**Artificial Intelligence**
Prof. Anupam Basu
Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur
Lecture - 12
Interface in Propositional Logic

In the last lecture we introduced the basic idea of propositional logic and we talked about representation of different sentences in propositional logic. In this lecture we will look in a formal way as to how we make inferences when some statements are represented in the form of propositional logic. As it has been proved in the last lecture that the objective of a knowledge representation scheme is not only representing the facts or some rules but also we have to be able to make the inferences. Unless we can make inferences, in whatever form we represent the knowledge that we have will be of no use. So we will be talking about inferencing in propositional logic representation today. But before we proceed with that let us have a quick look at some of the answers to the Quizzes that were given in the last lecture.

Instead of going into all the problems that were given I will rather give you some examples. Like the first problem was it rains in July. Now how do we represent this in propositional logic? It is really a simple case; we can say that rains July. That is sufficient to say that it rains in July. Similarly, if the statement was that it does not rain in November then in that case we could have written it as not rains November etc.

(Refer Slide Time 03:06)
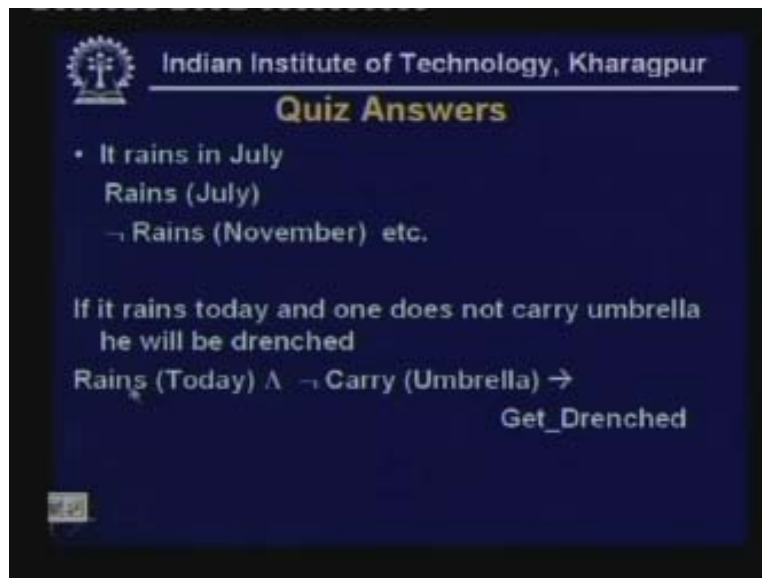
This is rather simple but the next example we have was also one of the problems given. If it rains today and one does not carry umbrella he will be drenched.
How do we represent this?

There is some implication here if and then. If it rains today and one does not carry umbrella then he will be drenched, then is implicit here. So, for this part of the clause we can say rains today and does not carry and not carry umbrella. If these two things happen then get drenched. This is one way. Here there is a word one and I have some how escaped using the statement one does not carry umbrella, I have simplified that because in order to capture this sort of thing one does not carry umbrella, no one carries umbrella, somebody does not carry umbrella etc require some more power than what is provided in the case of propositional logic. So let us neglect or ignore this one part here for the time being. So, if we neglect that we will get rains today and carry umbrella and does not carry umbrella implies that somebody gets drenched. Now if we do away with the problem of one here then let us take another example rains today and Tom does not carry umbrella implies that Tom will get drenched. So, rains today, AND NOT carry umbrella Tom, get drenched Tom.

(Refer Slide Time 04:27)
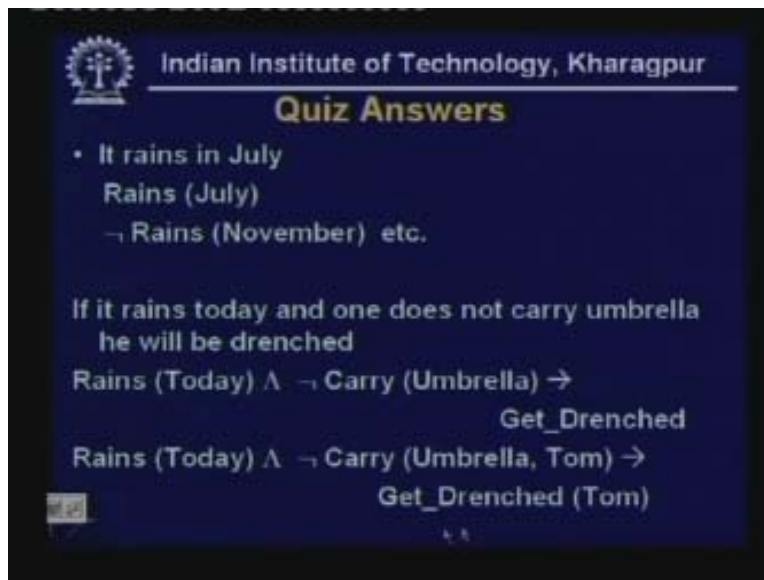
Now let us come back to today's lecture. In today's lecture our objective is to put you to the position where you will be able to infer the truth values of a proposition. You know that any proposition can be true or false. The truth value of the proposition will be evaluated. How to evaluate that? And you will be able to do reasoning towards arriving at new facts when you are given a new set of propositions. Also this lecture will help you to prove even a set of propositions. This means when a set of propositions are given and another proposition is given then you have to prove this later proposition using the existing propositions which are known to be true. In today's lecture we will also see the mechanism of doing that.

(Refer Slide Time 05:20)
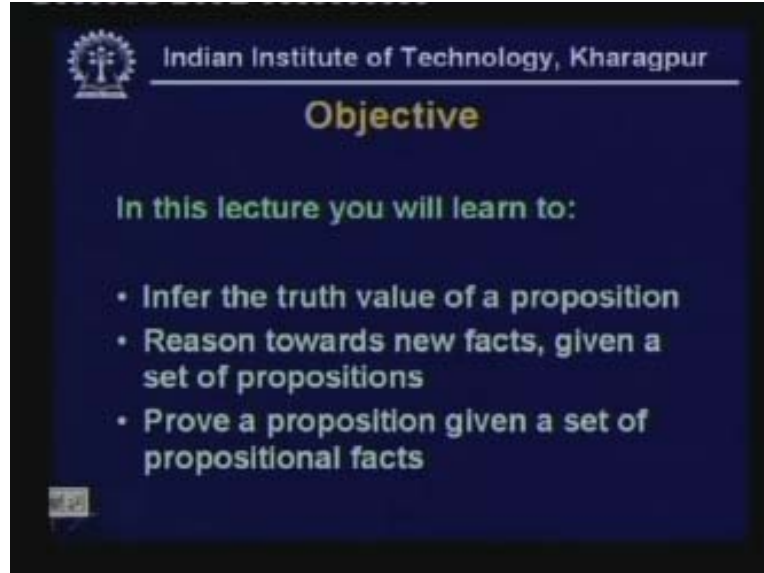
I had given two parts of the quiz, one I have answered but the other one I have not. <mark>But I assume that you have been able to do that some way or the other. But since we did not introduce any formal method for doing it I will try to give the answers of those a little later.</mark> But before that let us look at the procedure that is formally adopted to arrive at the truth value of compound propositions. Suppose we have got a proposition P and proposition Q the objective is to arrive at the truth value of the composite propositions like P AND Q P OR Q etc. Now the standard method for doing that is using the truth table.

Now here you see that we are deriving a truth table for the AND logical operation. So each of them P OR Q can have a value true or false. So we can have four combinations like both of them are false, one is false one is true, one is true one is false and both of them are true. So we make a truth table like this. We usually try to follow a system while you enumerate them false false then we make one of them true that is false true then we make the other one true the other one false true false and then true true. This is something to be noted here. This is a typical order in which we usually write.

Now, when we talk of P AND Q we know that AND is true if both of the propositions are true. Therefore when this is false false P AND Q will be false, if P is false and Q is true then P AND Q will be false because it requires both of them to be true, true and false will become false and only when both of them are true it will become true. Similarly if you go over here now it is again the same combination but OR operation P OR Q will be true when any one of them either P or Q is true.

(Refer Slide Time 08:22)

Therefore if it is false false it will be false but in false true case since one of them is true the result will be true, true OR false one of them is true so it will be true, if both of them are true then also it will be true. That is how we can find out the truth value of these composite propositions P OR Q. A few more examples: Here we are looking at the implication logical operation. Now what does implication mean?

If P is true then Q will certainly be true. So P is a sufficient condition for Q being true. But if P is false then also Q can be true. And if P being false still Q is true then also P implies Q this statement is not valid.

(Refer Slide Time 10:24)

So the truth table will be, both of them are false then obviously this is true because it is telling the truth that if it is false it may be false that is true this entire thing is true. If P is false and still Q is true yes that can happen because P is a sufficient condition P is truth it is a sufficient condition but not a necessary condition for Qs being true. If it is false and Q is true P implies Q then this statement is still true. If P is true and Q is false now we have a problem because P is a sufficient condition. Therefore if P is true Q has to be true if this is true. But P is true but still Q is false then obviously P implies Q this statement does not hold. Therefore P implies Q becomes false. But if both of them are true then also is true. So, when we try to derive the formula for one such composite statement in proposition calculus we can use this method called […..].10:29.

Here we have taken the example of a little more complicated thing that we want to find out the truth value of the statement NOT P OR Q implies P AND Q. Is this statement true or when will it be true?
Again we have to start with the standard method of enumerating the truth value of P AND Q. Now again we will have these four combinations false false, true true, true false, true true. Now, again since we have got NOT P here so if P is false NOT P is true we make a column for NOT P. P is false NOT P is true; P is true NOT P is false; P is true NOT P is false. And now we make another column for this entire NOT P OR Q and that column is here P OR Q. So NOT P OR Q how do you compute? It is an OR so NOT P is true Q is false NOT P OR Q is true, NOT P is true we have to just look at these two columns because we need only these two. Both of them are false so this is false, this is false this one is true so this will be true so I have got the truth value of this.

(Refer Slide Time 11:48)

And on this side we have got P AND Q. Now we have got P here Q here so P false Q false makes it false, P false Q true makes it false and since it is AND only when both of them are true it becomes true.

Now I have got these two. Now I want to see whether this statement which is an implication between this implies this, this implies this is true? What is the truth scenario for that? If the antecedent is true if this part is true then this has to be true because this is a sufficient condition. Now we see here this is true but this is false so this combination makes this entire statement false. There is a mistake here, this should be false, this should be false and this should be true there is a mistake here. This should be false and this should be true then this should be true. So, if this is false and this is true then this should be true. If both of them are false then also this is true, if both of them are true then this is true.

(Refer Slide Time 13:16)

De Morgan's Theorem

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$
$$\neg(P \vee Q) = \neg P \wedge \neg Q$$

P	Q	PVQ	¬(P V Q)	¬P	¬Q	¬P ∧ ¬Q
F	F	F	T	T	T	T
F	T	T	F	T	F	F
T	F	T	F	F	T	F
T	T	T	F	F	F	F

Now we come to another very important theorem called the De Morgan's theorem. De Morgan's theorem states that, if I make a negation, for example P AND Q this entire thing is negated then each of these literals each of these propositions like P AND Q will be individually negated and this operation would be complimented, this dual one will be taken so the dual of AND is OR and the dual of OR is AND. Therefore NOT of whole thing P AND Q will be NOT P and this one will be complementary and will become OR NOT Q. Similarly NOT of P OR Q will be NOT P and this will be complimented AND NOT Q.

Now this is De Morgan's theorem so if we just write down to prove the truth table then we can see that P AND Q will have these combinations again so P OR Q will have this value NOT of P OR Q will be compliment true, false or true becomes false true becomes false. Now we look at the right hand side of this, NOT P here P was false and NOT P is true, Q was false so NOT Q is true, so we get these things NOT P AND NOT Q and so NOT P AND NOT Q true true will become true so I am trying to compute this part, I am trying to prove De Morgan's theorem. So I have found the column for this value and now I am trying for this value. So, as soon as this becomes true false false false I can compare

this column and this column and I can see they are identical. Therefore these two are equal proving De Morgan's theorem.

The next thing we can do is retrieve the earlier quiz. What was given here is, if P is true and Q is true then is the following are true or false? First thing that was given P implies Q is it true or false if P is true and Q is true. Now we have got a formal method at our disposal so let us make a truth table for that. We see that P and Q and P implies Q if we make this picture we will find if P is true and Q is true P implies Q is true. Now let us take the second example.

(Refer Slide Time 15:40)

(Refer Slide Time 15:57)

Another example is NOT P OR Q implies P, how to deal with that? Again let us see if P AND Q are true. Here I have used another notation NOT P can be written in this or P prime can also be used to denote NOT P. Here you see P is true because it has been told in the problem if P is true then P prime is false negation of P is false and Q is true then what happens is S is this entire statement, is it true? NOT P is false so this is false Q is true therefore this part is true. And this part is true implies that P and P is also true.

(Refer Slide Time 17:40)

So this is true implies that this is true so this entire statement is true. Therefore you can use the truth table method to derive the validity of correctness of any propositional statement. Now let us come to reasoning. How do we carry out reasoning in the case of propositional logic? So, what we mean by reasoning that is using the given propositions which are assumed to be true you are trying to derive new facts which will also be true. Suppose there is a proposition it is the month of July and there is another proposition it rains and there is a rule that is told that P implies Q that means if it is the month of July then it rains is the statement.

Now another statement is given, it is the month of July that means P is true then what can you infer? Using your common sense you can say that you can conclude that it rains. Now how we did it? What was the mechanism that went behind this? We can write this in this form: P implies Q is a true proposition, if it is the month of July then it rains. I was also told it is the month of July so if P implies Q AND P then we can infer Q. So P implies Q is given AND P is given so we can infer Q. This is a very common deduction technique we often use in our day to day life and it has got one name and that name is Modus ponens. This is one typical inference rule which is very popularly used for carrying out inferences. Now was it true? Is Modus ponens valid? Is it a correct inference rule?

Let us look at that, P implies Q can certainly be written as NOT P OR Q that you have seen in the last class, NOT P OR Q result in the same truth table so we have got NOT P OR Q and P is true. So, if we conjoin it with P then we can write P AND NOT P OR Q therefore P AND NOT P by associativity OR Q so P AND NOT P is false OR Q anything false OR Q will be [….] so that is the proof of the Modus ponens. That means if P is given and P implies Q is given then you can certainly infer Q and that is a very common and usual inference rule. And when we talk of a valid inference rule it is irrespective of the interpretation.

(Refer Slide Time 19:41)
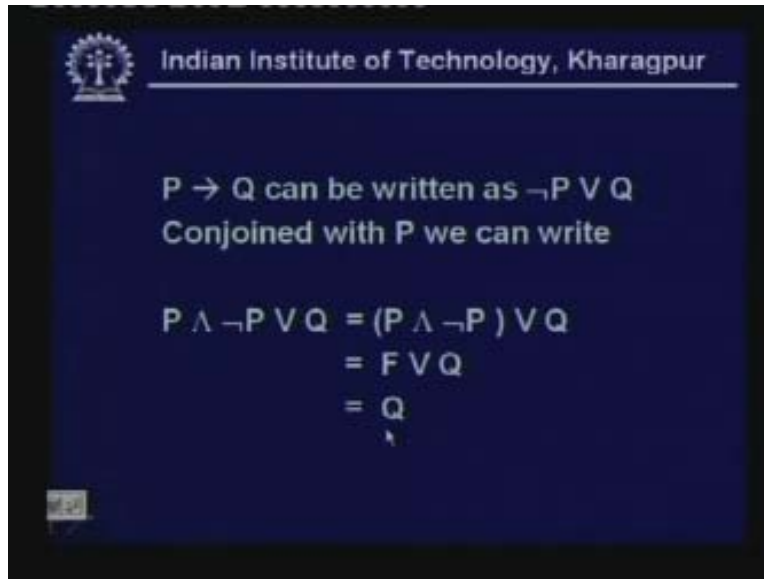
We are talking of inference rules which can be mechanically applied and these are irrespective of the meaning which is the interpretation we give.

So, irrespective of whatever the interpretation is Modus ponens allows us to infer the truth of Q. So Modus ponens is an inference rule that allows us to deduce the truth of a consequent depending on the truth of the antecedents, it is just a rule.

Why are we so much bothered about inference rules?
We are so much bothered about inference rules because we want to develop some mechanical procedures using which we can make the machine infer new facts. Only those inference rules are of interest to us which can be mechanically applied.

(Refer Slide Time 20:16)
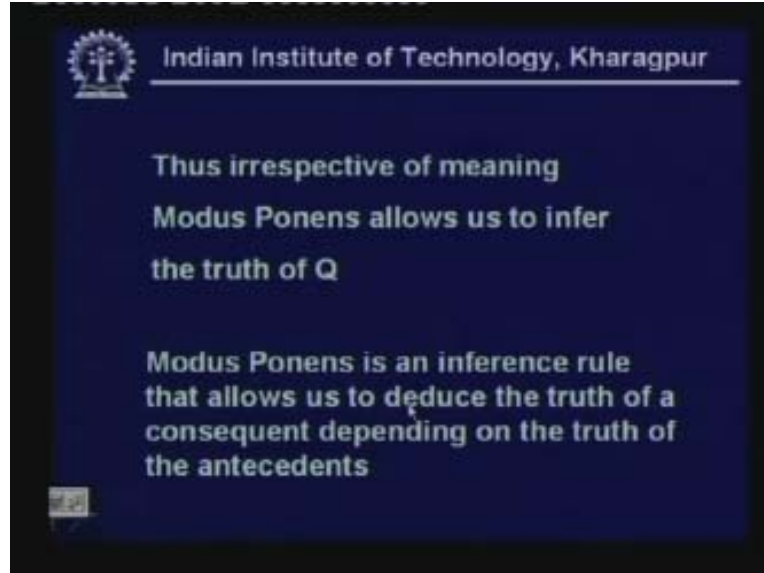
Besides Modus ponens there are couple of other inference rules which you can see through your common sense like P AND Q. If P is true and Q is true then obviously P is true. If P is true then P or Q is true. If NOT of NOT of P is true then P is true it is a double negation by De Morgan's law or any other law so these two cancel out each other and then we will get P. there is another interesting and useful rule called the chain rule.

For example, if we have one rule like if P is true then Q is true and I have got another rule if Q is true then R is true then we can certainly infer that, if P is true then R is true because Ps being true makes Q true and Qs being true makes R true so there is a chain. P is true makes Q true and Q true makes R true. Therefore by transitivity we can say if P is true then R is true. This is called a chain rule in which we will find a lot of applications when we look at rule based systems.

Besides these inference rules now we come to a very important concept called satisfiability. What does satisfiability mean?
Let us remember that whenever we try to prove any proposition we are proving it with respect to a particular interpretation. And that interpretation is nothing but mapping to a world. And a sentence is satisfiable by an interpretation if that interpretation makes the sentence true. That means, if under that interpretation that sentence evaluates to true only then that interpretation satisfies the sentence.

(Refer Slide Time 21:55)

(Refer Slide Time 22:48)

Now, if no interpretation makes a sentence true then that sentence is called unsatisfiable or inconsistent. If there are no interpretations that make all the sentences true you have in a set then that set of sentences is inconsistent or unsatisfiable. This is a very important notion.

Briefly speaking satisfiability means that, if given an interpretation a sentence evaluates to true then that interpretation satisfies the sentence. The sentence is satisfied under that interpretation. This leads to a very important concept called logical entailment. In our day to day way of speaking we often say this is a logical consequence of that, this logically

follows from that. What does the term logically follows really mean? What does it mean by the term following logically? That is essentially captured in the idea of entailment. If something logically entails something else then we can say it is a logical consequence. But what I said now is equally vague so let us try to make it concrete.

Suppose there is a set of sentences S and let us suppose there is another sentence here shown in small green bubble and we are thinking of one particular interpretation I that has been given to these set of sentences and this interpretation obviously makes these set of sentences true. That means this interpretation I satisfies these set of sentences. Suppose we apply the same interpretation to this new candidate sentence and if the same interpretation also makes this sentence true that means the interpretation that made these set of sentences true also makes this candidate sentence true then we can say that these set of sentences logically entail this sentence.

(Refer Slide Time 25:24)
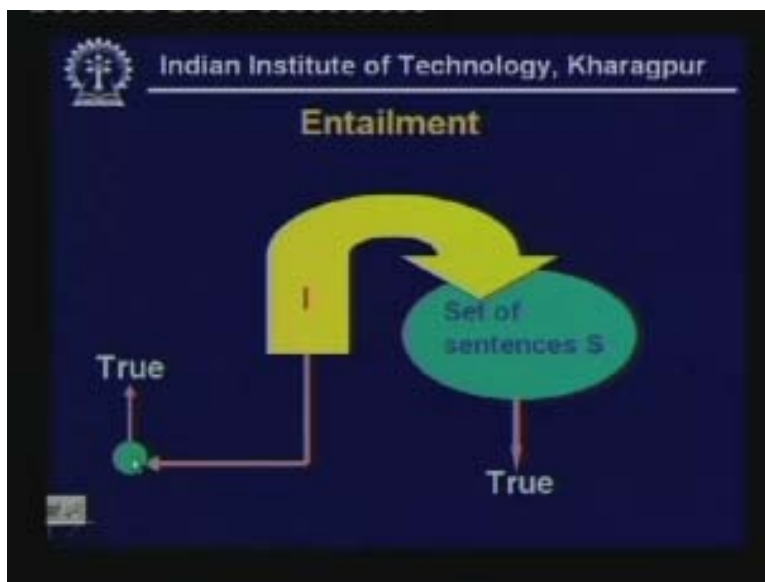
These set of sentences went through will also make this one true. That is called logical entailment. If put in words, there is a set of sentences and if a sentence S1 the small bubble here has a value true for all interpretations that make all sentences in a set of sentences S true then we can say that this set of sentences S logically entails S1. That means the set of sentences logically entails this new sentence or in other words S1 logically follows from S or S1 is a logical consequence of S etc.

Now we no longer remain vague when we say that this thing logically follows from this thing. That means a particular interpretation makes a set of sentences s true will also make the new candidate sentences s or S1 to be true. Now this is a very important consequence of our understanding of logical inference because when we try to infer we are actually trying to derive new sentences which are correct which are valid. Now by looking from another direction or from the angle of inference suppose we have got a set of sentences here and I have got a new proposition or a new sentence and I have got a

decision machine, the decision machine takes this set of sentences which are known to be true.

Suppose I have told some geometric theorems to the machine, now these geometric theorems are already proved and they are known to be true. And I have got a new theorem that I want to prove or a new problem that I want to prove. So I have got a set of sentences which are known to be true. Now, given that the decision machine takes this new proposition and asks this question is it true and the decision machine will come up with the answer yes or no. Therefore the answer will be yes if this new proposition is logically entailed by this set of sentences which are known to be true under a particular interpretation. That is what our proof mechanism is and nothing other than that.

(Refer Slide Time 28:12)
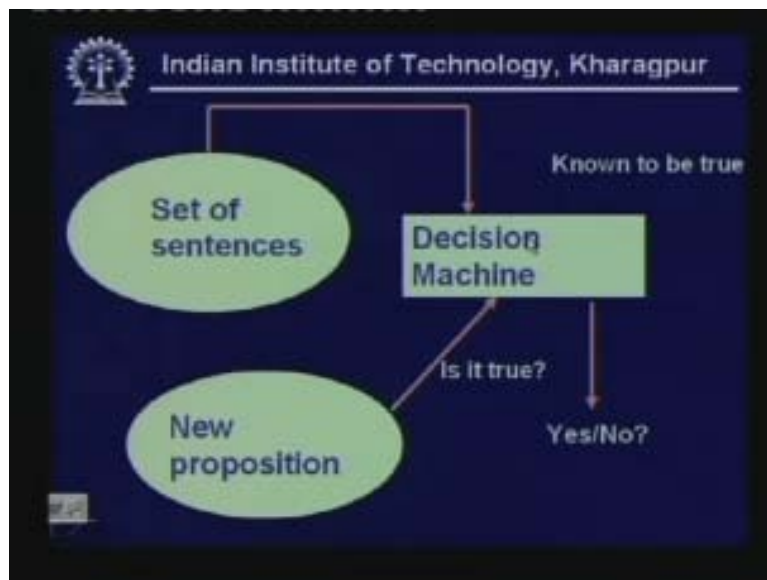
So if we have got a set of sentences which are known to be true and if a new proposition logically follows from that then the decision machine will be able to say that this is true. That is how our basic inference mechanism goes on. Now we have seen Modus ponens to be a very interesting and powerful way of inference that is an inference rule. Now we will look at another interesting inference mechanism known as resolution. But before going to resolution quickly we need to have a look at a special form of representation of propositions called the clause.

(Refer Slide Time 29:00)
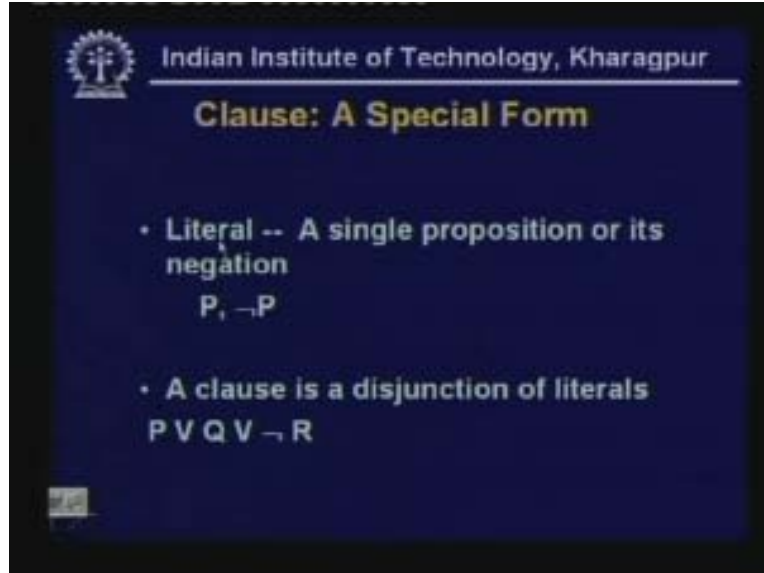
A clause is a special form. Before defining a clause we must know what a literal is. A single proposition which we often call an atom, a single proposition is always called an atom but a literal can be a single proposition or its negation. For example, P or NOT P, P is a literal NOT P is a literal. Now what is a clause? A clause is a disjunction of all these literals.

What does disjunction mean?
Disjunction means connection through OR not AND. When we connect it with AND we call it conjunction. But disjunction means that all these literals are connected with R. So you say P OR Q OR NOT R this is a valid clause. Now this is a particular form that is very useful for inferencing. Given any particular proposition can we convert it to the clausal form? The representation form in the form of clause is called a clausal form.

How can we convert a compound proposition to a clausal form?
There are some steps that we can follow and do that. Consider the sentence or a well formed formula A implies B OR C implies A. This is not in clausal form because we have got literals no doubt but we have got disjunction but we have got operators which are not disjunction. In a clause there will be only disjunction operator. This is a particular form of clause that I am talking of called Disjunctive Normal Form DNF that is of interest to us right now.

How can I convert this to a disjunctive normal form?
First of all I can eliminate these implication signs by the simple rule we know that if we write P implies Q then that thing can be written as NOT P OR Q we have already discussed that it can be written as NOT P OR Q. In the same way we can convert it in the clausal form. Any implication can be converted to this. My clause was NOT of A implies B OR C implies A so first I am trying to eliminate the implication sign. There was a NOT

here and let it be there. So A implies B will become NOT A OR B and C implies A will become NOT C OR A which is the first step.

(Refer Slide Time 32:15)

Now we have got this, still it is not a clausal form now how can we make it a clausal form?
We will have to eliminate the double negation sign and reduce the scope of the NOT signs using De Morgan's law. I had NOT of NOT A OR B so I will remove this double negation so if I apply De Morgan's law NOT applied here will negate this so this will become A and this will become NOT B but this operator will become AND which is the dual of that and similarly for this it will be NOT C OR A. But unfortunately as we have done this I have got an AND operator which is not allowed in a clausal form so I have to do something more. So what we got here is it is not a disjunctive form there are some ANDs also here.
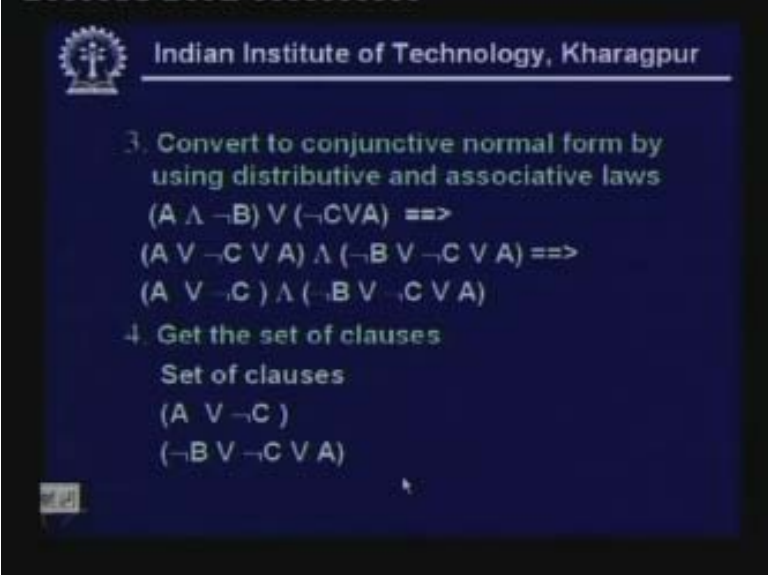
Therefore, as we go down we will have to conjunct this conjunctive normal form to disjunctive. I will first convert that to conjunctive normal form using these rules that I have got. Therefore by using the distributive and associative rules A AND B OR NOT C OR A can be converted to A OR NOT C OR A AND NOT B OR NOT C OR A. This can be further reduced to A OR NOT C this is one, AND NOT B OR NOT C OR A. Now each of them is a disjunction but they are connected with an AND operation. Since they are connected with AND this is called conjunctive normal form.

But our objective was to derive clauses and in the clauses there were only disjunctions allowed so what do we do?
We have got here individual clauses which are satisfying the definition of clause but they are connected with an AND. The answer is very simple. Each of these clauses can be looked upon separate clauses which are true. So we have to get the set of clauses A OR NOT C is one clause NOT B OR NOT C OR A is another clause. Thereby we derive the

conjunctive normal form but the conjunctive normal form consists of a conjunction of individual parts which are in disjunctive normal form, they are clauses so we have got two clauses over here. Therefore in this way we can convert any particular proposition into the clausal form.

(Refer Slide Time 34:54)
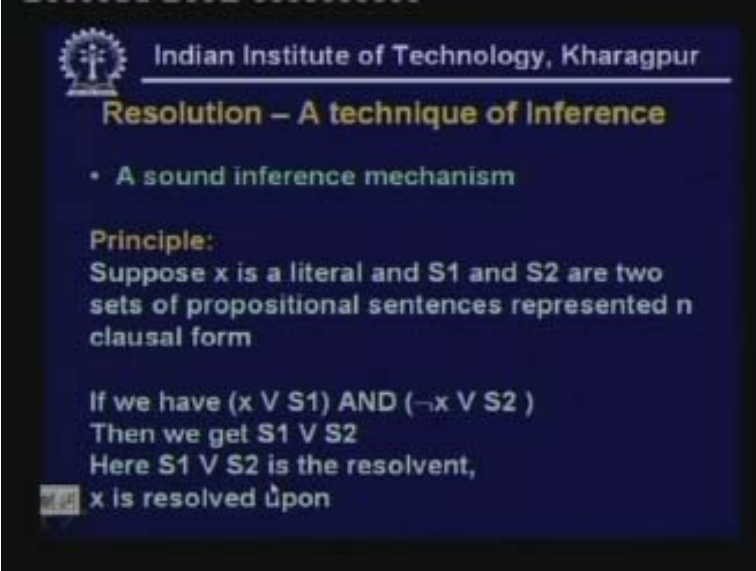

Why do we look at all these? Why are we so much interested into getting the clausal form?
It is because the clausal form will allow us to apply a very interesting inference mechanism called resolution. Resolution is another technique of inference that we learn. At the very beginning in the objective we said we learnt to prove new facts given a set of facts.

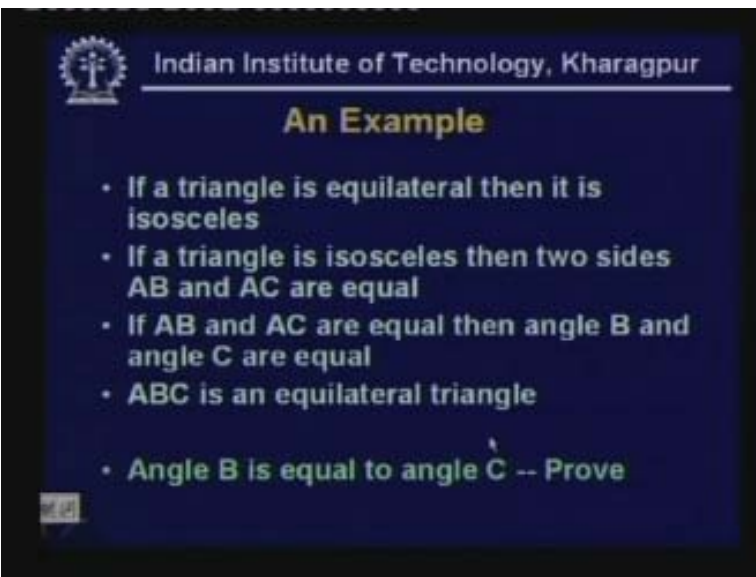Given a set of facts proving a fact means proving the logical entailment. We have also defined this. Modus ponens was one way but resolution is another way. Resolution is a sound inferencing mechanism. Suppose x is a literal and S1 and S2 are two sets of propositional sentences represented in clausal form then if we have x OR S1 AND this AND is a logical AND and NOT x OR S2 then we can get S1 OR S2. This is the basic principle. You can easily prove that x OR S1 and NOT x OR S2 if we apply the associativity over here then again we will have false OR S1 OR S2 then we get S1 OR S2 so that therefore we can eliminate a literal with its compliment and therefore here S1 OR S2 is the result of the resolution. This process is called the resolution.

(Refer Slide Time 36:54)

So we take two clauses and resolve one literal with its negation and the remaining part when combined gives us the resolvent. S1 OR S2 is the resolvent and x is a literal that is resolved upon and the entire process is resolution. Let us take an example. If a triangle is equilateral then it is isosceles. It is a rule a geometric theorem already proved by someone. And again if a triangle is isosceles then two sides AB and AC are equal and that is the definition. If AB and AC are equal then the base angles B and C are equal. ABC is an equilateral triangle and suppose you are asked to prove angle B is equal to angle C can you prove it? We can try to do these things in this way.

(Refer Slide Time 38:20)

Suppose I change this a little bit……. If now it is told that ABC is an equilateral triangle and I have been told to prove that these two angles B and C are equal how can I prove it? My task is to prove angle B is equal to angle C, how do you prove it? There are different ways of proving, you can try Modus ponens or let us try with resolution. The first thing we will be doing is, convert each of these English like sentences into a propositional form. So, if a triangle is equilateral then it is isosceles. I can write this equilateral ABC implies isosceles ABC. If a triangle is isosceles then two sides AB and AC are equal.

I can write that isosceles ABC implies equal AB and AC. If AB and AC are equal then angle B and angle C are equal. I can also write this statement as equal AB AC implies equal BC. These were some if then type of implications or rules and there is a statement ABC is an equilateral triangle. The proposition corresponding to that is simple equilateral ABC. Therefore from these statements we have got a set of propositions. Now we have to convert each of these propositions into the clausal form. So equilateral ABC implies isosceles ABC that is turning out with NOT equilateral ABC OR isosceles ABC. Isosceles ABC implies equal ABC that will equivalently be NOT isosceles ABC OR equal AB AC. equal AB AC implies equal BC that will be converted to NOT equal AB AC OR equal BC.

(Refer Slide Time 40:32)
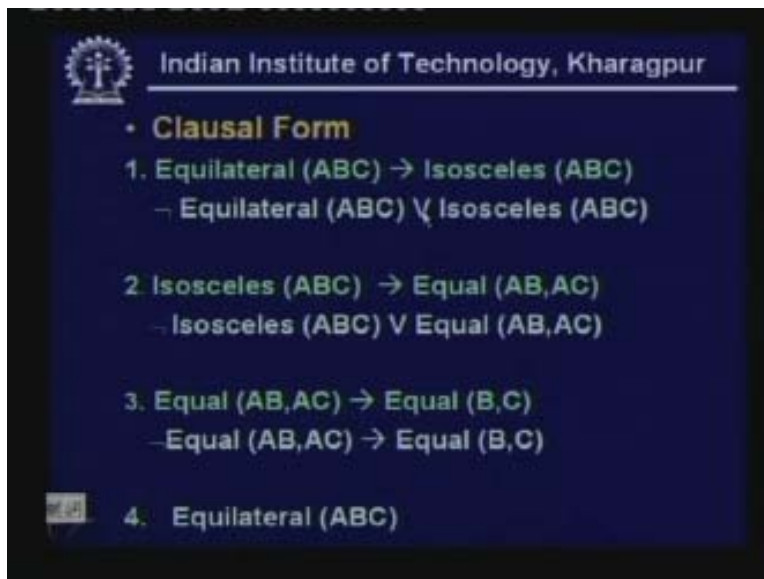
Here this implication sign is changed to OR. And the fourth was originally in the clausal form so I did not do anything. Therefore I have got four clauses that were given but I had another thing to prove, angle B is equal to angle C and when we represent that in a proposition equal BC therefore equal BC is a proposition AND it is in the clausal form itself so I need not convert it to clausal form any further.

Now, one approach that all of you must have done in the course of mathematics sometime is called proof by refutation sometimes it is also called reductio ad absurdum. So in order to prove equal BC what we can do is, we can also disprove not equal BC. If

we can prove that not equal BC is false that implies equal BC is true. So resolution actually tries to prove in that way.

We were supposed to prove equal B C and what I will be doing here is I will take the negation of this NOT equal BC and I will try to disprove it. That means I will try to prove that NOT equal BC is not a logical entailment of the set of propositions that has been given. That is the method of resolution. Here I have enumerated all the clauses I had NOT equilateral ABC etc all the clauses including that ABC is an equilateral triangle except the clause that I want to prove or actually the clause that I want to refute. So I take it on this side NOT equal BC which I want to disprove by this resolution.

Now we will start the method of resolution, how can I do it?
I will select one of the clauses from here. I need to be little intelligent and find out that here is a clause which has got the compliment of this literal NOT equal BC and here is equal BC. So I will select this clause, I will take this clause and will try to resolve between these two clauses.

Why did I select this clause?
It is because these are the negation of some part of the clause I am going to disprove. So when I select these two clauses I resolve them and when I resolve them what I get is these two will resolve out and the resolvent will be NOT equal AB AC. Now when I get NOT equal AB AC again I look over here and I will try to find out another clause which has got the compliment of that. Equal AB AC is here so I take this clause and try to resolve between these two. As I resolve these two will resolve out so I will get NOT isosceles ABC. So, if you just have a careful look you will see NOT isosceles this clause was actually meaning isosceles ABC implies equal AB AC.

(Refer Slide Time 44:19)
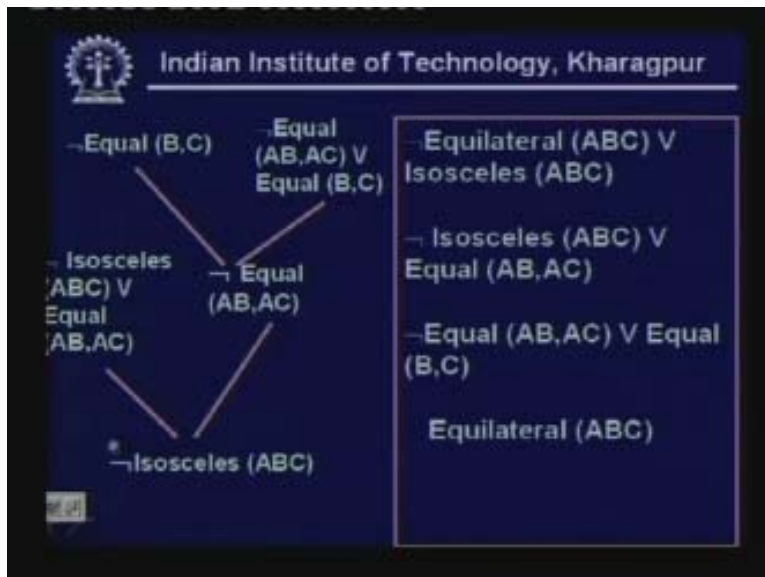
So from that what I have got is if it is equal ABC then I get isosceles ABC. So you just form it in the clausal form. I have got the NOT isosceles ABC. Again next step is, I start with this NOT isosceles ABC and I will try to resolve, so which clause have got isosceles ABC? This clause has got so I select this clause so this clause comes in I resolve between this resolvent and this what shall I get? I will get NOT equilateral ABC. Now I get Not equilateral ABC so I resolve further, here there is an equilateral ABC so what will happen if I resolve between these two? One is stating its equilateral ABC, another is saying NOT equilateral ABC so when I resolve them I will get a null clause.

(Refer Slide Time 45:05)
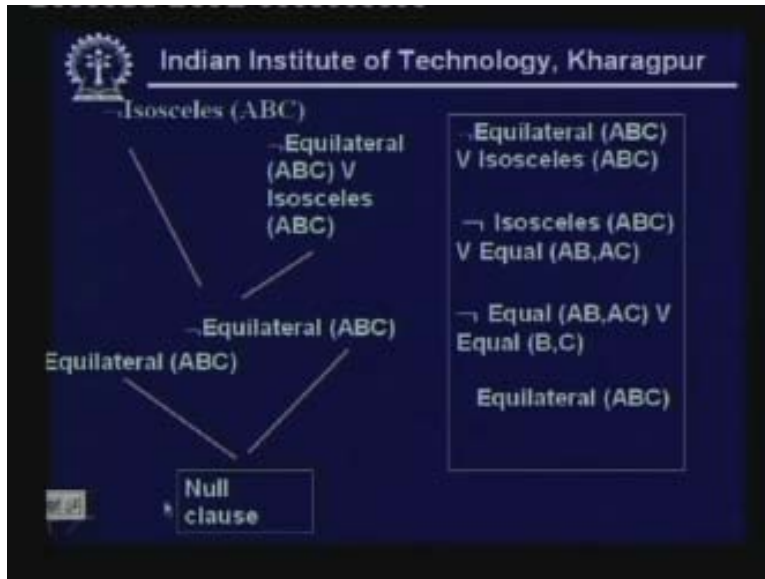
What is the significance of this null clause?
The significance of this null clause is that I have arrived at a contradictory situation that is a situation not supported by the given set of facts. There were a given set of facts like this and I started with NOT equal BC and from there by resolving I have come to a situation where I have got a null clause and the null clause essentially means that here is a contradiction. Therefore, the premise with which I started is false.

And what was my premise?
My premise was negation of the goal to be proved and that is disproved that means the goal is proved. This is the method of resolution. So the basic procedure for resolution if you enumerate will be like this: Convert a given proposition into a clausal form. Convert the negation of the sentence to be proved into clausal form. So first take the sentence to be true, take its negation then take it to the clausal form and combine all these clauses together into a set and iteratively apply resolution to the set as we were doing, and adding the resolvent to the set. So, if we go up a couple of slides earlier we tried to resolve this and once we resolved this we got this.

Now my resolution is taking place between this clause and this entire set of clauses and we are resolving between this element and this element and getting a null clause. So the

fourth step is, iteratively apply resolution to the set and add the resolvent to the set and we will continue until no further resolvents can be obtained or a null clause is obtained. That is the complete step of resolution. Here is a quiz.

(Refer Slide Time 47:29)
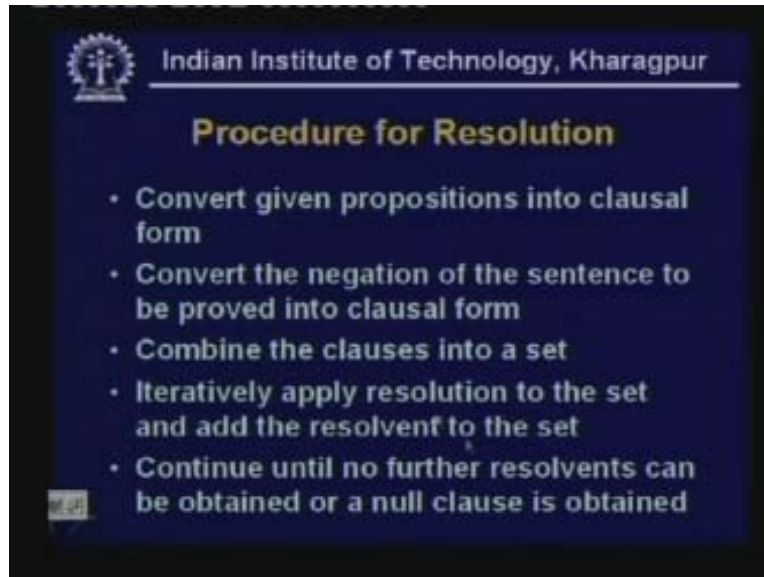
Consider the following sentences: mammals drink milk. Assume whether in the real world this is true or not, let us not bother about that. We are thinking of logical world so when we are making a statement I am assuming that this statement is true. Mammals drink milk, man is mortal, man is a mammal, Tom is a man so these are four statements that we have said. Prove that Tom drinks milk, prove that Tom is mortal it is very simple. Actually you can add in more implications but what you can try to do is, represent all these sentences in the clausal form. Now prove these two statements 5 and 6 using Modus ponens and again try to prove 5 and 6 using resolution. Therefore try to prove them using Modus ponens and resolution. This is the assignment.

Now let us look into the objective of today's lecture that is the formal way of inferencing using propositional logic. We have discussed about a couple of things. One is the truth table method. Using the truth table method we can find out the truth of any compound proposition when you know the truth values of the individual propositions. We have also seen that there are some inference rules which are not dependent on any interpretation. The propositions will evaluate to true or false based on some interpretation. So we have got a set of propositions we will give you the interpretation we will have some to be true some to be false but the inference rule is independent, it is true for all interpretations. Modus ponens is one such inference rule.

We have also seen a couple of inference rules like P OR Q implies P that is also another inference rule. But Modus ponens that if P is true then Q is true and we also know that P is true then we can derive that Q is true is a very powerful method of inference. This is typically the deductive method of inference. The other rule and a very powerful technique

we learnt today is resolution. Resolution requires that propositions can be converted into clausal form.

(Refer Slide Time 48:22)

We have also seen the technique of how we can convert a proposition into its clausal form. The steps are, first we will have to do away with the implication sign which you can easily do by converting P implies Q to the technique NOT P OR Q and after that we will eliminate the double negation. Then we will apply the distributive and associative properties and after that we will get a conjunctive normal form. And now we will decompose this conjunctive normal form into each of those components which are disjuncts so each of them will be clauses. Hence we start with a set of clauses.

Next when we want to do resolution we take the negation of the goal and convert into clausal form and then we take all these known things as well as the goal to be proved and resolve it in the manner that has been described, we iteratively apply that until we come to a null clause.

(Refer Slide Time 49:11)

If we get a null clause then certainly the negation of the goal has been disproved and therefore the goal has been proved.