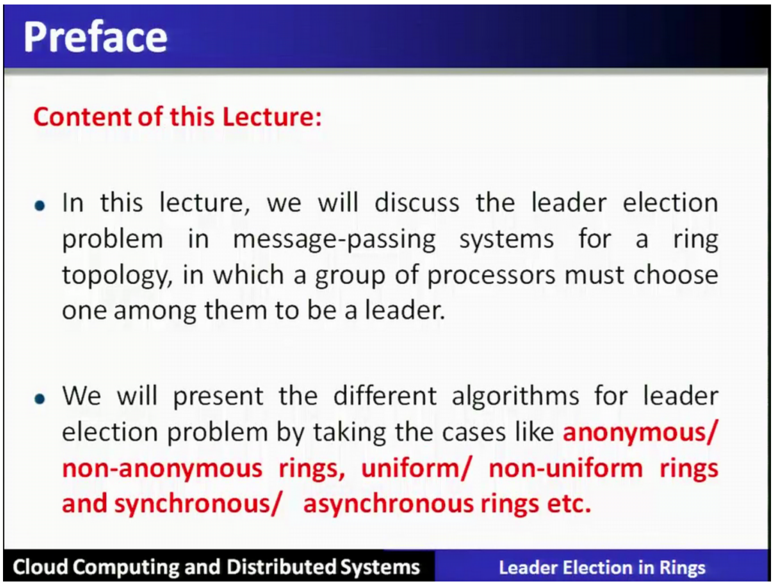
Cloud Computing and Distributed Systems
Dr. Rajiv Misra
Department of Computer Science and Engineering
Indian Institute of Technology, Patna

Lecture - 07
Leader Election in Rings (Classical Distributed Algorithms)

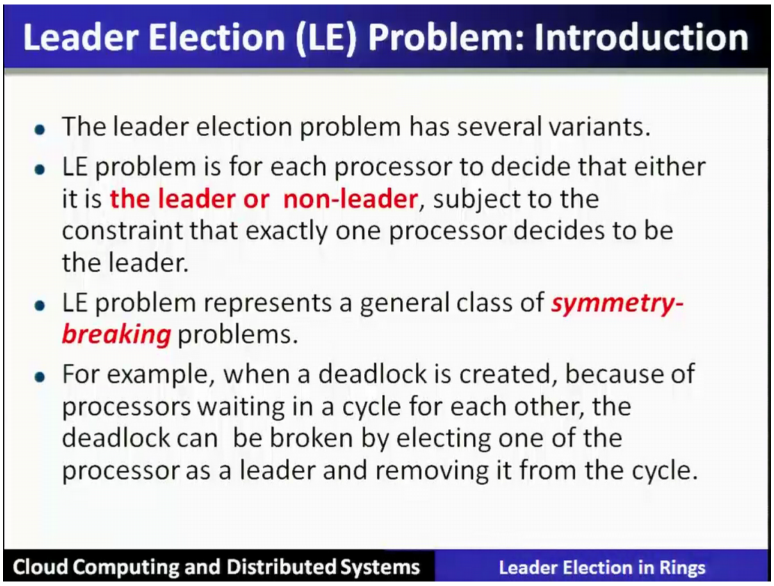
Leader Election in the Rings Classical Distributed Algorithms. In this lecture we will discuss leader election problem in a message passing systems.

(Refer Slide Time: 00:24)



Especially, for the ring topology in which the group of processors must choose one of them to be the leader. Different algorithms for leader election, different scenarios such as anonymous non-anonymous rings, uniform non-uniform rings, synchronous and asynchronous rings.

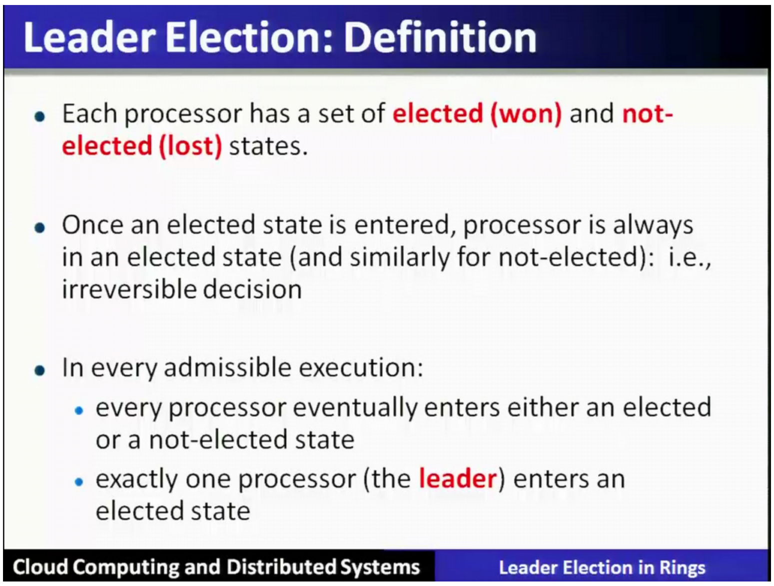
(Refer Slide Time: 00:49)



Leader election problem; the leader election problem has several variants, leader election is for each process to decide either it is the leader or a non-leader; subject to the constraint that exactly one processor decides to be a leader.

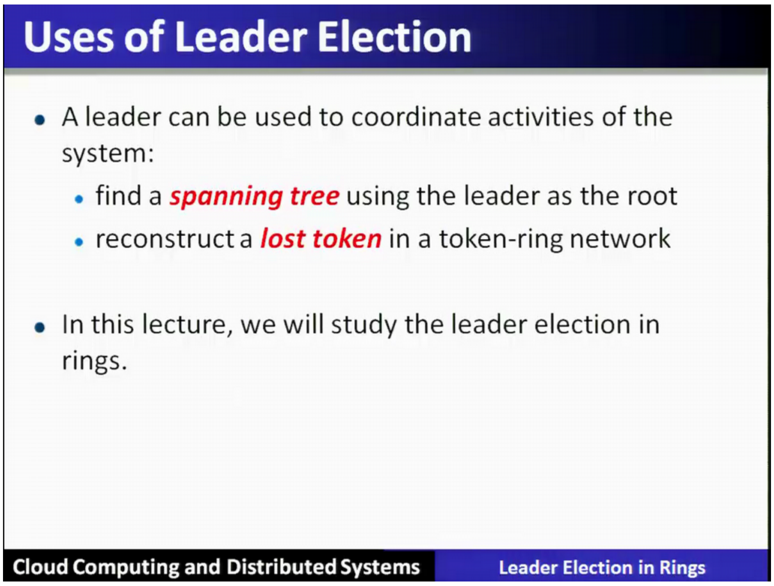
So, leader election problem represents a general class of symmetry breaking problems. For example, when a deadlock is created one of the processors waiting in a cycle for each other, the deadlock can be broken by electing one of the waiting processes as a leader and removing it from the cycle; that is breaking up the deadlock.

(Refer Slide Time: 01:23)



So, the leader election problem definitions, definition each process or each processor has a set of elected and a non-elected states. Once an elected state is entered the processor always in the elected state. And similarly for non-elected, that is irreversible decisions. So, in every admissible execution every processor eventually enters either as elected or non-elected state exactly one processors; are exactly one processor that is the leader enters into the elected state.

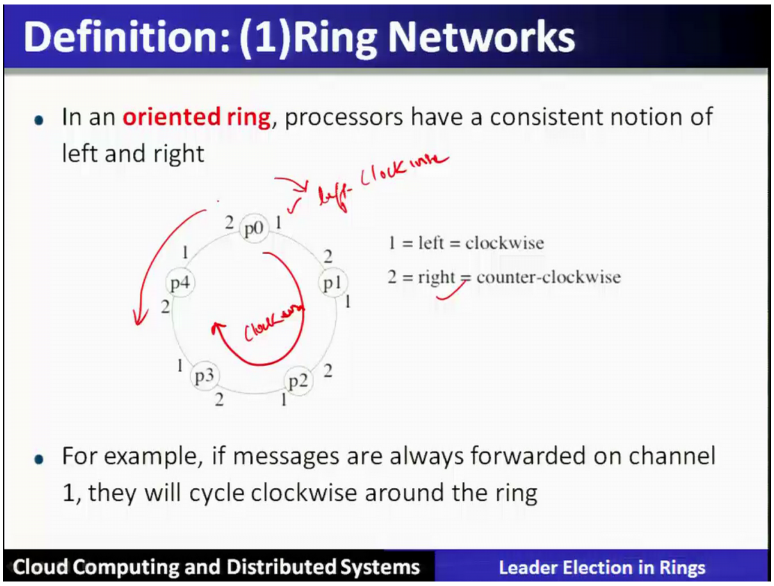
(Refer Slide Time: 01:59)



Now there are different uses of leader election algorithm. So, leader election can be used to coordinate the activities in the system. For example, to find out the spanning tree in a system requires a leader to be known which is called a root. Hence, if a root is given finding a spanning tree becomes easier.

Similarly, in a token ring system if a token is lost; that means, there is no leader so, it can elect a leader and restart the system with a token.

(Refer Slide Time: 02:42)

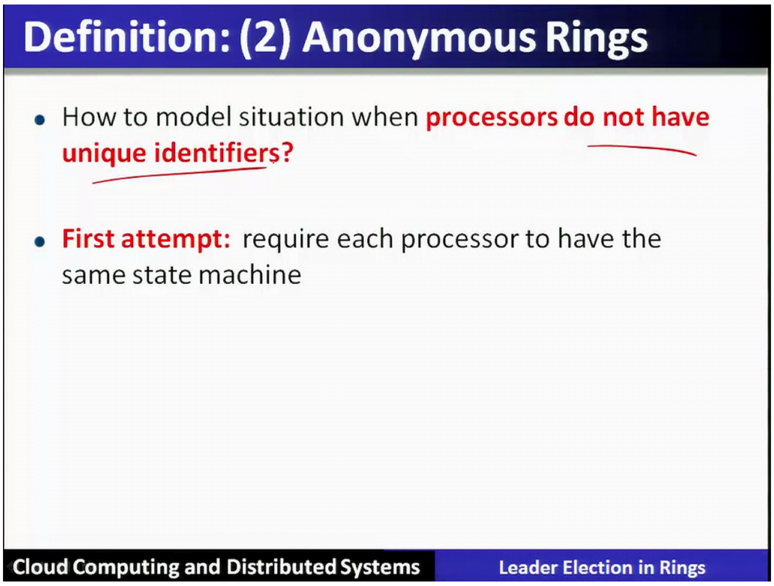


So, in this lecture we will study the leader election in a ring. So, let us define different terms which are used in the leader election problem; that is the ring problem. So, the first definition is about the ring networks.

So, in an oriented ring, the processors have a consistent notion of left and right. So, in this particular example, we can see here the label one called as a left side, if it is used to forward the message in this particular direction that is called a clockwise direction. Then this particular way the ring will be oriented in a clockwise manner. Similarly, if let us say that if p 0 and other processes they basically use the right side, that is the label number 2, always then it will be a counter clockwise and the ring will be oriented in that manner such rings are called oriented rings.

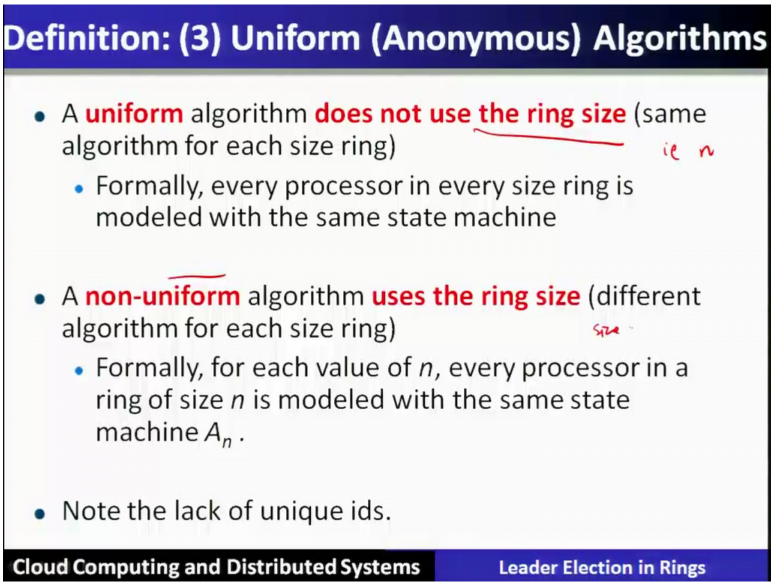
Now another definition is about anonymous rings.

(Refer Slide Time: 04:02)



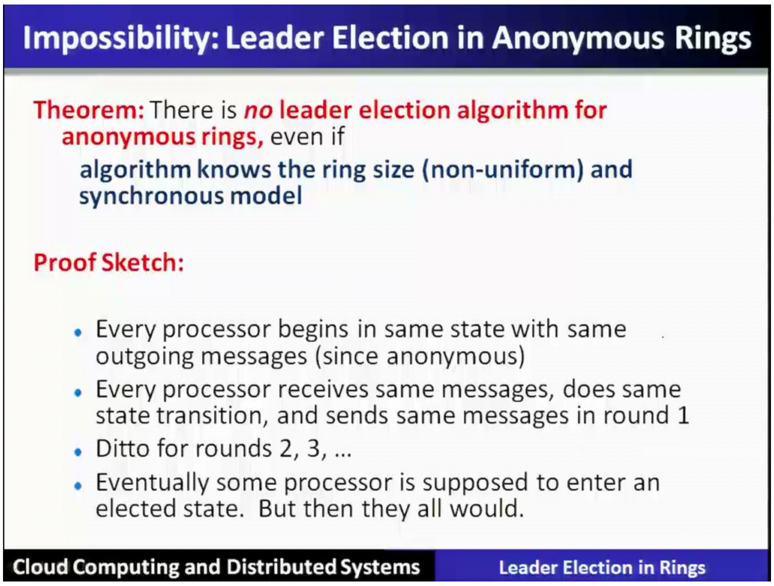
So, if the processors do not have the unique ids; that means they are not given an ids, then that particular ring is called anonymous rings. So, in that situation each processor is like same running state machine there is no distinction.

(Refer Slide Time: 04:24)



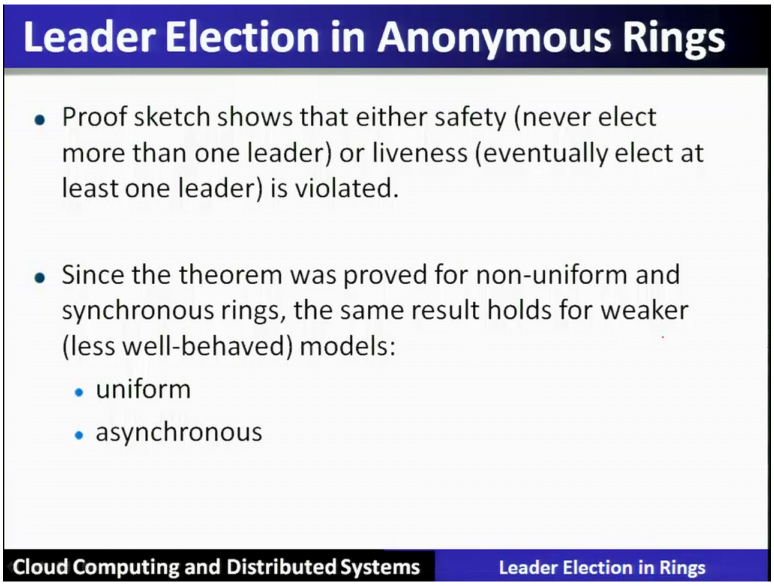
Third definition is about uniform anonymous algorithms. So, uniform algorithm means that it does not use the information of the ring size that is n; the number of nodes in the ring. So, formally every processor in every size ring is modeled with the same state machine. So, the algorithm which is called a non-uniform algorithm we will use the size of the ring that is an n the algorithm.

(Refer Slide Time: 05:03)



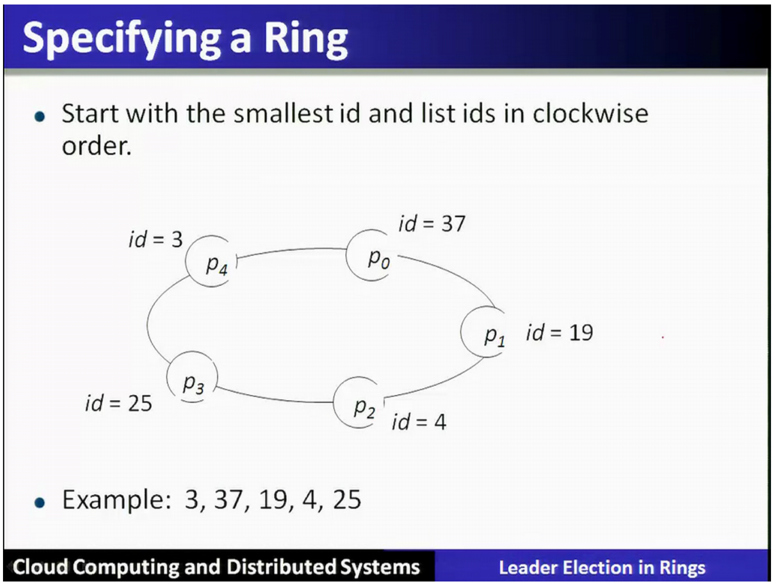
There is a impossibility result which says that about the leader election anonymous rings theorem. There is no leader election algorithm for anonymous rings even if the algorithm knows the ring size that is for the non-uniform. And also the model is synchronous model, why; because every processor begins in the same state with the same outgoing message. Since it is anonymous, and each processor when receive the message will also be in the same state transition, and sends the message in a round one.

(Refer Slide Time: 05:48)



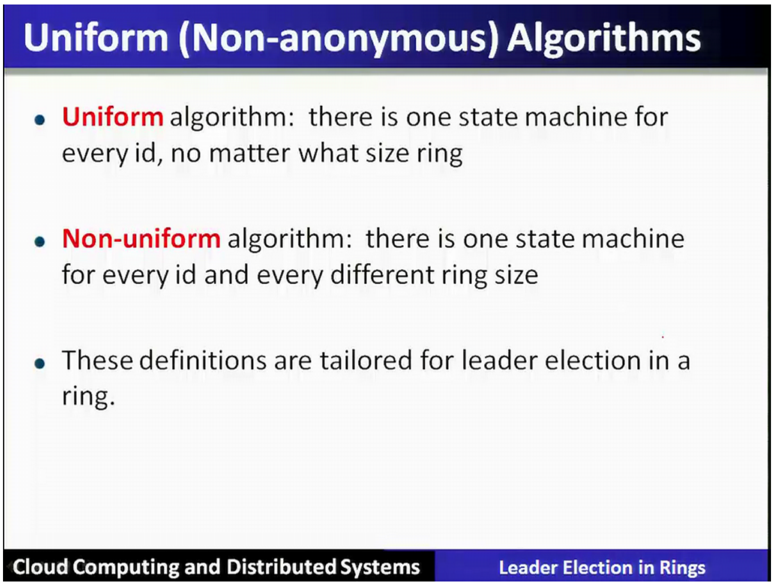
So, there is no distinction and there is all symmetry looking up in this particular way. So, it shows that either the safety or the liveness is violated here in this case. Hence the theorem was proved for non-uniform and synchronous rings. The same result hold for a weaker model that is for the uniform and the asynchronous model. So, rings with the ids. So, we will assume that each processor has the unique id, ok. Let us see a example of a ring.

(Refer Slide Time: 06:33)



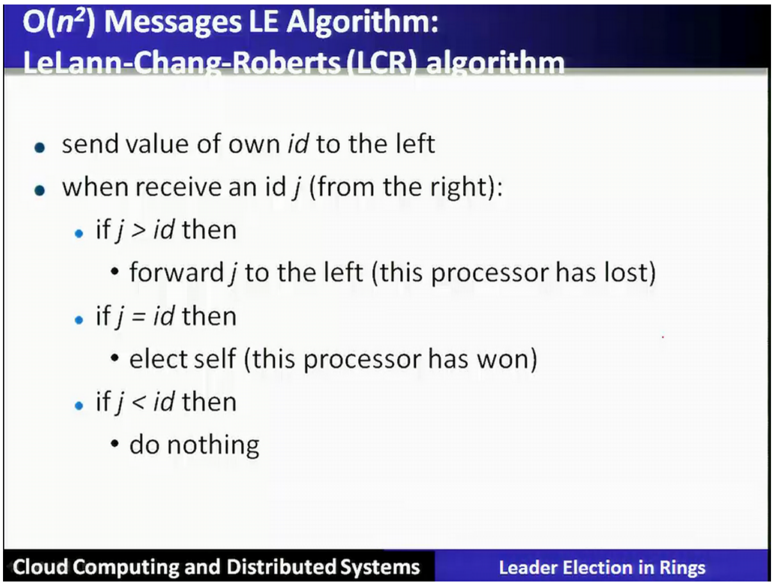
Here we see that every processor in the ring is having different ids for example, 3, 37, 19, 4 and 25.

(Refer Slide Time: 06:46)



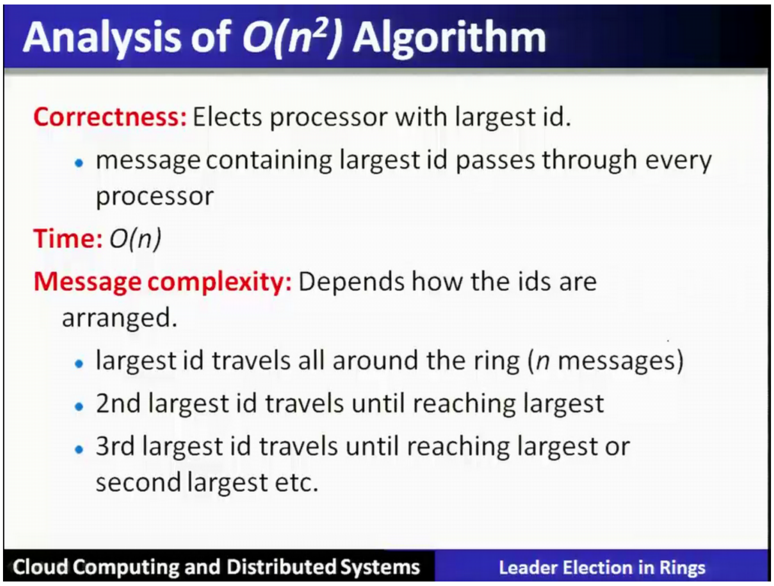
So, uniform algorithms means that the algorithm is not using the size of the ring. So, no matter what is the size of a ring the algorithm in the algorithm. Similarly, non-uniform rings shows that it uses the size of the ring.

(Refer Slide Time: 07:04)



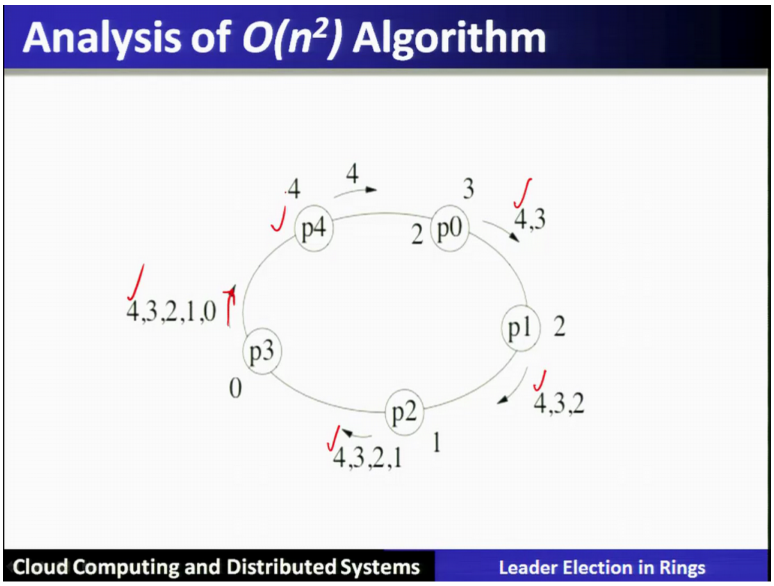
Let us see the algorithm which is called a Lelann-Chang-Roberts algorithm; LCR algorithm for leader election problem. This is also called as a order n square leader election algorithm. Here algorithm says that every processor will send its id to the left. When it receives an id from the right, and if j is greater than it is id, then it will forward to the left. If less than id, then it will not do anything it will not forward it will swallow. And if id is equal to the j then it will elect itself as the leader.

(Refer Slide Time: 07:53)



We will see through an example. As far as correctness is concerned, it will always elect a processor with a largest id in the time of the order n and the message complexity is order n square, let us see through an example.

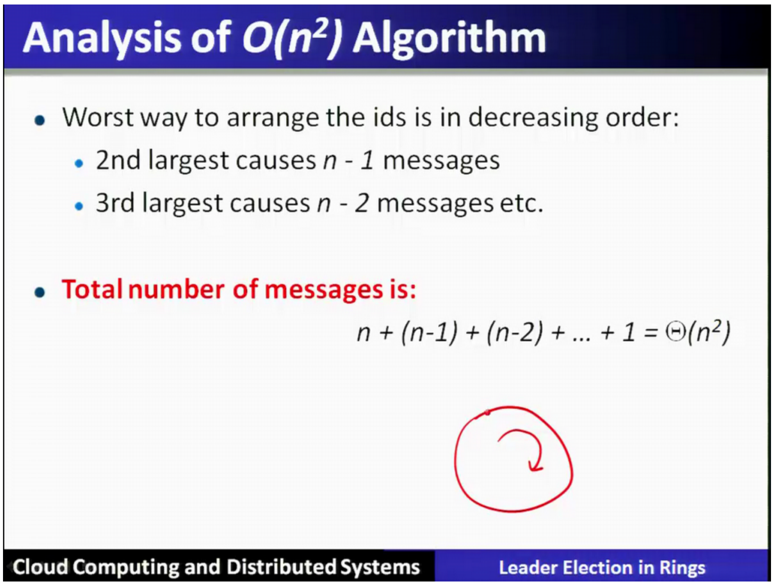
(Refer Slide Time: 08:07)



Now let us see that the processor p 3 will send it is id to the left p 4 on receiving 0 it will not forward it further. Whereas, p 4 will send it is own id, and when it reaches to p 0, it is id is greater than 3 so, it will be forwarded further. When it reaches to p 1 it will also be forwarded. And it will be forwarded, and when it reaches to this particular point because this is the highest. So, it will see its own message so, hence this will be elected as a leader.

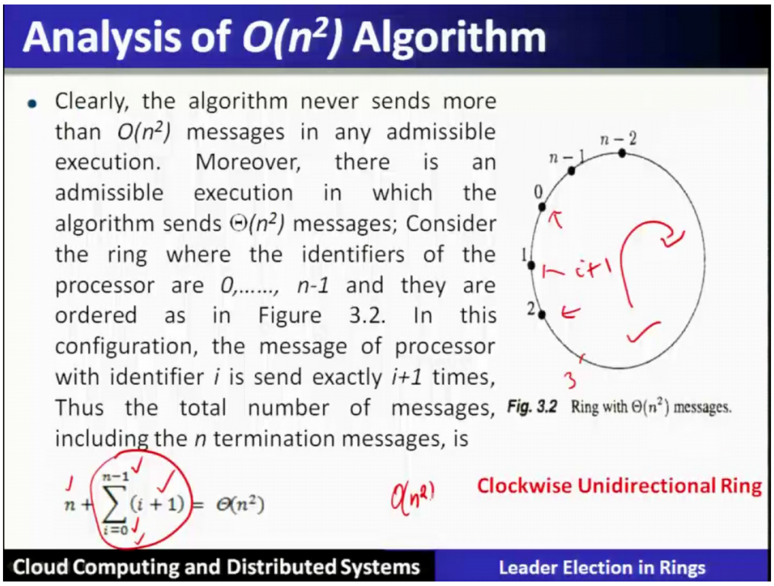
Whereas, the other messages having lower ids will be absorbed so, let us see when this order n square algorithm will arise.

(Refer Slide Time: 09:21)



Let us see that the messages are being forwarded here in this particular scenario. Let us see this particular method of analyzing the worst case scenario, that is of the order n square.

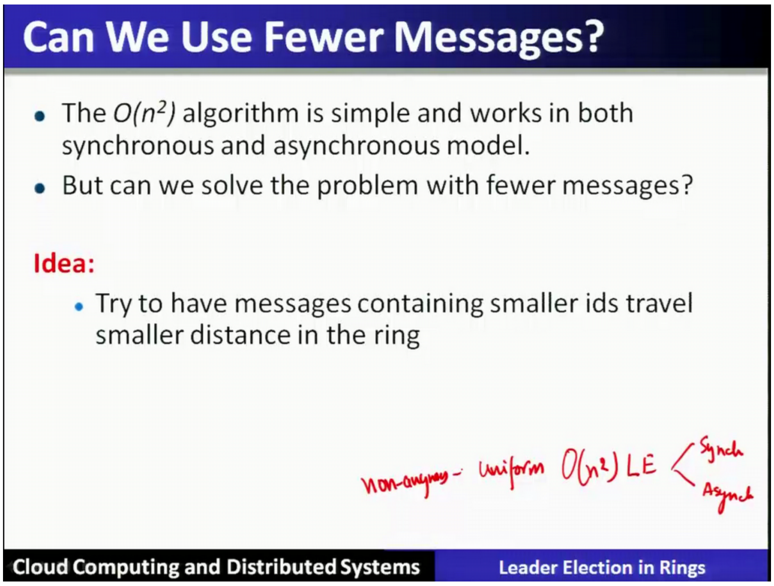
(Refer Slide Time: 09:46)



We will consider the ring where the identifiers of the processor 0 1 2 and so on up to n minus 1 and are ordered in this particular manner. Here the identifier i is send exactly i plus 1 times. So, for example, 0 will be send 0 plus 1 that is only one time it will be observed here. 1 that is i plus 1 times it will forwarded. 1 will be forwarded by 0; that means, it will be forwarded 2 times and so on. So, 2 will be forwarded 3 times, 3 will be forwarded 4 times and so on. So, this is the orientation of the ring.

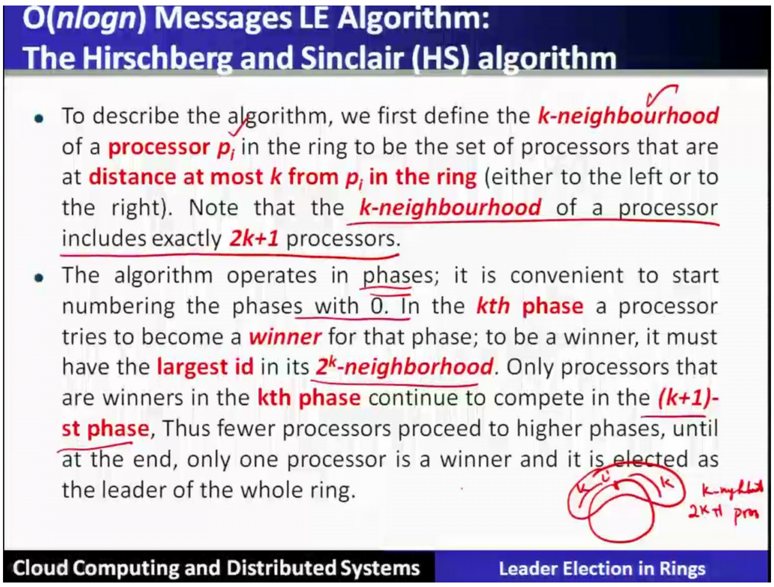
How many number of messages are being forwarded? Here so, we will see that for i it will be forwarded i plus 1 times. So, if we sum for the nodes from 0 to n minus 1. So, these are the total number of messages which will be forwarded in this particular worst case scenario in the ring. And when a leader is elected then the highest id will circulate it is message that is n. Now, this particular summation is of the order n square.

(Refer Slide Time: 11:49)



So, it requires n square number of messages in the worst situation we have shown in this analysis. Algorithm which is of the order n square algorithm is simple and works in both synchronous and asynchronous model. Another good thing about this algorithm is does not use the value of n or the size of n. Hence, it is non-uniform, hence it is uniform. So, as it is shown that it will work in both the situation either for the synchronous model or for asynchronous model, anonymous order n square leader election algorithm.

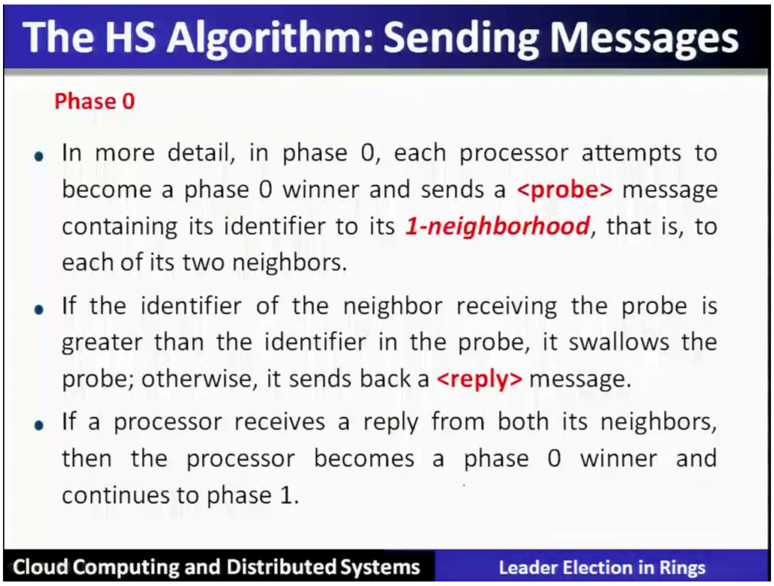
(Refer Slide Time: 12:44)



Now, can we do with a lesser message complexity then order n square? We will see an algorithm which is given by Hirschberg and Sinclair HS algorithm, it is called it is order n log n leader election algorithm. This algorithm uses the concept of k neighbourhood for any processor p i in the ring. That is nothing but distance of at most k. It is (Refer Time: 13:09) and this is k this is k and this is one that is 2 k plus 1 nodes for the processors in the k neighbourhood of a processor.

So, k neighbourhood of a processor includes exactly 2 k plus 1 processors. This algorithm operates in the phases. Therefore, it is convenient to start numbering the phases starting from the phase 0. So, the kth phase of a process will try to become the winner, for that phase to be the winner it must have the largest id in its 2 k neighbourhood. So, only the processor that are the winners of kth phase will continue to participate in k plus 1th phase. Thus, fewer processor proceeds to a higher phase until at the end only one processor is the winner and is elected as the leader of the whole ring.

(Refer Slide Time: 14:29)



So, let us see the phase 0, in more detail phase 0 each processor attempts to become of a phase 0 winner, and sends a probe message containing its id to its one of neighbourhood to each of the 2 neighbours so for example if this is the neighbourhood and to one neighbourhood on both the sides in phase 0 if the [FL].

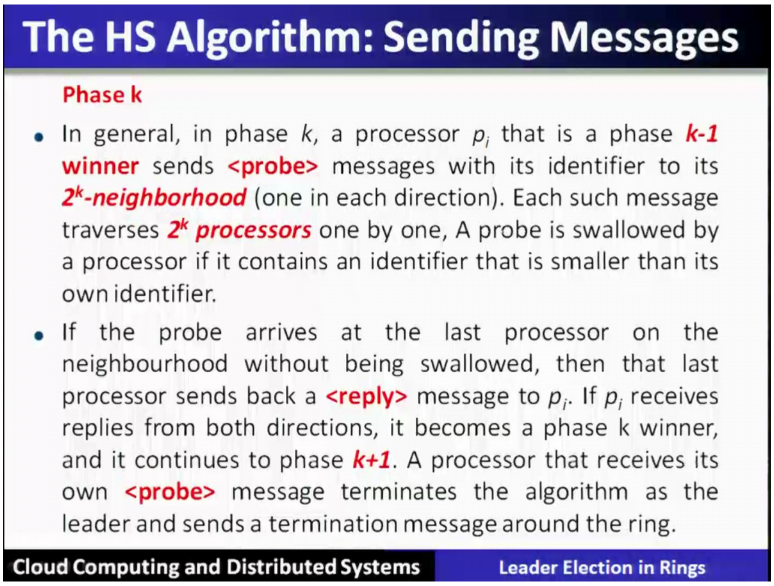
[FL].

[FL], If the id identifier of the neighbour receiving the probe is greater than the identifier in the probe, it swallows the probe, otherwise it sends back the reply.

So, if it gets the replies back from both the end; that means, it has become the winner of phase 0 and it will continue to the phase 1. So, in the phase 1 the neighbourhood size will basically double. If it is let us say that it is phase k; that means, all the winners of phase k minus 1 will send it is probe message with its id to its 2 raised power k neighbourhood one in each direction. Each message traverses 2 raised power k processor one by one and the probe is swallowed by the processor if it contains an id that is smaller than it is own id.

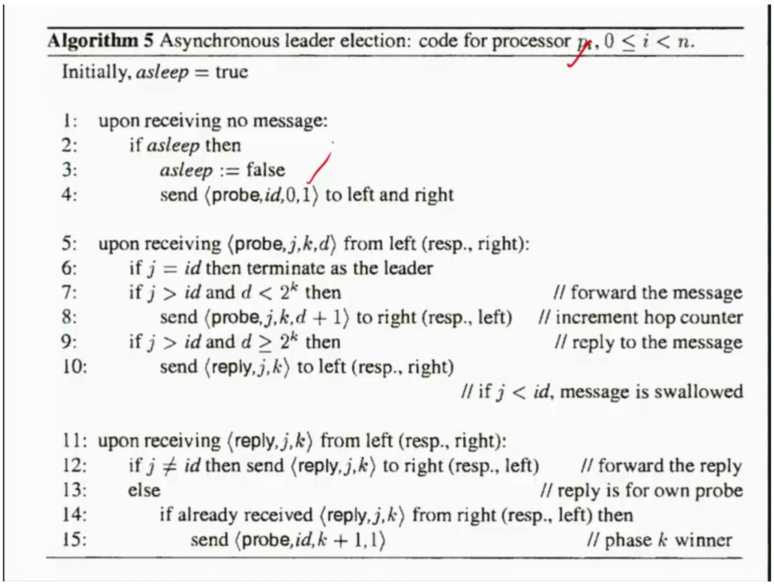
If the probe arrives at the last processor on the neighbourhood without being swallowed, then the last processor will send back the reply as we have seen in the first or the 0th phase.

(Refer Slide Time: 16:17)



So, this is the complete algorithm for the processor i and this algorithm will be for all the processor that is from 0 to n minus 1.

(Refer Slide Time: 16:28)



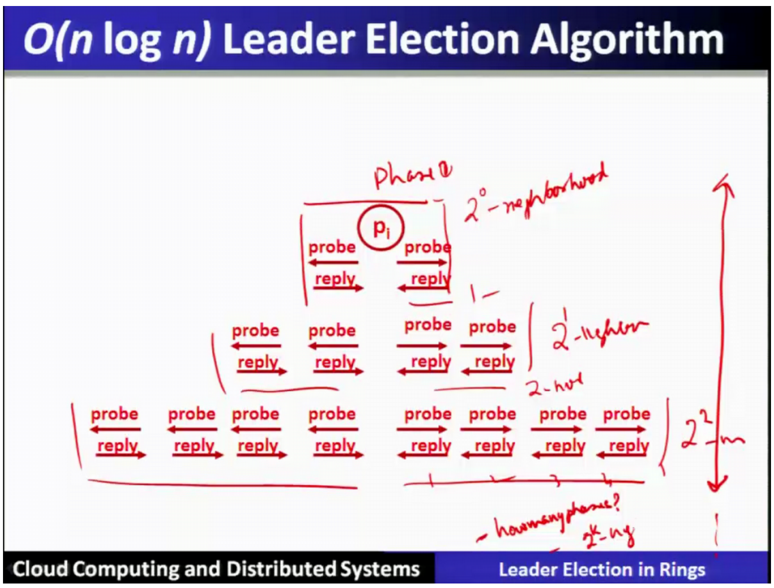
Algorithm 5 Asynchronous leader election: code for processor p_i, $0 \le i < n$.

```
Initially, asleep = true

upon receiving no message:
    if asleep then
        asleep := false
        send ⟨probe,id,0,1⟩ to left and right

upon receiving ⟨probe,j,k,d⟩ from left (resp., right):
    if j = id then terminate as the leader
    if j > id and d < 2^k then                        // forward the message
        send ⟨probe,j,k,d + 1⟩ to right (resp., left)  // increment hop counter
    if j > id and d ≥ 2^k then                        // reply to the message
        send ⟨reply,j,k⟩ to left (resp., right)
                                                      // if j < id, message is swallowed

upon receiving ⟨reply,j,k⟩ from left (resp., right):
    if j ≠ id then send ⟨reply,j,k⟩ to right (resp., left)   // forward the reply
    else                                                     // reply is for own probe
        if already received ⟨reply,j,k⟩ from right (resp., left) then
            send ⟨probe,id,k + 1,1⟩                          // phase k winner
```

So, here we will see that the algorithm, it will send this particular probe to the left and right, and it will do the election it will initiate multiple elections in the k neighbourhood; that means, the entire ring is divided into k neighbourhood and this elections will parallely run for the kth neighbourhood.

So, we see that that the size of the neighbourhood will double in each phase. So, if a probe reaches a node with a largest larger id the probe will stop.

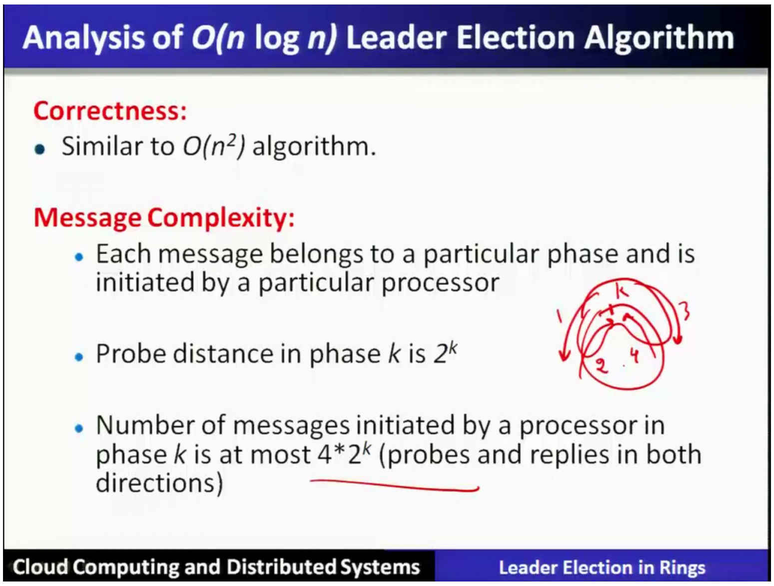
(Refer Slide Time: 17:18)



If the probe reaches the end of the neighborhood, then the reply will be sent back to the initiator. So, this can be seen through this particular diagram that this is phase 0; that means this particular one half neighbourhood will perform the elections. And whose over will be the winner then the size will be doubled that is 2 raised power one neighbourhood. So, that means, 2 this was earlier 1 node neighbourhood, now it is having 2 nodes in the neighbourhood.

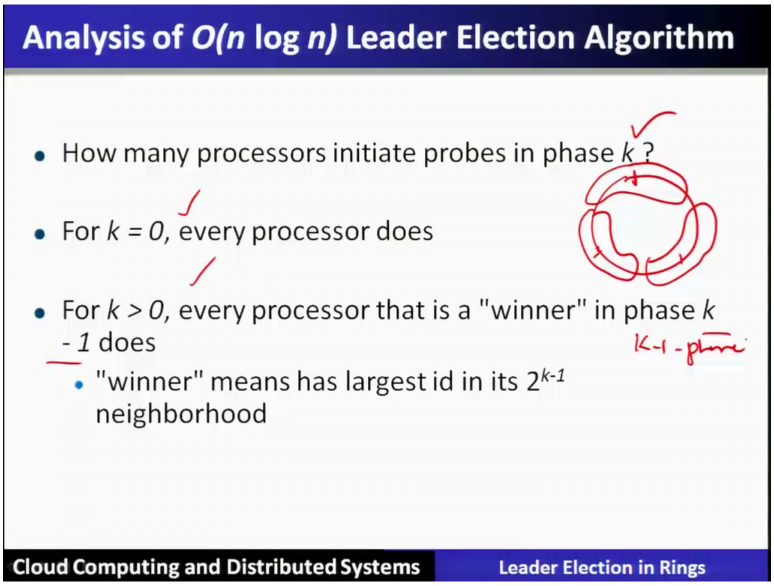
And whosever and if it is able to win for 2 raised power 1 neighbourhood, then the neighbourhood size will double, that is 2 raised power 2 neighbourhood; that means, 1 2 3 4 different so, the neighbourhood size will keeps on growing in this particular manner. Now we will count what is the depth; that means how long how many different phases will be there, how many phases will be there and in each phase, you know that it will be 2 raised power k neighbourhood. So, in each phase how many such elections will happen? And these particular information is required to compute the message complexity.

(Refer Slide Time: 19:10)



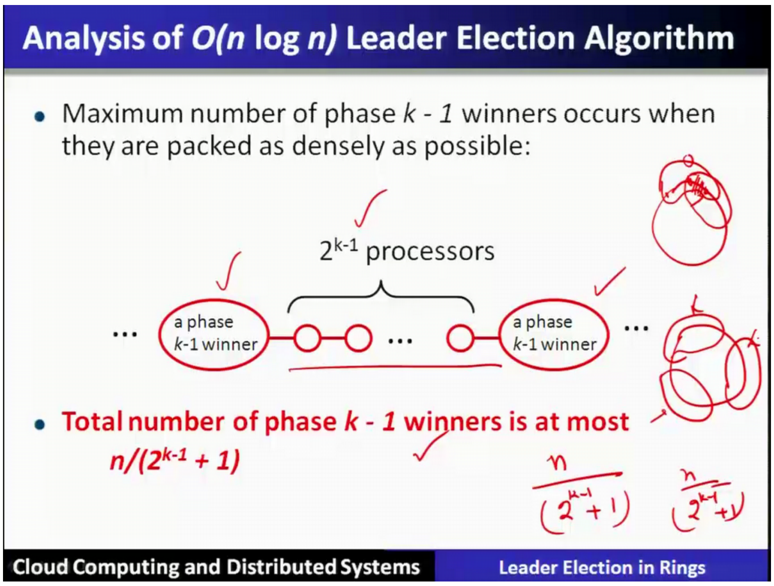
So, the message complexity, that means, here the probe distance is in a phase k is 2 raised power k. So, the number of messages initiated by a process in a phase k is at most 4 times 2 raised power k; that means if this is the k phase it is neighbourhood. So, the message will go one times, the reply will come back 2 times, then message will go on the right side third time and it will come back to the 4 times. So, 4 time 2 raised power k in every phase 4 times these probes will be sent in counting the message complexity.

(Refer Slide Time: 19:55)



Now, the question is how many such processor will initiate the probe in the phase k. Meaning to say that if this is the k hop so, how many such packing of k hops will be initiated at a particular phase k. Now for k is equal to 0 every processor will be participating in this particular election process when k is greater than 0; that means, the winner of phase k minus 1 only participate in kth phase.

(Refer Slide Time: 20:58)

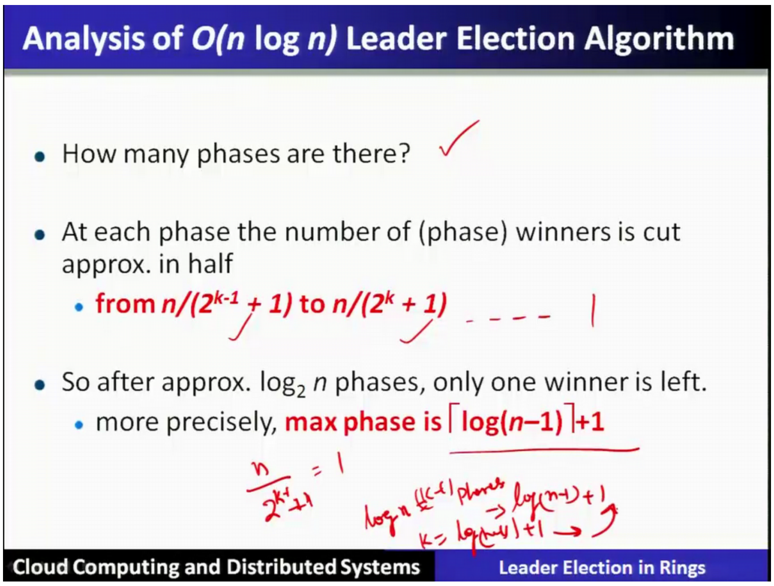


So, the winner of k minus 1th phase means that the largest id in 2 raised power k minus 1 neighbourhood which is being seen. So, the maximum number of phase k minus 1 winner occurs when they are packed as densely as possible.

So, let us see that here it is phase k minus 1 winner and another phase k minus 1 winner. So, how many number of processors which are in between 2 extreme phase k minus 1 winner is nothing but 2 raised power k minus 1 winner. We can see through an example that if this is phase 0 so, it will have the one neighbourhood. So, this particular 0, this particular another node will also have this kind of neighbourhood. So, the number of nodes involved is basically 2 raised power k minus 1. So, the total number of phase k minus 1 winner will be n divided by 2 raised power k minus 1 plus 1.

That means this is the packing, this is the packing of so many k minus 1. So, n divided by 2 raised power k minus 1 plus 1, this will be total number of such k minus 1 winners who are participating in kth round. Now the question is how many phases are there. Now you know that every phase the number of winners is cut in a half that is from n raised power 2 k minus 1 plus 1.

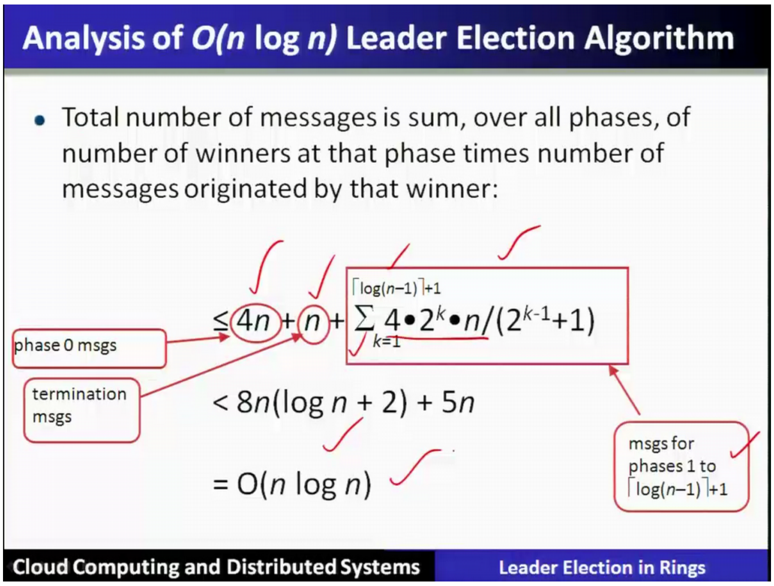
(Refer Slide Time: 22:40)



It will now go to n divided by 2 k plus 1 in the kth phase. So, we will see that we will continue in this manner so that finally, there will be only one such neighbourhood remains that is the entire ring. And if we compute then it becomes it comes out to be log of n minus 1 plus 1 total number of phases.

So, again we can see n divided by 2 k 2 k minus 1 plus 1. Let us say this becomes 1 if we take the log of n, then it will become k minus 1. So, this k is equal to log n minus 1 plus 1. So, this is how this particular formula is being obtained.

(Refer Slide Time: 24:14)

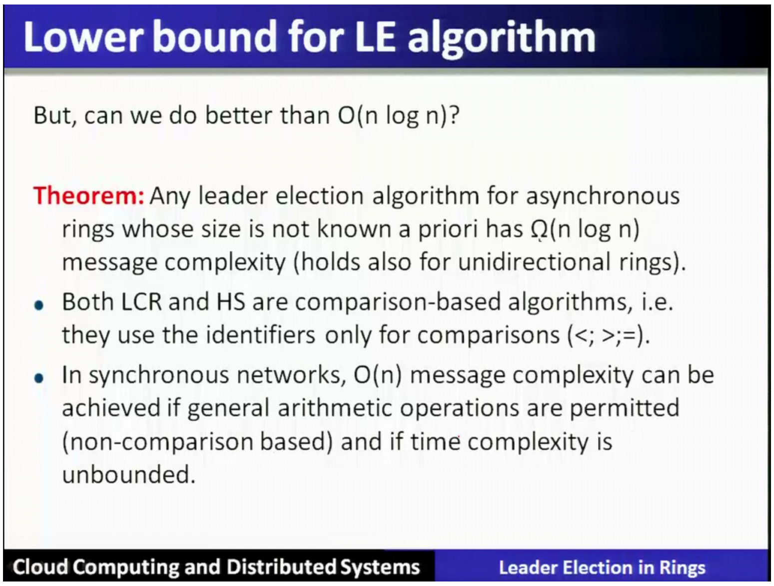


Now plugging up all the values we will find out the total number of messages in all the phases, that comes out to be this is the phase 0 messages, 4 times n and this is the termination message. And as far as this formula is concerned, this formula says that how many different messages are there for the phases 1 to log n minus 1.

So, this is the summation of different phases, and every phase will require 4 times 2 raised power k times n raised power n divided by 2 raised to power k minus 1 plus 1. So, if you compute it comes out to be 8 n log n plus 5 plus 2 plus 5 n. That comes out to be of the order n log n. So, we have seen that this particular algorithm will work both for synchronous and asynchronous case. So, the question is can we reduce the number of messages even further than order n log n.

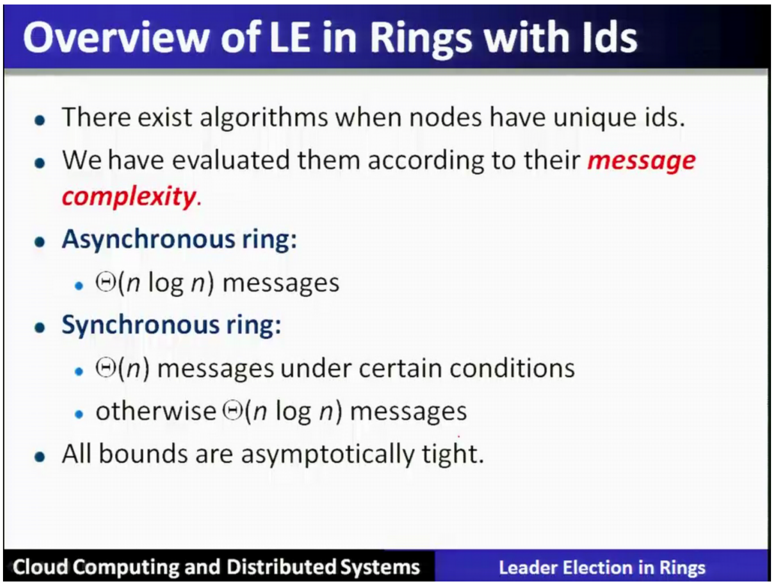
So, we will we have seen that not in asynchronous model you can do this better than order n log n; that for that we can show that this is the lower bound for the leader election problem that is of the order n log n.

(Refer Slide Time: 25:38)



So, the theorem says that any leader election algorithm for asynchronous rings whose size is not known a priori has the lower bound of n log n message complexity, holds also for the unidirectional rings. Both LCR and HS are comparison based algorithm; that is, they use identifiers only for the comparison. In synchronous algorithms order of n message complexity can be achieved if general arithmetic operations are permitted, non-comparison based and if the time complexity is unbounded.

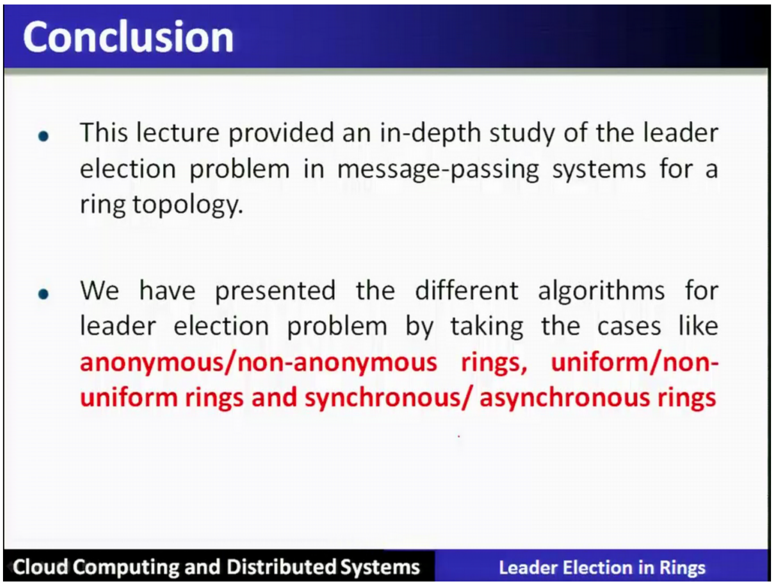
(Refer Slide Time: 26:29)



So, overview of leader election in rings with the ids, there exist algorithm when the nodes have unique ids. We have evaluated them according to their message complexity, and we found out that in the case of asynchronous ring the message complexity of the algorithm the best known algorithm gives n log n messages. Similarly, for synchronous rings we have seen that of the order n messages under certain conditions. Otherwise, the complexity is the same that is n log n messages in the comparison based algorithms.

So, all these bounds are asymptotically tight conclusion. This lecture provided an in depth study of the leader election problem in the message passing system for the ring topology.

(Refer Slide Time: 27:23)



We are also presented different algorithm for the leader election by taking different cases. Such as anonymous oblique non-anonymous rings, uniform oblique non-uniform rings, synchronous oblique asynchronous rings.

Thank you.