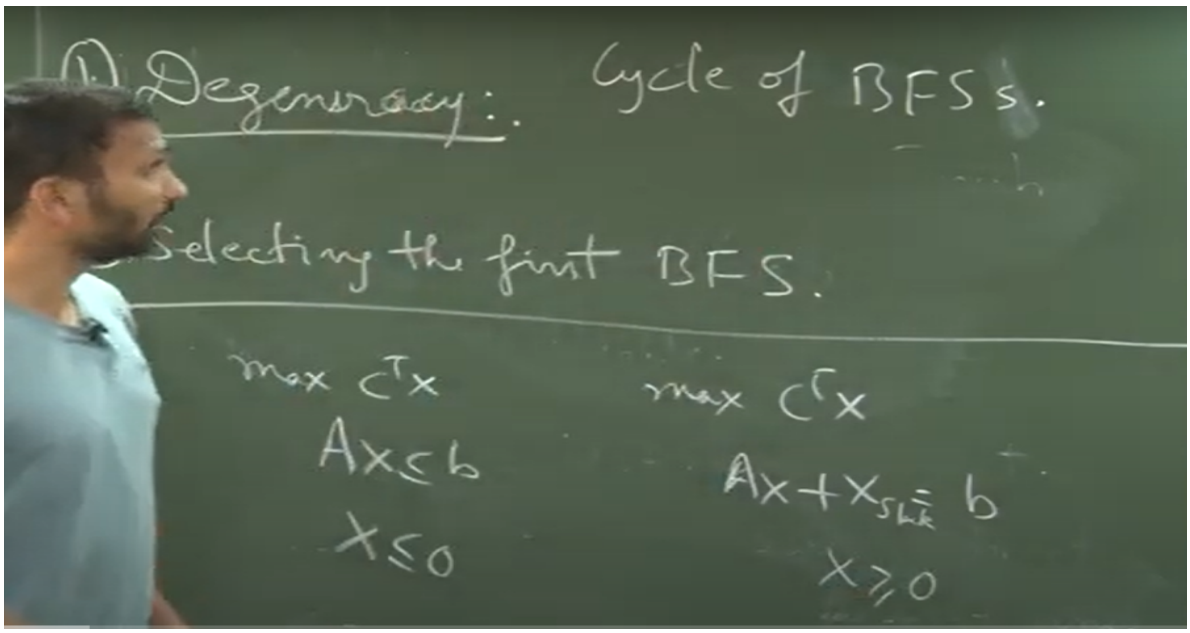
Linear Programming and its Applications to Computer Science
Prof. Rajat Mittal
Department of Computer Science and Engineering
Indian Institute Of Technology, Kanpur

Lecture – 21
Starting BFS

Let us look at selecting the first BFS change things. This I am doing it first because, it is kind of a neat idea. I really like again the way this solve this problem. So You can always assume that your program is in this form. Actually, do you remember how did you find the BFS in the first BFS in the simple example we did last time? You were lucky.

So, can assume this is the standard form.



And now, we may put slack variables so that these become typed. So Then max and now if B was entry wise positive, done. That is my basic feasible solution. I will just take all the slack variables number of slack variables equal to n happy. Problem is what if some of them are negative. Ok.

So, then what I need to do now is this trick of surplus variables. What this does is A X minus X S this is the surplus variables. So, assume the other slack variables they took care of the remaining ones. Now, surplus variables X greater than equal to 0. Right. Then ultimately what do I want? Ultimately I want these surpluses to become 0.

Then whatever were my and this is kind of a nice change. So, what do I now need to see is if my initial thing was feasible right. Then there is a solution here where all X S are 0. If I get a solution here that can make me go here. Now, but the good thing about this is that I have a BFS.

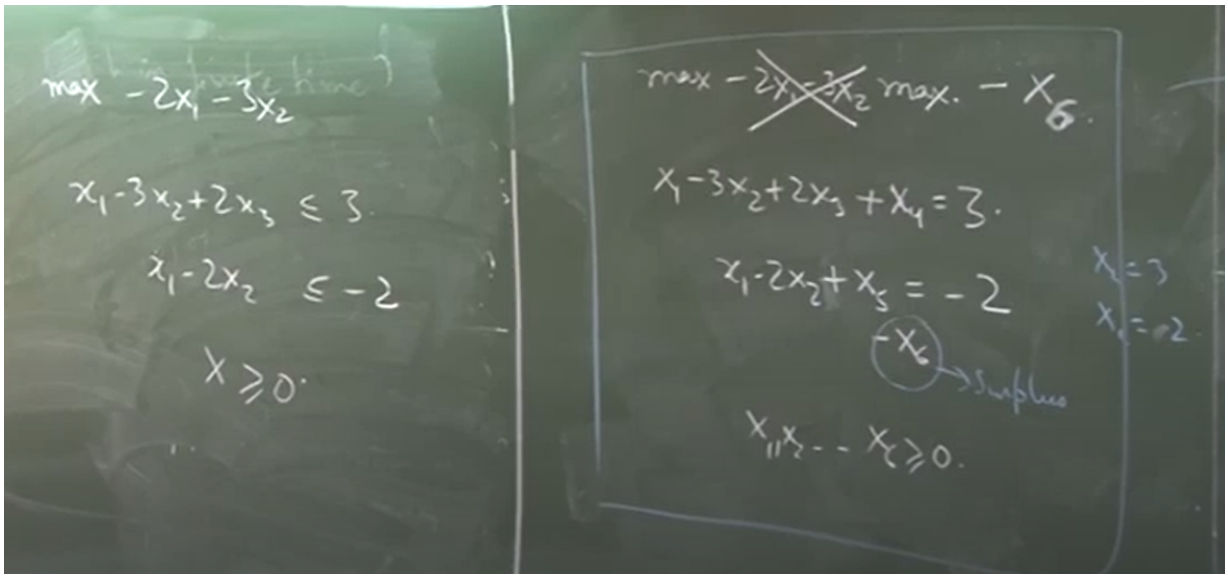
So, I will take an example obviously a small example because this is going to take some time. But let me take an example and then you can tell me if things are clear to you. Suppose, this is the LP, I wanted to solve. Then this is good this is my slack variable this is nice. Here I am introducing this extra variable which allows to absorb you know for whatever solution if it was greater than minus 2 still I can come up with a solution by keeping X 5.

So, if X 1 minus 2 X 2 was 0 I can still make that 2 and have that out here right. So, it is not clear that a solution here is the solution of my original thing right. This is not true I do not want that. What I do now is so, instead of this I want to say that I want to maximize some of that or minimize yeah you want to say did I have more variables. I made a mistake sorry yeah first yeah minus 2.

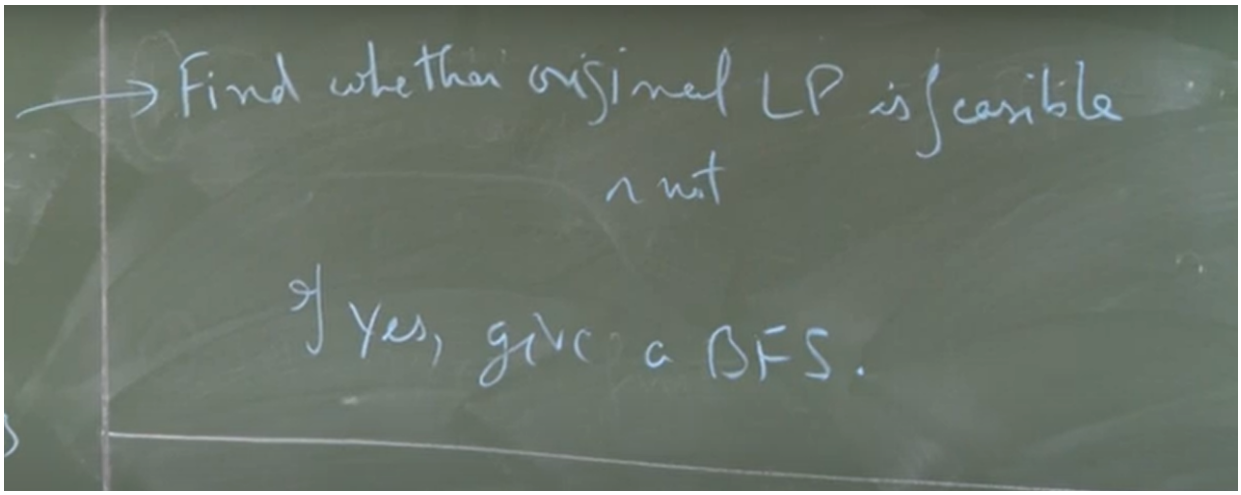
This is all the slack variables and then I add minus X 6. So, and then I have to have all these things right X 1 and X 2 right. So, X 6 is allowing me to make up a solution, but the problem here is that if X 6 is positive or negative then I am in deep trouble right. So, ultimately finally, I want to make X 6 0 right. So, when X 6 is 0 then a solution here is a good solution here correct.

So, since I want X 6 to be 0 I am maximizing minus X 6 remember X 6 is greater than equal to 0. Since it is greater than equal to 0 this maximum is at most 0. So, this is the time this is how I change it. So, what you want to convince yourself now is that if there is a feasible solution here I can take that feasible solution I can add X 6 equal to 0. It will remain a feasible solution it is 0.

And there can be multiple surplus variables here just a second there can be multiple surplus variables 3 4 equations S X X plus X 7 plus X 8. But if that maximization turns out to be 0 that means, all those surplus variables are 0. So, if this is feasible this has a solution at 0 and if this has a solution at 0 I can actually get my BFS here. Because what I will be doing was I will be going through BFS is here correct I will go through BFS is here finally, at the optimal one I will know that all my surplus variables are 0. The remaining thing will become my basic feasible solution that is the nice idea surplus variable this is in which wherever my B was negative.



So, see the problem was that if I did not have X 6 I would try to have my basic feasible solution as X 4 X 5. But then X 5 is equal to minus 2 that is not a feasible solution. So, what I do is I put a minus X 6 surplus variable now I have a basic solution what is my basic solution sorry. So, this trick has allowed me to find a basic feasible solution to start with. But then I also wanted to make sure that this becomes 0.

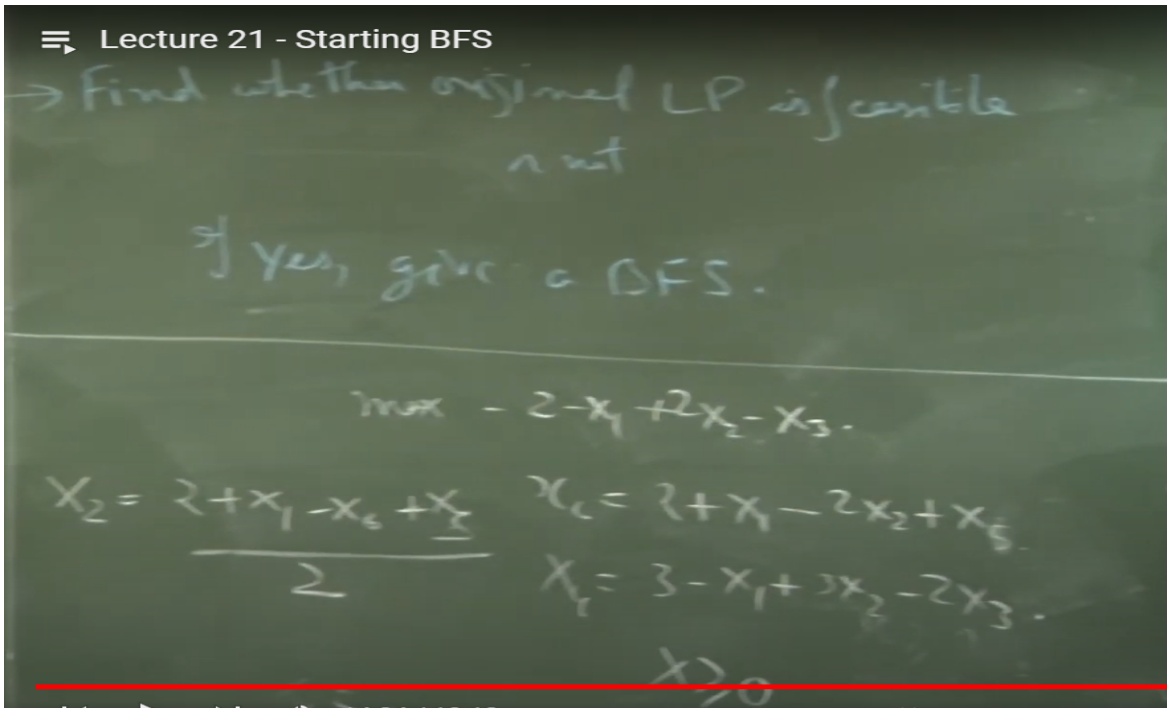


So, I change the objective function. So, this is probably the thing yes. I am not worried about any optimal of this linear program the entire objective of this LP is to find whether original LP is feasible or not. If yes give a BFS this is the reason why I have invented this LP. And since this is the reason objective function is redundant if I want to just do this I can through this in the sink and forget all about it that is what I have done.

Questions so I think this is a nice trick right just by applying this surplus variable and this smartly changing the maximization you have changed the problem. So, you can call

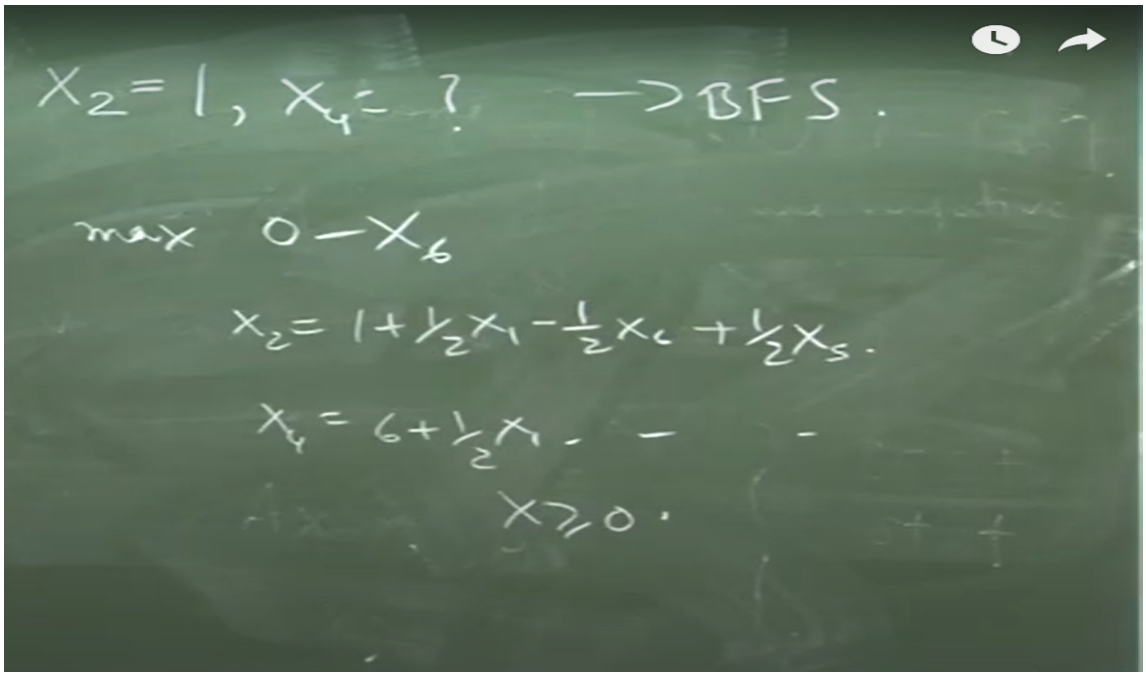
them surplus auxiliary whatever is I do not I think probably I call it an auxiliary in the notes right. So yeah I will save some time now these are my basic variables . So, that means I want to write X 4 equal to in terms of non basic X 6 equal to in terms of non basic and my this optimization function. Now I have forgotten all about this I am just worried about this right.

So, then sorry. Sorry. And now for at least for this LP you look at this obviously I wanted to make a 0. So, that means X 2 is my entering variable which is my leaving variable sorry X 6 because this is positive. So, there is no problem here this is the one. So, that means I can change it to now X 2 will become 1 I will figure that out.



So, my this is my new BSF and if you change according to this. So, what will I do I will substitute. So, X 2 in terms of X 6 X 2 in terms of X 6 is 2 plus X 1 minus X 6 plus X 5 divided by 2. And at least you can see that when I put this here this minus 2 gets cancelled.

So, I get sorry. 0 right you are saying that this objective function has to match that objective function then it has to be X 6. Actually why am I writing this here right. So, now I have a basic feasible solution to start for this sorry. Now I get back to this LP X 2 equal to 1 and X 4 equal to 6. Now I can represent I can write X 4 in terms of non basic and then change and you can write everything in terms of X 2 and X 4.



And that is possible because you know that the columns of X 2 and columns X 4 are going to be invertible. So, there is a small thing here. So what do I want to show the final BFS I am going to see is going to give me a BFS of the original program. Why that is what is the proof? You saw the problem what I want to now show is the BFS I will get a BFS here what I know is that my optimal value is 0. I have reached that value now what saying is that BFS should give me a BFS to start with in my original LP right.

But what is my BFS? There is a BFS here there are some variables how do you transfer them to some variables there it is exactly the same. What if there are some surplus variables here in the exactly. So now if you look at the basic feasible set here it will have some non surplus variables non auxiliary variables some normal variables just keep those normal variables you know rest of them are 0. Whatever are remaining the columns are linearly independent because the entire thing was linearly independent this is linearly independent. Now if they are not if they are not m you can add some extra columns to make them all this trick I have done the second time right.

You have some linearly independent columns, but they are not making up the rank of m. How do you convert them into a basic feasible solution? There has to be some few columns here which will make the rank m right that is by the property linear algebra right. So, just pick whatever columns you need to make these linearly independent and the. So, it is for your basic variables to be 0 non basic cannot be non 0 and this is the reason of degeneracy right this is when degeneracy happens. And your basic variables are 0 and they do circulator clear.

So, this is how we take care of starting with the basic feasible solution. This trick of having a new objective function solving an entirely new LP.