**Linear Programming and its Applications to Computer Science**
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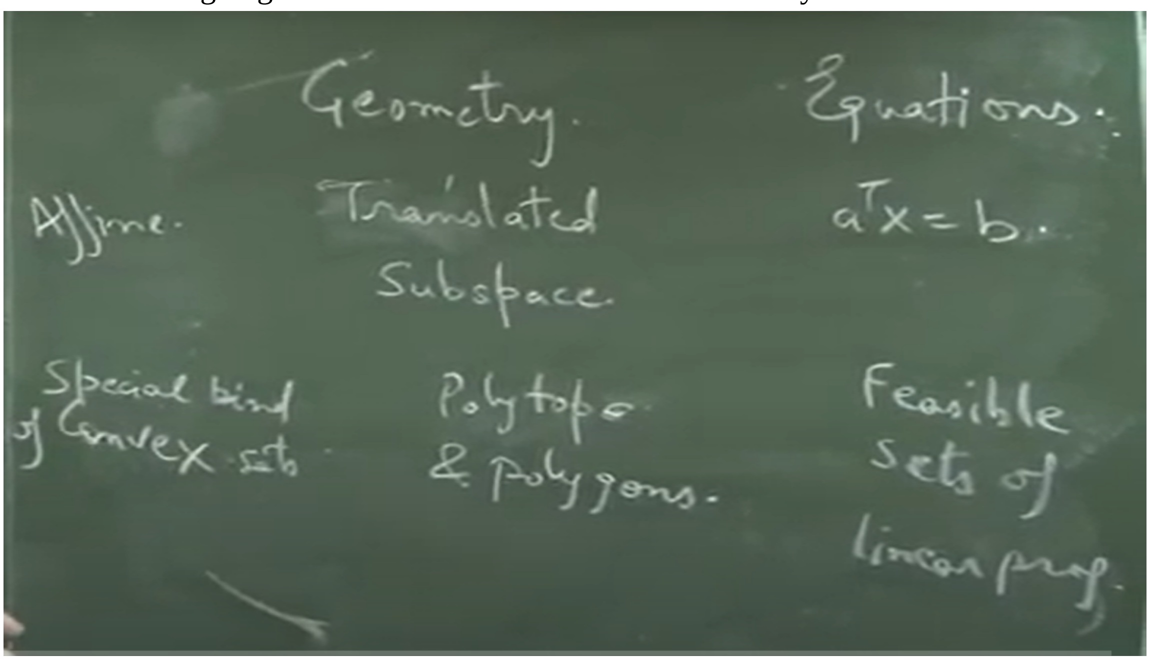
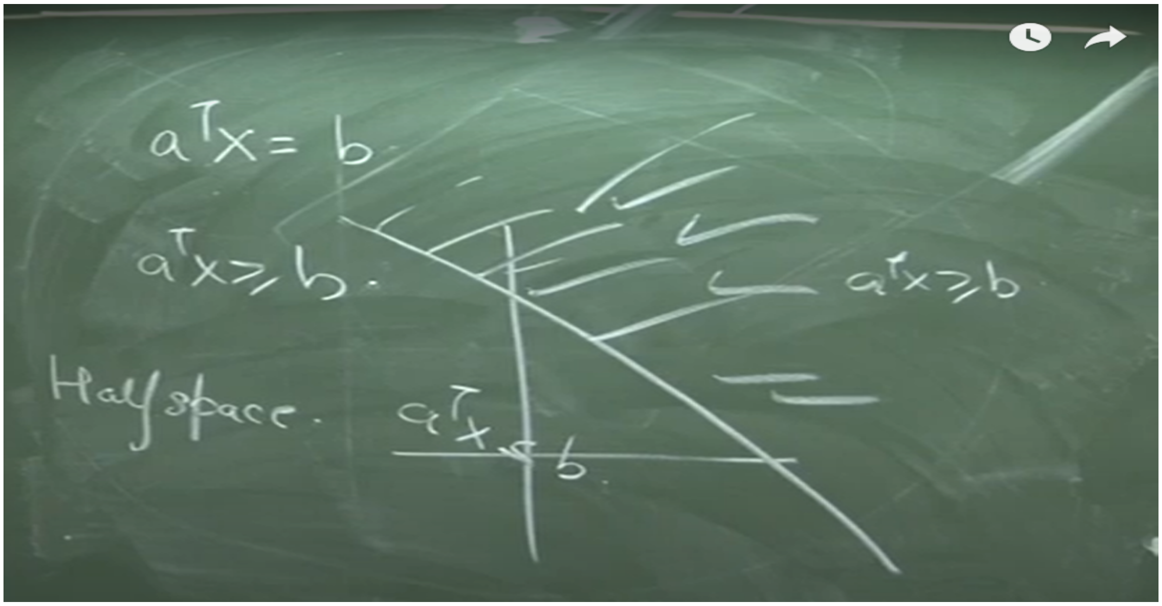
**Lecture – 13**
**Feasible Region of LP**

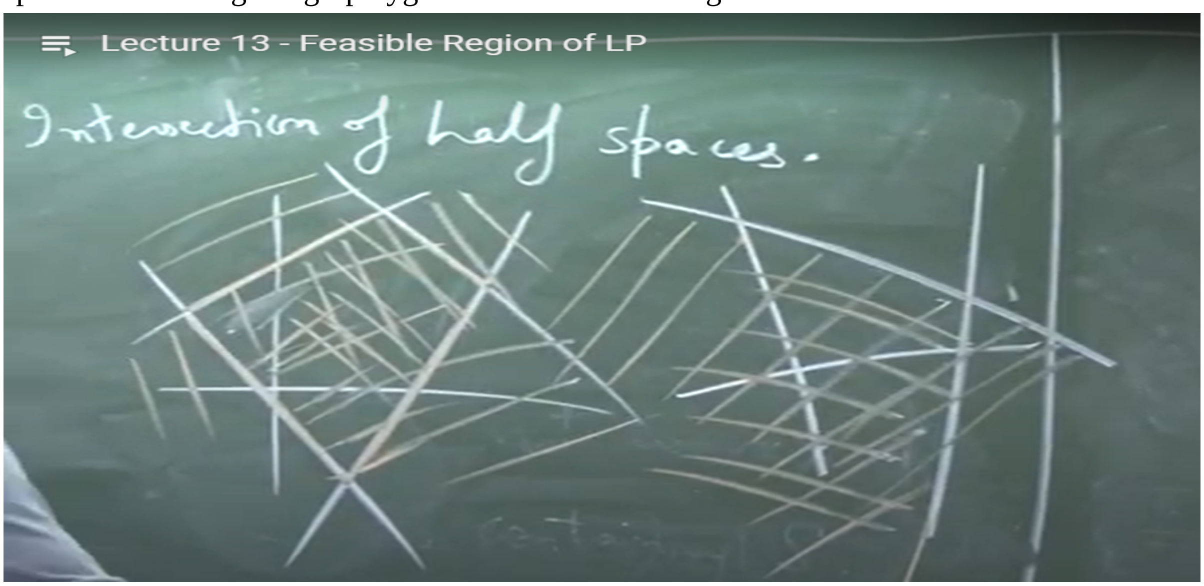
This was correct convex sets again this turned out to be different right. So, we will only describe a subset of these ok. So, special what is happen to my spelling skills polytope and polygons right. And this is going to be your not just for the standard words right. This is going to be intersection of your constraints ok.




And let us look at this. So, let us make this picture complete. This was a affine set or a hyper plane geometrically it was a hyper plane. It looks like a plane moved from the origin to some other place fine.

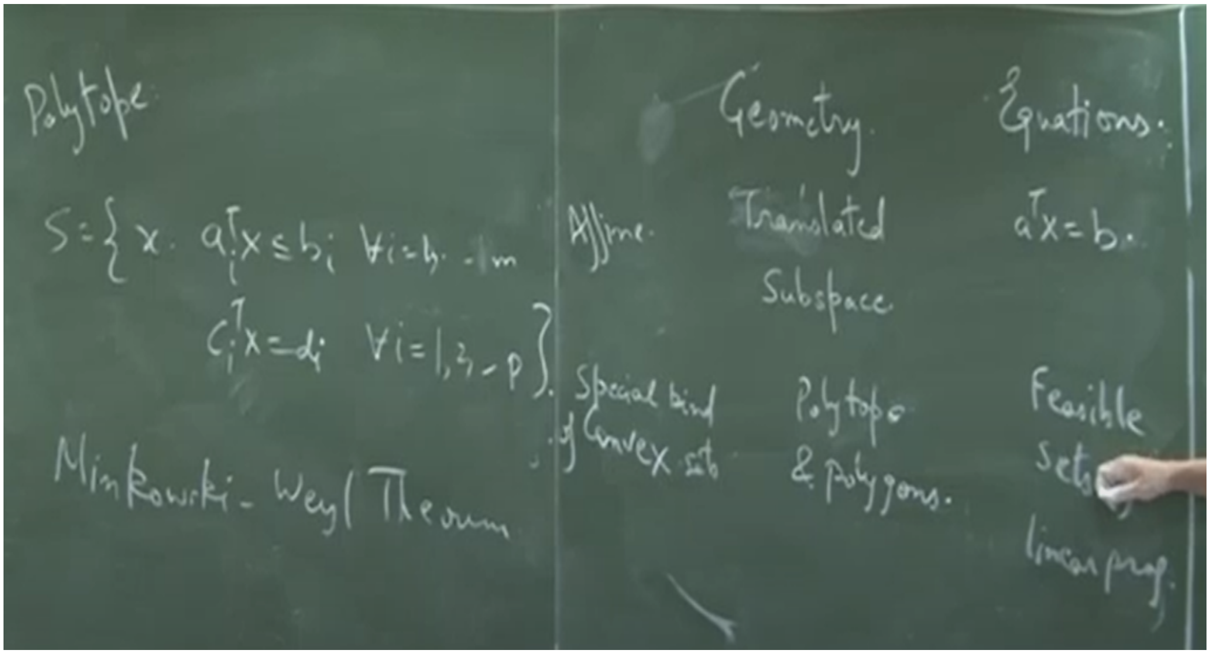



What is this? This is not a plane this is a half plane we call it half space. And yes you take a line this is the line describing A transpose X equal to B either this side or that side depends on what A and B are. One part of this is going to be A transpose X greater than equal to B other is less than equal to B. Cool? Ok and now if I take intersection of half spaces I start getting polygons. How do I get it? I need more colors.




This is an inequality this is an equality this is an equality this is an equality right. If I take the intersection I get this polygon. But now you also see that it need not be bounded right. For example, if I take intersection of this is a but it does not bounded. But whatever boundaries are there they are lines.

This is a polygon this is a polytope this is also a polytope this is also a polygon right clear. So, this is what we achieve with this intersection of A transpose X intersection of A transpose X less than equal to B. Now, I can give you a mathematical definition of polytope which is a set S which is described by some inequalities. So, any set which is kind of intersection of our constraints is going to be a polytope. And to prove that this is same as the geometric picture is called Minkowski Weyl theorem and we are not going to worry about it.




We just thank we just thank Minkowski and Weyl that they proved it is going to be very useful but kind of not that easy to prove. So, you will skip it, but what we will remember is this dual picture. This is saying that these two definitions are the same. So, that I have to be much more careful this is going to be a polytope this is how I did the polytope. This is going to be a convex hull of finite number of points with some rays like you have this infinite directions right.

So, then you have to precisely define this, but I want to just do it geometrically. But there is a kind of a cleaner way that you have finite points and then you have convex hull of that and you are allowed to put extend them in lines. Ok, And that is going to be a polytope or polygonal and that is same as these equation description. In two dimension hard to visualize, but if you take it take them in higher dimension you will see that they will give you some kind of. So, it is easy to see that these intersections will be convex right.

And then their boundaries would be linear. So, and yeah, but I cannot draw four dimension and I cannot imagine also visualize also. So, when even when I say that these intersections are going to be convex set polygonal polytope, I believe Minkowski and

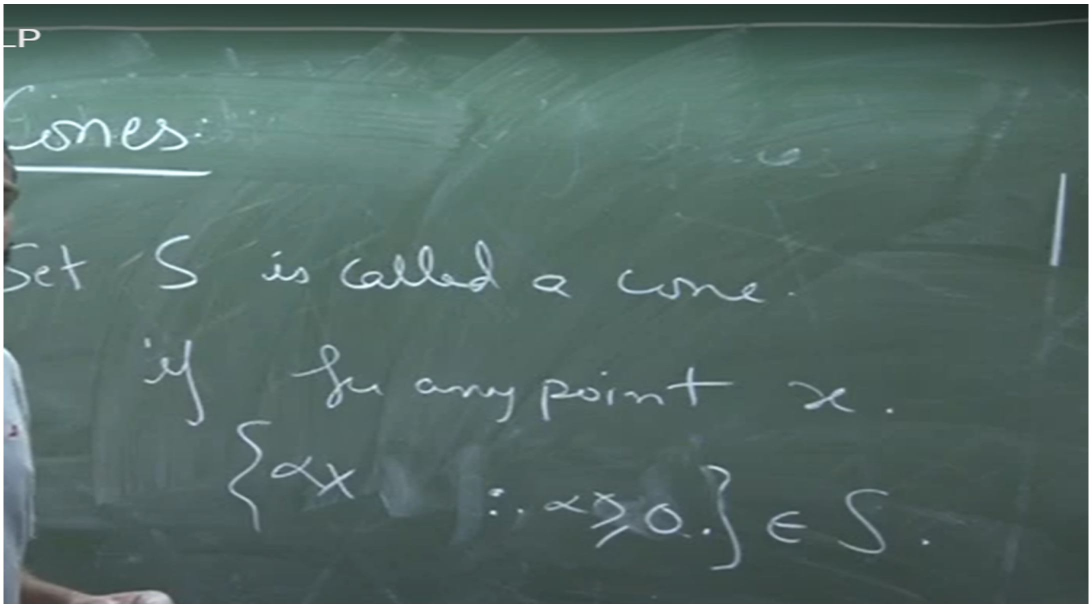
Weyl right. So, they look something like that, but you know yeah I understand it is weird because this looks like a linear subspace, but then we are taking intersection. So, that that linear subspace will be cut into parts, ok. But the whole thing was done.

So, that we understand convex sets are feasible region nicely. This is feasible region. So, our feasible region is going to be polytope or polygonal. No, no, no this theorem is telling us that our feasible region will look something like this. But where did you prove that this set looks like or or it let us say if it is bounded it is a convex hull of finite number of points.

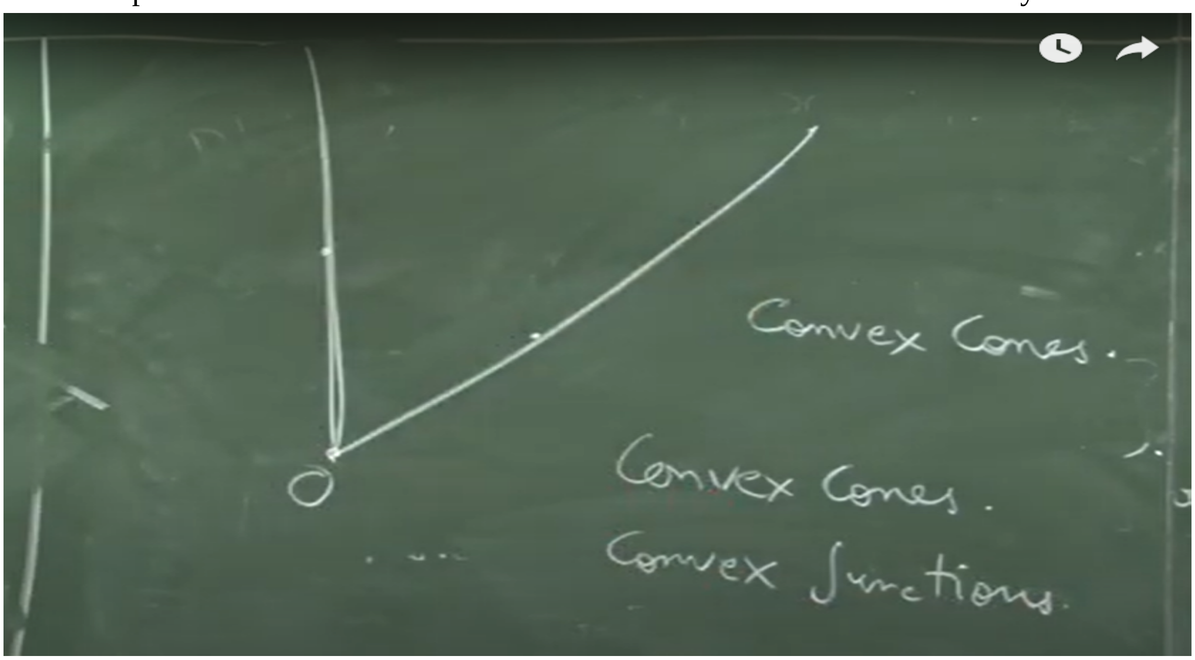
What are those points? Mathematically two ways this is one and other as I was pointing out to him you say it is a you take few points and few directions. Ok. So, it is convex combination of points and then you can extend them in any direction, but that is not going to be useful that is why I am not doing it. You can read about that on internet, but it is. So, if it is finite if it is bounded then it is clearly convex combination of finite sets, but now we want to allow infinite rays also. So, it is convex combination of points and then you can extend them with some rays.

So, there are vertices and then there are a ray that is how you can define them geometrically. I am just giving you a picture for that. Is that okay. So, x 1 to x n, v 1 to v k then it is convex combination of this plus I have not defined cones for you yet, but you can extend them in this direction v 1 to v k. Here there is no this the coefficients of this need not sum up to 1 just go in that direction as much as you like. Ok so for example, if you remember this it is convex combination of this extended in this direction there is one direction where it can lead to. Ok

This is the picture we will have in mind. The thing which we have not talked about is cones. Yes circle that is why I draw those weird shapes which looked like convex, but you are not sure all those shapes are sorry no this extension is also there right. So, that is why if you want to be mathematically precise then you have to appeal to this Minskowski Weyl Theorem. So, we have seen affine combinations, we have seen convex combinations and then the thing which is left is what we call conic combinations right.




And your question should be that we defined a fine we defined convex sets what about cones conic sets or something. First I define cone a set S is called a cone if for any point X this is a line from origin to alpha is contained in S. So, it basically looks like a cone it is a cone if it looks like a cone. And now you might be asking what if is it true that if I take two points their conic combination is inside the cone or not what do you think.




Half line. Yes. What do you think? Does the cone contain all possible conic combinations of points inside it? That is because you do not use pen and paper. These two lines is that a core or not? Yeah this is origin yes, but it definitely does not contain the conic combination of these two points right. So, yeah there are reasons why we study cones and then we study convex cones which are going to be you know more like containing all possible convex combinations also, but that is for the next class. So, now our plan is learn about convex cones then convex functions and once we learn that a nice algorithm for linear programming will pop up. And you might think oh we did this all this linear algebra and our efforts went to waste no.

The things which you learnt you know remember the solution space looks like this all that all that intuition will be helpful while implementing solution based on this, ok. This is a good reminder nothing goes to waste especially in Math right. So, we did linear algebra it did not give us it gave a simple solution, but now we will see with this extra conditions our solution becomes slightly difficult, but that can be obtained from what we have seen by modifying them appropriately. And that is the plan for future lectures.