antecedent is a conjunction then we will take the minimum of those. Now, out of the rules which are applicable out of this minimum I will take the maximum and that rule I will apply and that consequent will give me some fuzzy membership and with that fuzzy membership positive medium it is saying or turn the knob slightly to the left, in that slightly left you are getting a membership 0.4. Now you defuzzify and 0.4 membership is leading to which particular crisp value and that is the shift we will give. Now this approach is rather effective and it is being applied in several applications now-a-days.