

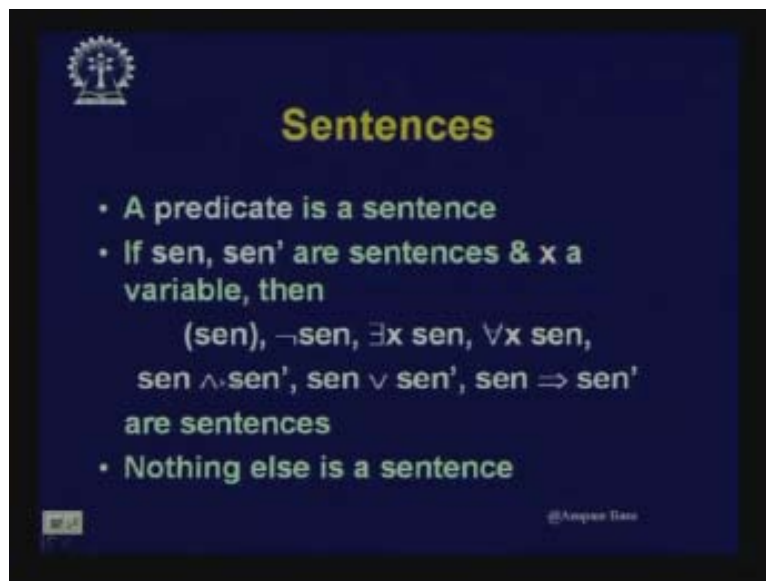
**Artificial Intelligence**  
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**Lecture - 14**  
**Reasoning using First -Order Logic**

In the last lecture we introduced the first order logic also known as predicate logic to express the different domain specific knowledge we require in order to build an integrated system. In today's lecture we will be talking about different ways and means for reasoning using such a language that is the first order logic. How we can use different sentences of first order logic to reason out a particular problem is what we will start looking at from today. Before we go into the inferencing process we would like to first show or rather revisit some of the notions we presented in the last lecture. To start with we introduced sentences in first order logic.

What a sentence is?

A predicate is a sentence and we had already seen what a predicate is. And if  $\text{sen}$  is a short form of any sentence and say  $\text{sen}'$  is a negation of  $\text{sen}$ , so suppose there is a sentence or the negation of a sentence these are sentences **on the whole**.

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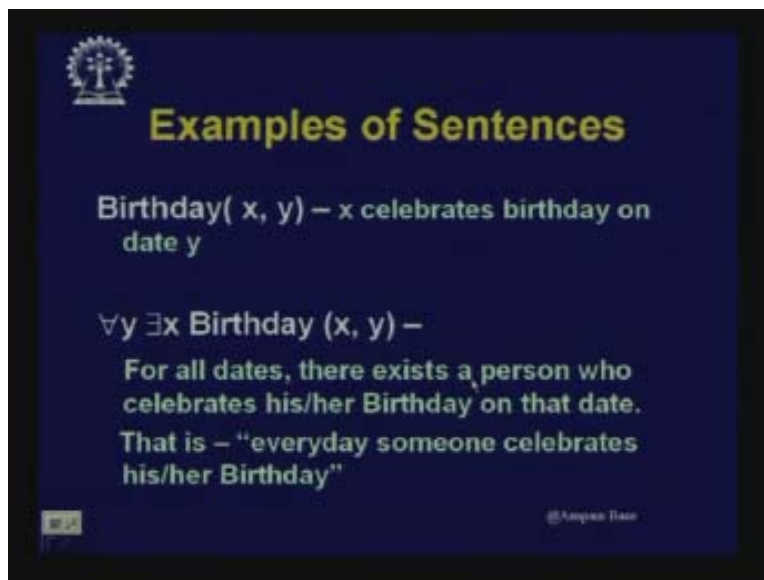
If these are sentences and  $x$  is a variable then  $\text{sen}$  is a sentence, NOT of  $\text{sen}$  is a sentence, there exists  $x$  such that the sentence is true is a sentence in first order logic, for all  $x$   $\text{sen}$  is a sentence, now these two are equivalent. If you look I have sometimes written the NOT symbol as this and sometimes as this prime. But essentially these two are meaning the same thing. So sentence and the negation of the sentence is a sentence, sentence OR the negation of the sentence is also a sentence. Suppose we are considering two sentences this is not the proper way, say  $\text{sen}$  and  $\text{sen}'$  are two sentences suppose there are two

sentences and NOT sen is the compliment of sentence, if sen is a sentence and sen prime is another sentence and x is a variable then sen is a sentence, NOT of sen is a sentence, there exists x sen is a sentence, for all x sen is a sentence, sentence, sen and sen prime the conjunction of these two sentences is a sentence, sen OR sen prime is a disjunction of these two sentences is also a sentence, sen implies sen prime is one sentence implying the other is also a sentence and in proper predicate logic nothing else is considered as a sentence.

So what we really mean is, we start with a predicate say  $P(x)$  tall(x) that is a sentence and say strong(y) is another sentence. So tall(x) tall(x) is a predicate and hence it is a sentence. there exists some x such that tall(x) is a sentence, for all x tall(x) may not be true, that is for all people they are tall, for all x tall(x) is a sentence may not be true, may not be satisfied in a particular domain. So tall(x) and strong(y) is a sentence, tall(x) strong(y) is a sentence, tall(x) implies strong(y) is another sentence. So these are sentences. That means if we take a predicate and the negation of the predicate is a sentence since the predicate itself is a sentence existential quantifier which we had introduced in the last lecture existentially quantified sentence is a sentence, universally quantified sentence is a sentence, the conjunction of two sentences are sentence, the disjunction of two sentences is a sentence, implication one sentence implying the other is also a sentence and nothing else is a sentence.

Now let us look at some example sentences. If we write the predicate birthday (x, y) so this is a binary predicate and why is it a binary predicate? It is because it has got two variables as parameters, what is the meaning of this in a particular domain? It means that x celebrates birthday on the date y.

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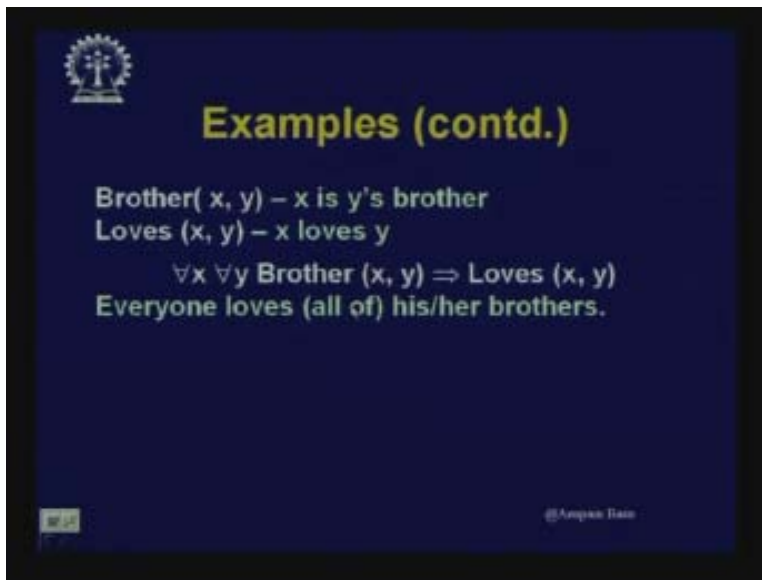


So for example, Tom celebrates birthday on 1<sup>st</sup> January or Julie celebrates birthday on 1<sup>st</sup> March etc. So in general I can say x celebrates birthday on date y and can be written as

this predicate, so this is a sentence. Now If we write for all  $y$  there exists  $x$  such that  $\text{birthday}(x, y)$  what does it mean? Looking at this predicate we can see  $x$  celebrates birthday on the date  $y$ . And what does this sentence mean? This is a sentence according to our last definition. This is also a sentence because it is existentially quantified. You said there exists  $x$  sen is a sentence but again you are saying for all  $y$ .

Let us go back a little bit and let us have a look at that again. Here it is said that an existentially quantified sentence is a sentence so this whole thing is a sentence. Therefore if I apply a universal quantifier over a sentence then that also remains a sentence. So here we can see that  $x$  celebrates birthday on date  $y$  and what does it mean? That is, for all  $y$  that means for all dates, for all possible dates there exists some person who has got a birthday on that date.

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Now if I express this in English then it would mean, that for all dates there exists a person who celebrates his or her birthday on that date. That is, everyday someone celebrates his or her birthday. Now this relatively complicated notion can be very succinctly, in a very expressive manner, in a very brief and packed manner can be expressed in the language of first order predicate logic as is shown here. So this is a sentence and when I quantify this with an existential quantifier that is a sentence and this whole sentence when quantified with a universal quantifier is also a sentence.

Next let us go to another example; when I write  $\text{brother}(x, y)$  that means that  $x$  is the brother of  $y$ . Now, when I write  $\text{loves}(x, y)$  that means  $x$  loves  $y$ . These two are predicates, both binary predicates and these two are also sentences. Now I want to make a more expressive statement;  $\text{brother}(x, y)$  implies  $\text{loves}(x, y)$  and that is universally quantified over all  $x$  and all  $y$ . That means for all  $x$  and for all  $y$  if  $x$  is the brother of  $y$  then  $x$  will love  $y$ , whether in real world that is true or not that is a different issue but this is a proper sentence in terms of language of first order predicate logic. So what is the

meaning of this? The meaning of this sentence is; everyone loves, all of his brothers or her brothers, now this is a correct sentence.

Now you might have noted that often I am saying valid and then I am just changing that word valid because I am using this word as a valid sentence in normal English but in terms of logic since we have already discussed the interpretation of sentence in a particular domain this word valid has got a special meaning.

When a sentence is evaluated to be true in a particular domain then the sentence is said to be satisfied. But when we say a sentence is valid strictly in terms of first order logic then actually we mean that for all possible domains this sentence will evaluate to be true. So it has got a special logical connotation although in our normal day to day English we often use the word valid in a much looser sense. That is why whenever I am talking of logic and normally the word valid is being used but I am trying to retract from it. So it is a correct sentence in terms of the grammar of first order logic. So, everyone loves, all of his or her brothers is expressed in this sentence.

Now let  $m(x)$  represent mother of  $x$  then I want to express this English sentence everyone loves his or her mother. How would I express it? Suppose by  $m(x)$  I denote  $m$  is a predicate this predicate will denote mother of  $x$  and I want to express everyone loves his or her mother what would be the correct first order predicate logic sentence in order to express this? It will be for all  $x$  loves  $x$   $m(x)$  why is it for all  $x$ ? Everyone so I can take any  $x$  and whatever  $x$  it be Tom, Dick, Harry, Ram, Shyam, Jagjeet then  $x$  will love mother of  $x$  because  $m(x)$  denotes this. So these are some examples of first order predicate logic sentences.

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**Examples (contd.)**

Brother(  $x, y$  ) –  $x$  is  $y$ 's brother  
Loves(  $x, y$  ) –  $x$  loves  $y$   
 $\forall x \forall y$  Brother(  $x, y$  )  $\Rightarrow$  Loves(  $x, y$  )  
Everyone loves (all of) his/her brothers.

Let  $m(x)$  represent mother of  $x$  then  
"everyone loves his/her mother" is  
 $\forall x$  Loves(  $x, m(x)$  )

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Till now we were giving examples from our day to day usage but when we come to a little more mathematical domain then also we will find that first order logic can express a

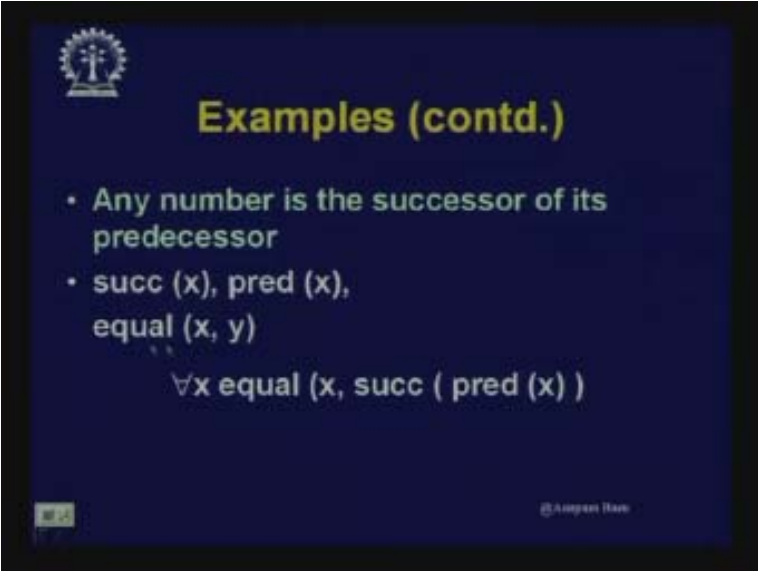
lot more things in a powerful and succinct manner. Here is an example; Suppose any number is the successor of its predecessor, I take any number 7 what is the predecessor of 7? It is 6 and what is the successor of 6? It is 7 so these are very true sentences that any number is the successor of its predecessor. So in order to express this I choose two predicates successor  $x$  and predecessor  $x$  that is  $\text{succ } x$   $\text{pred } x$ . Now with these two I also introduce another predicate equal  $(x, y)$  which means  $x$  and  $y$  are equal.

Using these three predicates I would like to express this English sentence that any number is the successor of its predecessor how can I do it? For all  $x$  equal  $x$  successor predecessor  $x$ , now let us try to see what it means? For any number if  $x$  be the number then predecessor of  $x$  will return me the predecessor of that number and the successor of that number will return me the successor of predecessor of  $x$ . And this number, and this number, and this number should be equal.

Say  $x$  is 20 then predecessor  $x$  will return me 19 then I am computing successor of 19 what is that? It is 20. So 20 is my  $y$  and the  $x$  was my 20 so these two are equal. So these two are equal and the same thing can be expressed in this way. It is for all  $x$  so you can take any number and this will hold true. Now you can think that I could have represented that in a better manner why I did not use the equality sign? I tried an alternative way of representing it. So the previous example can be represented very succinctly as, for all  $x$  successor of predecessor  $x$  equals  $x$ .

Now if you are careful you can note that I have put in this equality in a different color. Unfortunately the definition of sentences and the valid operators we mentioned in the case of first order logic did not have any equality operator. There were predicates, there were quantifiers but this equality operator was not there. Therefore we had to take recourse to this sort of predicate equal  $(x, y)$  which was essentially denoting  $x$  equals  $y$ .

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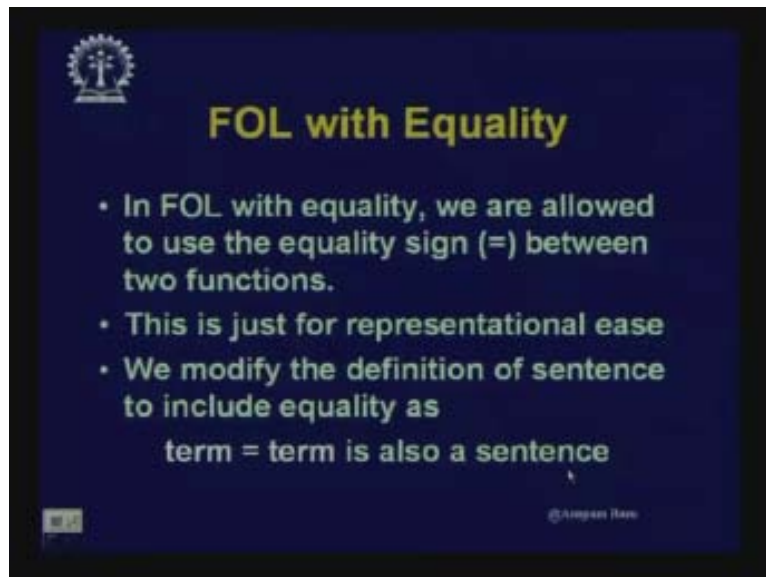


The slide features a dark blue background with a logo in the top left corner. The title 'Examples (contd.)' is written in yellow. Below it, there is a list of items in light blue text: 'Any number is the successor of its predecessor', 'succ (x), pred (x),', and 'equal (x, y)'. Below the list, the logical expression  $\forall x \text{ equal } (x, \text{succ } (\text{pred } (x)))$  is displayed in light blue. In the bottom left corner, there is a small green box with the number '4.6'. In the bottom right corner, there is a small white box with the text '© Arjun Rao'.

In the standard predicate logic as defined this equality operator does not exist therefore this is not a grammatically correct predicate sentence. However it was found that this limitation was really becoming a little difficult to maintain because often if we had the equality operator allowed in predicate logic we could have expressed many sentences in a better way. Therefore first order logic was extended with first order logic with equality; it was extended to first order logic with equality.

In first order logic with equality we are allowed to use the equality sign between two functions. This is for representational ease and so we can now modify the definition of sentences we have provided where we had written  $\text{sen}$ ,  $\text{sen prime}$  as two sentences we can call them as terms. Each of these  $\text{sen}$  can be called terms and there are different ways of defining this. We said  $\text{sen}$ ,  $\text{sen prime}$  are sentences therefore for all  $x$   $\text{sen}$  is a sentence. Now whenever it becomes a sentence we can call that small sentence a term. So now we extend this definition as  $\text{term} = \text{term}$  which is also a valid sentence.

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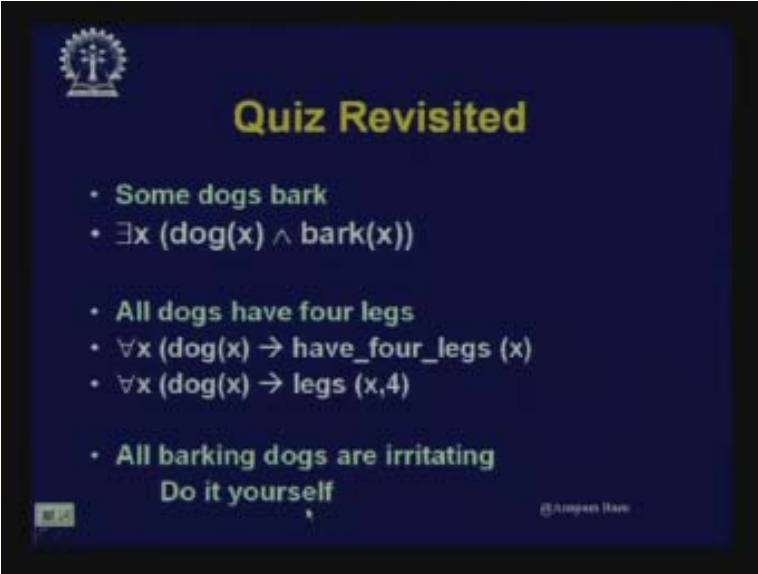
So we can now use this equality operator as a part of the first order predicate logic [r...] 20:20 valid operator. In the last quiz that was given to you now with this revision of predicate logic let us work that out a little bit. The first sentence we wanted to represent in predicate logic was; some dogs bark, you must have tried this and this one we can represent as a first order predicate sentence as there exists  $x$  because it is such that some dog  $x$  AND bark  $x$ , why do I need this existential operator? It is because some dogs bark and I did not say that all dogs bark. But if all dogs bark then also this will be true because in order to prove its truth I just need to find one dog one  $x$  which is a dog and that which barks then this sentence will be proved true.

The next example is, all dogs have 4 legs. Now this can be represented as for all  $x$  dog  $x$  implies have 4 legs  $x$  here I should have put another parenthesis, here there should be another bracket because the scope of this for all  $x$  is over this entire sentence so there

should be a corresponding matching parenthesis with this parenthesis. Actually there should be another parenthesis here. So, for all  $x$  dog  $x$  implies have 4 legs  $x$ . now there could be another alternative representation to this, I could have said here in the first one I have used this both these are unary predicates because have 4 legs is a predicate which is having only one variable parameter whereas here for all  $x$  dog  $x$  implies legs  $x$  4 so legs is a binary predicate which has got one which is the variable  $x$  and here is a constant term.

Here again I have missed one parenthesis, so for all  $x$  dog  $x$  implies legs  $x$  4. So this is another way we can do it. Now just a question here, could I write this as, for all  $x$  dog  $x$  and have 4 legs  $x$ ? Would it be correct or is this a better representation? Here I have said for all  $x$  dog  $x$  implies have 4 legs  $x$ . If I had written it as for all  $x$  dog  $x$  and have 4 legs  $x$ , would there be any problem? That is another issue. Now another example that was given in the last quiz was, all barking dogs are irritating. Now with this discussion we had in revision of first order logic you should be able to work on it. The next sentence that was asked to converted was, no dogs purr, one possible representation of that is, there does not exist an  $x$  such that dog  $x$  AND purr  $x$ .

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The slide is titled "Quiz Revisited" and features a list of statements and their corresponding logical formulas. At the top left is a small logo. The text is as follows:

- Some dogs bark  
 $\exists x (\text{dog}(x) \wedge \text{bark}(x))$
- All dogs have four legs  
 $\forall x (\text{dog}(x) \rightarrow \text{have\_four\_legs}(x))$   
 $\forall x (\text{dog}(x) \rightarrow \text{legs}(x,4))$
- All barking dogs are irritating

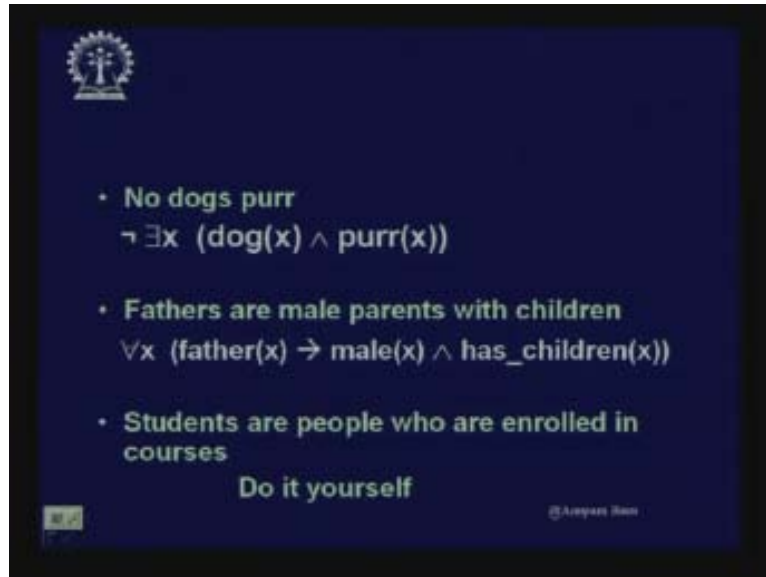
At the bottom of the slide, it says "Do it yourself" and "© Arjun Rao".

Now this is one way of representing it. Could there be another alternative way?

If I had started this as, suppose for all  $x$  then how would I have written this? For all  $x$  dog  $x$  implies not purr  $x$  or could I write it for all  $x$  dog  $x$  AND not purr  $x$ , it is also possible. Here we bring in the duality nature that any existential quantification can also be converted to universal quantification. Therefore no dogs bark can be represented in this way. Now here is the next example; fathers are male parents with children. Now this is the solution to that; for all  $x$  if  $x$  is a father implies that  $x$  is a male and  $x$  has children. So I can represent this as father  $x$  implies male  $x$  AND has children  $x$ .

The next example is; students are people who are enrolled in courses. May be you should be able to do that now. Try something like this; student  $x$  implies people  $x$  etc and put in the proper quantifier to that. Although it was given in the last lecture try out whether you can write them down in the first order logic.

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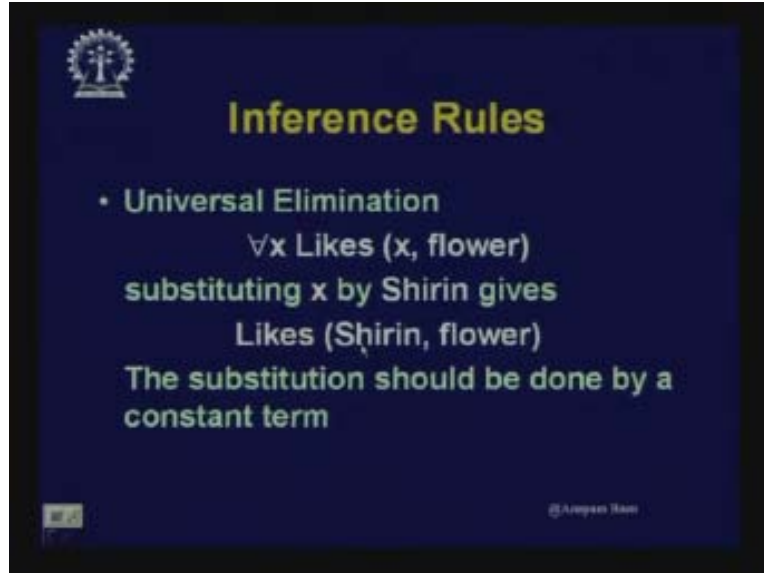


Also try another thing whether you can represent some of these in the form of propositional logic and when you cannot think why you cannot do it in propositional logic then try to do it in predicate logic. May be there will be some sentences which you cannot really capture in predicate logic also but those are different cases. **But you will find many sentences that we use in our day to day conversation that you can represent it in predicate logic.** This is all about the revision we intended to do about the first order logic.

The next thing that we will be doing is inference rules because given the sentences which are represented in first order logic our ultimate objective is to reason using them. And in the inference rules there are some basic techniques which we must learn. One is universal elimination. That means, if we have a sentence like this for all  $x$  Likes ( $x$ , flower) then Likes ( $x$ , flower) will hold for all  $x$ . Therefore I can substitute this  $x$  by some constant like a particular name Shirin, Shirin likes flower, there will be nothing wrong that I am doing because you have said it is for all  $x$ . So whenever there is a universal quantifier I can drop it and represent this with a constant term. So eliminate this for all  $x$  part and the substitution of that  $x$  in this part of the parameter should be done with a constant term and that is technique one and that is universal elimination, the elimination of universal quantifiers.



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The next thing is which is little more complex which is elimination of the existential quantifiers now why are we doing this? We are trying to come to a state where we can take these predicate logic sentences and use them in our reasoning in a mechanical way and that is the purpose. So we are trying to give them a different form where we are removing the universal quantifier but what will happen in the case of existential quantifier. For universal quantifiers it was so simple. For all  $x$   $P(x)$  so I can always drop that  $x$  and put in any constant because it will hold for all  $x$ . But what happens in the case of existential quantifier there exists some  $x$  for which  $P(x)$  is true.

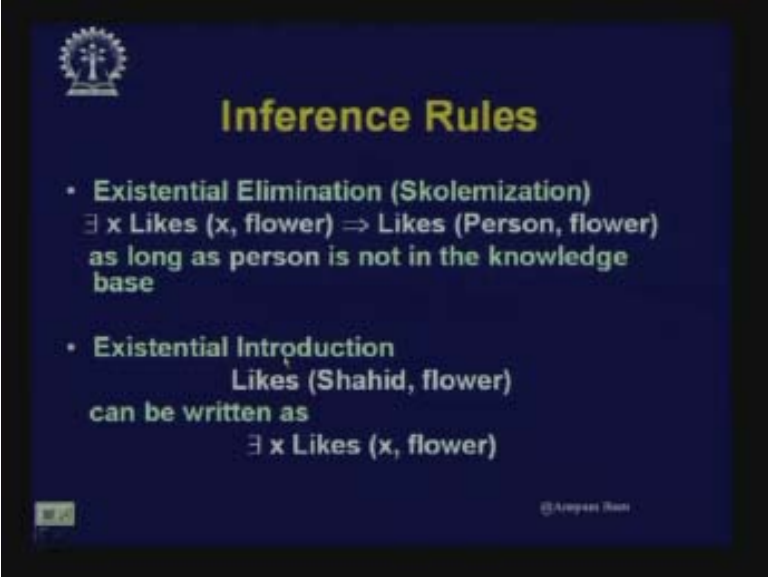
Now with which  $x$  can I replace that  $x$  part of  $P(x)$ ?

This method is also known as skolemization. Here for all  $x$  there exists  $x$  such that Likes ( $x$ , flower). This one means Likes (person, flower). Now this person is some particular person which is not there in the knowledge base and if I can find out that person I can put that in and that particular person say there is a sentence there exists  $x$  such that likes John  $x$  and food  $x$  say  $x$  is a food and there exists  $x$  such that John likes  $x$ . now then there is some  $x$  which John likes. Suppose I find out apple then I can represent that particular  $x$  parameter with apple and remove that existential quantifier. This method of finding out a particular constant that satisfies the predicate there exists  $x$  such that john likes  $x$ .

Finding out that particular constant satisfies, this is known as skolemization and when we can find that out we can represent it. Thus by eliminating this existential quantifier we can get such a sentence where the existential quantifier is not there. Just the opposite, in some cases we may like to introduce existential quantifiers. That is existential introduction. Suppose I am given a sentence Likes (Shahid, flower) now I know that Shahid likes flower then I can certainly write it as there exists  $x$  such that Likes ( $x$ , flower) because I know that it will always be true because I can replace this  $x$  with Shahid and there will be no problem. This is just the opposite of existential elimination

and this is existential introduction. Now with this let us go into the reasoning and start with a particular problem.

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The slide is titled "Inference Rules" in yellow text on a dark blue background. It features a logo in the top left corner and a small "CC BY" icon in the bottom left. The content is as follows:

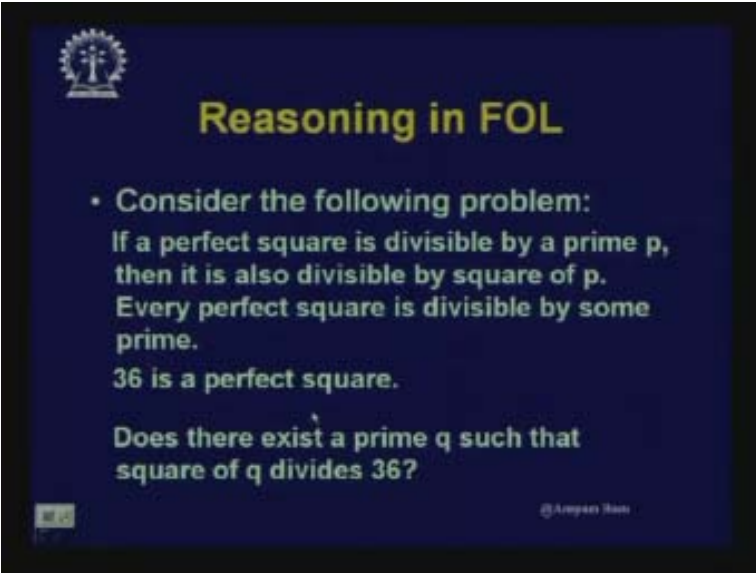
- **Existential Elimination (Skolemization)**  
 $\exists x \text{ Likes } (x, \text{flower}) \Rightarrow \text{Likes } (\text{Person}, \text{flower})$   
as long as person is not in the knowledge base
- **Existential Introduction**  
 $\text{Likes } (\text{Shahid}, \text{flower})$   
can be written as  
 $\exists x \text{ Likes } (x, \text{flower})$

And the problem is defined here as we are making a statement; a perfect square is nothing but 16, 20, 5 etc, so, if a perfect square is divisible by a prime number  $p$  say 25 is a perfect square and is divisible by a prime  $p$  5 then it is also divisible by the square of  $p$  this is one sentence I state and in mathematics that is right. The next statement I make is; every perfect square is divisible by some prime number. Think **about a square** 16 it is divisible by 2 it is a prime number. I also say 36 is a perfect square.

What are the three statements that have been made?

If a number is a perfect square then it is divisible by a prime number and if a perfect square is divisible by prime number then it is also divisible by the square of that prime number. And every perfect square is divisible by some prime, 36 is a perfect square. Now with these three statements now I want to reason to find out does there exist a particular prime number  $q$  such that the square of  $q$  divides 36. That is what we want to find out. The next step will be to convert these into predicate sentences.

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**Reasoning in FOL**

- Consider the following problem:  
If a perfect square is divisible by a prime  $p$ , then it is also divisible by square of  $p$ .  
Every perfect square is divisible by some prime.  
36 is a perfect square.  
Does there exist a prime  $q$  such that square of  $q$  divides 36?

@Anjan Ray

The next thing we will be doing is convert them into the first order predicate logic form. So here is a sentence; the method is you have to take the sentences one by one and convert them into predicate form. So if a perfect square is divisible by a prime  $p$  then it is also divisible by the square of  $p$ . So I write it as, for all  $x$  and  $y$  or for all  $x$  for all  $y$  also which means the same thing the perfect square  $x$  if  $x$  is a perfect square and prime  $y$  so  $y$  is a prime and divides  $(x, y)$  that is this prime  $y$  divides  $x$  that takes care of this if a perfect square is divisible by a prime  $p$ .

Although over here I have said  $p$  in my sentence I need not write  $p$  I can write  $x, y$  anything but I must be consistent in this sentence then it is also divisible by the square of  $p$ . This implies divides  $x$  which is a perfect square by the square of  $y$ . Now I did not use  $p$  in my logic sentence, I have used  $y$  so I have used  $y$  here also. So if a perfect square is divisible by a prime  $p$  then it is also divisible by the square of  $p$ . This is the conversion of this sentence into the predicate form. Next is, every perfect square is divisible by some prime. Since that is every perfect square so for all  $x$  perfect square  $x$  there is some prime number which divides it. So I put in for all  $x$  perfect square  $x$  there exists  $y$  some prime is there such that  $y$  is a prime so perfect square  $x$  AND prime  $y$  AND divides  $(x, y)$   $x$  is divided by  $y$ . Therefore  $x$  has been universally quantified by  $y$  it is the same for every perfect square and  $y$  has been quantified with existential quantifier  $y$  because there is some prime which divides it, not that all prime numbers will divide it.

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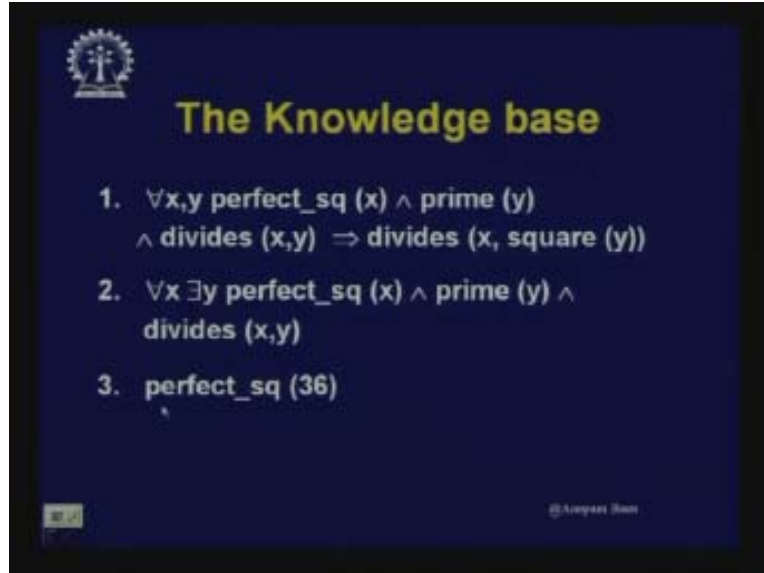
**Representation in FOL**

- If a perfect square is divisible by a prime  $p$ , then it is also divisible by square of  $p$ .  
 $\forall x,y \text{ perfect\_sq } (x) \wedge \text{prime } (y) \wedge \text{divides } (x,y) \Rightarrow \text{divides } (x, \text{square } (y))$
- Every perfect square is divisible by some prime.  
 $\forall x \exists y \text{ perfect\_sq } (x) \wedge \text{prime } (y) \wedge \text{divides } (x,y)$

There exists  $y$  and  $y$  is a prime number AND divides  $(x, y)$ . So these are two sentences, what was the next sentence? It is very simple, 36 is a perfect square so I can write down perfect square 36 there is no problem with this. Does there exist some prime number  $q$  such that the square of  $q$  divides 36. In order to answer this question does there exist I can also pose it as a theorem which I will be proving. If I can prove it I can say I just make a statement there exists a prime  $q$  such that the square of  $q$  divides 36 I make that statement and then I try to prove it by inference.

If I can prove it then there exists an  $x$  and if I cannot prove it then it does not exist. So this particular query shown here can be converted in the form of a predicate which I shall try to prove using my reasoning mechanism of first order logic. Therefore I can write it down as there exists  $y$  such that  $y$  is a prime AND divides 36, I can divide 36 by square of  $y$ . These are my sentences. With these I have got a knowledge base and what is that knowledge base? These statements; for all  $x, y$ , perfect square  $x$  AND prime  $y$ , AND divides  $(x, y)$  implies, divides  $(x, \text{square } (y))$  that was the first sentence, the second sentence was this, the third sentence was this which we just now did.

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Now comes reasoning the inference from two and universal elimination. What was two? Two was this for all x perfect square x AND prime y AND divides (x, y). Now when I eliminate this I want to prove it that I am dealing with a particular perfect square that is 36 that is of interest to me. Therefore I can put 36 in place of x and remove this for all x part. That was I put 36 here as well as I put 36 here that was our inference method of elimination of universal quantifier. So I eliminate that then I get a new clause, now the part for all x is gone so there exists y perfect square 36 AND prime y AND divides 36 y that is just the thing that was here and from here I have replaced these two x's with 36 and I have removed this universal quantifier and I have got the 4th clause.

Now from this 4 I want to eliminate this existential quantifier. Now existential quantifier elimination is by skolemization that means I will find out some constant which is a prime. So let me put some constant P and remove this existential quantifier and get this new fifth clause, perfect square 36 AND prime p AND p divides 36. This is the fifth one and what was the one? One was this. I have eliminated from two but here for all x, y perfect square (x) AND prime (y) AND divides (x, y) implies divides (x, square (y)).

Now 1 and 5 what is my 5? It is this one I can get. So I get perfect square 36 prime p divides 36 p. Now this was also universal quantifier so I put 36 here and in place of y I put p and I can remove all these quantifications straight away. So I get perfect square 36 and prime p and divides p 36 that was this part clause 5 implies divides 36 square of p. So that is the 6th one that I am getting.

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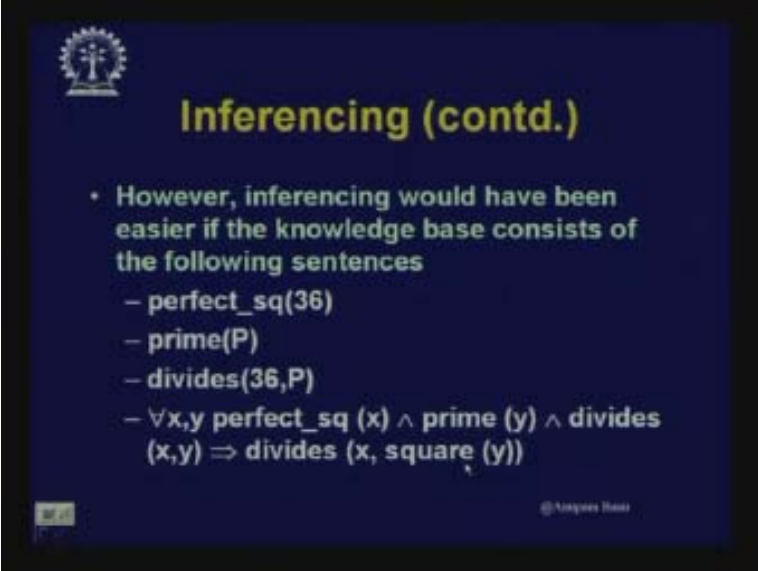
**Inferencing**

- From 2 and Universal Elimination  
(4)  $\exists y \text{ perfect\_sq } (36) \wedge \text{prime } (y) \wedge \text{divides } (36, y)$
- From 4 and Existential Elimination  
(5)  $\text{perfect\_sq } (36) \wedge \text{prime } (P) \wedge \text{divides } (36, P)$
- From (1) and (5)  
(6)  $\text{divides } (36, \text{square}(P))$

Now this is the 6<sup>th</sup> one. So I get divides 36 square p. Now from 5 and 6 prime p AND divides 36 by square of p what was 5 and what was 6? Here 5 was this perfect square 36 prime p and divides 36 p and I get p as divides 36 square of p. so in place of p I can simply put square p, here also square p so what I get is perfect square 36 and prime square p, prime p and divides 36 square p. Now you can see the use of existential introduction. I introduce this existential quantifier so now all these are constants and in place of p I put a variable y, here also for p I put a variable y and introduce this existential quantifier and I get there exists y such that prime y and divides 36 square y.

Now, if you recall, that was my thing to be proven. My objective was to prove this; does there exist a prime q such that the square of q divides 36? That was this thing; prime y divides 36 square of y and that is the thing I wanted to prove. And through the technique of universal elimination, existential elimination and again existential introduction and replacement of variables I get the same thing that means I could prove this I could infer this. However inferencing would have been easier if the knowledge base consisted of the following sentences. Suppose the knowledge base consisted of the sentences like this; perfect square 36 which was there, prime p divides 36 p and for all x, y prime perfect square (x) AND prime (y) divides (x, y) then the inferencing would have been easier.

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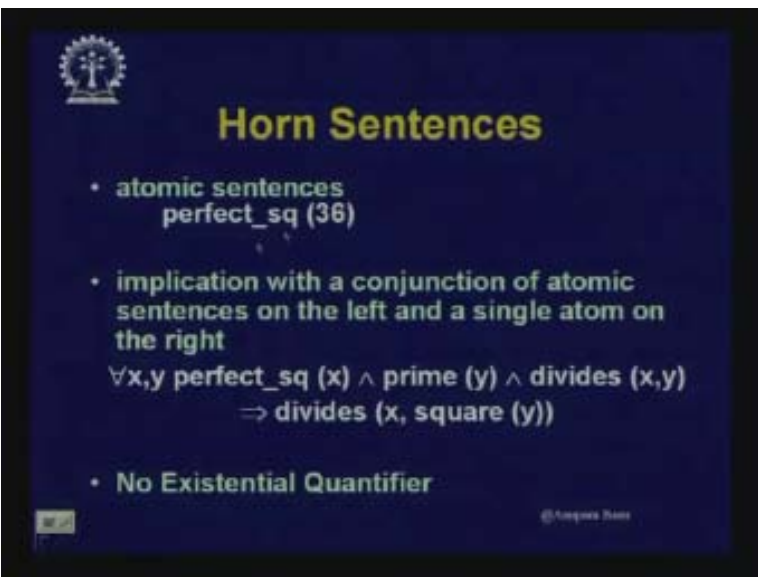


**Inferencing (contd.)**

- However, inferencing would have been easier if the knowledge base consists of the following sentences
  - perfect\_sq(36)
  - prime(P)
  - divides(36,P)
  - $\forall x,y \text{ perfect\_sq}(x) \wedge \text{prime}(y) \wedge \text{divides}(x,y) \Rightarrow \text{divides}(x, \text{square}(y))$

Now I would like to introduce a very important notion called a particular type of sentences which is known as horn sentences or horn clauses. These horn sentences are also known as horn clauses and this has got a tremendous application in the case of inferencing in predicate calculus.

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**Horn Sentences**

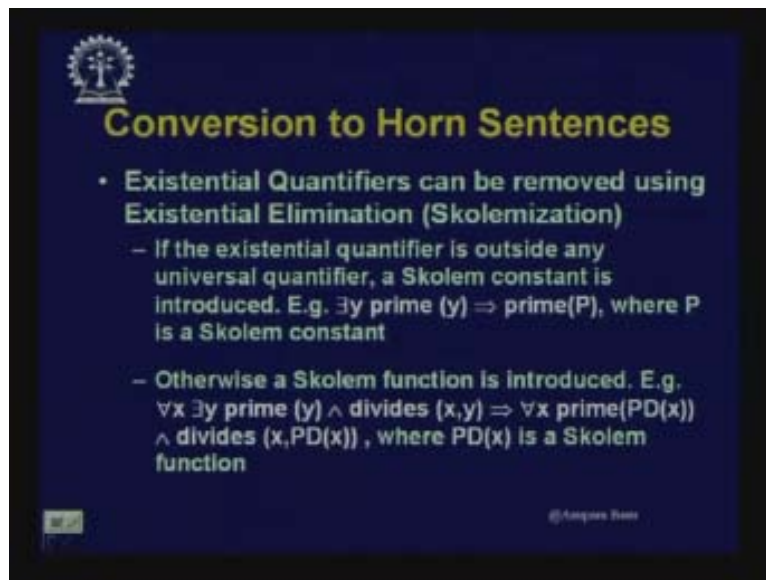
- atomic sentences  
perfect\_sq(36)
- implication with a conjunction of atomic sentences on the left and a single atom on the right  
 $\forall x,y \text{ perfect\_sq}(x) \wedge \text{prime}(y) \wedge \text{divides}(x,y) \Rightarrow \text{divides}(x, \text{square}(y))$
- No Existential Quantifier

What are horn clauses? What are the possible horn sentences?

Any atomic sentence is a horn sentence. So, sentence like perfect square 36 is a horn sentence. Implications with a conjunction of atomic sentences where atomic sentences are joined together **conjoined** and a single atom is on the right so a sentence like this; for all

$x, y$  perfect square ( $x$ ) AND prime ( $y$ ) AND divides ( $x, y$ ) implies divides ( $x$ , square ( $y$ )) that is a horn sentence and there is no existential quantifier. There should not be any existential quantifier. On the left hand side of this implication there are atomic sentences, there can be negations on that side also and on the right hand side we have a single atom and there are no existential quantifiers. So, if this be our horn sentence and this side the conjunction there is no **ORing** and here also there is only one unit now if a particular sentence given to you is not a horn sentence then we will have to convert it to the horn sentence.

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How can I convert it to a horn sentence?

We know the fact that existential quantifiers can be removed using existential elimination. You can eliminate the existential quantifiers by the process of skolemization. If the existential quantifier is outside any universal quantifier a skolem constant is introduced. You know a skolem constant is a particular constant that I find out from the knowledge base and replace it in place of that particular variable  $y$  for example which was quantified with there exists. So, if I have something like this there exists  $y$  prime  $y$  now  $y$  being existentially quantified I can put a skolem constant  $p$  in place of this. This is a horn sentence. Otherwise the skolem function is introduced.

When I do not find the constant I can still leave the responsibility as if I am leaving a responsibility to a function that function is a magic function which will some how find out a particular constant value which will satisfy the clause of the sentence. It will find out if there exists something that skolem function will find out that constant. For example here, for all  $x$  there exists  $y$  prime  $y$  and divides ( $x, y$ ) here  $y$  is universally quantified so I replace there exists  $y$  with for all  $x$  prime say  $PD$  is a function say prime detector or whatever you can think of  $x$ . That divides  $x$  so in the given  $x$  I find out some prime  $PD$  AND divides ( $x, PD(x)$ ) where  $PD(x)$  is a skolem function, this is a skolem function. So using that I can get rid of the universal quantifiers.



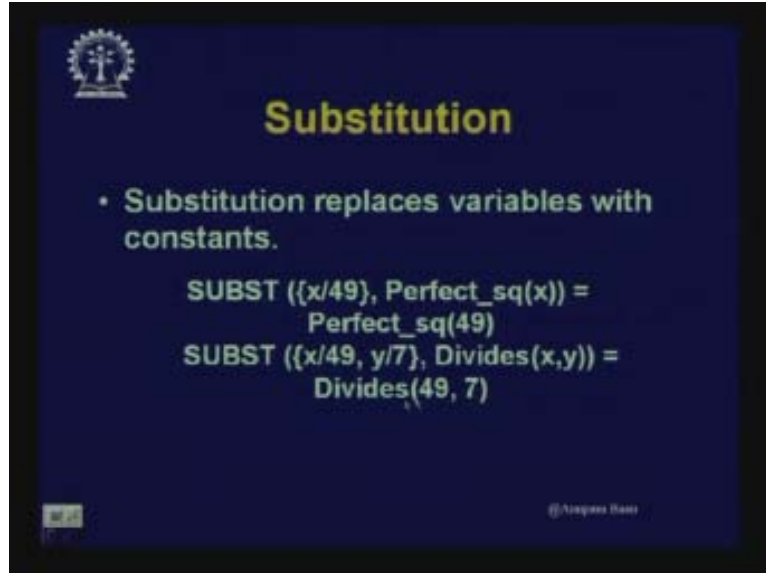
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The slide features a dark blue background with a white logo in the top left corner. The title 'Conversion to Horn Sentences' is written in yellow. Below it, a bullet point '• And-Elimination' is shown in white. The main text, also in white, states: 'prime (P)  $\wedge$  divides (x,P) can be written as two clauses', followed by 'prime (P)' and 'divides (x,P)' on separate lines. A small logo is in the bottom left, and the text '@Angeles Ross' is in the bottom right.

And the AND part can always be eliminated on the other side if a conjunction here on the right hand side I can always do say prime P AND divides (x, P) can always be written as two clauses like prime p and divides (x, P) it is possible. I can break them up into two different clauses and each of them remains a horn clause because any atomic sentence is a horn clause.

The other thing which is often required in inferencing in first order logic is substitution. Substitution replaces variables with constants. Variables are represented with constants. If substitution is an operator x is replaced with 49 then if I do this substitution then perfect square x will become perfect square 49. Substitution of x with (49, x) is substituted with 49 and y is substituted with 7 then divides (x, y) with this particular substitution it will reduce to 49 replaces x so divides 49 and y is 7 so (7 divides 49). This is a substitution operator where I have to define a consistent substitution and then from this divides (x, y) I can come to this.

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Now this gives us a very nice operator often used in predicate logic that is unification. Let us just look at some examples of unification. There are algorithms given in any artificial intelligence book where you can find unification algorithms. Unification is a process of finding a particular substitution that will make two atomic sentences identical. When two atomic sentences are identical then I can use one for the other and that is our attempt. That is often our attempt that helps out in inferencing.

So what is a substitution?

All substitutions cannot unify. For example, there are prime  $x$  prime 7 and prime  $x$  now I apply the substitution  $x$  substituted by 7 then this substitution really makes these two identical so they can be unified. This means it is possible to unify prime 7 with prime  $x$  with the substitution of  $x$  with 7. It is possible to unify these two sentences divides (49 by  $x$ ,  $x$ ) divides 49 and divides (7,  $y$ ) these two can be made identical with the substitution of ( $y$  by 49) and ( $x$  by 7). So if I do this then what will happen is, this will become ( $x$  by 7) so divides (49, 7) and divides ( $y$  by 49) divides (49, 7). This particular substitution is possible which unifies these two sentences. On the other hand you see unify prime 7 and prime 17. It is not possible because there cannot be any substitution that will change these to constants.

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**Unification**

- Unification is the Process of finding a substitution that makes two atomic sentences identical.
  - UNIFY (Prime(7), Prime(x)) = {x/7}
  - UNIFY (Divides(49,x), Divides (y,7)) = {y/49, x/7}
  - UNIFY (Prime(7),Prime(17)) – Impossible!
  - UNIFY (Divides(49,x), Divides (x,7)) – Impossible!

@Angela Rose

On the other hand, here you see, the attempt to unify divides (49, x) and divides (x by 7). It is not possible because if I substitute x I can substitute x here with a particular variable. In that case I will have to again replace it here these two cannot be unified these are not possible. So it is not the case that always I can find a substitution but if I can get a substitution then it will be possible to make two clauses identical and that will help us in inferencing.

Let us quickly revisit what we had done in today's lecture. A revision of first order logic, how to make the sentences and we have seen the basic steps of universal elimination, existential quantification elimination, existential quantification introduction and substitution as well as we have seen horn clauses and how non horn clauses can be converted to horn clauses. So with this background we will move to the next lecture which will be farther reasoning techniques on first order logic.