Linear Programming and its Applications to Computer Science

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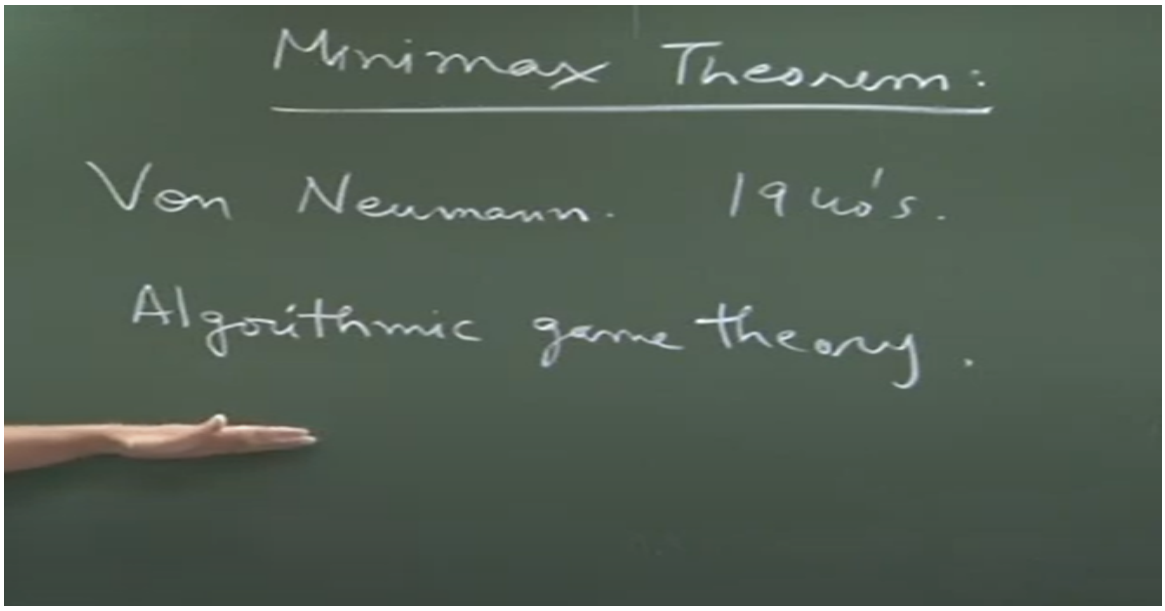
Lecture – 31
Introduction to Algorithmic Game Theory

Welcome to another class of linear programming. We have been talking about duality and as we know in case of linear programming strong duality exist. What it means is except these small boundary cases when one could be feasible, unfeasible one could be unbounded or something. If both have a feasible solution then their values agree. Not just that we have this very nice connection between the constraints here and variables there and the variables here and the constraints here right. And we have seen how to take dual using primal and you know all these things.

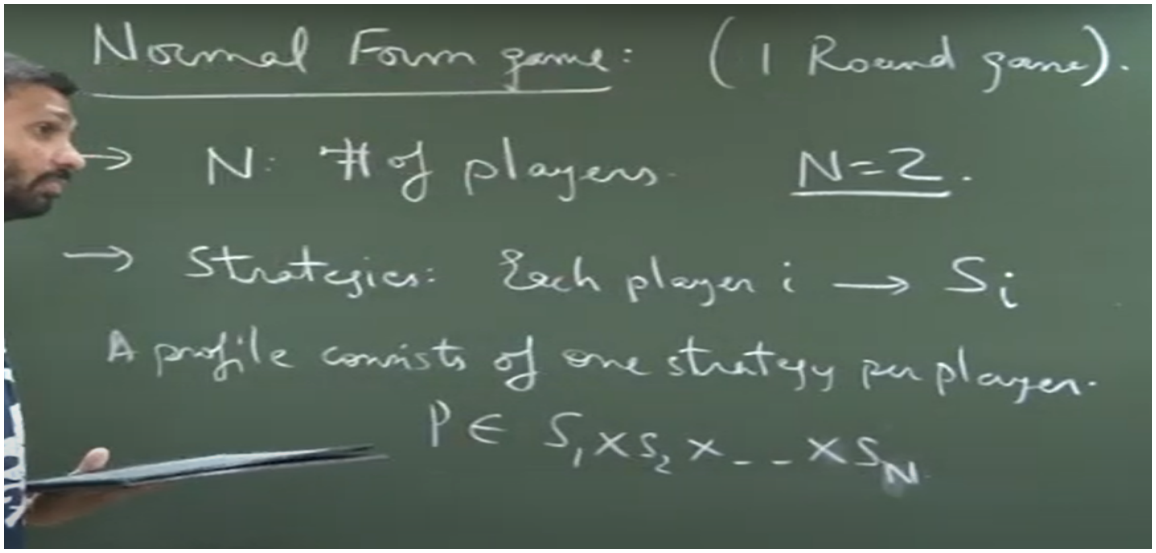


Now, the focus would be to give you nice application simple obvious kind of straight forward applications of strong duality. One of the very nice application is Minimax theorem. This was given by Von Neumann and if you do not know him then time to increase your general knowledge go home search in Wikipedia, he gave this Minimax theorem which we are going to talk about. And it is a straight forward application of strong duality and it is generally considered in the field of algorithmic game theory.

We will see its application even in communication complexity that for later first we would like to learn what Minimax theorem is. And it is presented in the setting of the form of algorithmic game theory. So, you will get some of the basics right for AGT. So, today's idea is get some very basic information about AGT then see what Minimax

theorem is and if time permits you will see a proof of it. Sounds good? How many people have taken a course in algorithmic game theory? Do not worry we need very very basic stuff.

So, what are games? So, what we will study we will call them normal form game we will abstract out most of the things. So, one thing is going to be a one round game. So, many games which you think of as multiple round can also be framed as this one round game. Idea is N is the number of players they are playing against each other right. They are everyone is trying to Maximize their payoff I will talk about payoff.



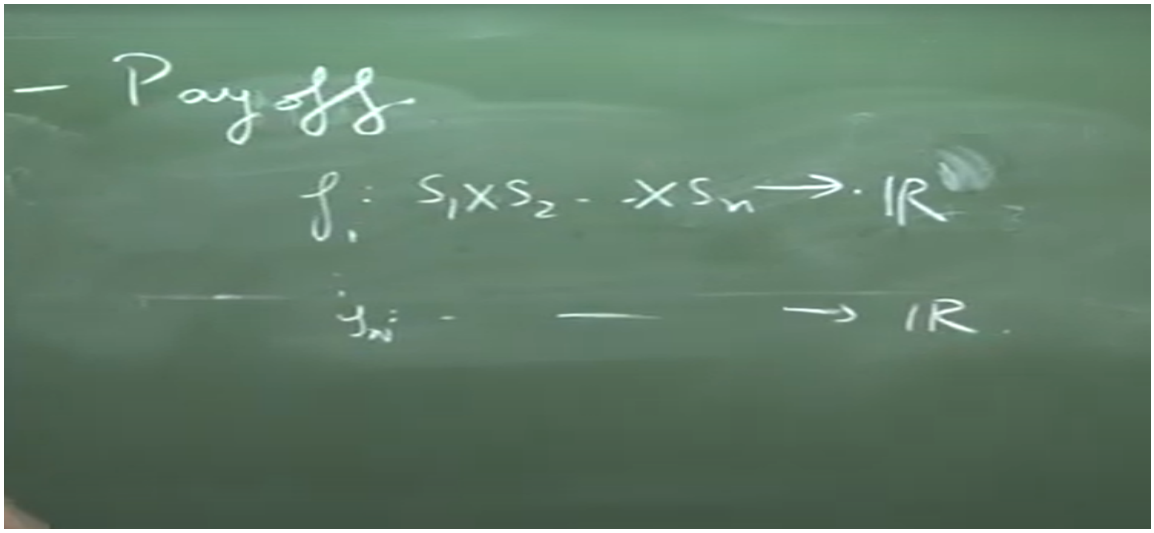
But they are playing against each other number of players for today you can safely assume N equal to 2 or for this course you can safely assume N equal to 2. Do not worry about rules that was just intuition I will describe the game mathematically what a game means I am going to describe now. And then we can take back your question if things are not clear. Then we have strategies each player I will have some set of strategies Si sounds good ok. A profile consists of 1 strategy per player.

So, think of a strategy as players move it is a one round game each player has to make a move the possible options of the move are written here. The exact description of the move is not relevant to us there this is the set of moves any play is a profile. That means, each player has made a move right. So, this profile P is an element of no the set is not the same for each player right. So, we are looking at right this is a particular instance of a game player I has played whatever is P in S1, S2 so on so forth.

And. . Strategy is not a function strategy is just basically strategy is an elements in this Si again think of strategy as a move. Again you are thinking of English idea of game forget about what you know of game in your mind as an English look at this definition. I will give you an example and then probably you would want to call it as a move or you

want to call it as a chance or whatever, but that we can debate upon. But think of strategy as a new Mathematical word for you which is which I am thinking of intuitively I am thinking of it as this is the act done by the player I in the game.

It is a one round game there is no adaptiveness there is no dependence on others. So, this is the action taken by player I in that game that is. On all players.



In when n equal to 2 it is describing on both sides exactly thank you. And then most important thing payoff this is basically a function from. So, you take any profile it gives you a function n numbers which are the payoffs given to each of the party. And each player wants to optimize his or her payoff sounds good. So, simply another way to think of it is every player S I think of it as a deck of cards red blue green player I can play one card from his or her deck depending on what cards are played I have the payoff function.

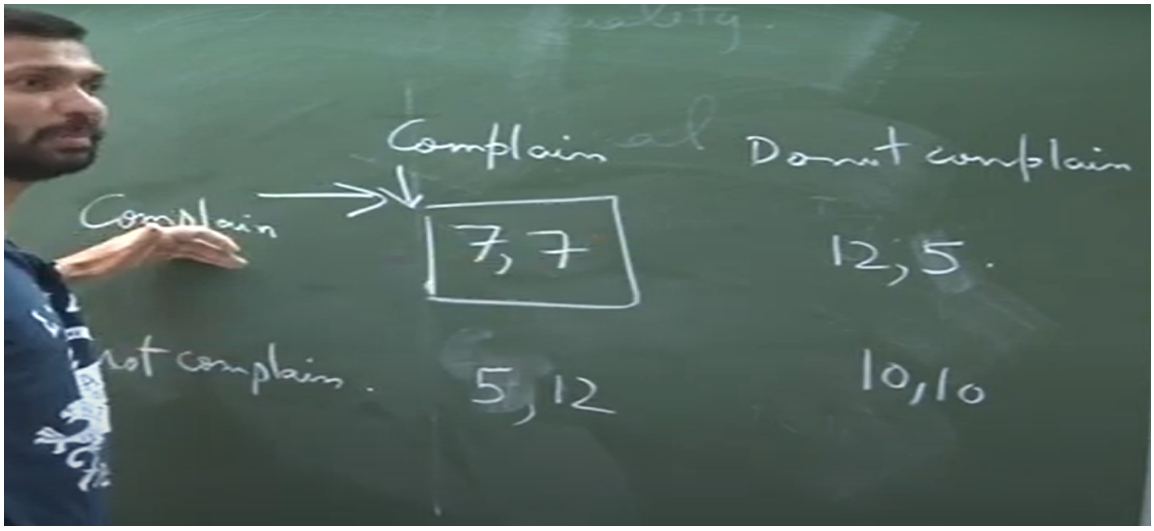
And each player is trying to optimize their payoff right this is the game what we want to do with the game I have not told you we will come to that ok, but this is a game. And again nothing explains better than an example. So, let us take an example. S I is the set yes.

Sorry. P is an n-tuple. Oh the their individual see payoff is a number here right. So, depending on this strategy I will get a n-tuple first one is the payoff or first player. So, you can think of it if you wish you can think of it as n functions.

And then first player is maximizing this second player is maximizing that is also fine ok. So, I had just combine everything and it has. So, yeah as we say nothing explains better than example right and now you will know why I made a team of 2 players in the project ok. So, whenever there are project partners at the end of the project it is a problem for me and how do I solve it I solve it in the mathematical way right.

So, there are 20 marks let us say assigned for the project and if nobody complains both get 10 10 each it is the maximum. But suppose someone comes and says sir what am I going to do I am going to say ok complaining is cost 3 marks anytime you complain to me and this is going to cost you 3 marks. So, if both partners complain both are null I like 7 each. But in case 1 percent complains and the other person does not complain then I assume that the complain is true. So, then 50 percent of the marks of the other player will be transferred to the first player.

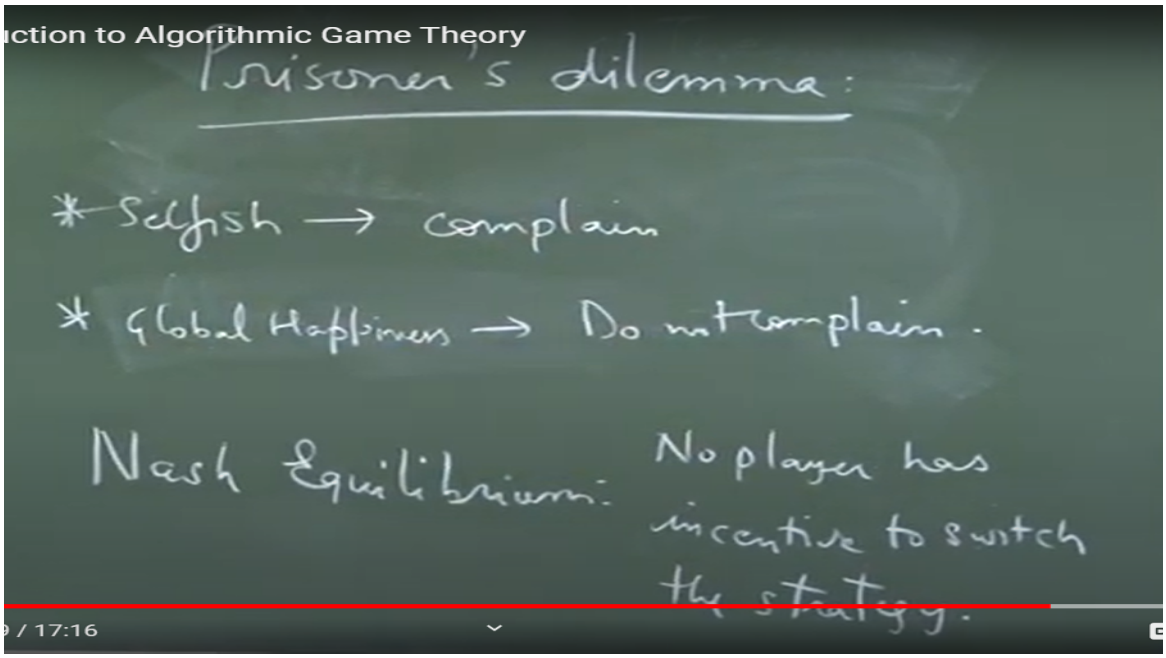
So, that means, this person did not complain this is the payoff should we keep this for our course? 3 marks for charge and then 10 marks for their total oh sorry there they get 5 yeah. So, 50 percent of the credit thank you completely correct. So, they did not complain to 7 has nothing to do with them they get transferred fine right. So, first question do you want to play this game for this course? And again if your computer science g k is very good this is called Prisoner's dilemma ok. But before I describe the dilemma does it make sense how this is a game and why this is the way you can write a game right.



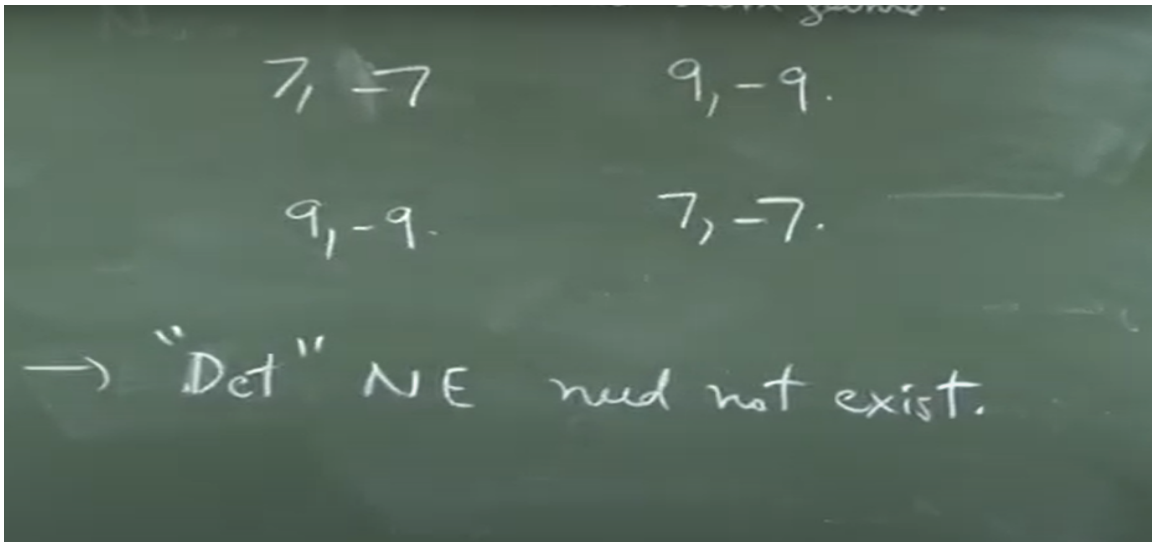
So, number of players possible strategies since they we have 2 players it is always a matrix. And then there will be 1 matrix for each payoff I am just writing it together here, but you can think of it as 2 matrices of 2 cross 2 sounds good. So, this is the way I can describe this game now it might seem like a 1 round game or anything, but you think about it obviously you can write rock paper scissor like this this is a good exercise how to write rock paper scissor very very easy you can write chess as this. You can consider all possible moves of 1 person all possible moves there will be entries where there will be a cross that you know not valid or something, but still you can describe it as a big matrix right. So, it might seem like a very very simple game, but yes you can describe still many things as this 2 player normal form game.

But yeah this is, but then it is not saying anything right the in some sense you do not

have any information about strategies inside them I am just taking it as a plain set probably there is a relation between them and things like that all those I am losing out. So, it is not a great description, but it captures lot of games sounds good ok. So, now what do you think player should do exactly right. So, if you look at this matrix if let us say if I am looking at the first player right first player irrespective of whether the second player complains or do not complain he is better off complaining right. So, in that case selfish reasoning suggest complain, but global happiness and actually your personal happiness is also maximized if you do not complain.



This kind of tells you the importance of trust which we lack in Indian society, but this exemplifies that fact right trust helps though since you are looking at it mathematically this is not our concern ok. So, this is called Prisoner's Dilemma because for a prisoner if both of them do not complain they get the best, but then what it will turn out is if they are just looking out for themselves they are going to complain and both get the worst possible answer ok. But getting back from prisoner's dilemma the strategy of both complaining is actually what we call Nash equilibrium. What do I mean by that no player has incentive to switch the strategy ok. What does it mean if I am here and I keep fixing my strategy other person cannot get a better payoff I cannot lose I other person does whatever this is this is going to be my best answer similarly for the other side.



So, Nash equilibrium is every player no player wants to switch because if they do not switch other player does whatever they will stay at the optimum ok. So, this is called a Nash equilibrium and here this is the Nash equilibrium. Does Nash equilibrium always exist what do you think yes no yeah. So, you can actually construct easy games where you do not have Nash equilibrium sorry I should say. So, this could be a payoff this is in some sense you are saying that you know there is in this case person will give you 7 rupees to the other player in this case 9 rupees to the other player you can think of it as these are called 0 sum games.

Because one player's loss is other player's gain or something. But so in this case it is very easy, but even in this case you can easily find this payoff matrices where none of the strategy is a Nash equilibrium because at least for one player it makes sense to switch ok. So, I will just change this and I will call it a deterministic Nash equilibrium and I will you will get the reason of why I am doing this and it is now it is clear that deterministic any need not exist.