Linear Programming and its Applications to Computer Science

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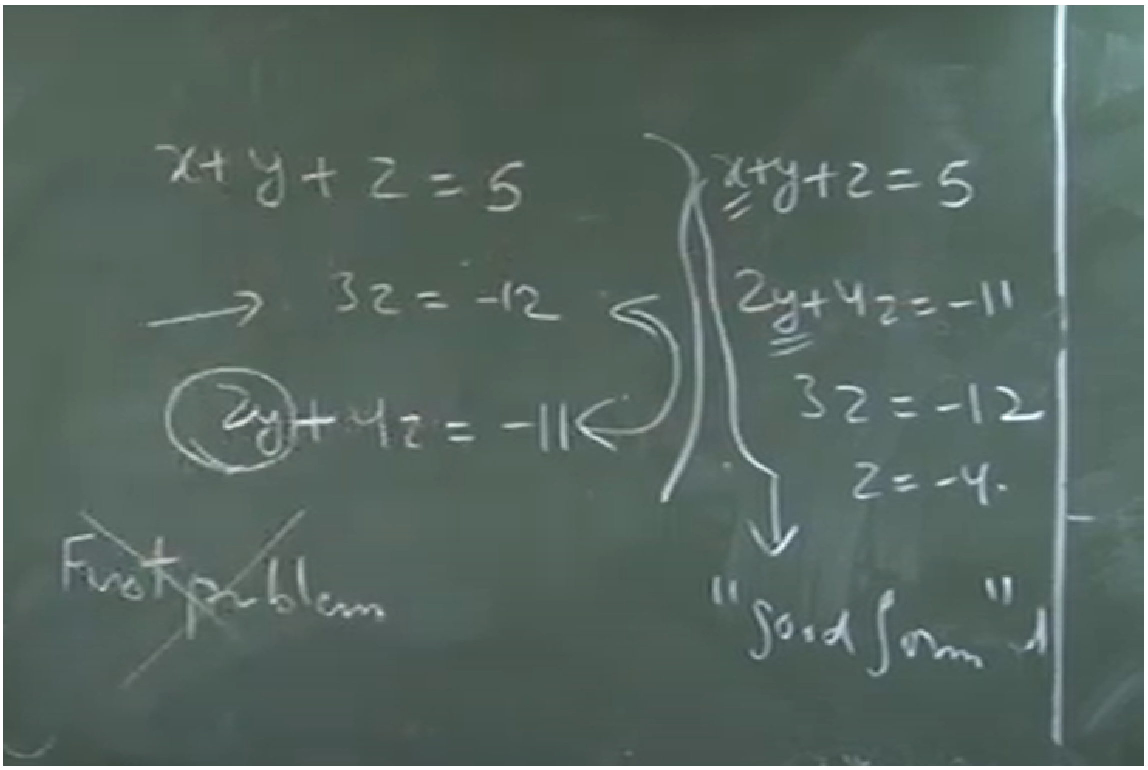
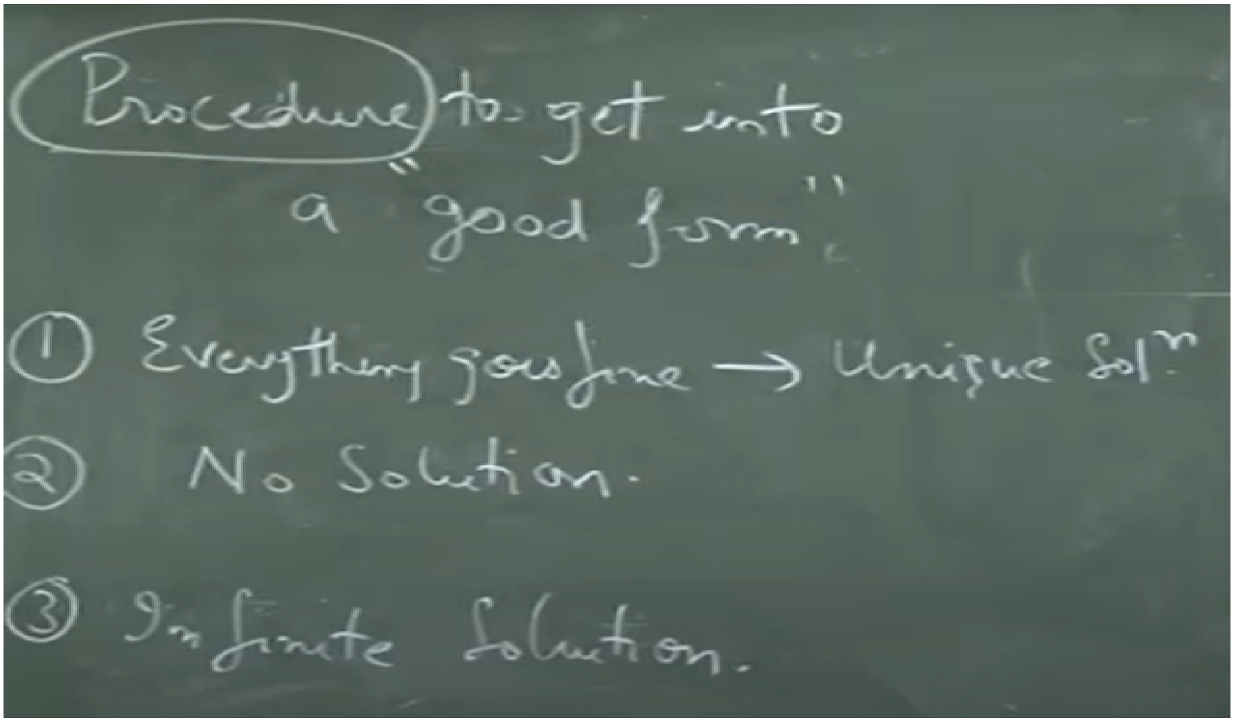
Lecture – 04
Summary of Gaussian Elimination

We had a procedure to get into a good form. The good form was the one where we can back substitute and get the solutions completely. I think I have an example of that on the board.

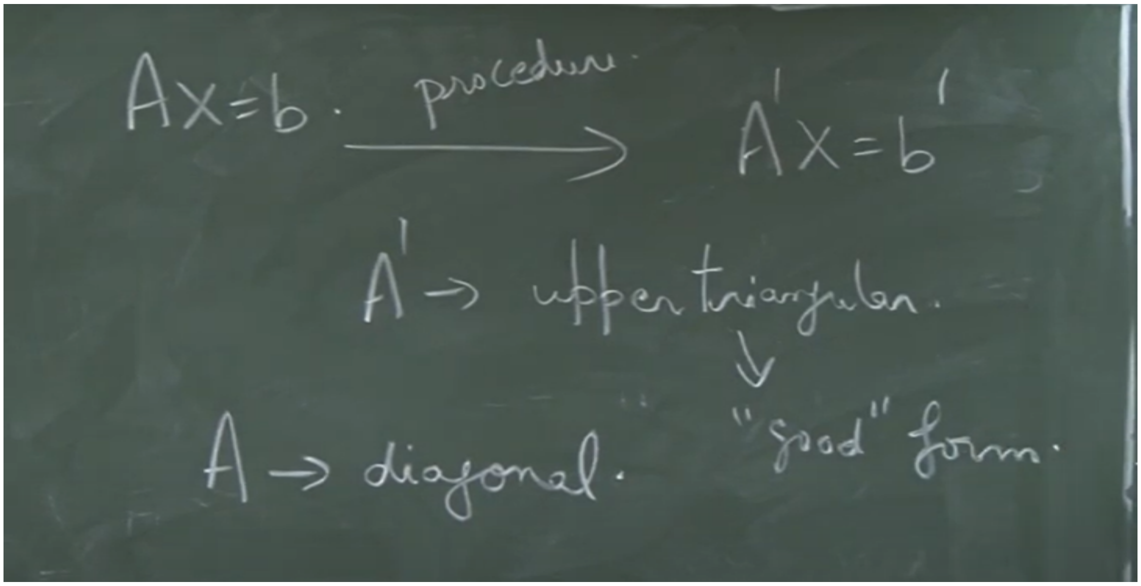


Oh yeah like this. This was in good form, I get Z equal to minus 4 then on Z I can get your Y once I fix Y once I fix Z I get the value of X2. This was the simple case. Now if everything goes fine we get into the good form we get a unique solution.



But there were weird cases we can get no solution or we can get finite solutions. This is one thing which we will remember that these are the 3 cases possible, but I also want to outline what do I mean by a procedure. The procedure was you are given Ax equal to b, you wanted to apply the procedure to get it into equivalent set of equations. A prime X equal to b prime such that A prime is kind of upper triangular.

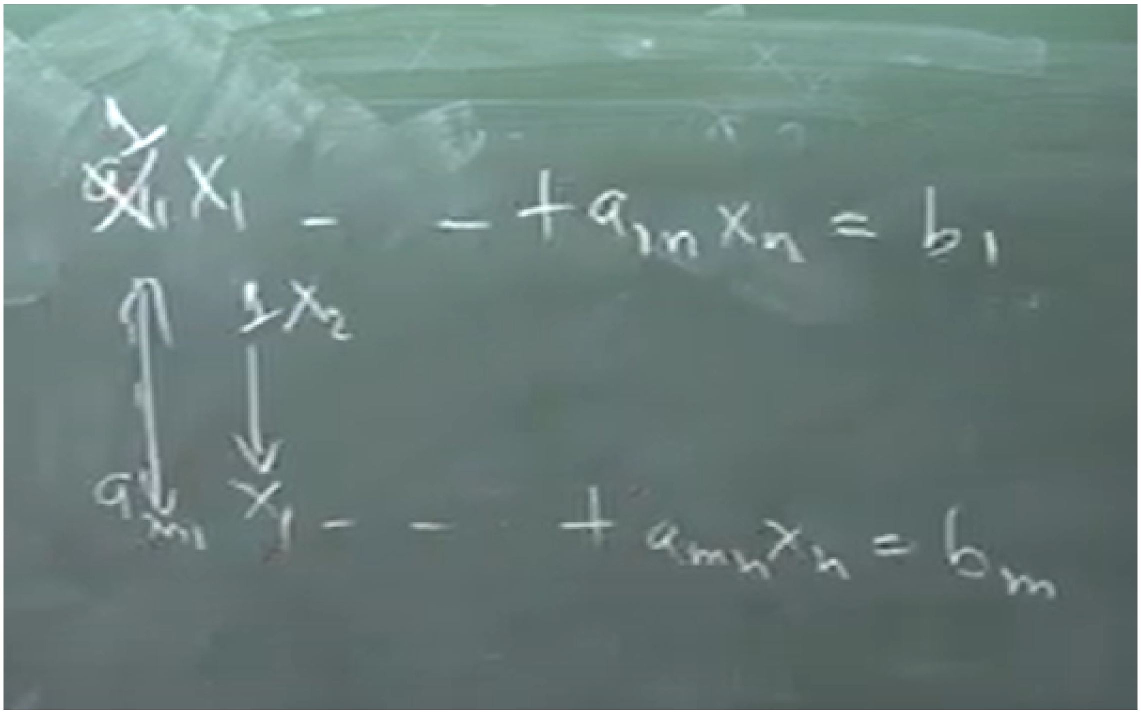
And to do this procedure we had multiple flexibilities, we had these operations which we can do among equations. You remember subtract some equation, multiply equation by a constant things like that. And once I do that my life becomes simple. If the idea of upper triangular makes you slightly vary notice what is the easiest case to solve linear equations. When your A is thank you yes as Tufan pointed out if A is diagonal.



This was the easiest case if A is diagonal then we have values of X equal to something, Y equal to something written for us. Ideally if we could change our equations to diagonal equations that would be great. But we realize that if we had an upper triangular matrix we can always convert it into a diagonal matrix by back substitution that is the good form. This is the good form. And to get to this good form we have some operations.

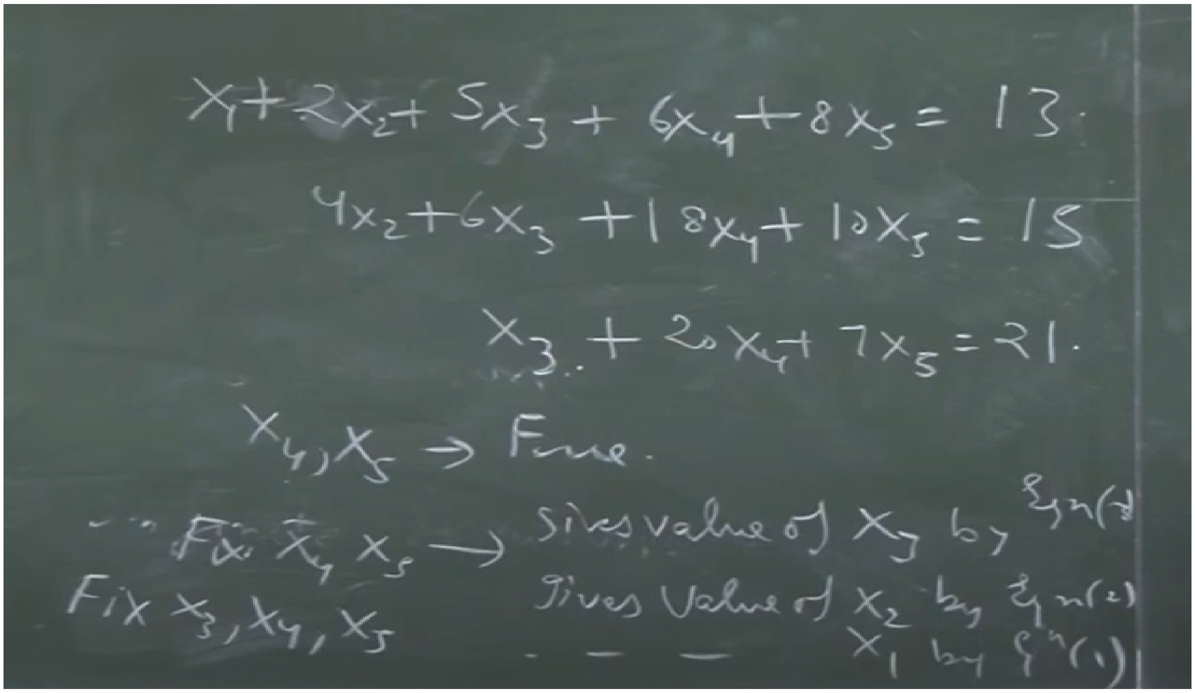
What are the operations? One is we can subtract a scalar multiple of an equation from another equation. This is something which we did multiple times E1 minus sorry E2 minus 2 times E1 E3 minus 4 times E1 so on and so forth. So this we can do sometimes when we got into a problem we could exchange two equations. Right, This has to make sure that variables which are still important they lie on the left hand side like. Changing the order.

Changing the order exactly this was like and third part third set is not really very very important. It's there So, as to make the equations look, beautiful is to be able to multiply an equation by a constant scalar or a real number. This helps us to look, at the equation if you want to make the coefficient of Z1 we can multiply it by 1 by 3. This gives us the solution of Z directly and also when you want to subtract this from the other equations it is very easy to figure out the multiplier. So, this is kind of for the aesthetics of the things not really really necessary, but this makes our life more seamless more clear.



So, that is why using these two of three operations we can keep doing this thing we can first we can eliminate the coefficient. So, let me write it out. Remember this was our set of equations we can first make this coefficient 1, then use this equation to eliminate all the constants of X1 from all the other equations. Then we multiply make sure that this coefficient becomes 1 use this anytime if you do not find the coefficient we just interchange equations or interchange variables to keep going. And this procedure finally, will give us a good form once you have a good form it is easy to read off the solutions.

This was the essential process. Let me emphasize slightly this I will once again talk about,

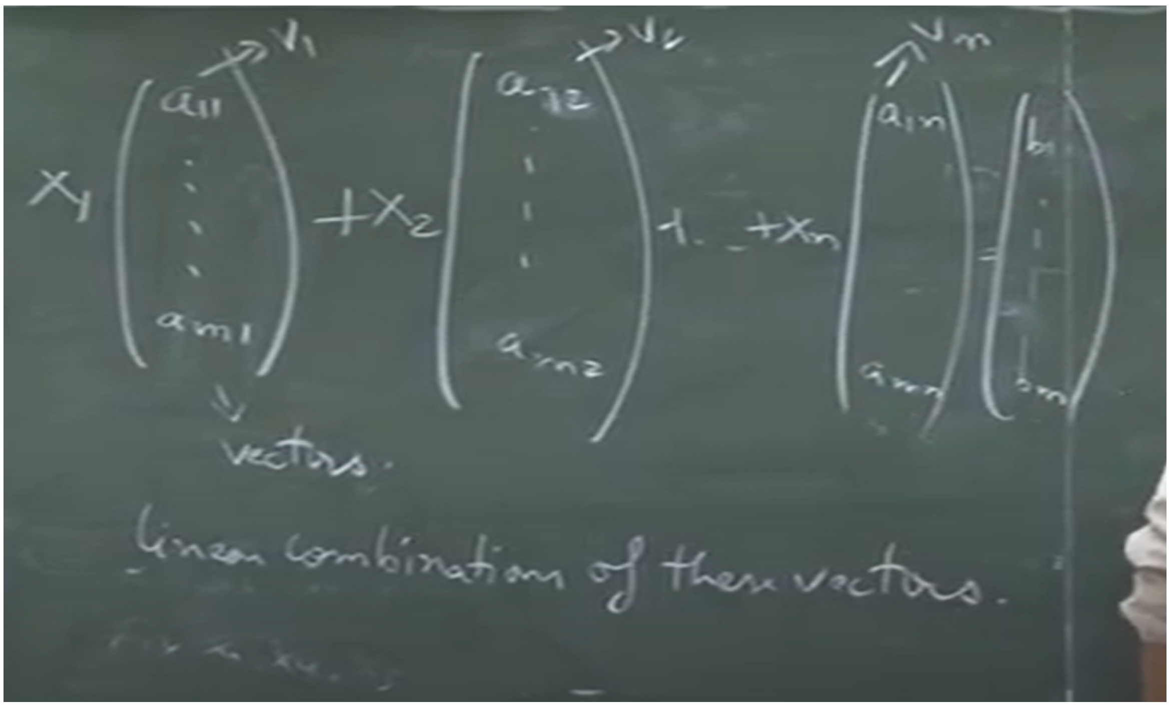


But let me just clarify this thing that sometimes our good form might look, like this. I should say I should use some constant let us say 13. Now, this look, s like in a good form the way to figure out this equation when you have such kind of thing when you have lot of variables and less number of equations. What you will realize is that what we call X4 and X5 to be free.

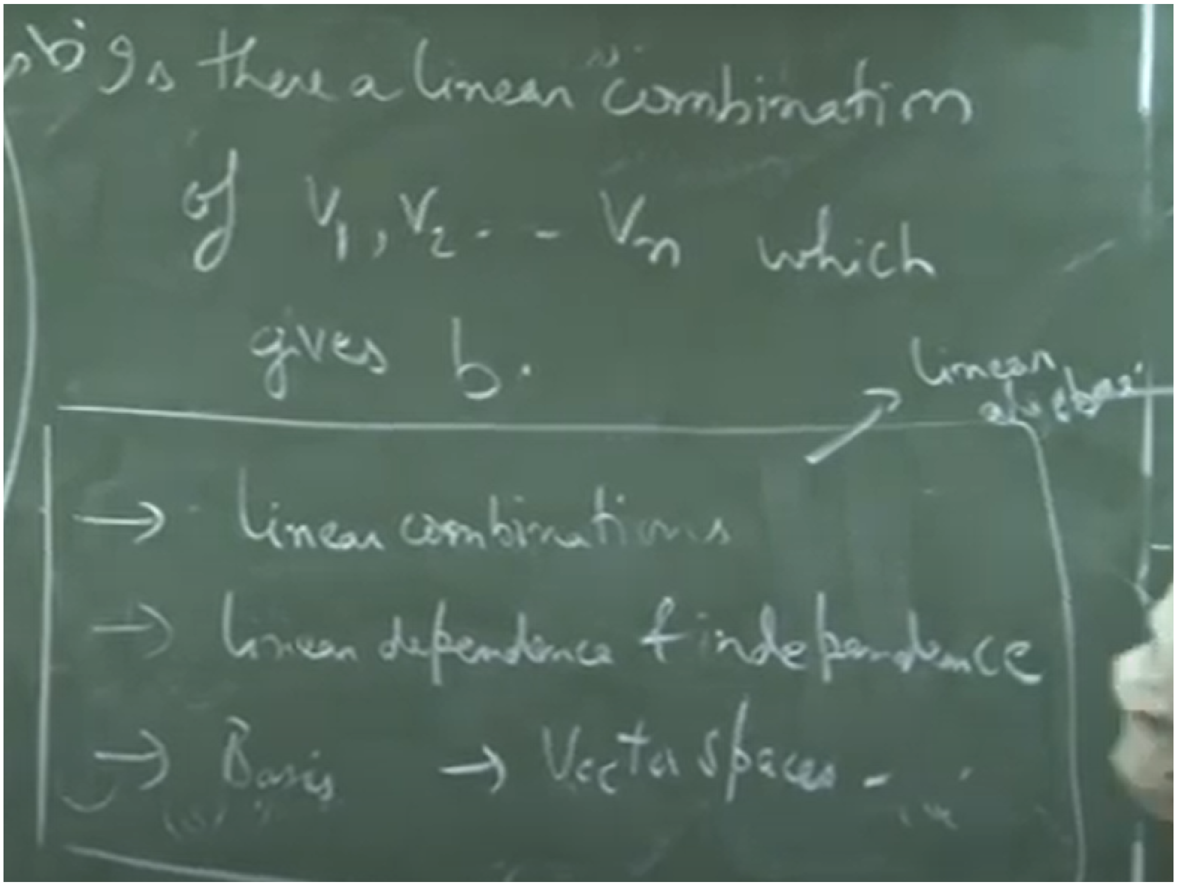
You can choose their values as you like depending on those values you will get the value of X3. So once you pick fix X4 and X5 gives value of X3 by equation 3. Then now you have X3, X4, X5 now gives using this we get the value of X2 by equation 2 similarly X1 by equation 1. So finally, you will have such kind of a system which you can solve easily and read off the solution. In this case you have infinite solutions.

So, this was a primer of solving set of linear equations by Gaussian elimination. So I am going to describe it just now the what Tufan asked was and it is a very pertinent question how to understand the structure of this free variables the bounded variables what does it mean how can figure out how many free variables do we have all that. So what we have seen at least till now is that given a set of linear equations we can we have this process called Gaussian elimination which is these steps after which we can kind of see the solution. And when we see the solution it can fall in categories we can have nice unique solution we can have no solution we can have infinite solution. It is good that we have an algorithm to solve it, but I would say we still do not understand it very nicely as Tufan pointed out how to exactly say how many free variables are going to remain what are the free variables what we can choose what will be the variables which we can set the value to all these are questions which need to be answered and that will be done by making a theory of linear algebra.

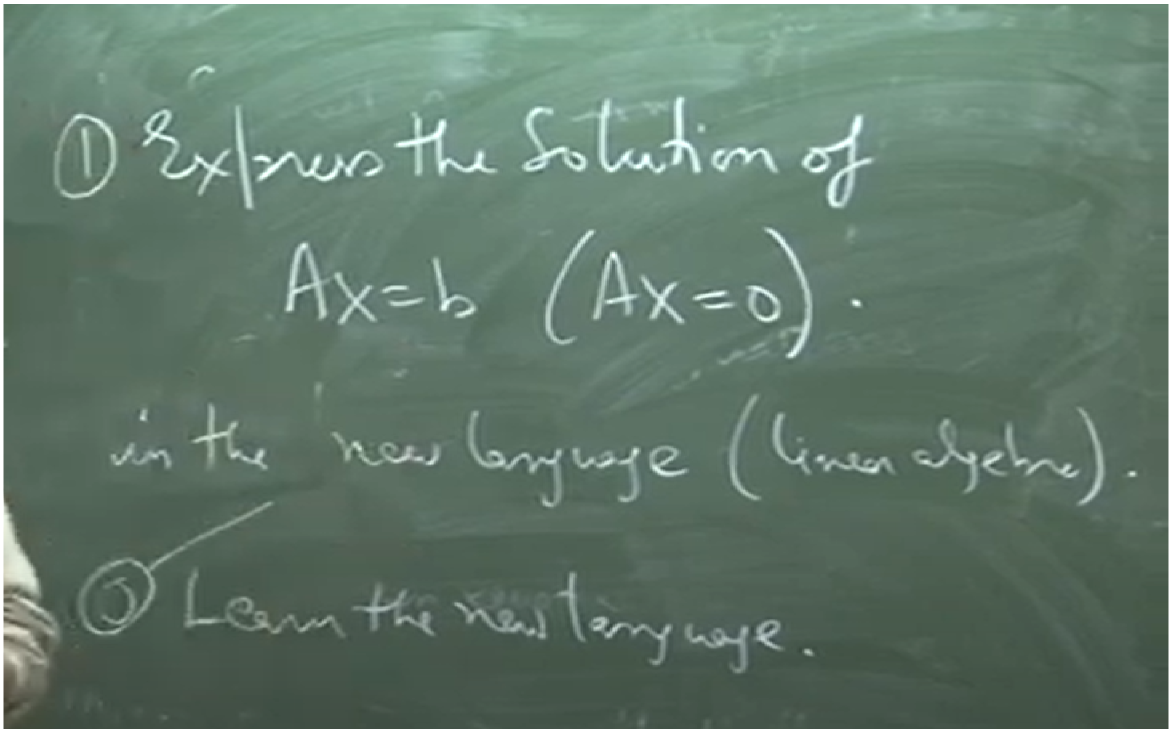
So we are going to see how to look at this equation in a slightly different way till this point we have seen this bunch of equations. So let me just write them as it is remember this was the set of linear equations and all the discussion we had done we had done by saying these are M set of linear equations We viewed them as M rows and did all the operations between these rows in mathematics in life in general it is always fruitful to see things in a different way. So now looking at this equation I can I need not look, at its row wise I can also look, at it column wise. So that means, I can say that x1 times. So, this equation now I will view it as column wise equation and you know these objects these are called vectors and x1 times this plus x2 times this is linear combination of these vectors.



What is the difference here to solve using Gaussian equations we Gaussian elimination we thought of them as set of linear equations as M rows we did operations between them. Now we are going to flip things over and say that now all these vectors let us say V1 V2 Vn then the question is is there a linear combination of V1 V2 Vn. So, call this vector V. So, is there a linear combination of these vectors which give me the vector on the right hand side this is other way to ask the same question. So, this takes us to linear combinations linear dependence and independence basis.



So, vector spaces so on and so forth and in most of the places this is what they call linear algebra. So, once we look, at it as row equations we found an ad hoc way Gaussian elimination to find solutions, but you wanted to understand it more. So, we flip it we think of it as linear combination of vectors and then ask what happens to linear combination of vectors. Once we take we apply linear combination of vectors what kind of vectors can we get in particular can we get the vector V1 V2. This idea generates the entire theory of linear combinations vector spaces basis linear dependence and linear independence.



Our task now would be to learn these concepts and then express the solutions we got by Gaussian elimination in this theory. So, the another refresher on linear algebra will be there and the idea would be express the solution of Ax equal to b or I would say simple more simply Ax equal to 0 in the new language of linear difference. But before that we have to learn the new language writing in the opposite way, but in the next lecture we will learn this new language I am sure many of you are already familiar with it, but again we will go through it we will learn the new language and then we will see how we can express the solution of these equations we really wanted. Remember this will give us the feasible solution of a linear program we wanted to we wanted to give solution feasible set of linear program and this was an important thing. So, to understand the solutions not just say oh we can give the solution to understand the solutions we have to write it in the new language that is going to be our purpose and that will be covered in the next lecture with that we will be ready to tackle linear programs and the solutions of linear programs.