Artificial Intelligence
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Lecture - 13
First Order Logic

In the last lecture we discussed about one form of logic called propositional logic. We also saw how reasoning is carried out in propositional logic. Today we will discuss about a little more powerful form of logic called first order logic also called as predicate logic. This form of logic can capture more variety of sentences than can be captured with propositional logic. But before moving into the tales of first order logic let us start with quiz's that we had given in the last class.

You can have a look at the statements we had already given: Mammals drink milk, man is mortal, man is a mammal, Tom is a man. These were the four sentences which were given and you were asked to try to prove these two statements. Tom drink or Tom drinks for grammatical part of English Tom drinks milk and Tom is mortal. Intuitively all of you can understand that mammals drink milk and man is a mammal and Tom is a man and Tom drinks milk that is obvious to common sense. But if you had tried the techniques that we had discussed like Modus ponens resolution probably you will not be able to prove this mechanism with the information that was provided.

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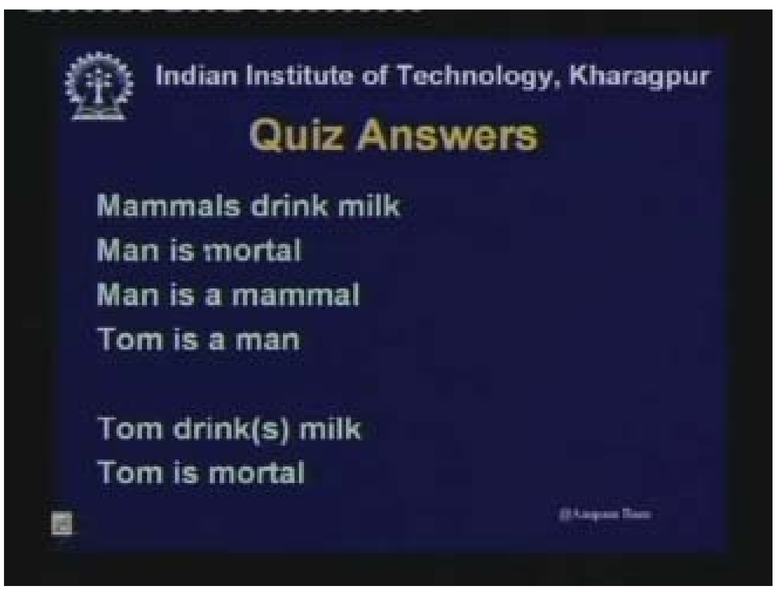

There is a subtle difference between these two statements I am making. One is that, intuitively you are being able to understand and derive Tom drinks milk. But what is the mechanism that is going behind this reasoning?
Unless we can understand that mechanism we cannot emulate that in a computer because a computer will mechanically do this. So the mechanical methods we discussed like

Modus ponens, resolution will not be able to prove that. So what is missing? Why cannot we do it mechanically although you can understand that?

We had these statements: mammals drink milk, man is mortal, man is a mammal, Tom is a man and we are supposed to prove these. Now mammals drink milk if we write that in this new form that there were some intuitive assumptions that we are making. Tom is a mammal so mammal (Tom) if Tom is a mammal then drink (Tom, Milk) is true. So the reasoning is possible and in order to make this reasoning possible we had to bring in this implication sign here and we have to write it as an implication proposition.

Similarly, man is mortal can be written as, if Tom is a man then Tom is mortal this is my second implication. Man is a mammal that can be written as, if Tom is a man then Tom is mammal because if Tom is a man then Tom is mammal which my third statement and Tom is a man. Now this is no implication but it is just an assertion Tom is a man. So what I want to really say is that, in my statement mammals drink milk I can say drink mammal milk that will be fine. But that would not help me in this particular type of reasoning and there is also a hidden implication within this mammals drink milk. That means, say a particular person say Tom in this case is a mammal then it is also true that it is that particular person Tom will drink milk.

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In this way if we write then let us see how we can prove our desired statements. I am now naming these as propositions P, Q, R and S. You can see that mammal (Tom) implies drink (Tom, Milk) is a proposition. Similarly, man (Tom) implies mortal, Tom is another proposition. So all these are propositions and these are implications and this S is just an assertion. I have also numbered them as 1, 2, 3, 4. Now, given these propositions if I select my favorite tool like Modus ponens if I apply Modus ponens in R and S, that is, man (Tom) implies mammal, Tom is true and man (Tom) is true then applying the knowledge of Modus ponens you can certainly arrive at the fact that mammal (Tom). So I can prove that Tom is a mammal.

Again if I apply Modus ponens I have derived mammal (Tom) and if I apply Modus ponens on this number 5 and number 1, that is, if Tom is a mammal then Tom drinks milk and if I apply Modus ponens on P and this new one which is mammal (Tom) then I can again derive drink (Tom, Milk). Thus the desired statement Tom drinks milk can be proved using Modus ponens. Until I wrote them down as implication I could not apply Modus ponens, I could not apply the inference rule of Modus ponens. Now let us try to do the same thing using the other proof technique we learnt. The other proof technique was resolution. And again in resolution there is a subtle difference between Modus ponens and in the application of resolution. In order to apply resolution I have to convert each of these propositions into their clausal form.

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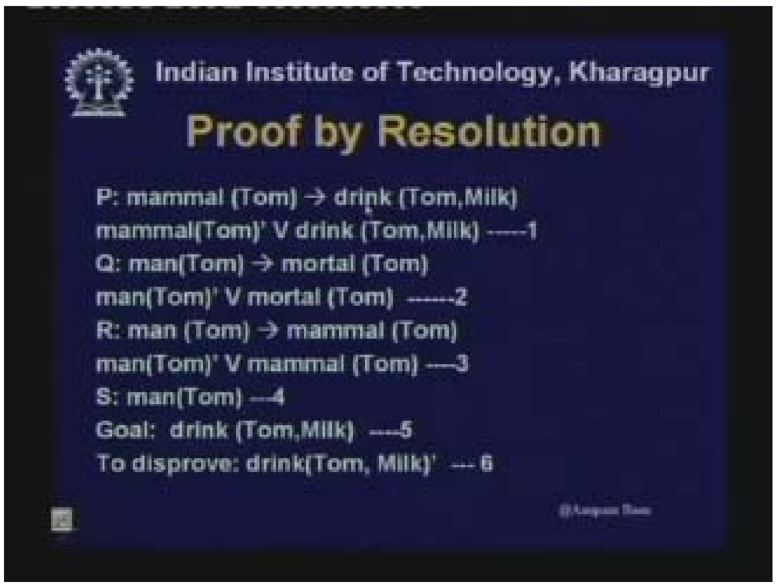

A clausal form is nothing but a disjunction of literals. So, mammal (Tom) prime implies drink (Tom, Milk) I have written it in different way mammal (Tom) prime that is NOT mammal so NOT mammal (Tom) OR drink (Tom, Milk) I can write it in that way. And Q man (Tom) implies mortal (Tom). When I convert that into clausal form it becomes NOT man (Tom) OR mortal (Tom). Similarly, the third one, man (Tom) implies mammal (Tom) says that man (Tom) prime of that NOT man (Tom) OR mammal (Tom) in this way I convert each of them in clausal form note that is last one Tom is a man, man (Tom) is itself in clausal form so I need not do anything with it. Next what do I have to prove? Suppose there is only one statement given is proven say I take a statement Tom drinks milk.

The clausal form of that is drink (Tom, Milk) and the propositional form of that is drink (Tom, Milk) but in resolution what we do? In resolution we try to disprove we proved by refutation so my starting point would be the negation of the goal which I should try to disprove. That was the basic approach of resolution. So here the goal is drink (Tom, Milk) so I convert my goal to disprove that Tom does not drink milk. The compliment of drink (Tom, Milk) is not drink (Tom, Milk) means Tom does not drink milk I have to disprove it, if I have to disprove it given these statements then I can certainly prove that Tom drinks milk.

Now let us try to do with resolution. I start with the negation of the goal, I select one clause which has got the compliment of this and if I resolve these two I will get a new statement mammal (Tom) prime that means not mammal (Tom). Now I will have to again look into the set of statements that I have to find out which clause has got the negation of not mammal (Tom) or which clause has got the literal mammal (Tom).

And here is one not man (Tom) OR mammal (Tom). So let me resolve these two. As I resolve what I would get, mammal (Tom) and not mammal (Tom) will resolve out and my resolvent will be not man (Tom). So not man (Tom) I get. Now I have to select the other clause that I have man (Tom). Now as I resolve these two what I get the contradiction with the null clause. That means this goal has been disproved. Now let me just go through this and state it in a different way. My job was to disprove that Tom does not drink milk, I had to disprove that Tom is not a mammal and in order to disprove that

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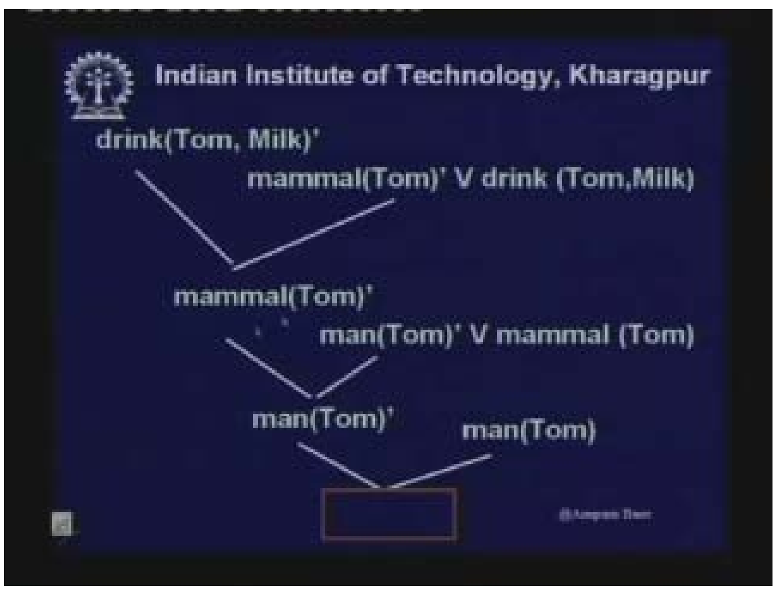

I had to show that man (Tom) man (Tom) is not a man but Tom is a man so there is a contradiction. And since there is a contradiction I am arriving at a null clause thereby refuting my basic assumption drink (Tom, Milk). So this is proven that Tom does not drink milk is refuted and Tom drinks milk is proven. Now that is all about propositional logic.

I also made a subtle adjustment in the formulation. When it was written that mammals drink milk I just said if Tom is a mammal then Tom drinks milk. I will show you later that was not very fair. The objective of this specific lecture is to enable you to formulate more variety of sentences using logic. And it will enable you to write correct predicate logic formulae.

We will discuss what predicate logic is. For that first let us look at the limitations of propositional logic. Why do we need another form of logic when you have propositional logic? The limitations of propositional logic will be evident from this statement. Consider the following argument: all dogs are faithful. Tommy is a dog therefore Tommy is faithful. This argument is quite understandable. All dogs are faithful. Tommy is a dog therefore Tommy is faithful.

Now, how to represent and infer this in propositional logic?
Suppose P is the proposition all dogs are faithful that stands for all dogs are faithful. And q is a proposition that says Tommy is a dog. But can I combine all dogs are faithful all dogs and how do I know that Tommy belongs to this all dogs. So if all dogs are faithful is true then Tommy is also covered over there all dogs is covering all dogs other than Tommy so p AND q can I say using Tommy is faithful? No, we cannot infer this in propositional logic because here there are two different constants two different things Tommy and all dogs they are different. I do not know what all dogs mean.

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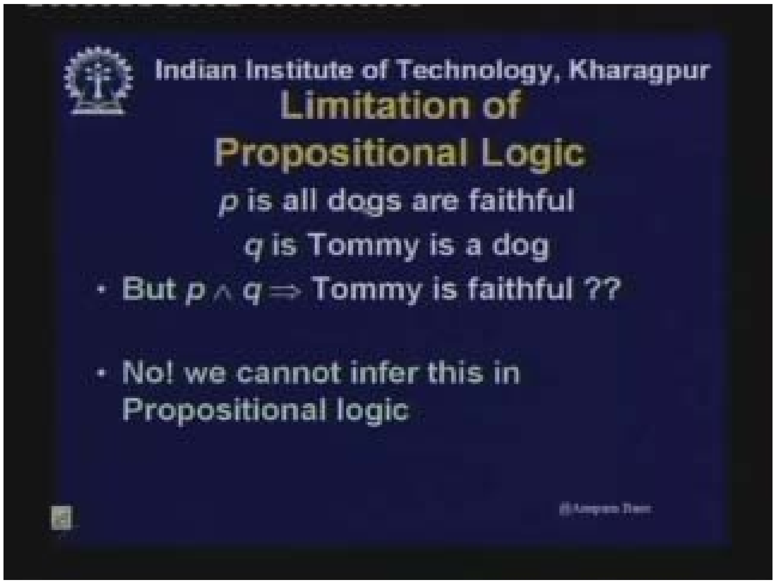

When we are talking of logic we are talking of mechanical methodology that will work irrespective of my subject of interpretation so we have to prove it mechanically. Let us see a couple of more examples. Tom is a hardworking student. I can certainly write that as hardworking Tom as a proposition. Tom is an intelligent student, I can write down intelligent Tom. If Tom is hardworking and Tom is intelligent then Tom scores high marks. So this is also an implication proposition and I can write it as hardworking Tom AND intelligent Tom implies that scores high marks Tom.

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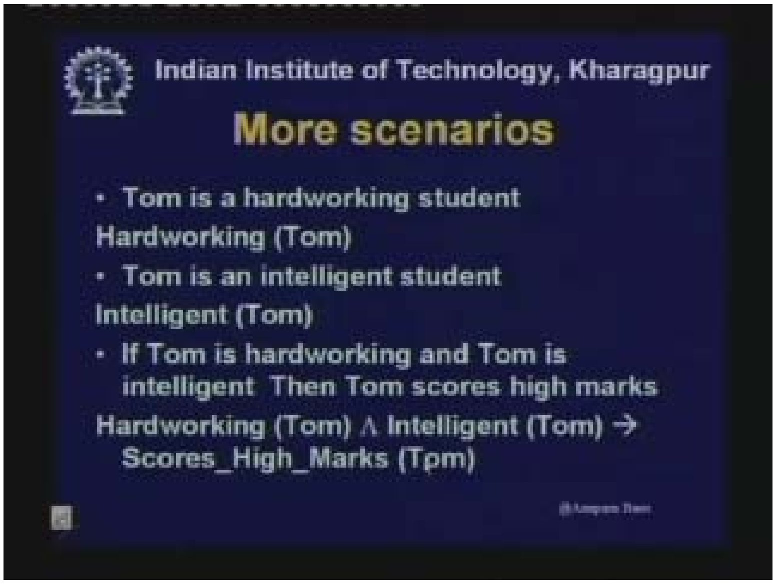

There is no problem up to this but what about John and Jill? Here what we have said is, I have made hardworking Tom, intelligent Tom and I have written one rule that, if Tom is hardworking and Tom is intelligent then Tom scores high marks.

Suppose John is another student so I have to write, if John is hardworking and John is intelligent then John scores high marks. If Jill is hardworking and Jill is intelligent then Jill scores high marks. Do I have to enumerate the rules for each of the students separately? Is it a feasible way of doing things? And a new student works in the class you have to know his or her name and you have to write down another rule? That is not the way to do it. You should have a general statement to make which should apply to all these. But unfortunately if we are restricted to propositional logic we are restricted to this sort of statements that we can make.

Instead of this if I could write, all students are hardworking and intelligent scores high marks we have to find that it covers Tom that it covers John that it covers Jill or any student who will come in. Unfortunately these sorts of statements all students who are hardworking and intelligent, scores high marks cannot be written in propositional logic because in propositional logic the structure of the proposition do not allow us to do that.

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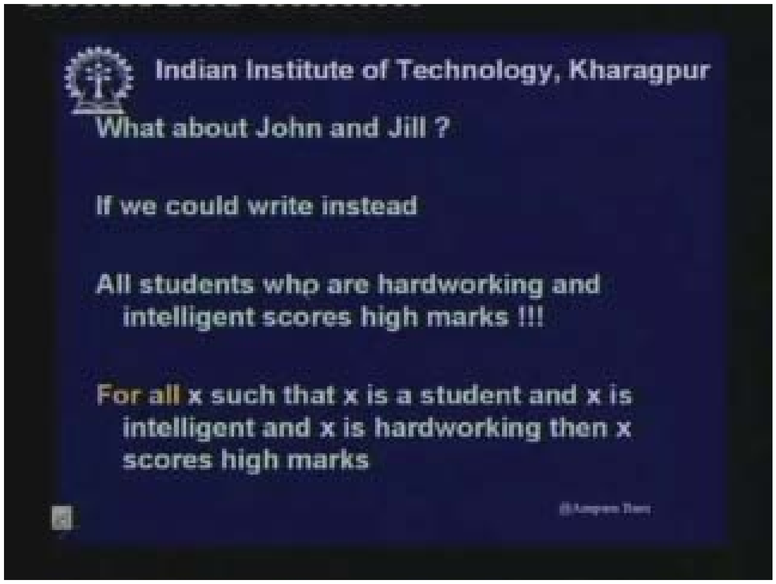

Therefore essentially when I want to write some statement like this; all students who are intelligent and hardworking I can write that in this way. For all x where (x) is a variable this is white this is variable, so for all x such that x is a student and x is intelligent and x is hardworking, first of all, all x, x can be a student, x can be a teacher, can be a visitor but I am restricting myself to only the students. So, for all x such that x is a student and x is intelligent and x is hardworking then x scores high marks. Now (x) being a variable I can put in a value for this variable and can make it Tom, john Jill or whatever I like. So what I need is the ability is the power to write such sentences which unfortunately was not possible in the case of propositional logic. That is why we need the predicate logic.

So essentially what was the problem?
The problem is therefore the problem of infinite model. In propositional logic we have to restrict ourselves to constants. But in general, propositional logic can deal with only a finite number of propositions. If there are only three dogs for example Tommy, Jimmy, Laika then I could have written T such that Tommy is faithful, J Jimmy is faithful, L Laika is faithful then all dogs are faithful will be T AND J AND L. If there be a new dog then how long should we go on that is not possible. So that is the finiteness of the statements and also even part of Tommy, Jimmy and Laika I have to write separate propositions and I have to do this conjunction and that is where the problem of propositional logic lies. So what if there were infinite number of dogs? I would have faced a problem.

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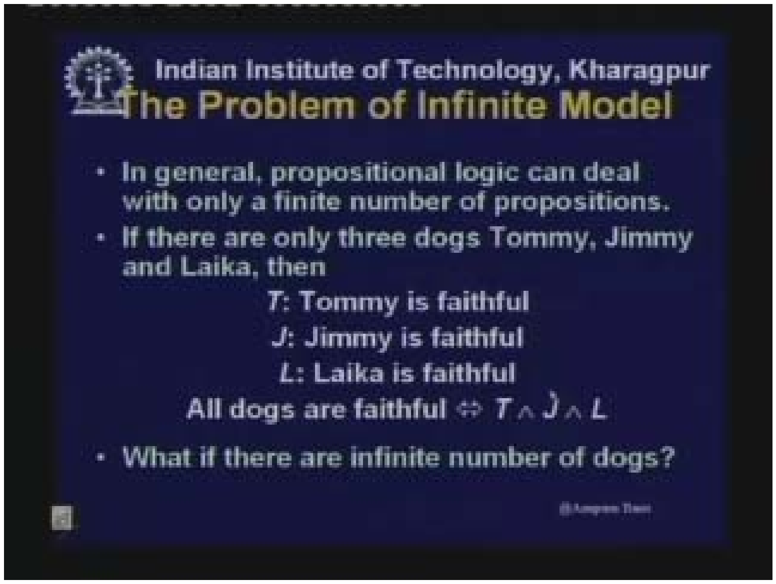

Therefore we move to a new thing called the first order logic. First order logic or predicate logic can be considered to be a generalization of whatever propositional logic you have understood and it allows us to express and infer facts over a large or infinite models.

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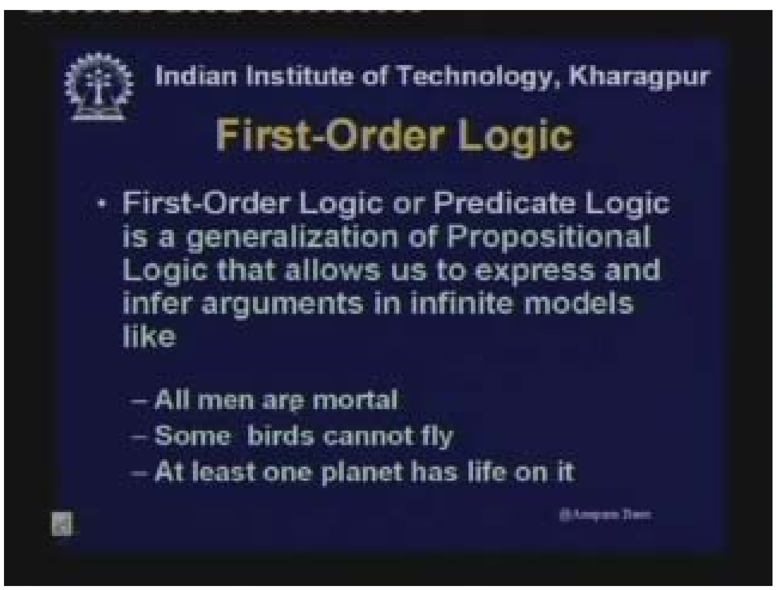

For example, all men are mortal, some birds cannot fly; at least one planet has life on it. Going back to the previous statement all mammals drink milk, all mammals are mortal. For these kinds of statements I can do first order logic or predicate logic but not the propositional logic. So what is added in first order logic? In first order logic in addition to propositional logic we have got the concept of variables and we have got concepts of quantifiers. The syntax of first order logic can be defined in terms of, terms, predicates and quantifiers.

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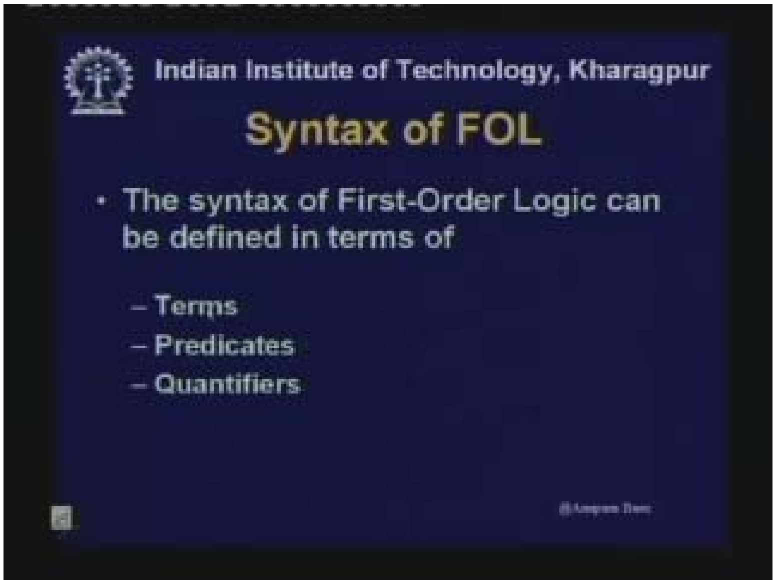

We can express them in terms of three things namely terms, predicates and quantifiers. I will explain each of them one by one now. What is a term? A term denotes some object other than the constant that we have already encountered true or false. Tommy is a dog, all men are mortal, men is a term very similar to what we talked in propositional logic lectures we mentioned them as objects so each of them is a term.

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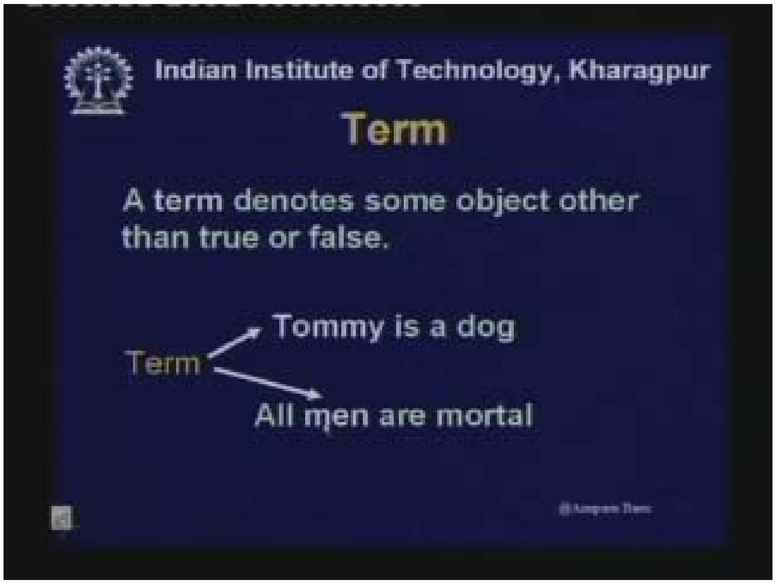

Now, terms in this case can be constants as well as variables. In the case of propositional logic it was only constants. Now here we can see a constant of type W any type is a name that denotes a particular object in a set W. When we are talking about types of objects then that is a set of objects of a particular type. Now when I am saying constant then I am particularly pointing at an individual object of that set. If I say any particular variable that belongs to that set belongs to that type then it can connect itself to any particular object of that set. But whenever we are saying constant then that is mapping to the particular object of that set. For example, if I say 5, 5 is a particular object in the set on natural numbers so it is a constant. Tommy, among all the names all the objects called dogs given their names Tommy is a particular dog.

On the other hand, a variable of type W is a name that can denote any element in the set W. For example, x belongs to N is a set of natural numbers. So x is a particular natural number, (x) is a variable that can connect to any particular natural number whereas if I say 5 then that is in a particular natural number, it is not meaning any element of that set of numbers. Similarly d denotes the name of a dog any dog so that is the basic difference between constants and variables.

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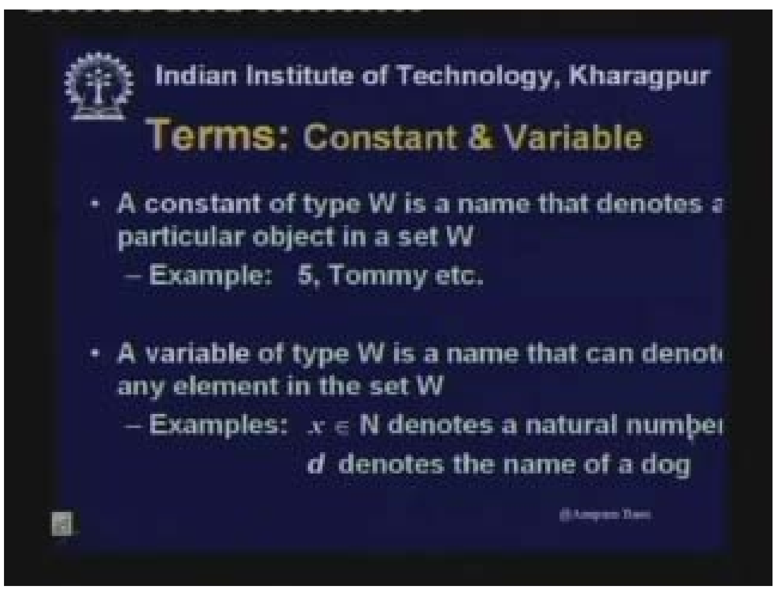

Next let us understand what is a function?
A functional term of arity n, how many parameters the function will have, takes n objects of type W_1, W_2 up to W_n may be of different types as inputs and returns an object of the type W so it takes parameters of objects which can be variables or constants and returns an object of type W. Now, for example, we are familiar with these statements, f is a function of W_1, W_2, W_n plus is a function which has taken two constants, two objects of type constant from the set of natural numbers, so this is a function. So here this is the functional term plus is the functional term, 3 and 4 are constant terms and 7 is also a constant term. So 7 is a value that is an object that is returned by this function.

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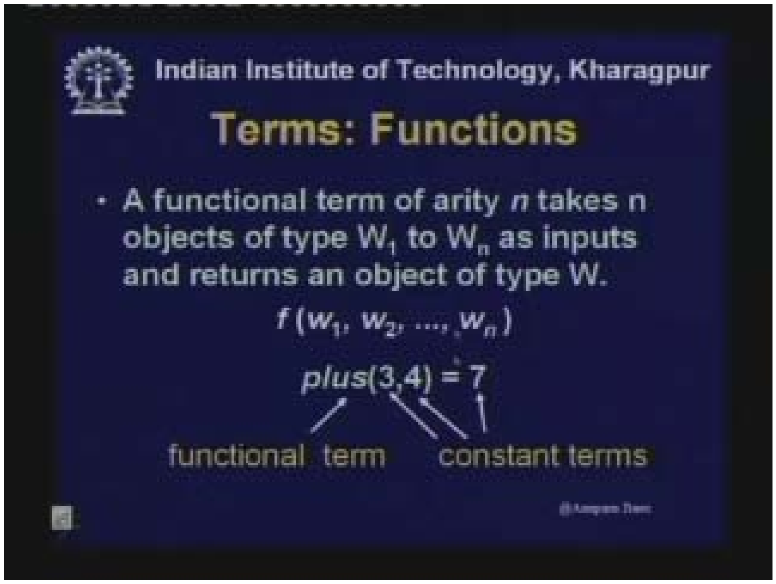

Now let plus be a function that takes two arguments of type natural number and returns a natural number. Now if that be my definition let us see, plus 2, 3 is a valid function. Plus 5 plus 7, 3 is also a valid function because plus 7, 3 will return an object of type natural number that is 10. Therefore plus 5 and 10 both belonging to type natural numbers we will add up and return a value of type W that is the natural number 15.

Similarly, plus plus 100 in that way I can go on writing in this order it is also valid although they are number of nested functions. And here based on the definition that I gave that plus is a function that takes two arguments of type natural numbers and returns a natural number based on that definition this is an invalid function because minus 1 is not a natural number and I have defined plus to be a function that takes two arguments that both arguments should be of type natural numbers and returns a natural number neither of these two satisfy. Similarly plus (1.2, 3.1) is not a valid function based on the definition of plus because 1.2 and 3.1 are not natural numbers they are belonging to the real numbers.

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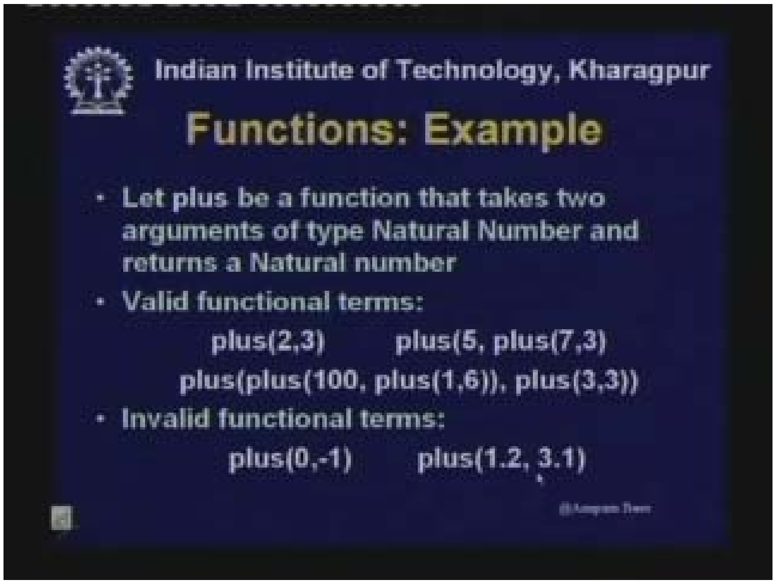

Similarly, we can have functions with variable arguments like plus (x, y) product prod (x, y, z) any of these things we can have. It is not necessary although in the examples we have shown with constants it can also have variables. Now that is what function is.

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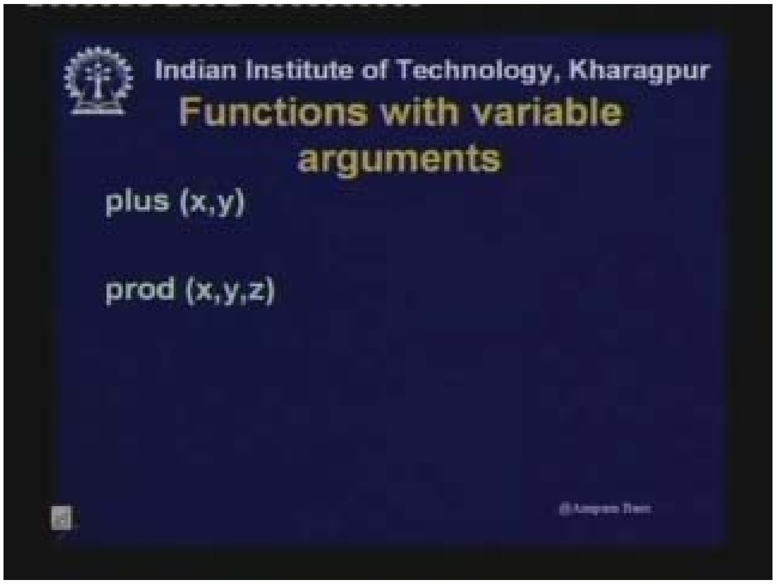

What are predicates?
Let us very easily understand that predicates are like functions except that they can only return, the return type can be only true or false. These two constants can be true. Other things are same as functions. A function can be any type of values but here it is a little

restricted that a predicate will either be true or false. So predicate is a function except that return type is only true or false.

Now let us look at these examples. Gt stands for greater than, greater than (x, y) variables is true if (x) is greater than y. In propositional logic we could not deal with variables but here in predicate logic in functions we can have variables and predicates being nothing but functions and we can have variables in predicates as well and that is giving us a lot more expressive power.

So, greater than (x) y is true if (x) is greater than y. If (x) is instantiated with a value 5 and y is instantiated with a value 10 then this entire predicate value will be false because 5 is not greater than 10. Therefore here greater than is a predicate symbol that is taking two arguments of type natural numbers but it is returning a value true or false. So greater than (3, 4) is a valid predicate but greater than 3 minus 4 is not a valid predicate because greater than is a predicate that has been defined. Suppose you take two arguments of type natural number minus 4 is not a natural number therefore this one is not a valid predicate.
The validity of the predicate now has got one relationship with it being evaluated true or false. The valid predicate gt (x, y) can sometimes evaluate to true sometimes evaluate to false depending on what values are being given to these variables (x) and (y).

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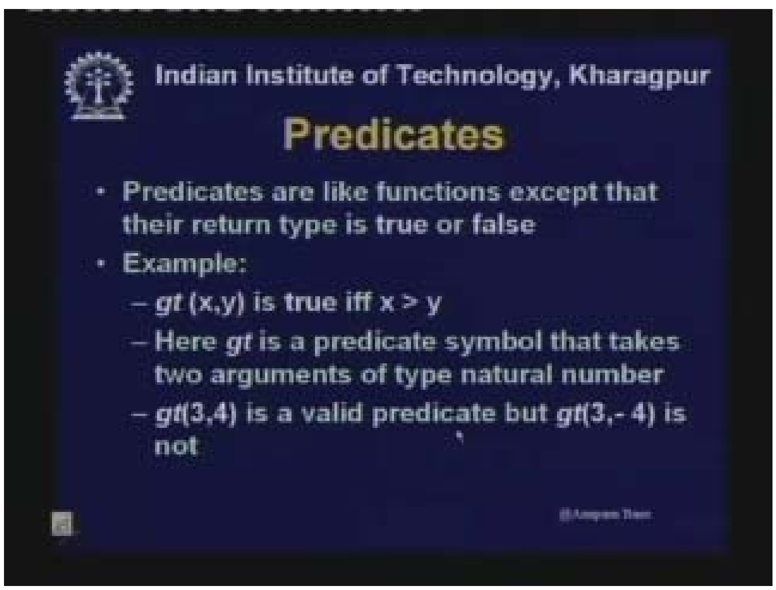

Now let us see the different types of predicates. The simplest predicate is a predicate with no variable. If we have no variable then I will get what we had done earlier that is a proposition. So Tommy is a dog is a proposition and this thing is also predicate. So predicate is more general. So predicates can capture everything that proposition can but not the other way round. So predicate with one variable is called a property. Dog (x) that is (x) is a dog. This is true if and only if (x) is a dog. Mortal (y) is true if and only if (y) is mortal.

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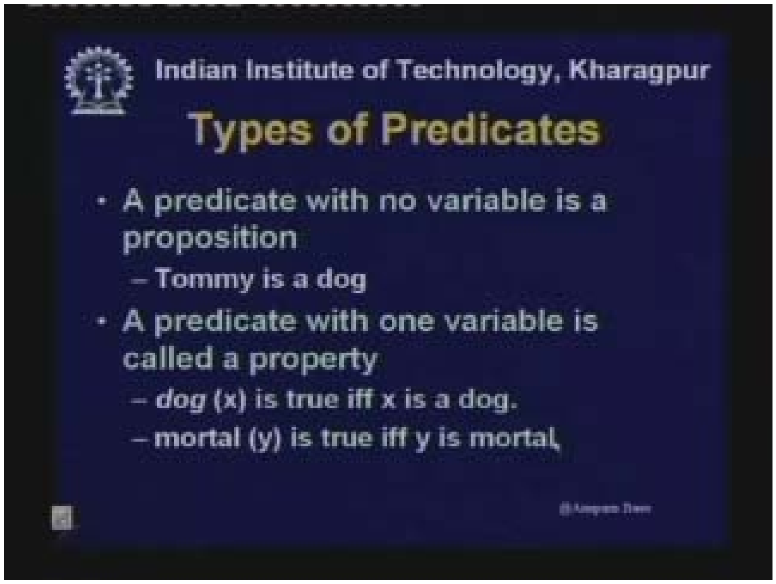

So a predicate with one variable is called a property. Now how do we formulate it? Let P having arguments (x, y) etc and Q having arguments (x, y) etc are two predicates. Remember how we defined well formed formula or valid sentences in case of propositional logic. We have got exactly similar type of definitions in the case of predicates also. So, in predicate logic if P and Q are two predicates then P OR Q is a valid predicate well formed formula, P AND Q is a valid predicate, NOT P is a valid predicate, P implies Q is a valid predicate. Exactly these similar things we had in the case of propositional logic. So let us have some more examples. If (x) is a man then (x) is mortal, this is a statement that I want to make.

So I can write, man (x) implies mortal (x), this is a valid predicate it is no longer a proposition because I have got variables here. If anybody is a man that is any man is mortal. So in the clausal form it will be NOT man (x) OR mortal (x). If (n) is a natural number then (n) is either even or odd. If (n) is a natural number that means (n) is a variable that must be a natural number then n is either even or odd. How do I express it? I can express it natural n if n is a natural number then even (n) OR odd (n). Now even (n) OR odd n any one of them will be true.

Let us once again look at the last example here that looks like a complicated sentence but as soon as we write in the form of predicate logic and if you understand that then it becomes really subtle. Now we have said that predicate logic requires variables, it allows you to have variables and in addition to variables there is another thing that predicate logic is introduced and that is called quantifiers.

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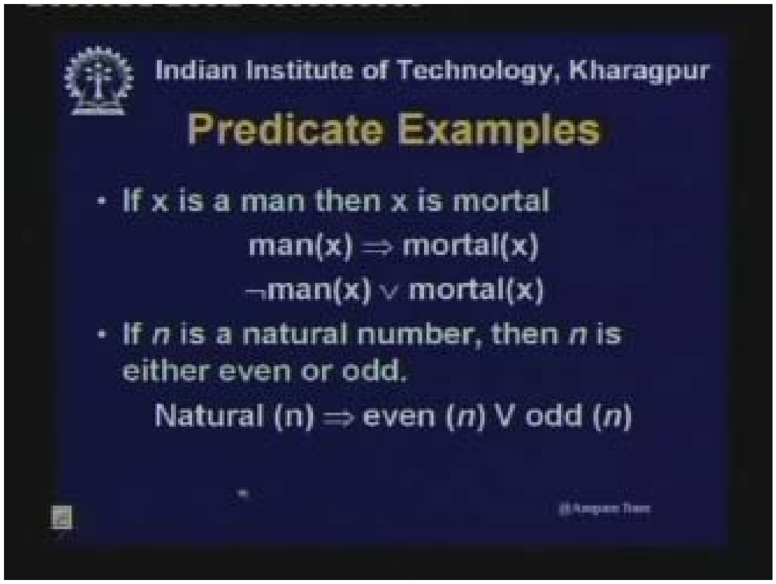

Now there are two basic quantifiers in first order logic or predicate logic for all which is denoted as this V with a horizontal line across it. Now for all is also known as universal quantifier. For example, if we say for all (x) then I am talking of all for any value that (x) can take the proposition to be true. Another one is; there exists which is also called existential quantifier that something exists. There is a subtle difference again between these two. So, for all (x) let us just think of a set, suppose I have got a set and in that set I have got a number of elements.

Now when I write for all (x) something is I write P(x) I just say that P(x) is true for all x. Now this need not be denoted as (x) so I can just put thick dots here, these are just elements of the set. When I write this then what I mean is that this P(x) will be true for any element that I choose from this set. Now, suppose let me choose another caller here and let me put this color and mark it they are some special elements, and suppose I write there exists (x) such that Q(x). That means this Q(x) should be true there should be at least one for these two elements in this set for which Q(x) is true.

Now here I have shown the elements Q(x) is true. Let me just write down an example, there exists (x) such that holiday(x). Now it is 1, 2, 3, 4, 5, 6 and let me put in another one, suppose these are the seven days in a week now is this statement true? There must be an element in this set suppose this is the days of the week. Now, obviously here lies Saturday and Sunday two days for which holiday(x) is true. Therefore there exists an (x) there exist a day for which holiday(x) is true. At least one would have sufficed. Suppose Saturday is not a holiday, only Sunday is a holiday so this one is not a holiday this is just an ordinary dot but at least I have got one then this statement will be true. This entire predicate will be true only if there is at least one (x).

On the other hand, in order that this statement is true all these elements in this set must satisfy P(x) that is the basic difference.

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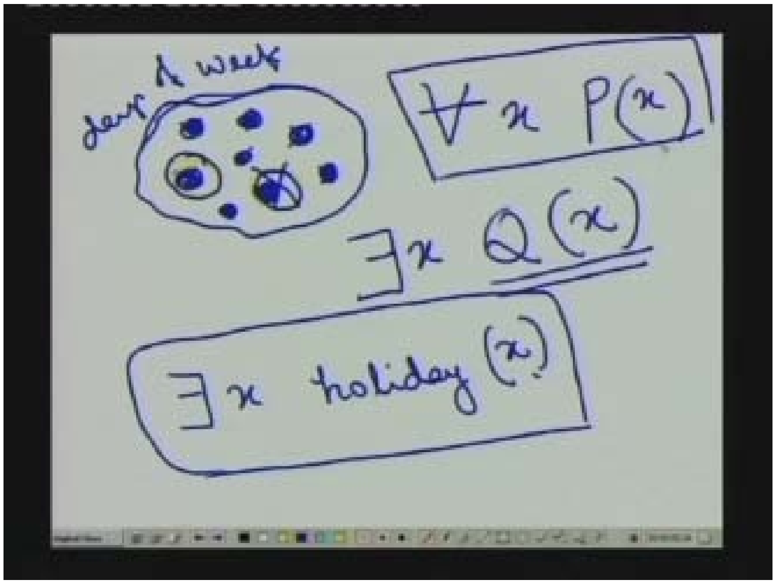

So, that is the essential difference between existential quantifier and universal quantifier. Now let us look at some more universal quantifiers. All dogs are faithful. Now I write, (x) is faithful is being denoted as faithful x, (x) is a dog is denoted by dog (x) so I can write for all (x) for all dogs in that set for all dogs dog (x) implies faithful (x) and that is true for any (x) which will make dog (x) true. That means for any (x) that is a dog this faithful (x) will be true. Now, how can we do this?

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The next statement is all birds cannot fly. Suppose we decide that with fly (x) we will denote (x) can fly and (x) is a bird denoted by bird (x). All birds cannot fly, therefore it is not the case that for all (x) bird (x) implies fly (x). Forget about this NOT part right now. In that case if I had written this for all (x) if (x) is a bird then (x) can fly. Now that is not true. I am saying all birds cannot fly so this entire statement for all (x) bird (x) implies fly (x) is not true. So this is the predicate logic expression with quantifier and variable for this statement all birds cannot fly.

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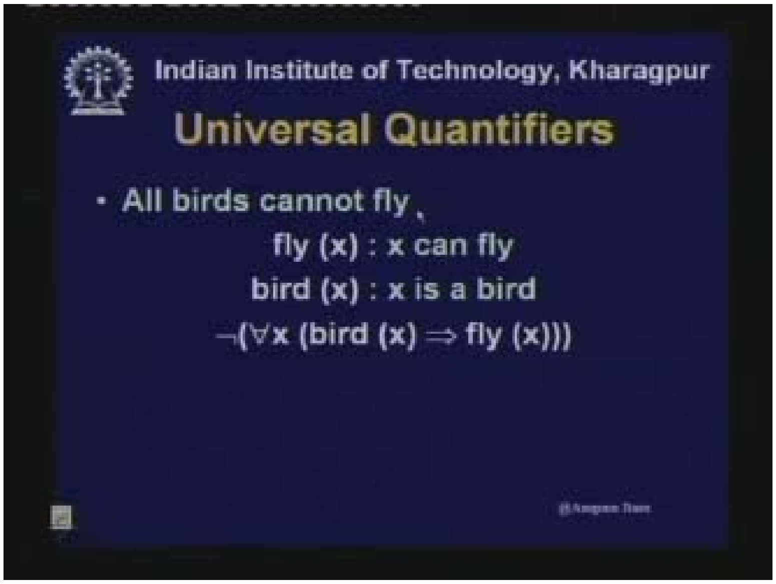

Now let us quickly revisit the quiz we have done and I said that I did play a little trick I did a little bit of adjustment. The statements which are given were; mammals drink milk, man is mortal, man is a mammal, Tom is a man these were given and I had written them as mammal (Tom) implies drink (Tom, Milk). But this statement is unfortunately not a statement made for Tom alone. So I should write it in a little different way. How should I write it? I can write it in this way now. Now I know predicate calculus so why should I do that sort of adjustment that I did.

So I can write; for all (x) mammal (x) implies drink (x) milk. Note here, milk is a constant object, among all the food milk is one, (x) is a variable so for all x mammal (x), if (x) is a mammal I take one animal and see whether that is a mammal and if that is the case then (x) will drink milk. Man is mortal, for all (x) if (x) is a man then mortal (x). man is a mammal, here I should have written for all (x) with some color.

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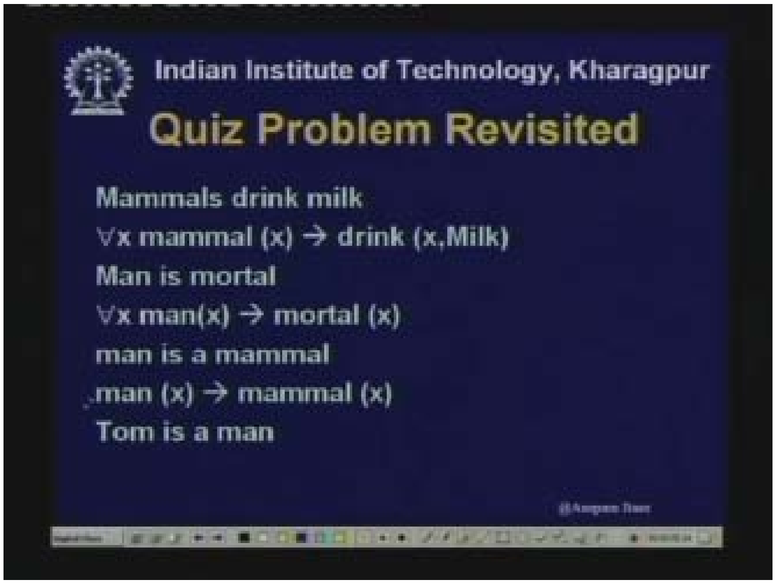

So it should be for all (x) man (x) mammal (x) and Tom is a man was as before man (Tom) there was no problem with that at all. Now we can do it in a much easier and natural way instead of making adjustments and that is the power that predicate logic has given us. In the examples given to you the universal quantifier was sufficient to deal with it.

Now let us look at existential quantifiers. If I make a statement at least one planet has life on it that means here I say planet (x) will denote (x) as a planet has life (x) will denote (x) as life on it. Then my statement would be, there exists an (x) such that planet (x) AND has life (x) that means there must be one planet there must be one element of that set of planets such that it is a planet and it has got life. So this is where I mean universal quantifier would not do. If I had written here for all (x) planet (x) AND has life (x) then this would be true only if all planets have life. But that is not what I want to do, what I want to do is to find whether there is at least one planet which has got life.

I am not going into stars I am just going into planets that is why I put in this predicate planet x. Now let us look into another statement; all birds cannot fly this statement is just equivalent to the statement there exists at least a bird which cannot fly. If you say all birds cannot fly then go ahead and prove it and you can prove it just by selecting at least one bird might be ostrich and you can show that this bird cannot fly. So what you need to really say is; there exists a bird which cannot fly or I can say there exists (x) such that (x) is a bird and (x) cannot fly. So we can straight away say all birds cannot fly and by bidirectional implication that is equivalent to there exists a bird that cannot fly there exists an (x) such that (x) is a bird and NOT fly x. Both these things are true for a particular (x) at least. In that case I can prove that all birds cannot fly.

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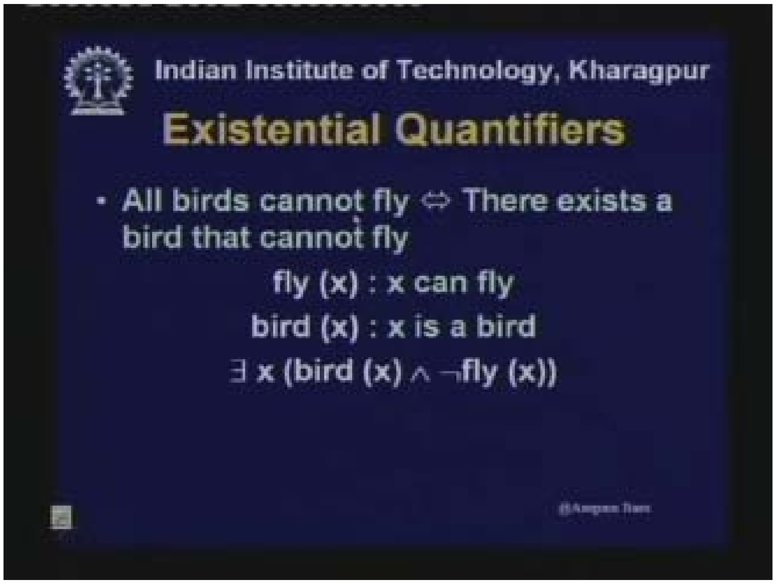

Now let's look at another interesting aspect called the duality of quantifiers. There is a duality between this existential quantifier and universal quantifier. You must have observed if you have noted in the earlier example that all birds cannot fly, it plays a duality, I am saying all but I am translating that into some sort of existential statement. Now say if we have, all men are mortal that is equivalent to no man is immortal. Similar to De Morgan's double negation no man is immortal.

Now formally we can say for all (x) men (x) this is the notation for a particular type, for all (x) men (x) implies mortal x. That means it is equivalent to there does not exist an (x) such that men (x) and NOT mortal (x) so I can always do that. Similarly there exists birds that can fly. I can always make it make the statement like this it is not the case that all birds cannot fly. Now try to read it a little carefully. There exists birds that can fly.

Again going back to the earlier type, if I have a set here like this in which there are elements and my statement was, is not the case that all birds cannot fly then it is again sufficient for me to show one particular element and show that it can fly so there exists an (x) that can fly. So there exists birds that can fly, one is sufficient. That means it is not the case that all birds cannot fly. Now how to express that? Formally there exist a bird which can fly, if I can prove that then I can certainly prove that for all (x) birds cannot fly is not true. So that shows the duality of such statements.

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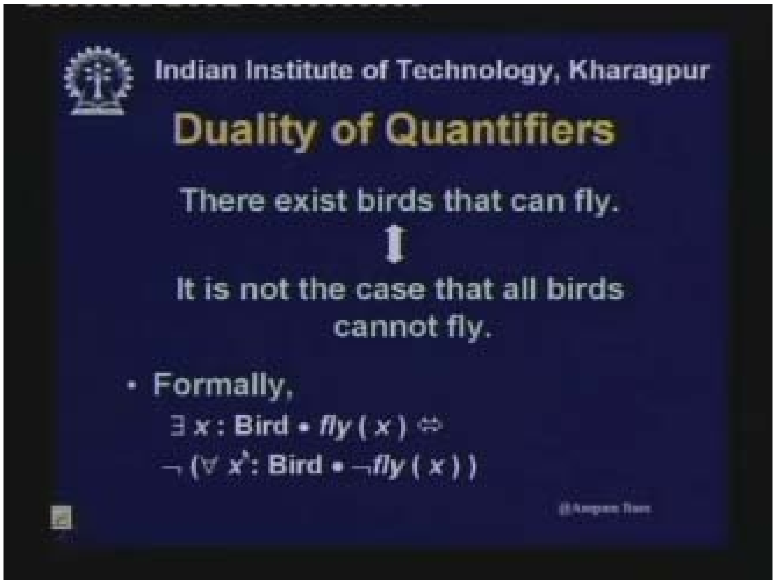

Universal quantifiers can again be expressed in a different way with existential quantifiers. Any predicate is a sentence if sen is a sentence then sen compliment is a sentence. And (x) is a variable then within parenthesis this sentence is a sentence, complement of a sentence is a sentence, there exists (x) sentence is a sentence, for all (x) sentence is a sentence, sentence AND all sentences are connected with AND OR implication are sentences. This varies and nothing else is a sentence. Now if you recall what we discussed in the case of propositional calculus we had also given sort of such strict definitions for a well formed formula and we had given a couple of examples in some sort of hard definitions.

Predicate logic is an extension of propositional logic. It is a more powerful version of propositional logic with incorporation of variables and with the incorporation of quantifiers. Therefore with these incorporations we can capture sentences much more naturally which we either could not capture it propositional logic or we had to play tricks and do some artificial maneuvering to capture those as I did in the first solution of the quiz. But as soon as we have got the power of predicate logic we can do that.

What are the two types of quantifiers?
The two types of quantifiers we have are universal quantifiers and existential quantifiers. Again there is a duality between these quantifiers. In the case of propositions we said if P is a proposition and Q is a proposition then if P is a proposition then what was the compound proposition P OR Q? NOT P is a proposition, P within parenthesis is a proposition, P OR Q is a proposition, P AND Q is a proposition, P implies Q is a proposition and nothing else is a proposition. In the case of predicates we also said that if P is a predicate and Q is a predicate then so is not P that is also a predicate. Then for all (x) P is a predicate, there exists (x) P is a predicate. Any predicate implying another predicate is a predicate. Predicate within parenthesis is a predicate and nothing else is a predicate. Now propositional logic and predicate logic we can put them side by side and

see the similarity and also you have to observe the extra power we have got in the case of predicate logic because of incorporation of variables and quantifiers. Here are some sentences which you should convert in the form of predicate logic.

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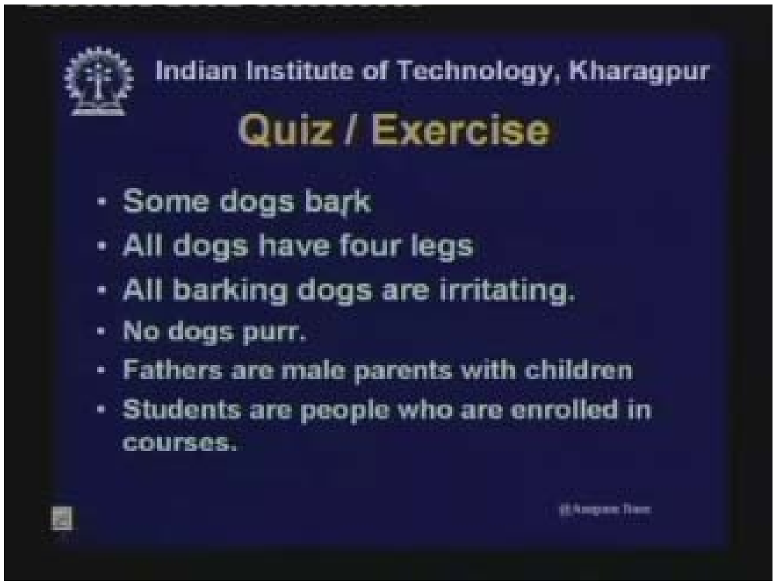

Some dogs bark, all dogs have four legs, all barking dogs are irritating, no dogs purr, fathers are male parents with children, students are people who are enrolled in courses. These are some of the exercises to work out.