Linear Programming and its Applications to Computer Science

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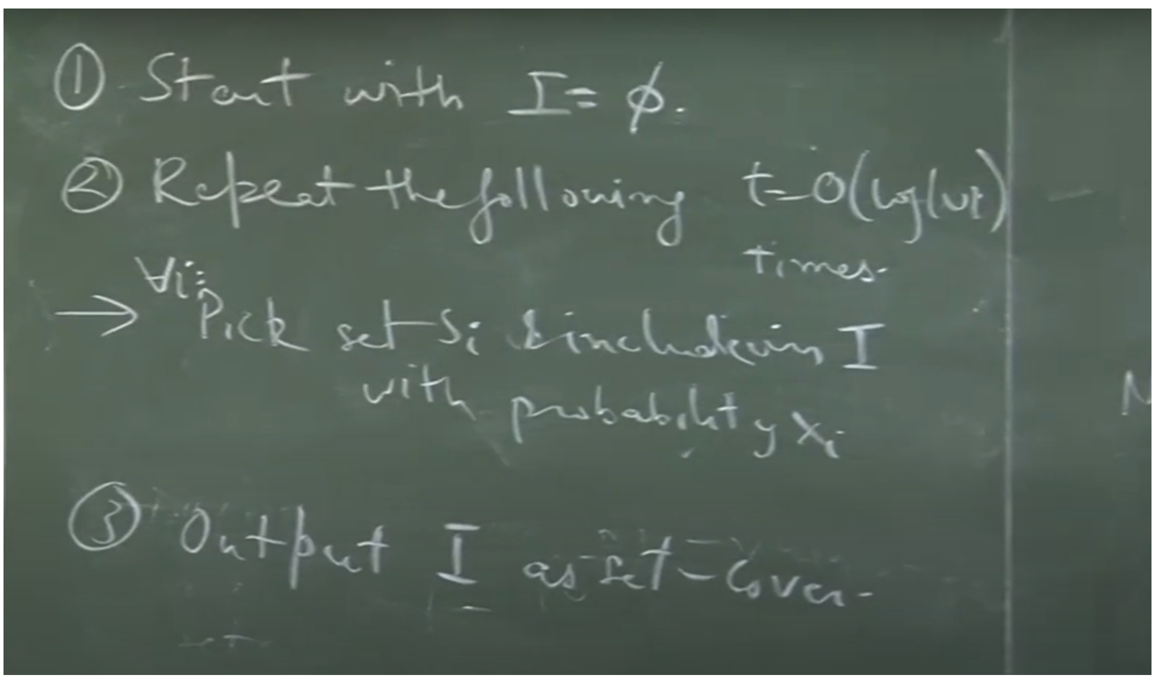
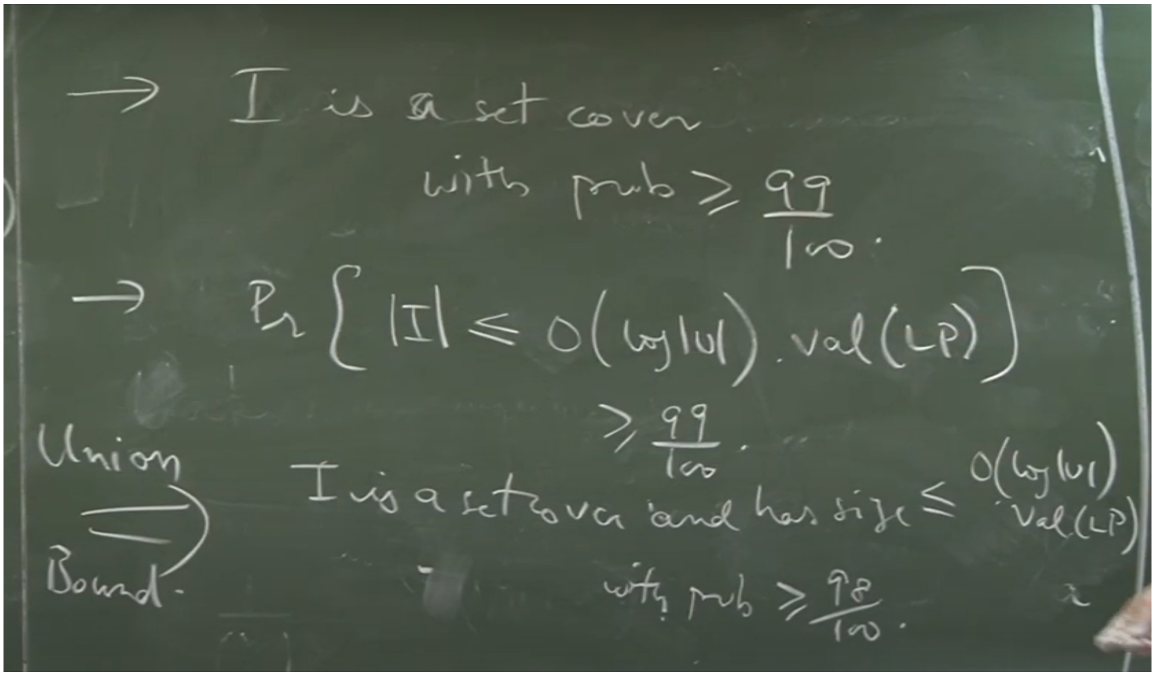
Lecture – 47
Algorithm for set cover

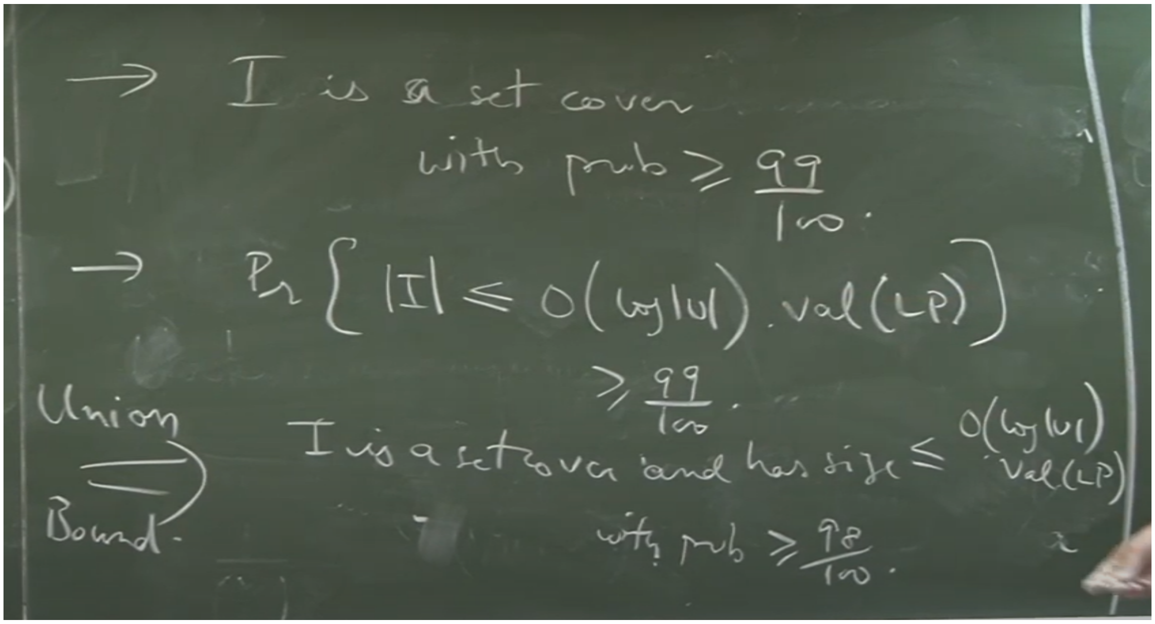
So, let us just recap our analysis and our algorithm. So, that we write it very cleanly what we have done. Start with my collection of set which is going to be empty ok. Some 100 times log u or whatever or 10 times log u whatever repeat the following t times what is going to be. And this once I have done it once I have gotten some sets in the next iteration it might happen that I will end up on the same set specially if x i is very high right. You want to keep it in eye it is your choice if you do not want to keep it in eye it is your choice our analysis will still work.



We know x i we do not know x i was the solution of the LP we have solved the LP we have solutions to the whole thing. You agree this was the algorithm which we talked about repeat this t times what results did we get? Correct. You also got. So, you can put again ILP there also the actual solution it does not matter right this is this one this is the exact thing which I am saying correct. So then, this is a simple union bound we had 2 cases of failure for both of which we have said that the probability of failure is ultra small it is 1 by 100. So, at max the probability that we fail in 1 or other is 2 by 100 that means the probability that we not we do not fail in any of those is 90 by 100 it is a simple union bound.



So, this gives us a this gives us an approximate randomized for set cover. So, this was a result by this guy from Chicago I do not remember the other authors ok and again as I said this was kind of a big result in the set cover literature people had worked on these things for a long time these were very hard, but suddenly now. So, these are all commutative problems right max flow min cut your commutative problems you are trying commutative techniques. And suddenly when you take come to this LP linear programming kind of techniques optimization techniques you get new results that is the excitement. You have a commutative field people had tools for that you apply linear programming tools there and you get nice good results good example of this cross border terrorism.



Any questions about this particular algorithm for set cover right the and the thing which I like is the ideas are pretty intuitive right we described I L the integer linear program when we come to LP the idea was oh now this xi is not 0 and 1, but if it is 99 by 100 then probably I should keep that I in my set cover if it is 1 by 100 then I should ignore it. That was the idea which we had. And this is exactly what we did in the rounding we said let us pick this set with probability xi with probability 99 by 100 or 1 by 100. The harder part in almost all these cases is the analysis giving the rounding is generally ok. Relaxation we saw relaxation was very obvious you just say oh between 0 and 1 you convert into this continuous 0 and 1. Even rounding some rounding you will see oh it does not work other natural rounding the hard part is to show exactly that this works coming up with rounding generally is not the hard part.

And in this case as probably should say that oh the analysis was also like easy yes depending upon whether you compare with constant or you great. So, in all of this again I would like to emphasize this general technique relax. Relax convert. So, let me make this joke more prominent even if your ILP is not equal to ILP relax convert your let us write again is ILP relax convert your ILP is not equal to ILP into LP. This is for optimization people as computer scientist we say solve it and let us know the solution do all the analysis numerical whatever we do not care give the solution.

ILP or the actual problem solution and in this entire area the most work is done in these three words. This is generally obvious this we have offloaded our work is generally showing that what you get is not bad. And there are LPs where all your rounding are generally bad that is where inapproximability comes up where you show that no relax no rounding algorithm will work. So, this might feel like a big constant for you, but to give an example like for maximum independent set we believe that there is no approximation algorithm even of N to the power 0.5 or something like that.

So, this might seem big, but no for commutative problems this is not big the approximations do not reduce the exact problem reduces, but the approximation factor changes. So, you cannot say that if you can find an approximation algorithm of max set max set with factor three that does not imply you get a set cover solution of approximation factor three. So, that does not hold again that would have been great let me know if you can prove it.