Advanced Graph Theory

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Lecture - 06
Spanning Tree and Enumeration

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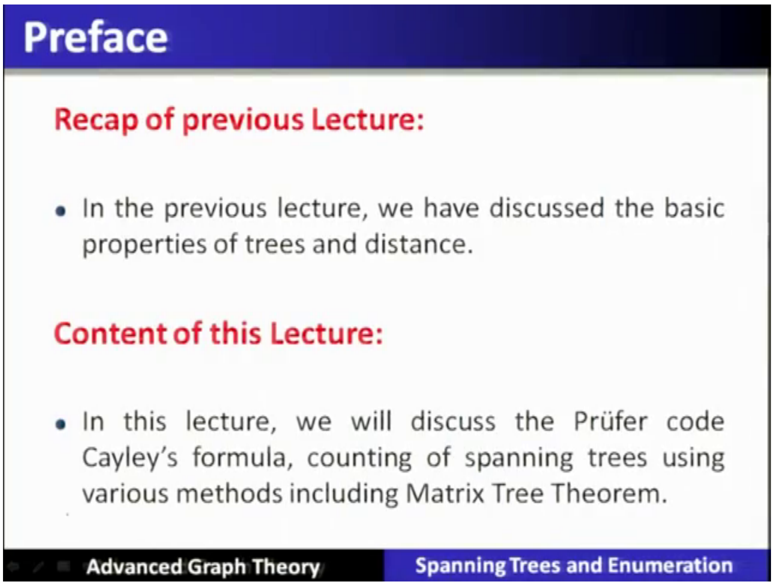

Lecture 6 spanning trees and enumeration. Recap of previous lecture previous lecture, we have discussed the basic properties of trees and distance content of this lecture in this lecture we will discuss the Prufer code, Cayley's theorem counting of spanning trees using various methods including matrix tree theorem.

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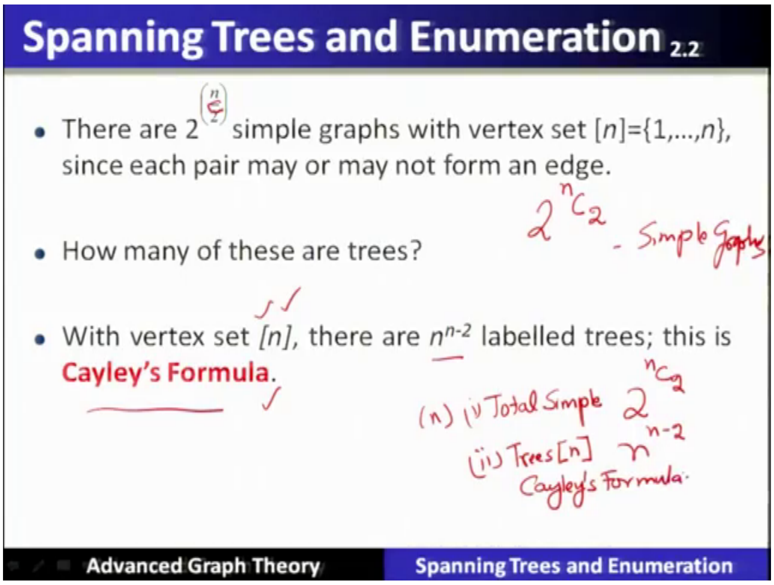

Spanning tress and enumeration there are 2 raise power n C 2 simple graphs with vertex set, which are Labeled from 1 to n since, each pair may or may not form an edge therefore, total number of simple graphs which are required or which will be formed is 2 raise power n C 2. Now, out of this simple graphs how many are basically the trees? How many are the trees? So, that is given by Cayley's formula. So, with the set vertices, which are labeled as from 1 to n , that is denoted by this symbol. So, then they are n raise power n minus 2 different Labeled trees and that is called the Cayley's formula. So, the total number of possible simple graphs of n vertices is 2 raise power n C 2 and out of these graphs how many are basically different Labeled trees ? Which are Labeled the vertices from 1 to n they are n raise power n minus 2 different trees are there, that is given by Cayley's formula.

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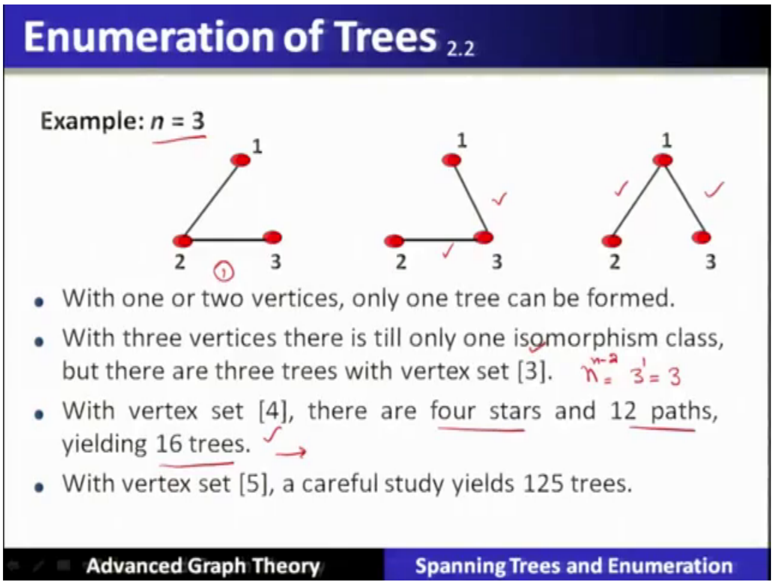

Now, using Cayley's formula we can see that, if there are 3 different vertices and we label them as 1 2 3. So, how many different trees will be possible? So, 3 raise power 1, that is 3 different trees are possible. So, this is 1 possible tree then, these the node 1 and 2 and 2 and 3 they have got an edge in another possibility if 1 and 3 and 2 and 3 they have got an edge, that possibility when 1 on 2 and 1 and 3 have got an edge. So, there are 3 possibilities only with the Labeled trees. Similarly, if we take a vertex set 4 vertices and they are Labeled out of this set 4 then, there are 4 stars and 12 different paths in turn they comes out to be 16 different trees as per as Cayley's formula is concerned and so on.

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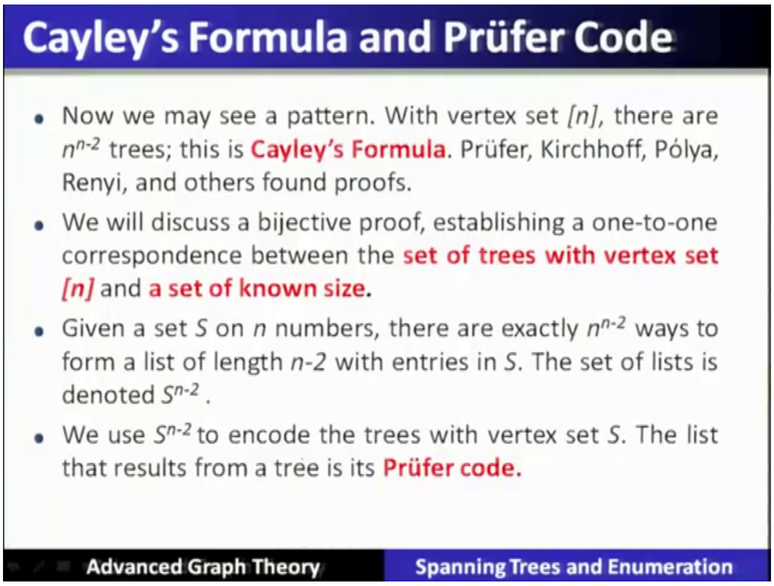

So now we can see a pattern with the given vertex set n, and using this particular formula that particular pattern is basically covered, that is n raise to the power n minus 2 and that is called Cayley's formula. Prufer, Kirchhoff, Polya and Renyi they have formed out the proofs we are going to discuss all this in this part of this lecture, we will discuss bijective proof establishing one to one correspondence between the set of trees with the vertex set n and another set of known size. So, given a set n set on n numbers there are exactly n raise power n minus 2 different ways to form a list of length n minus 2 with the entries in S. So, the set of list is denoted by S raise power n minus 2 we use S raise power n minus 2 to encode the trees with the vertex set S the list, that results from the tree is it is Prufer code.

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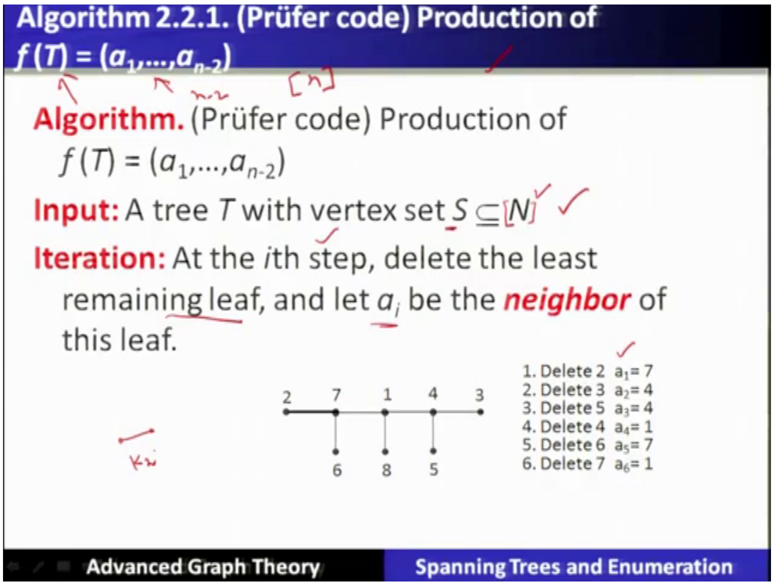

Let us see the algorithm for Prufer code. So, the Prufer code will produce code of length n minus 2 , that is these numbers a 1, a 2 and so on up to a n minus 2 these numbers are drawn out of this particular set n and so given a tree this particular function f will produce a code and that is called a Prufer code. So, for a given tree there is code which will be of length n minus 2 that is called a Prufer code. So, the input to this algorithm is a tress with the vertex set S, which is a subset of this particular label, and which is given as per as this algorithm is concerned at ith step it will delete the last remaining leaf and a i is basically the neighbor of this particular leaf, that will be noted down. This particular iteration keeps on iterating finally, it will terminate when there will be only one edge joining between 2 vertices remains or you can also say up K 2 graphs will remain then only the algorithm terminates with a successful Prufer code of Labeled tree.

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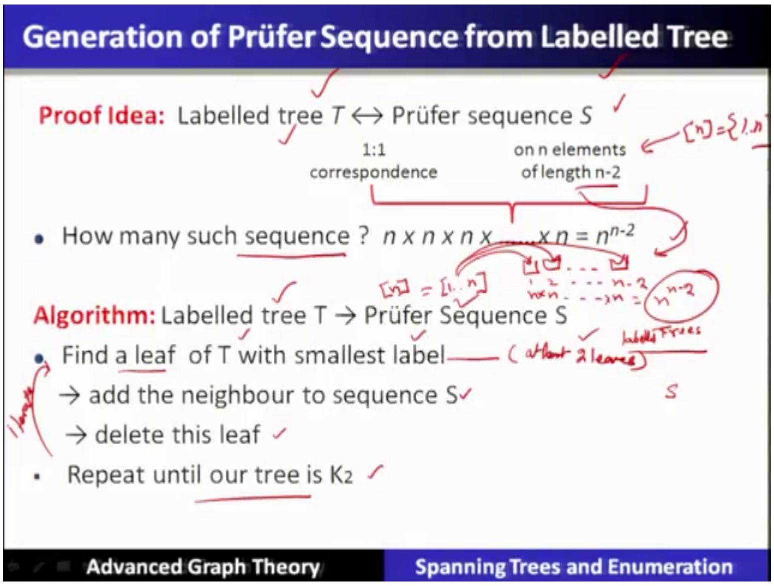

So, let us see the different aspects involved in this Prufer code algorithm. So, first we will see one aspect that is, how to generate a Prufer sequence from a given Labeled tree? So, the so; that means, we are given a Labeled tree and we have to generate a Prufer sequence S and on the other side also if a Prufer sequence S is given we can generate that, unique tree which is Labeled tree. So, there is one to one correspondence from tree to a particular Prufer sequence and Prufer sequence to a tree we can construct. Now, if that is the case for a given tree , we are going to produce a Prufer sequence and these sequence will be drawn from each element of this Prufer sequence will be from 1 to n and total length of this particular code S will be n minus 2. So, how many such sequences are possible? So, let us see that, if the length of this particular sequence is n minus 2; that means, there are n minus 2 different containers and each element of this container you can fill it, with the number which are ranging from 1 to n, that is the label of these vertices. Similarly, in the other container also we can replace or we can choose any of these n numbers and so on so; that means, n multiplied by n and so on up to n minus n times if you multiply that becomes, n raise power n minus 2. So, how many such sequences are possible? They are basically in this number, that is n raise power n minus 2 at every sequence represents a specific Labeled trees. So, how many trees will be there that is these number of spanning trees are possible.

Now, we have to see this particular algorithm, that given Labeled tree how we are going to generate the Prufer sequence? So, the algorithm the first step of the algorithm is find a leaf of a tree with the smallest label this particular; that means, whenever a tree is given always there are 2 leafs that, we have seen in the previous theorem 2 leafs whenever there is a in the tree. So, this particular assumption to find out a leaf is possible whenever a tree is given.

So, find a leaf of T with the smallest label. So, having identified a smallest label leaf what we will do? We will delete this particular leaf and the neighbor of this particular leaf will be added to that particular sequence. So, we are building up that sequence this particular step is repeated we have to keep on repeating iterate till this condition arises that, our tree becomes only K2; that means, only one edge connecting to different vertices basically is left out at, that time this particular algorithm will terminate.

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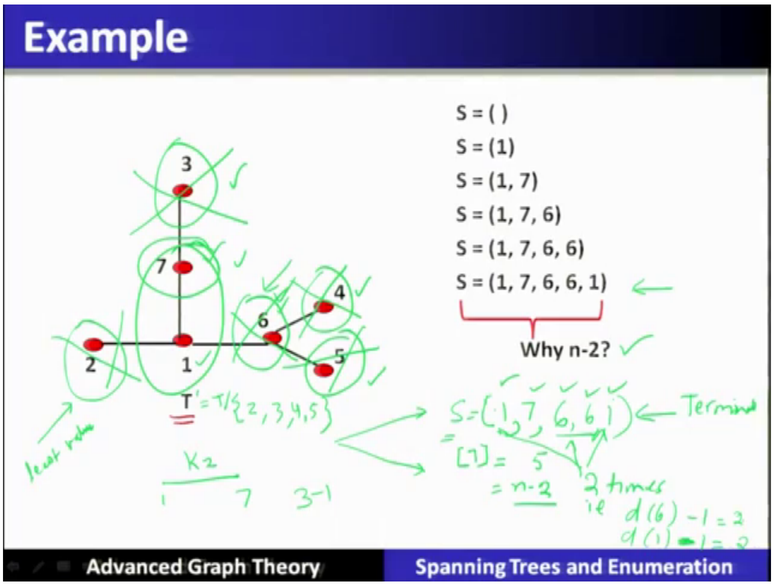

Let us see this particular example, let us say this T is given to us and we want to generate a Prufer sequence. So, first we have to see in this particular tree how many different we have to see this tree? How many different leafs are there? This is one leaf another leaf. So, there are 4 set of leafs and out of this 4 this is one is the leaf with a least value 2. So, what we are going to do? We will take this particular neighbor, that is this particular neighbor we are going to take it and this will be removed and the remaining tree, that is let us say after one iteration that becomes, T prime T minus the vertex, which is label to will be deleted.

So, this particular tree T prime will have how many other leafs? 3 4 5 there are 3 different leafs, which are remaining and the one with a least value is 3. So, 3 will be removed, but the neighbor of 3 is 7, 7 will be added in the sequence S. So, 3 also will be removed from this particular tree. So, 2 and 3 they will be removed. So, the in the resulting tree there are how many leafs are there? 1 2 3 different leafs are there and out of them the least value is 4. So, the neighbor of 4 is 6. So, 6 will be included and now, 4 will be removed from the tree and the resulting tree will basically get 4 removed out of it.

Now, the remaining leaf notes will be 7 and 5 out of them 5 is having the least value. So, the neighbor of 5 is 6. So, 6 will be added again. So, you see some of the vertices, some of the labels are repeated in the sequence S or the Prufer code and then we will remove it from. So, the resulting tree will be the T minus 2 3 4 and 5. So, the leafs which will be remaining is 6 and 7 in that resulting tree. So, out of 6 and 7 the least value is 6. So, it will be picked up and the neighbor of 6 is 1.

So, 1 will be put and 6 will be deleted. So, after that deletion there will be only one edge remains, which is connecting 1 and 7 , and that is nothing but 2 K 2. So, this particular algorithm will terminate at this end terminate with a sequence S. So, if we count how many elements are there in the particular sequence? 1, 2, 3, 4, 5 and how many notes were there? They were 7. So, that 5 is nothing but n minus 2. So, the length of the Prufer sequence as we have seen in the algorithm that, it will generate for a particular tree a Prufer sequence of length Prufer sequence S of length n minus 2. Let us see, why this n minus 2 is basically coming up as the Prufer sequence length.

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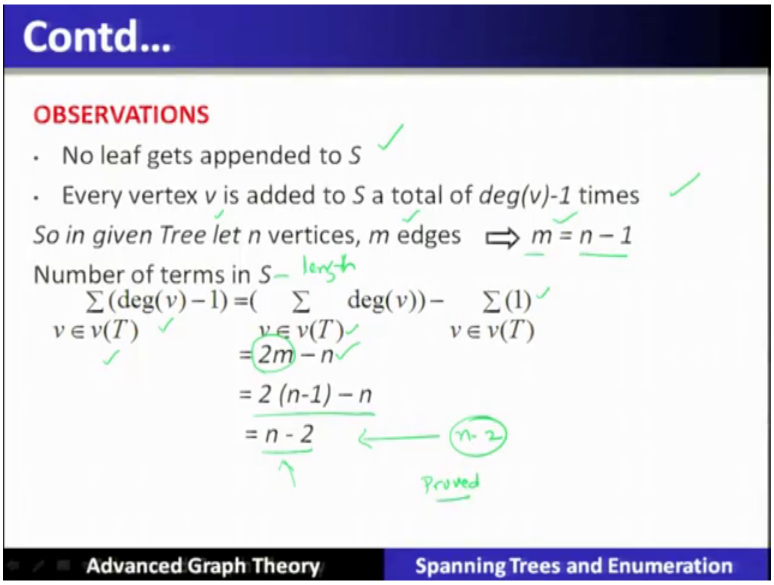

So, in the observation we have seen there is no leaf note, which is appended to that particular sequence S and also, we have seen another observation that, every vertex v is added to S a total of how many times? Degree of that vertex minus 1 times. So, we can see. So, 6 is appearing twice, why? Because, the degree of 6 note number 6 is having 3. So, 3 minus 1 that is 2 times, that is the degree of this note 6 minus 1 that is 2 times it is repeating. Similarly, here, the note number 1 is also repeated 2 times because, of the same condition that the degree of is equal to degree of 1 minus 1 becomes 2. So, 2 times it is appearing.

So, with these 2 observations what we can see that, for a given tree T having n different vertices or the labels and m different edges since, it is a tree we know that this m is equal to n minus 1. Now, let us see how many terms are there in S? So, number of terms which will appear in the Prufer sequence S is nothing but, degree minus 1 of that vertex and for all the vertices we are going to make a sum, that will adapt to the length of the Prufer sequence. So, with this particular summation if we sum the degrees for all the vertices this is nothing but, a hand shaking lemma this will result in twice the number of edges similarly, one if we add how many times if we add n times it will become n. So, 2 m if we substitute the value of m is n minus 1. So, this will gives you the length as n minus 2. So, the length of the Prufer code is n minus 2 that we have proved.

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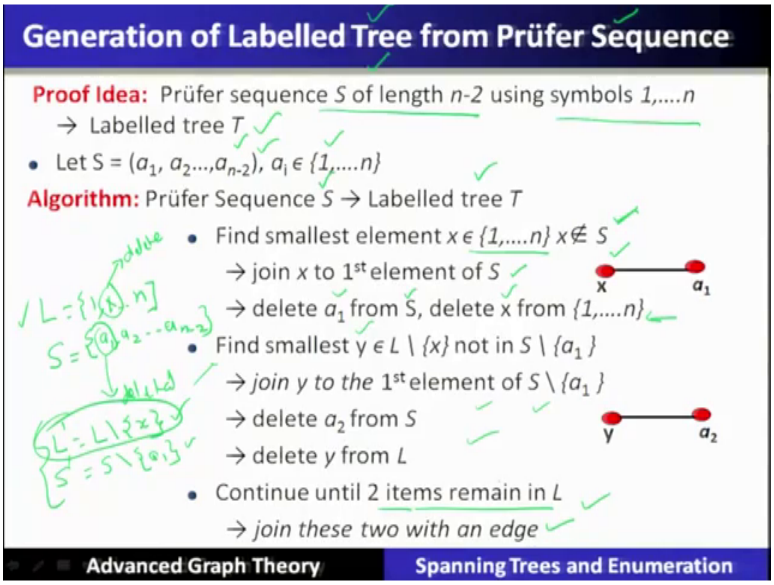

So, before we go ahead let us see that, if you are given Labeled if you are given a Prufer sequence how we are going to generate a label tree from the Prufer sequence? Let us see the proof idea, let us assume that the Prufer sequence of n minus 2 length is given using this symbols 1 to n then we are going to generate a unique label tree T. Let us assume that, S contains a1, a 2 and so on up to a n minus 2, why? Because, only the length of the code is n minus 2 and these values of a i is drawn from 1 to n. Let us see the steps of this algorithm that once the Prufer sequence is given how we are going to generate a label tree from a Prufer sequence of length n minus 2 which is having the symbols from 1 to n.

So, the first step is to find out the smallest element x from this particular set of label that is 1 to n, which is not present in that particular Prufer sequence S which is given to us. So, let us assume that, it is that particular label S which is not present in S. So, join this particular element to the first element of S and delete a1 from S and delete x also from the set of labels. So, we will run 2 different 2 different sets one set let us call it as L , the other set is a Prufer code that is a1 a 2 and so on up to a n minus 2. So, we will take this particular value of x and we will delete this x. Similarly, here, the first element also will be deleted. So, the resulting list L of labels, and that the sequence S will be after removing the element a 1.

So, now the next iteration will began from this particular modified list L prime and S prime again we are going to find out a smallest particular label here, which is not present in the modified sequences prime and we are going to join this particular element, which is having the smallest label with the first element of S prime and we add an edge and then, we will remove these 2 elements from this particular list again we are modifying and we continue to iterate until 2 items remain in this particular L then, these 2 items when they will remain we are going to add an edge and we conclude generating all label tree out of a given Prufer code.

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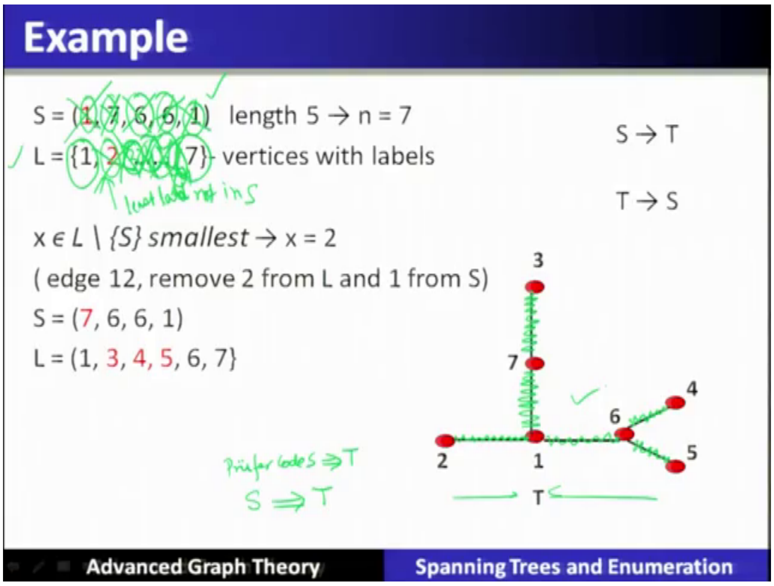

Let us take an example, if the Prufer code let us say 1 7 6 6 1 is given. Here, we can see that length is 5 and the label set is L 1, 2 and so on up to 7. So, the smallest label here is 2, which is not present in S. So, what we will do? We will take and we will take the first element of S1 S and we will add n edge between 1 and 2 then, we will remove it and that resulting list S and L is go will be considered for the next iteration, in the next iteration what we will see that, 3 is the label which is not present in S. So, let us consider 3 and we are going to basically join it with 7.

So, 3 and 7 will contribute to add an edge and now, they will be removed from both L and S. Similarly, let us take the label number 4, 4 is not present in S. So, 4 is the least label, which we will consider and we will add an edge with 6. So, 6 and 4 will be placed and now, we are going to remove these 2 notes from this particular list. So, the remaining elements in list L, let us say, the label number 5 is not present in 6 and 1. So, 5 will be chosen up and the first element in S is 6 both will be joining with an edge and now, they will be removed then we are going to consider the label number 6, 6 is not present in S.

So, 6 and 1; they will get an edge. So, 6 will be removed and 1 also will be removed. Now finally, we are left with 1 and 7. So, 1 and 7 will be placed an edge and we terminate. So, when we terminate we see that, we have generated out of S we have produced a tree. So, given a Prufer code S, we are we have produced a tree corresponding tree , that we have already seen in the in the previous proof that, if let us say a tree is given how we are going to produce a Prufer code and now, we have seen given a Prufer code we have to generate we can generate a unique tree out of that code, that becomes a Cayley's formula.

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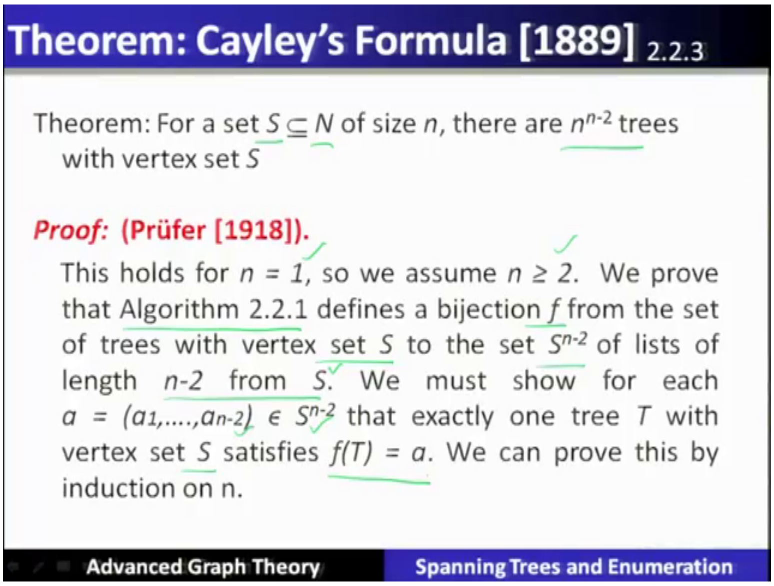

So, Cayley's theorem says that for a given set S, which is basically of containing the elements from that label n. So, there are n raise to power n minus 2 different trees let us see the proof of this particular theorem. Now, this will hold for the number of vertices is 1. So, let us assume that n is greater than 2. So now, we will prove that this particular algorithm defines the bijection the algorithm which, we have seen earlier that will define a bijection function f from this set of tress with the vertex set, which is given as S to that particular set S raise to power n minus 2 of the list of the length n minus 2 from S, that we have already seen that, if let us say, tree is given how we can generate a Prufer sequence of length and minus 1? So, we must show that for each Prufer sequence a 1 to a n minus 2, which is a member of this S raise to power n minus 2, that exactly only one tree with that vertex set satisfies this particular mapping function f and we can prove this by the induction on n.

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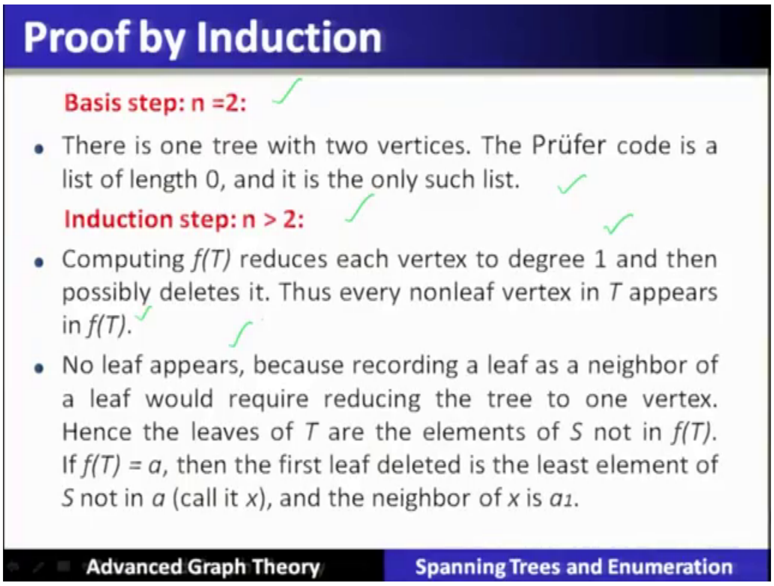

Let us take the induction on n, when n is equal to 2 there is only one tree with the vertices the Prufer code is a length n minus 2, that is length 0 and it is the only such list. When induction hypothesis, let us assume n is greater than 2 now, computing f T will reduce each vertex to a degree one; that means, degree of T minus 1 and then possibly it will it will get deleted, thus every non-leaf vertex in T appears in this particular f of T, that is in the Prufer sequence.

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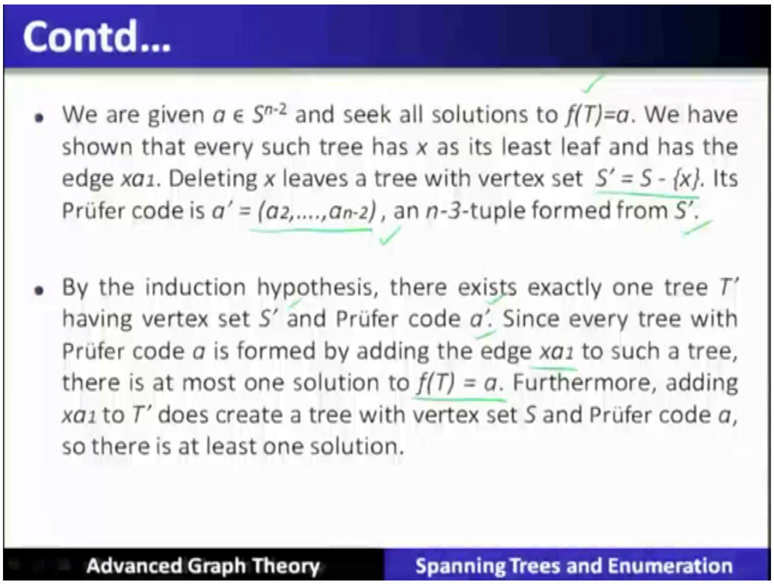

Now, here we also see that there is no leaf appears in S because, recording of the leaf as a neighbor of a leaf would require reducing the tree to one vertex. Hence, the leafs of tree are the elements of S not in f T. So, if f T is equal to a then the first leaf gets deleted is the least element of S, which is not in a call it as x and the neighbor of x is a1. Now, we are given a which is an element in S raise to power n minus 2 and we see all the solutions to this particular mapping of T to a and we have shown that, every such tree has x as it is least leaf and has the edge x a1 deleting x leafs a tree with the vertex set as prime, that we have already seen and it is Prufer code is a prime a 2 to a n minus 3 and n minus 3 tuple formed from S prime and so on. So, by the induction hypothesis there exist exactly 1 T tree prime having the vertex set S prime, that we have constructed and having the Prufer code a prime. Since, every tree with the Prufer code a is formed by adding an extra edge x a 1 to such a tree there is at most one solution to f T is equal to a furthermore adding x a1 to T prime does not create a tree with the vertex set x and the Prufer code a. So, there is at least one solution. So, that gets proved.

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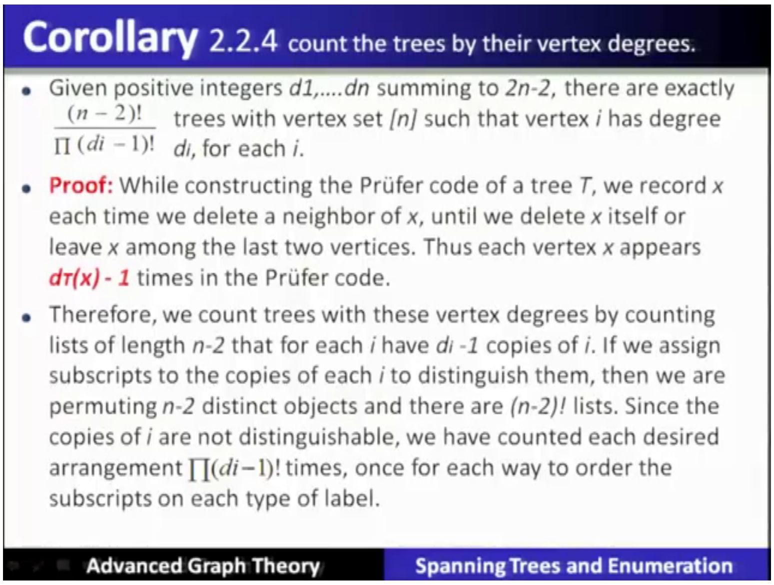

Now, corollary this particular corollary will count the trees by their vertex degrees. So, this is an application of a Cayley's or a Prufer code or Cayley's formula. So, given a positive integers d 1, d 2 and so on up to d n they are nothing but the degrees of all n vertices and if we sum up these degrees to n minus 2 n minus 2, there is exactly there is there are exactly n minus 2 factorial divided by d i minus 1 factorials the multiplication of all for all i s. Let us see the proof. So, while constructing the Prufer code of a tree T we record x each time we delete the neighbor of x until, we delete x itself or leave x among the last 2 vertices thus each vertex appears how many times? This degree of x minus 1 times in the Prufer code therefore, we count the trees with these vertex degrees by counting the list of length n minus 2, that is for each i have d i minus 1 different copies of i so; that means, a particular label is appearing d i minus 1 times. So, if we assign subscripts to this copies of i to distinguish them then we are permuting n minus 2 different distinct objects and there are n minus 2 factorial such list since, the copies of i are distinguishable we have counted each desired arrangement how many times d i minus 1 factorial and multiply for all different i s once each way to order the subscript on each type of label.

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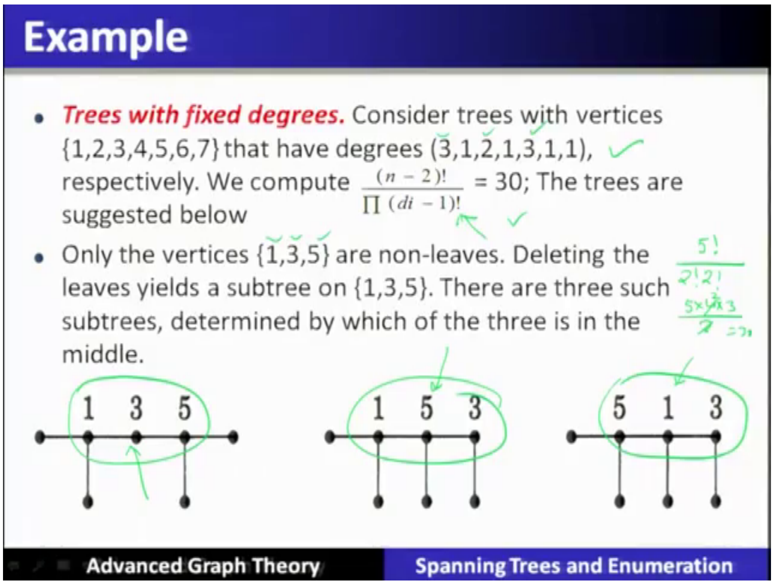

Let us see through an example. So, let us see the we are given the trees with the fixed degrees consider tree with the vertices 1, 2, 3 and so on up to 7 different vertices and the degrees of these vertices are given as 3 1 2 1 3 1 1 respectively. So, let us use this particular formula to find out how many different trees are there? So, that will be n minus 2 factorial divided by all the degrees. So, we can use in this particular equation if we apply. So, n means 7, 7 minus 2 is 5 factorial divided by all these degrees minus 1. So, first one is 3 minus 1 2 factorial then, is 0 factorial, 1 factorial, 0 one and again 2 factorial. So, 5 factorial divided by that becomes 30.

So, 30 different trees are possible. Now, out of these 30 trees we can see here that, only the vertices 1 3 and 5 1 3 and 5 they are non leaves they will be the internal notes. So, the deleting leaves will yield the tree on 1 3 5 and there are 3 such subtrees possible where, in the first tree we see that, in the center the 3 is there in the other one the center is 5 and the third one the center is 1, there are 3 different possibilities where, in these 5 1 3 and becomes the internal notes.

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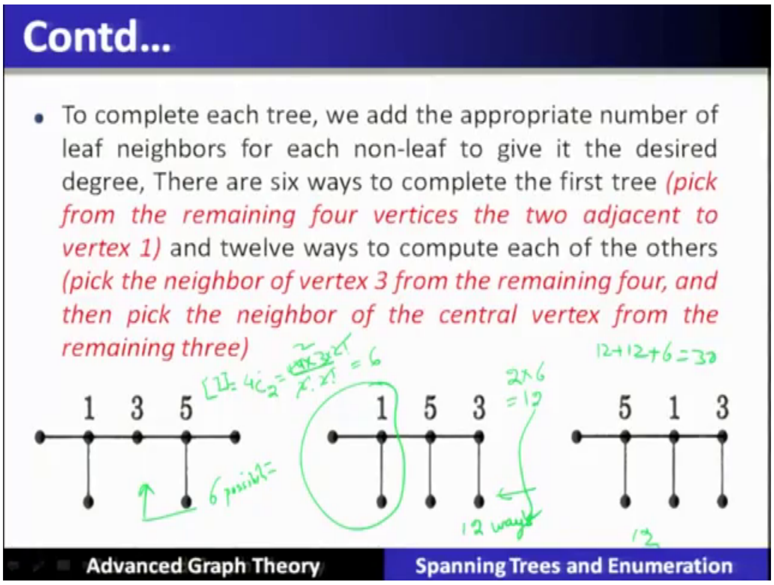

So, let us count them there are 3 such subtrees by which, 3 is in middle and so on to complete each tree we add the appropriate number of leaf notes leaf neighbors to each of these non-leaf to give it the desired degree. So, there are 6 different ways to complete the first tree here. So, so 3 different labels are already used how many more are there? 4 different labels are remaining out of this particular 7 labels because, 3 are already used. So, 4 are remaining so; that means, if we choose out of 4 these 2 labels, which are going to be assigned to 1.

So, in turn it will and then 2 will be attached to the 5. So, 4 C 2 that becomes, so that becomes 6 different possibilities of this particular tree. Similarly, as per as this is concerned there are 12 possible ways here in this particular case, why? Because, 6 is choosing this possibilities now, both 2 the remaining 2 notes the remaining 2 different labels are not going to be attached to 5, but they are being distributed to 5 and 3. So, they are how many possibilities 2 possibilities are there 2 into 6 there are 12 different possibilities. Similarly, here, it is also having same thing that is 12 possibilities. So, total possibilities 12 plus 12 plus 6, that becomes 30. So, 30 different trees are being generated by this way.

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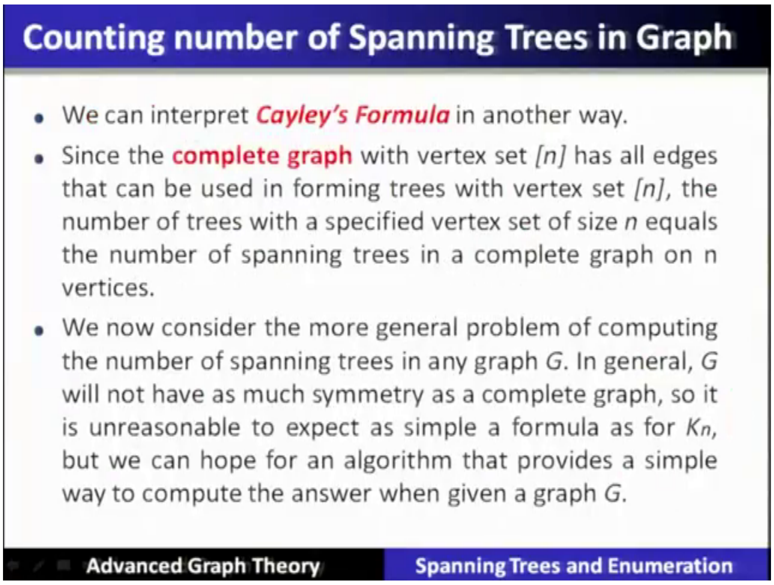

Now, counting the number of spanning trees in a graph now, we can interpret the Cayley's formula in another way since, the complete graph with the vertex set n has all the edges, that can be used in forming the trees with the vertex set n the number of trees with the specified vertex set of size n equal the number of spanning trees in the complete graph on n vertices. Now, consider more general problem of computing the number wise spanning trees in a graph in general G will not have much symmetry as a complete graph. So, it is unreasonable to expect a simple formula as K n, but we hope for an algorithm that provide the simple way to compute the answer when a given graph is G.

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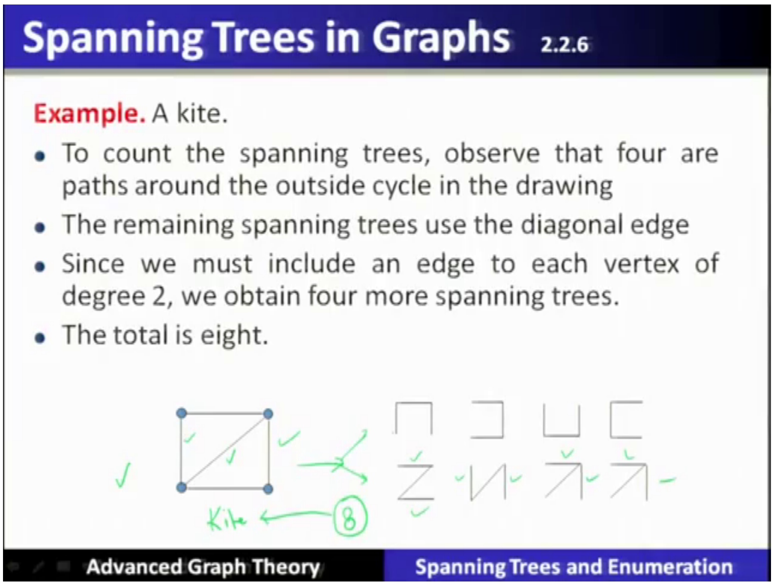

Let us, see in this particular example the same thing. So, if the graph is not complete graph let us, say a kite is given. So, this graph is called a kite the name of this graph is kite, kite is given and now we have to count how many spanning trees are there? Now, we if we observe that how many spanning tree? That means; we can form this spanning tree without this diagonal edge and we can form the spanning tree with diagonal edge there are only 2 possibilities.

So, if we form the spanning trees without this diagonal edge how many different possibilities are there? So; that means, these particular vertices are going to be attached with the edge an one set of vertices will not have an edge. So, there are only 4 different possibilities. Now, when the spanning trees when where the diagonal is used, that also is having 4 possibilities, why because, there are 2 ends who are not going to get the edges. So, you see that who are going to get the edges. So, this is one possibility, this is another possibility, this is another and so on. So, there are 4 different possibilities total 8 different spanning trees is possible from this particular graph, which is called a kite.

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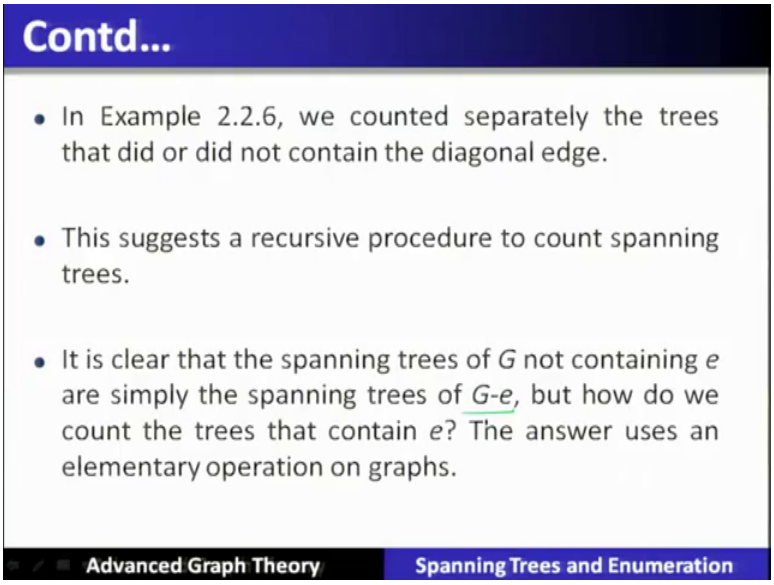

So, let us see in this example how using a systematic formula we are we can basically do this. So, it is clear that the spanning trees of graph G not containing e are simply the spanning trees of G minus e; that means, if you remove that particular edge that, is nothing but a diagonal edge and when we basically include, that diagonal edge, so there are only 2 possibilities and we can count these 2 possibilities 4 and 4 that becomes 8.

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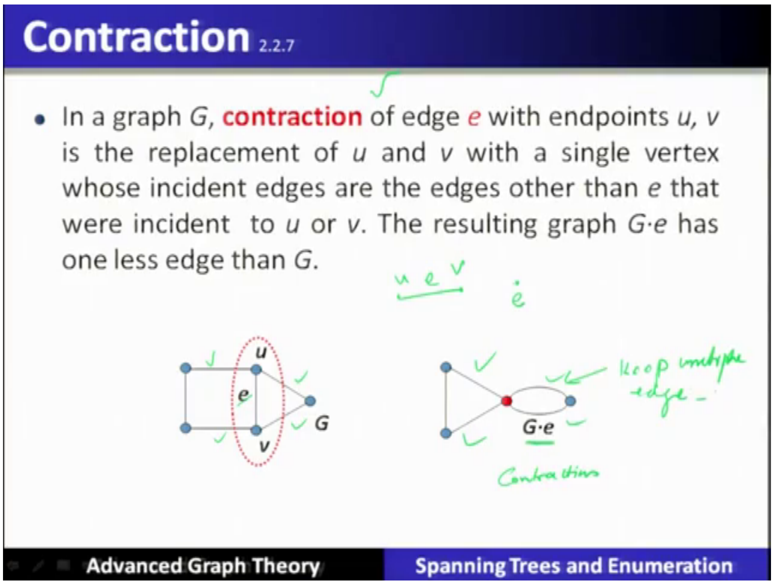

Now, we for that we are going to define an operation, which is called a contraction of an edge e having the end point u v. So, we can contract means, we can join on a same what on a same end and basically this edge is will be removed. So, for example, if we contract u and v as the same note all other edges will be preserved you can see all the other edges are preserved and this particular edge will be removed. So, this becomes G minus e G dot e, why? Because, it is a contraction we have not removed, but it is a contraction; that means, that edge is basically contracted this is called contraction operation.

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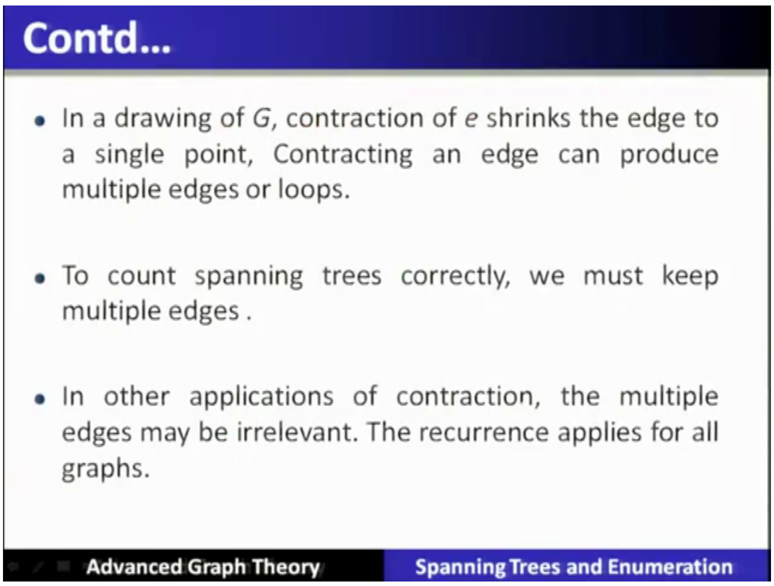

So, to count the spanning tree correctly we must keep the multiple edges like here, this multiple keep the multiple edges resulting out of that contraction.

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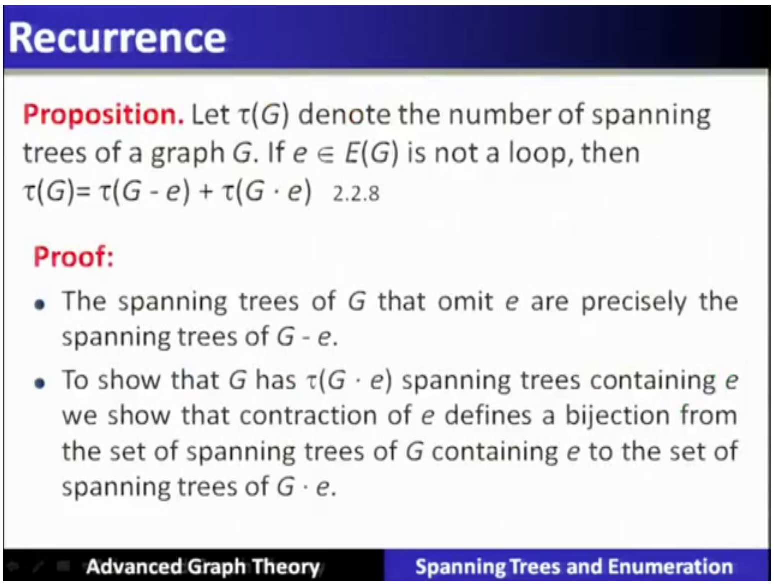

So, with this we have basically come up with a recurrence equation and this recurrence equation will help us in finding out how many different spanning trees will be there for a given general graph G

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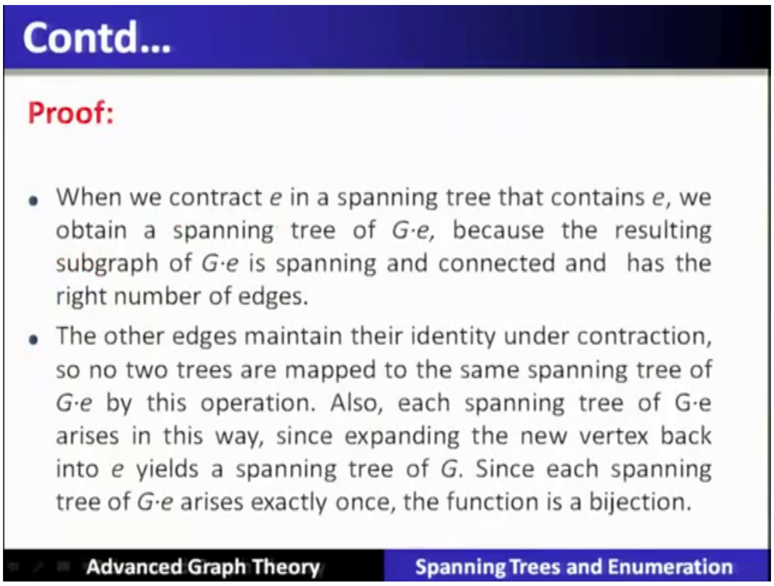

So, let us see the recurrence equation; that means, if we remove that particular middle edge e that, we have seen the diagonal edge when it is removed we have to count how many are there? And then when we contract it then how many are there? And some of them will become that, total number of different spanning trees.

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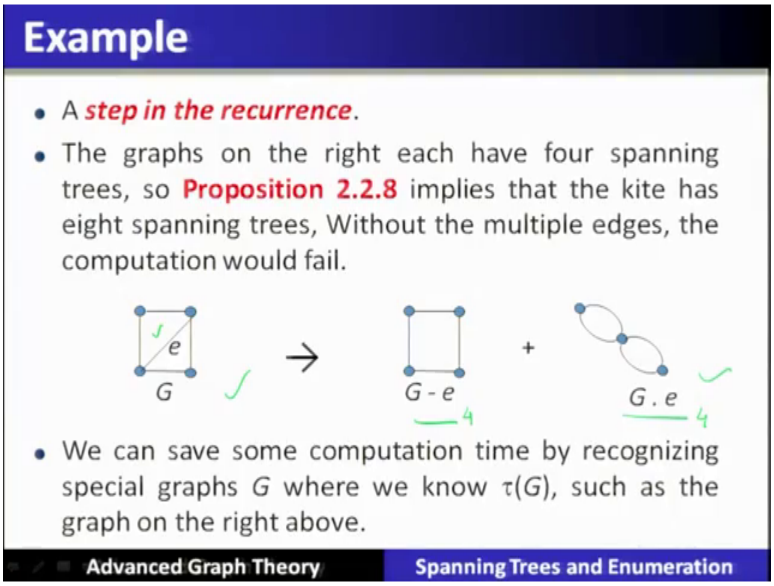

So, let us take this particular example. So, if this is the graph which is a kite graph. So, when we remove this particular edge. So, this will be the graph without e and then this will be when we contract then, this will be graph when we have contracted the edge and now, if we count how many are the possibilities in this particular graph G minus e? We have seen there are 4 possibilities and how many possibilities are there here? Again 4 4 plus 4 will become 8.

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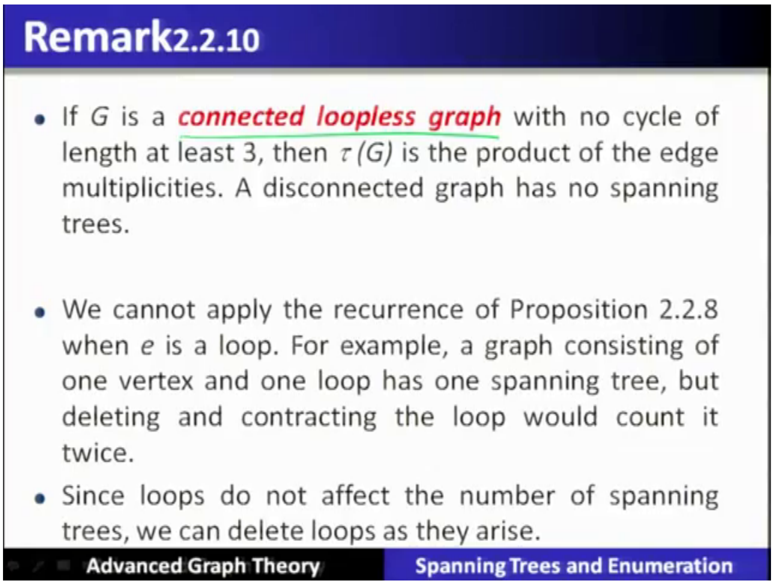

So, basically here, the importance of the loop is not going to effect the number of spanning trees. So, if we can delete the loop, which is there in before we start then, also the number of spanning trees count will be correct. So, that is why? We consider always the connected loop less graph for computing the recurrence equation.

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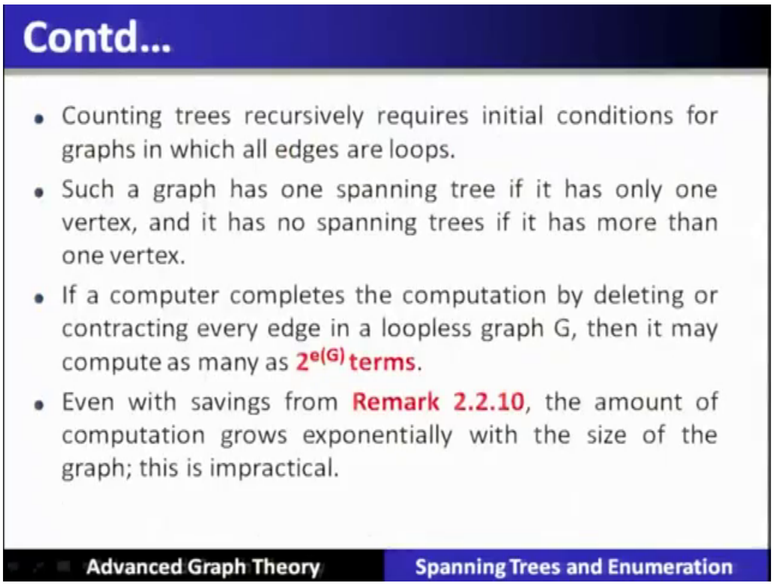

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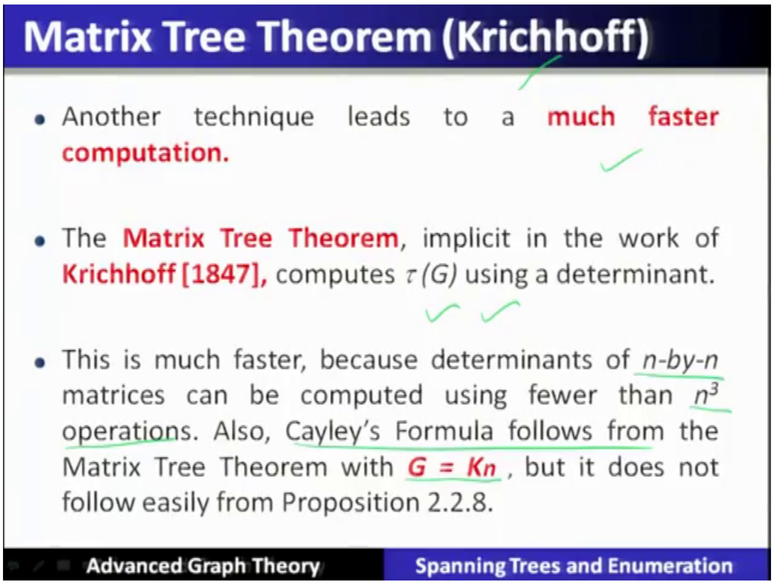

Now, we are going to see another very generic technique, and which is must faster in compare to the previous once and this method is called a matrix tree theorem or it is given by the Kirchhoff. So, basically for a given graph instead of computing the recurrence equation, we will apply this matrix tree theorem and this particular algorithm, which basically uses to compute the determinant and this computation of determinant of the adjacency matrix is basically quite efficient hence, this particular algorithm will compute the number of spanning trees for a very large values of n a much faster method. So, let us see how this all can be done.

So, this matrix 3 theorem is much faster because, the determinants of n by n matrix can be computed using fewer operation than n cube operation compare to the previous methods also the Cayleys formula follows from the matrix tree theorem, when G is equal to Kn, but it does not follow easily from this preposition that we have seen earlier.

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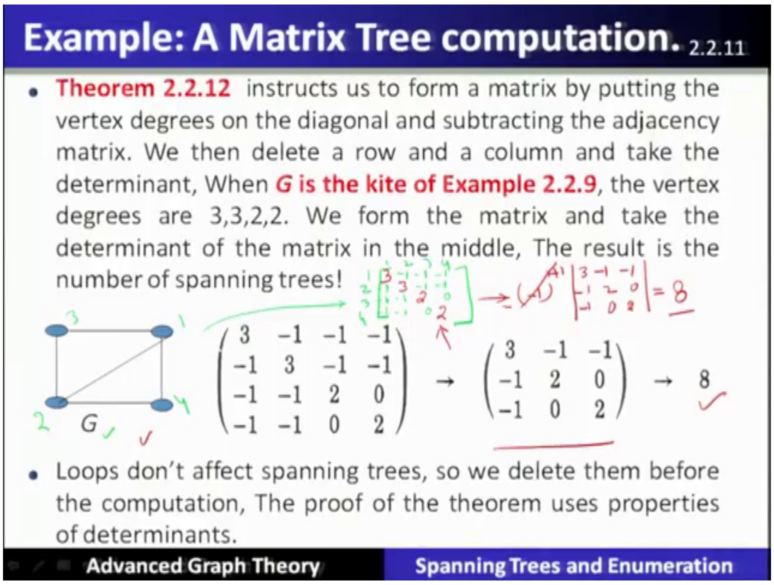

Let us take the example. So, in this example how this particular method will quickly compute that, we are going to see. Now, let us see that this particular graph is given and we want to find out how many different spanning trees are there? We have already seen using the recurrence now, we have to see this particular matrix tree computation how it does it? So, what we have to do? We have to find out first we have to draw a graph some matrix, that is called adjacency matrix of this particular graph and let us take this particular vertex as 1.

So, here this particular vertex 1 2, then 3 and 4 these are number 1 2 3 4 1 2 3 4. Now, we will fill it up with the adjacency equation; that means, whenever there is an edge we will place 1 if it is not then 0. So, between 1 and 2 we are going to place 1 1 and 4 we are going to place 1 1 and 3 we are going to place 1 and this becomes 0. Similarly, as per as 2 is concerned 2 and 3, 2 and 3 there is an edge 2 and 4 there is an edge 2 and 1 there is an edge 2 and 2 there is no edge. Now, as far as 3 is concerned 3 and 1 is an edge 3 and 2 there is an edge and all other is not having an edge.

Now, note number 4 2 is having an edge, 1 is having an edge and all others are not having an edge. So, this is the adjacency matrix of this particular graph. Now, what we are going to do is? We have to multiply these values by minus 1. So, they becomes minus wherever this symbols are there they becomes, minus next thing is we are going to instead of 0s in the diagonal we have to place the degrees of that particular note. So, for example, the degree of 1 is 3. So, in the diagonal instead of 0 we will write down 3 over here, similarly as per as the degree of note 2 is 3. So, instead of 0, we are going to basically put 3 similarly, the degree of 3 is 2 instead of 0 we are going to put 2 and similarly, the degree of 4 is 2.

So, instead of 0 we are going to place 2 out of it. Now, then we are going to compute the determinant or a cofactor of this particular matrix. So, this particular way for example, minus 1 1 plus 1 and then, basically we have to find out this value, that is 3 minus minus 1 2 0 minus 1 0 2 we have to compute this value now, this becomes plus 1. So, that will go out now as per as this is concerned this you can compute easily, and that will be basically the value becomes 8. So, 8 we have seen this particular graph as given 8 using recurrence.

Now, this is very quiet efficient why because every big graph also we can convert in a form of the adjacency matrix and then, we will multiply by minus 1 and then, all the different degrees of the vertices are being placed here on the diagonal and then, we will compute the determinant or of this particular matrix, and that determinant will give the number of trees possible here, in that particular graph and that is called matrix tree computation.

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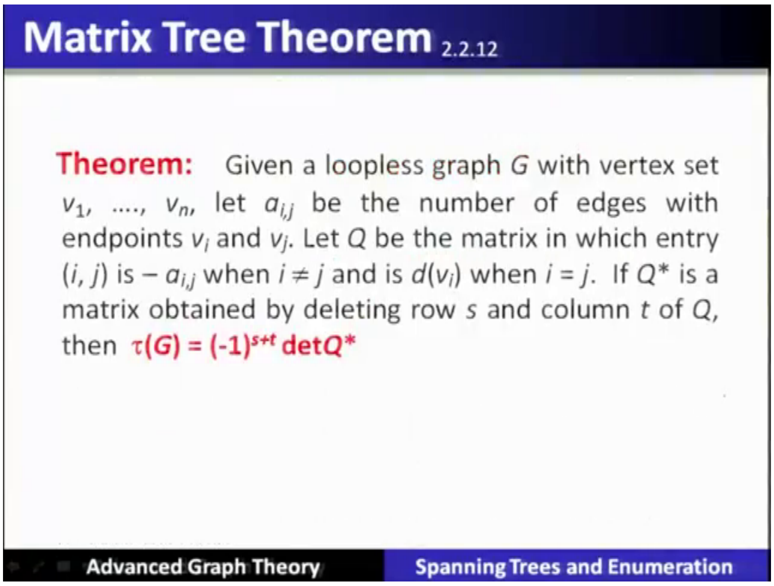

So, let us see the theorem statement of a matrix tree theorem says that, given a loopless graph G with a vertex set v 1 to v n, let a i j be the number of edges with the end point v i and v j let Q be the matrix in, which the entry of i j is minus of a i j when i is not equal to j; that means, there is an edge. So, we have to put minus of a i j and whenever, i is equal to j; that means, the diagonal element that we have talking about there, we have to place the degree of that particular vertex. Now, if Q star is a matrix which is obtained by deleting the row and the column of T of Q that we have already seen; so, we are going to get this cofactor that is minus 1 times S plus T that, we have already done types the determinant of Q.

So, this is how we are going to establish the matrix tree formula, matrix tree theorem? And we can solve this particular problem of finding the spanning tree on a given graph very efficiently using this particular matrix tree theorem.

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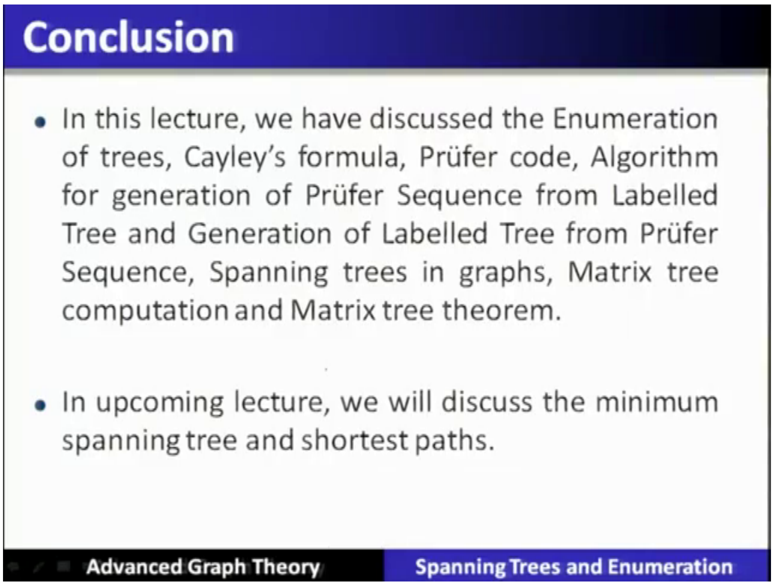

Conclusion, in this lecture we have discussed the enumeration of trees Cayleys formula, Prufer code algorithm for generating the Prufer sequence from Labeled tree, generation of Labeled tree from the Prufer sequence spanning trees in the graph matrix tree, computation matrix tree theorem. In upcoming lectures, we will discuss minimum spanning tree and shortest path.

Thank you.