Advanced Graph Theory

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Lecture – 14
K-Connected Graphs

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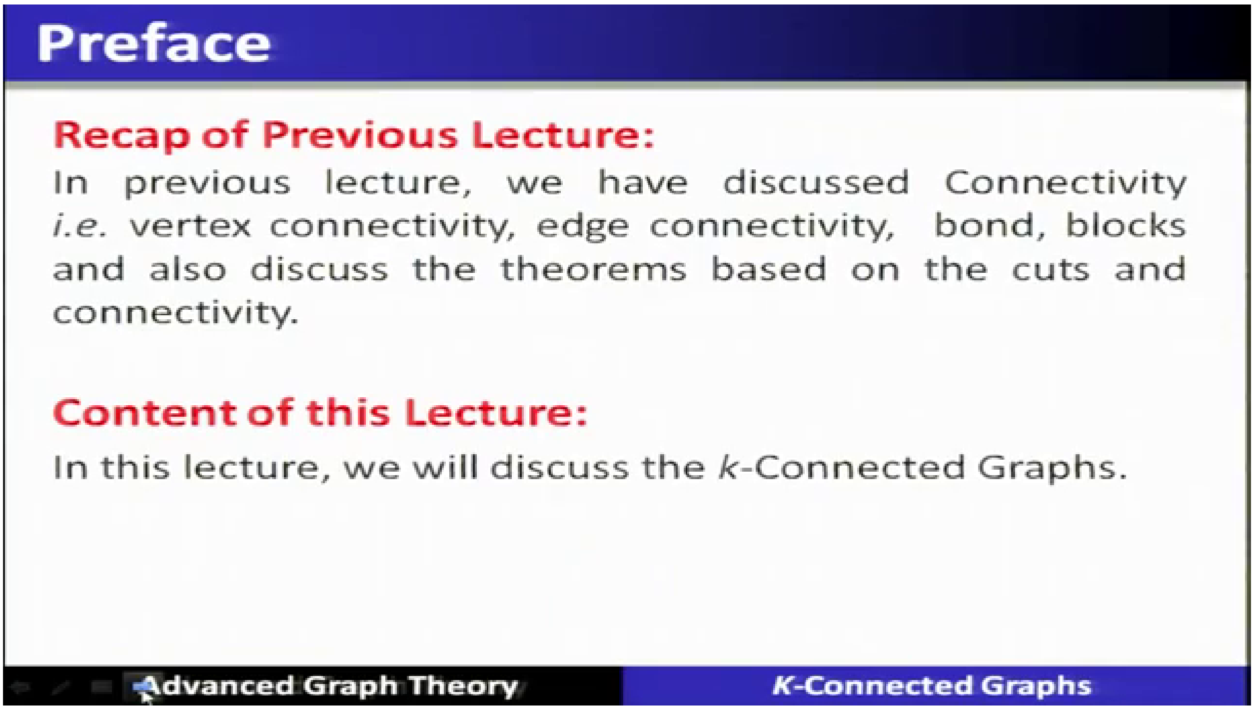

K Connected Graphs. Recap of previous lecture we have discussed connectivity that is what is connectivity, edge connectivity, we have also covered bonds blocks, and discuss the theorems are based on cuts and connectivity, content of this lecture.

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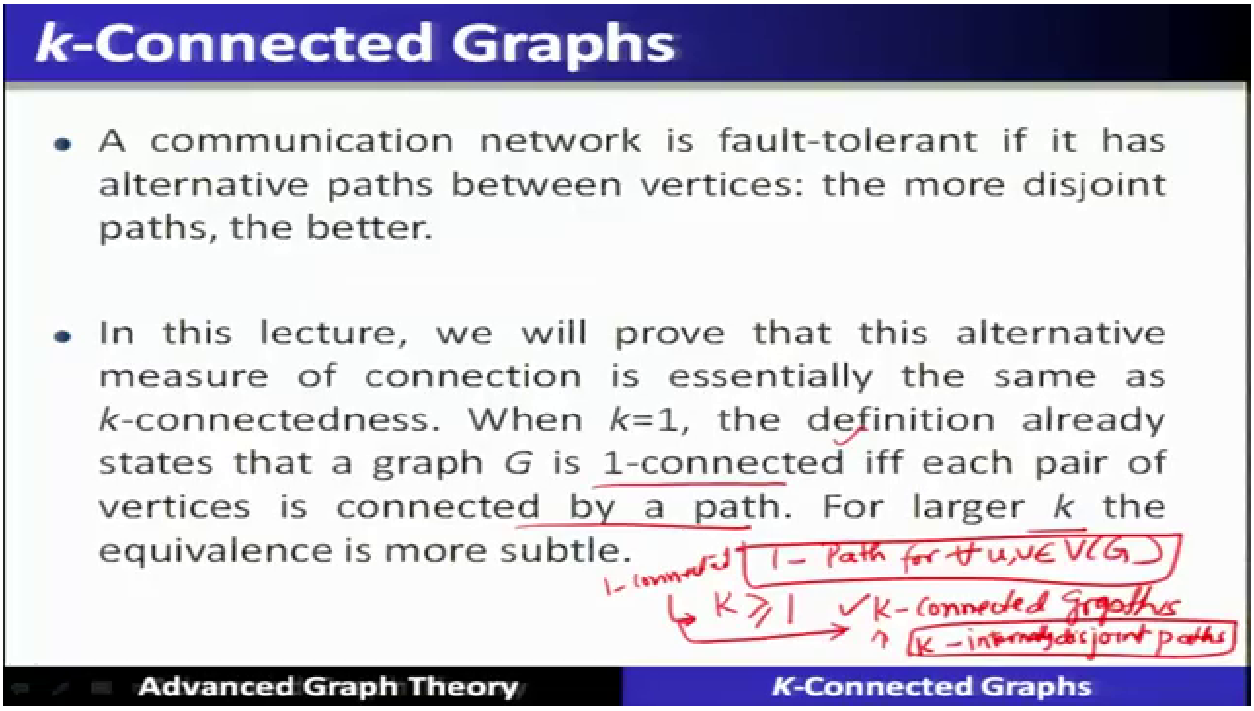

We will discuss k connected graphs, k connected graphs [noise] are used to ensure the fault tolerance in the communication network or similar such applications. [vocalized-noise]

So, a communication network is fault-tolerant out, and it has alternative paths between [noise] vertices, the more disjoint paths [noise] the better the network is in terms of reliability so, but it requires the extra redundancy in terms of the vertices, and the paths that we will see, [vocalized-noise] in this lecture we will prove [noise] that this alternative measure of connection is essentially the same as k connectedness. So, k connectedness will basically bring up [noise] the fault tolerance in the communication network graph or similar such applications.

Now, when k becomes 1 that is [noise] 1 connected graphs, we have already discussed. So, a graph is 1 connected if and only if each pair of vertices is connected by a path that definition we have already seen the connectedness. So, the if the entire graph is connected between any 2 pair of vertices, if there is a path then we can generalize this [noise] 1 connectedness in terms of k a larger value of k; that means, k is greater than or equal to 1 than [noise] we define [noise] k connected graphs. [noise]

So, like in 1 connected graph between any pair of vertices there is a path, [noise] there is 1 path for every [noise] pair of vertices [noise] in the graph [noise] that we have seen for [noise] 1 connected [noise] graph, we can extend it to k connected graphs, thus [noise] we can see how we can ensure k different paths that is vertex [noise] disjoint, [noise] or internally disjoint. [noise] Hence [noise] the discussion of k connected graphs for a larger value of k is more subtle, [noise] and these intricacies we will go and discuss in this particular lecture.

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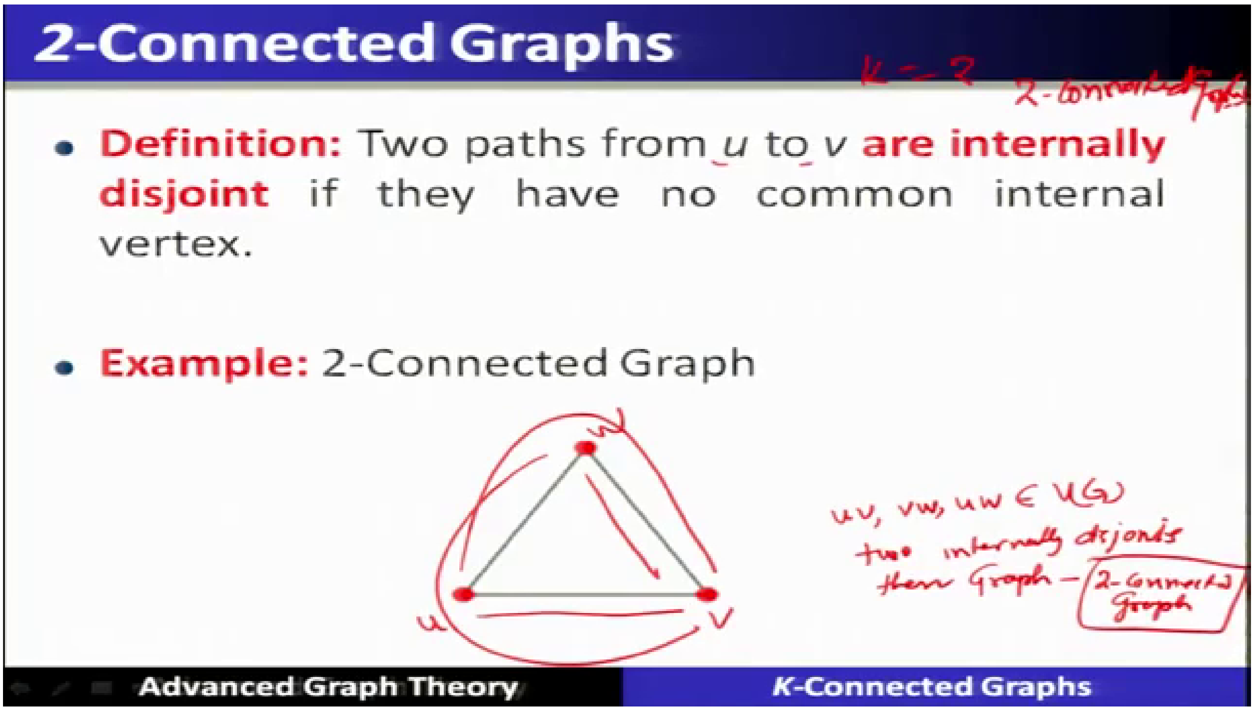

So, the starting with a more value of k that is more than 1 that is let us start with the 2 connected graphs. So, when k is equal to 2. So, it becomes [noise] 2 connected [noise] graphs [vocalized-noise] the definition says that two path from u to v are internally disjoint, if they have no common internal vertex. So, this definition is to provide the connectivity between a pair of vertices let us say u and v [noise] in a graph. So, we can see in this particular diagram that for u and v we have two different paths, 1 is from u to v the other is again internally disjoint two different paths are connecting u and v.

Similarly, if let us say this particular vertex is w [vocalized-noise] if w and v also can be connected with 2 internally disjoint paths. Similarly u and w also it can be connected so; that means, for every pair of vertices [noise] for every pair of vertices, [noise] if we can show that there exist two paths [noise] two [noise] internally disjoint [noise] paths, then for the entire graph G we can say that it is [noise] 2 connected [noise] graph. [noise]

So, hence the connectivity is very important. So, as we increase the value of k we have to ensure that that many number of internally disjoint paths exist between any 2 pair of vertices, then only the entire graph will take this particular property of that value of k connectedness, here in this example we have seen 2 connected graphs [noise] [vocalized-noise] with this particular [noise] simple example, we will go ahead and characterize 2 connected graphs later on. [vocalized-noise]

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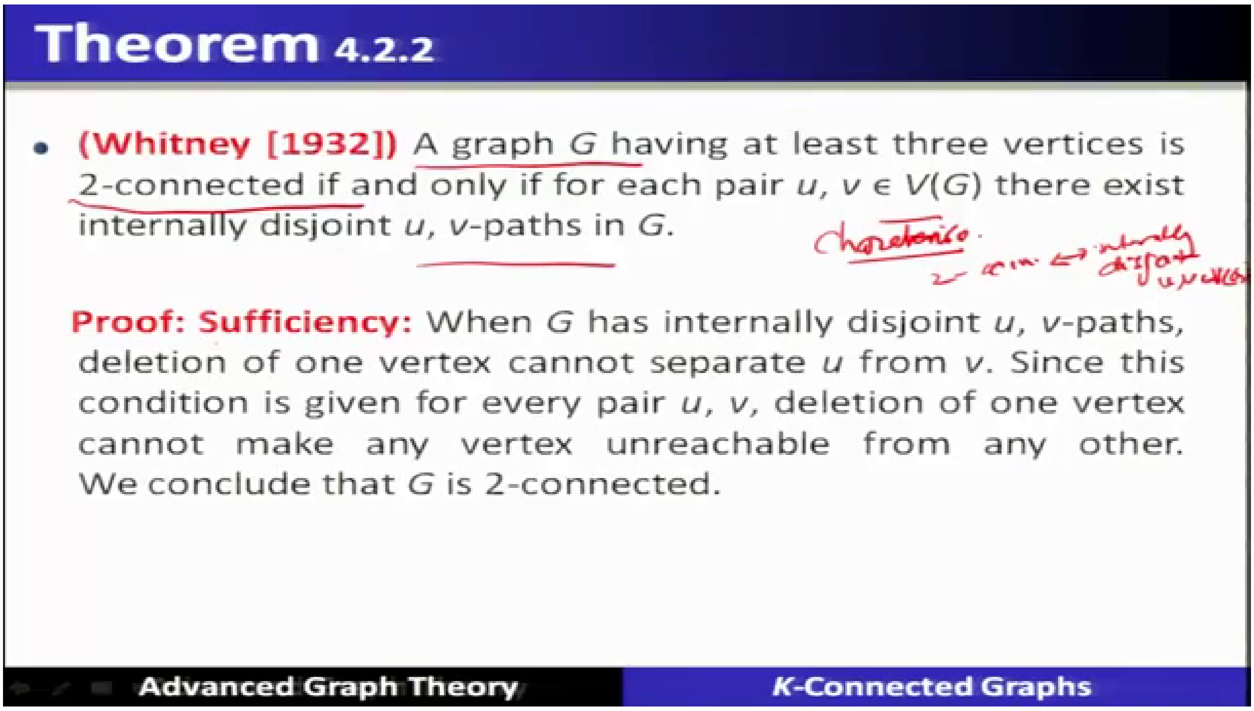

Whitney in 1932 has given a theorem for 2 connected graphs characterization, [noise] So, theorem is stated as a graph G having at least three vertices is 2 connected, if and only for each pair u v of a vertex set of a graph, there exist internally disjoint u v paths in G Whitney in 1932 has given this particular theorem, this is this will characterize the 2 connected graph; that means, 2 connected graphs for a graph G exist, if and only if each pair of this particular graph has internally disjoint u v pair of [noise] paths is totally disjoint here there are two different paths we are talking about.

Let us prove this theorem [noise] which will characterize [noise] [vocalized-noise] or we will make equivalent statement that to connected [vocalized-noise] graph means that they are exist internally disjoint u v path between any 2 pair of vertices of that particular graph. Let us see the proof [vocalized-noise] first we will see the sufficiency [noise] condition.

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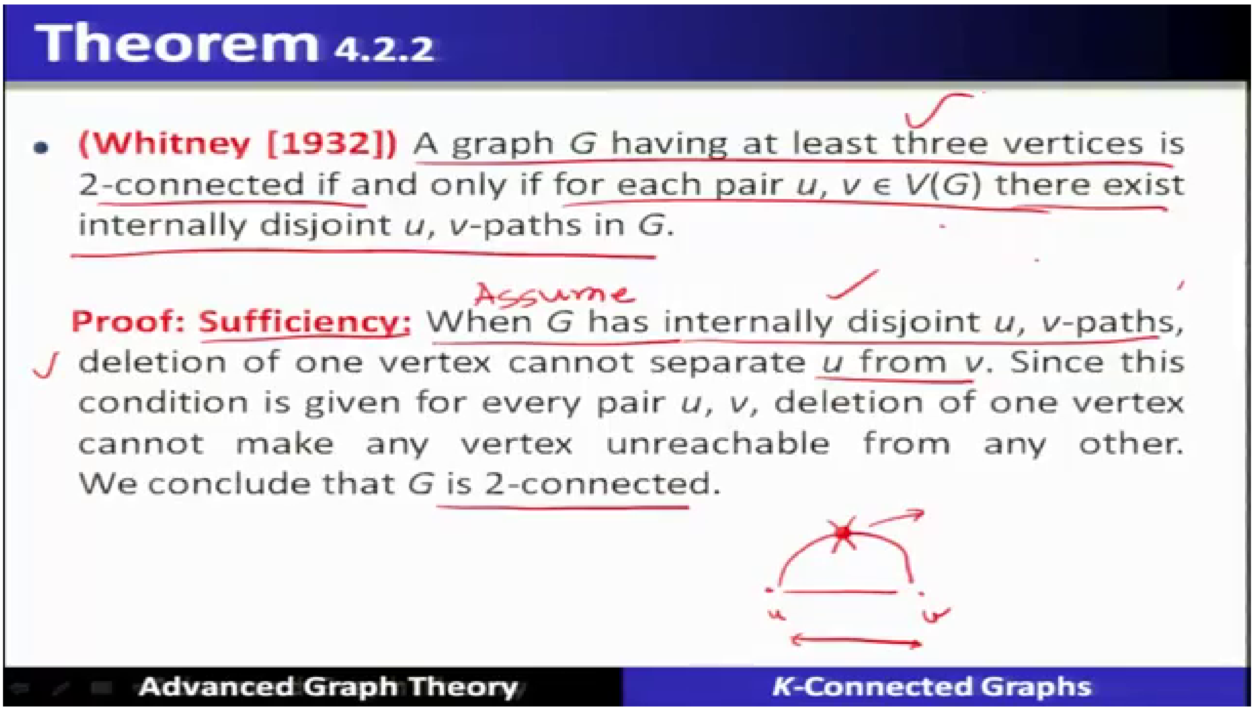

So, in sufficiency condition let us assume that there exist internally disjoint u v paths. [noise] Let us assume [noise] or we are given this [noise] that G has internally disjoint u v path so this part.

[vocalized-noise] So, now we can see that if we delete one vertex then [noise] we cannot separate u from v, [vocalized-noise] that means u and v [noise] they are internally disjoint paths, if one vertex is deleted or removed there will be an alternative path between u and v. So, u and we will not be disconnected, if we remove it since this condition is given for every pair [noise] u v. So, deletion of one vertex cannot make any vertex unreachable from each other, hence we conclude that the graph is to connect it [noise] given that G has the internally disjoint u v path between every two pair of vertices.

Now, we will see the necessary condition. So, necessary condition we will see that [noise] we will assume that a graph [noise] G is 2 connected and we have to prove that it, then that graph there exist internally disjoint u v paths for every pair of vertices.

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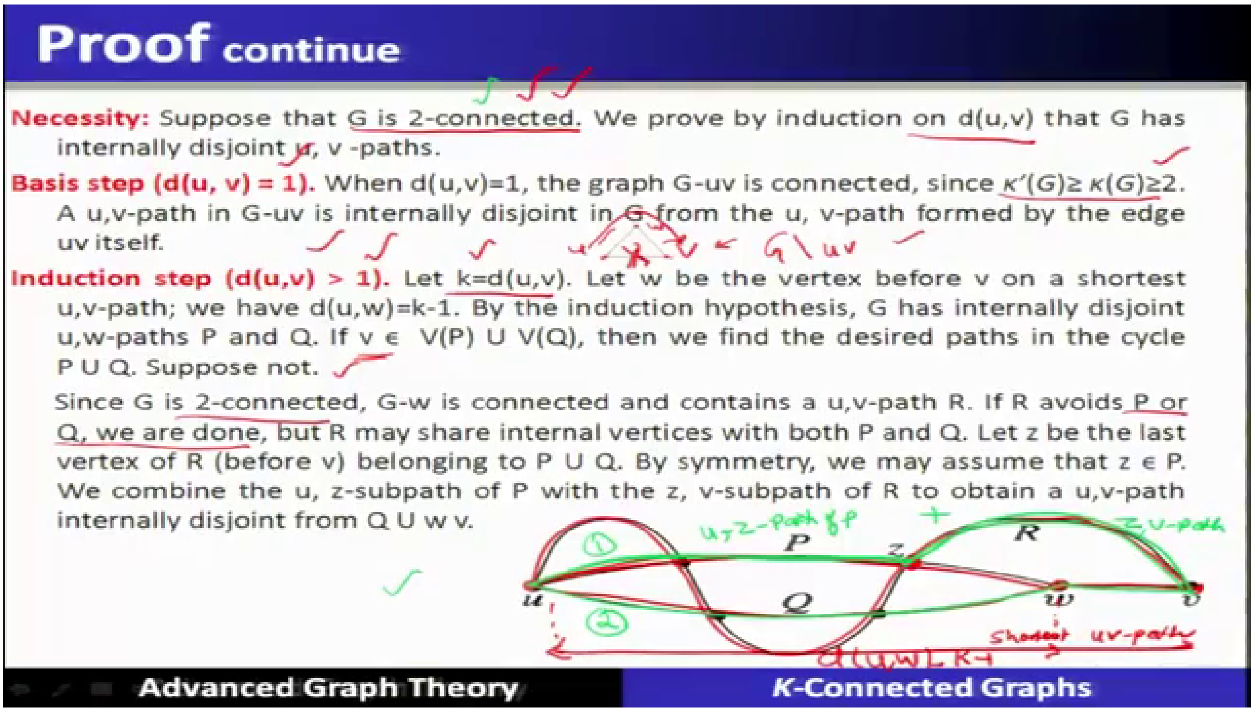

So, necessary condition we will assume that the graph is to connected, [vocalized-noise] to prove that the graph has internally disjoint paths [noise] between any pair of vertices u v between every pair of vertices u v, we will prove by induction on the distance between u v that G has internally disjoint u v paths.

So, the basis step [noise] assumes that the distance between u and v is 1. Now we have also seen that or we have already assume that the graph has at least three vertices. [vocalized-noise] So, if the distance between u and v is 1 there must exist another vertex, and if we delete this particular edge from the graph. So, the graph without u v will have another path. [noise] Hence the kappa prime is greater than kappa is greater than 2. So, the u v path in G minus u v is internally disjoint in G [noise] from u to v formed by an edge in u v itself. [noise] So, this particular base step is clear, because if we remove 1 particular edge even then these pair of vertices is connected through a internally disjoint paths which exist in the original graph G. [vocalized-noise]

Now we will [noise] see the induction step that D u v is greater than 1 assume that [noise] D u v is k; that means, and w be the vertex. [noise] [vocalized-noise] So, u and v they are basically separated by the distance k, and that [noise] is greater than 1. let us assume that there is the vertex [noise] w which is just before v on the shortest u v path, [noise] hence [noise] this part of the shortest path up to [noise] u to w will have the distance [noise] k minus 1 why because, w is closer to v. So, the remaining path will be having the distance of k minus 1. [noise]

[vocalized-noise] So, by induction hypothesis we can assume that this G has internally u w paths [noise] which are internally disjoint paths, internally disjoint u w paths let us call it as P U to w, and another path [noise] internally disjoint is let us say Q which will connect U to w having that distance k minus 1. Now if this particular vertex v is an element of or having an element v in it; that means, these path or these particular disjoint paths will also include v, then we can find the desired path in the cycle P union suppose it is not there in this particular example we are shown.

Then since G is to connected that we have assumed. So, G minus w is connected and contains ah u v path R, if R avoids P R Q, then we are done because this will be a one path w to v and u to w, we have disjoint paths. And another alternative path will be there R if it is not using if it avoids P or Q then we are done, but R means here by internal vertices with both P and Q let z be the the last vertex of our before b belonging to P union Q; that means, belonging to this particular cycle.

So, by symmetry we may assume that z is n P. So, we combine u z of P [noise] plus z v of R. [noise] So, if we combine them it will give the two internally disjoint paths, [noise] one is shown in this way the other is why a Q up to w and w to v, there are two internally disjoint paths this is one path, this is another [noise] path and no vertices is internally disjoint. Hence we have shown that if the graph is to connected, then there is we have shown that to the induction that for all values of 2 this will be internally disjoint paths exist in the graph.

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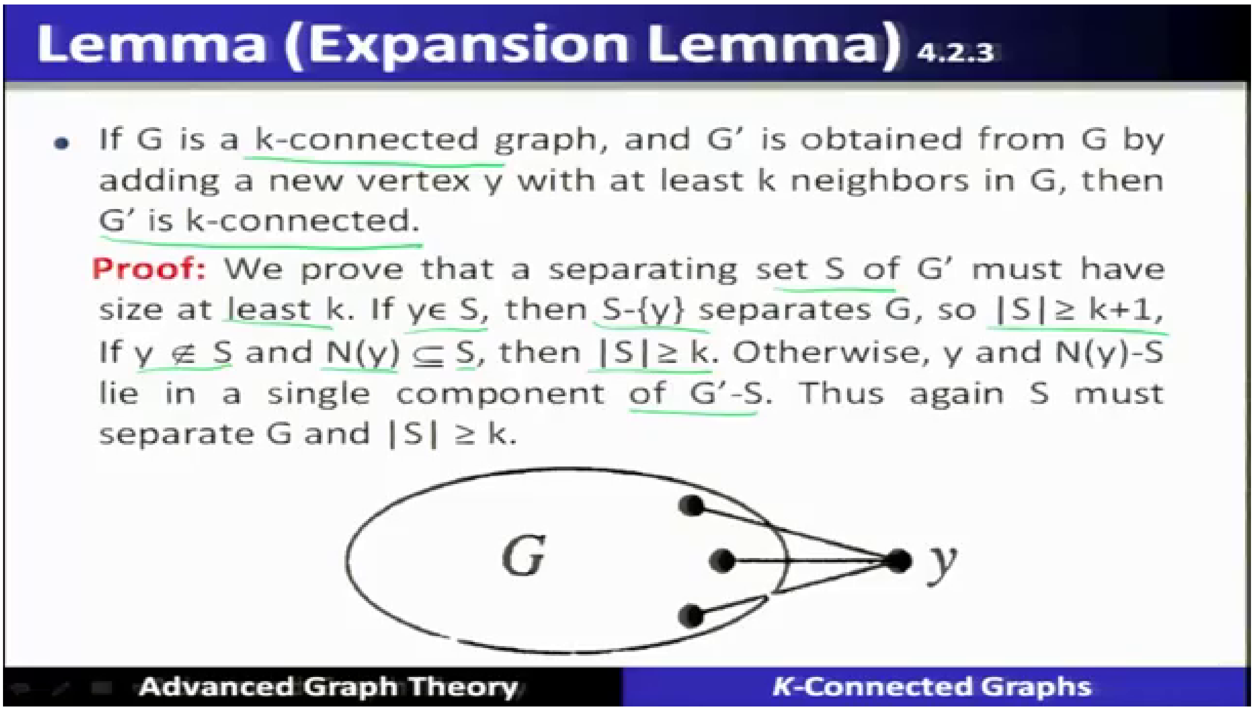

Expansion lemma if G is k connected graph, [noise] and G prime is obtained from G by [noise] adding a new vertex y with at least k neighbors in G, then [noise] G prime is k connected. [noise]

So, let us see the proof quickly will not go in more detail. So, in this particular proof we will see that [vocalized-noise] the separating set of G prime, must be have size at least k y because it is k connected graph. So, if y will be in that separating set, [vocalized-noise] then S minus y will separate G. So, the size of S will be k plus 1, if y is not in S and [noise] the neighbor of y is basically belongs to S, then S is at least k. Otherwise y [noise] and N y neighbor of y minus S lie in the same single component of G prime S, [noise] thus again S must be separate G and the separating set size is at least k hence it is k connected, hence G prime is also k connected.

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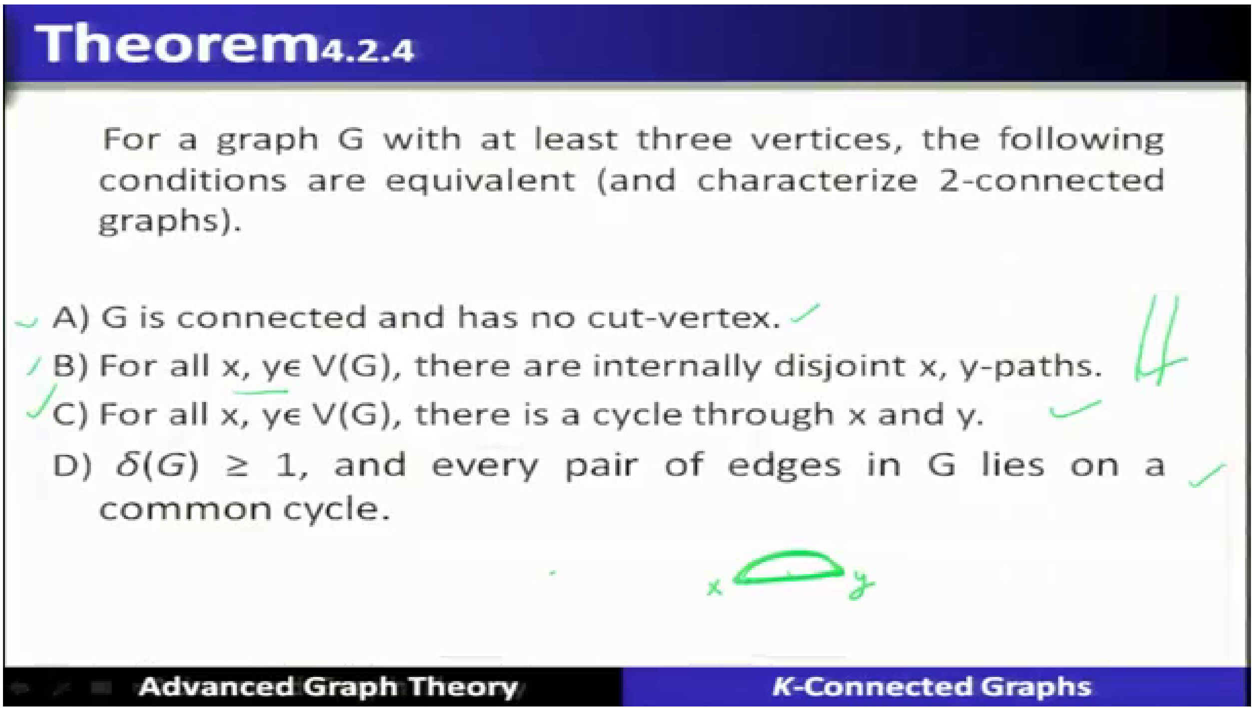

Now, we will see the theorem. So, this theorem will characterize to [vocalized-noise] connected graphs. So, for a graph G with at least three vertices the following conditions are equivalent, and characterize 2 connected graphs that is [noise] the first condition says that G is connected and has no cut vertex. The second condition says that for all x y pair of vertices which are there in the vertex set of G, there are internally disjoint x y path, for all x y there are vertices which are there in the vertex set of G there is a cycle, which passes through x and y. And finally, the last statement which will characterize to connected graphs, and equivalent to all the three is if little delta of G is greater than 1 greater than or equal to 1, and every pair of edges in G lies on a common cycle, then it will also characterize 2 connected graphs.

Let us see they are equivalence. So, A is equivalent to B that means, if a graph is connected and has no cut vertex, if a graph is connected and has is no cut vertex, this will be cut vertex if it has no cut vertex then there will be internally disjoint paths, hence from A we have proved the B and from B, if there is no if there are internally disjoint paths [noise] obviously, there will not be any cut vertex both are equivalent, then B and e C they are equivalent. So, B says that for all [noise] x y [noise] there are internally disjoint x y paths the two paths, and C says that for all x y there is a cycle so; that means, if this particular internally disjoint paths are there through passing through x and y, they will be forming a cycle through x and y. Hence B and C they are all [noise] equivalent that we have already seen.

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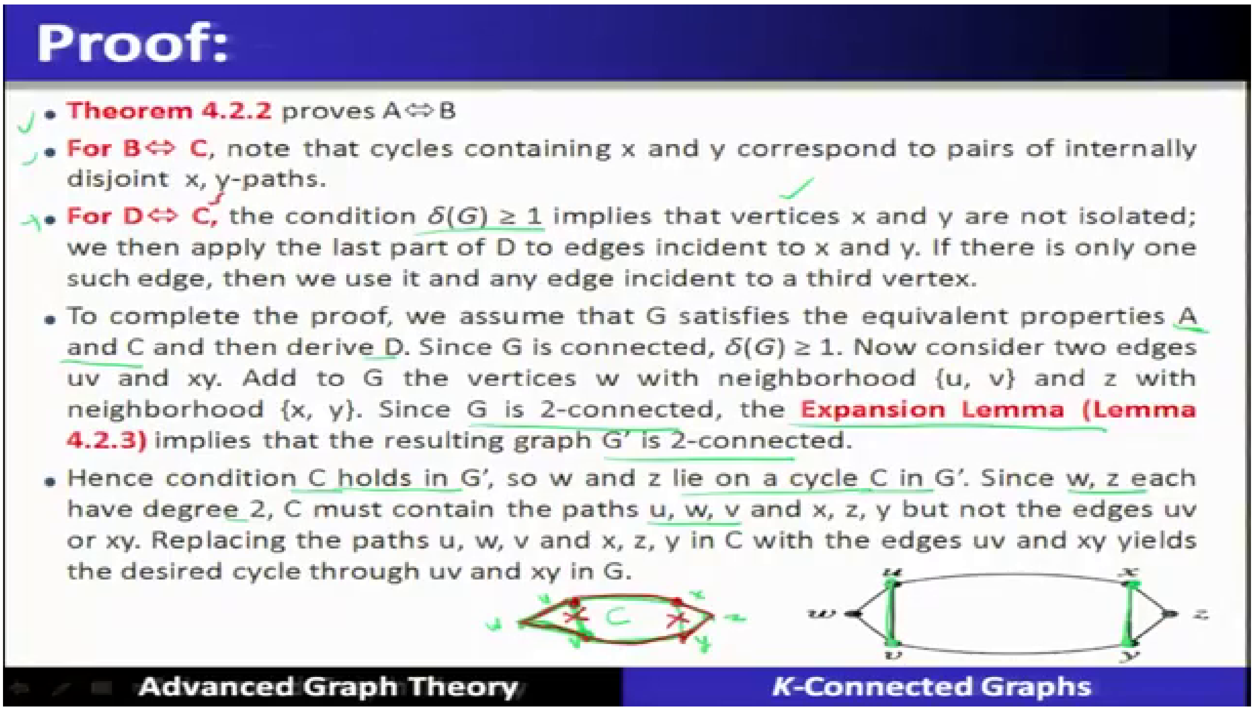

So, for equivalence from D to C let us assume that the condition little delta is greater than or equal to 1, which will imply that the vertices x and y are not isolated, we then apply the last part of D to the edges incident to x and y, if there is only 1 such edge then we use it and any edge incident to the third vertex to complete the proof we assume G satisfies the equivalence properties A and C, and then derive this particular D. Since G is connected little delta of G is greater than or equal to 1, now consider the 2 edges u v and x y 2 edges. Let us consider [vocalized-noise] so at 2 G the vertex w with the neighborhood of u v and z with the neighborhood of x y. Now since G is 2 connected by using the expansion lemma which will imply that the resulting graph G prime is also to connected.

Hence the condition C holds in G prime so w and z, they lie on a cycle and this cycle will be like this. Since w and z have the degrees 2. So, C must contains the path u w v, [vocalized-noise] and x z y and this will be added to the cycle C, but not the edges u [vocalized-noise] u v and x y. So, replacing u u v u w v, and x z y in C with the edges with the edges, u v and x y this will yield the desired cycle which is passing through u v and x and y. So, hence we have proved that if these conditions are given, then there exist cycle and hence from d v have proved the condition C which C says that there [noise] is a cycle which goes through x and y for all [noise] x and y pairs.

Hence all four conditions are equivalent as they characterized 2 connected graphs; that means, the 2 connected graph is a connected and has no cut vertex, a 2 connected graph for all x y pair of vertices of that particular graph, there are internally disjoint x y path into connected graphs [vocalized-noise] for every 2 connected graphs for all x y pair of vertices, there is a cycle which will pass through due to pair of vertices, and also to connected graph where little delta G is greater than or equal to 1, and every pair of edges in G [noise] lies on a common cycle. So, all four conditions characterizes the 2 connected graphs, and there all four conditions are equivalent, and we have stated that.

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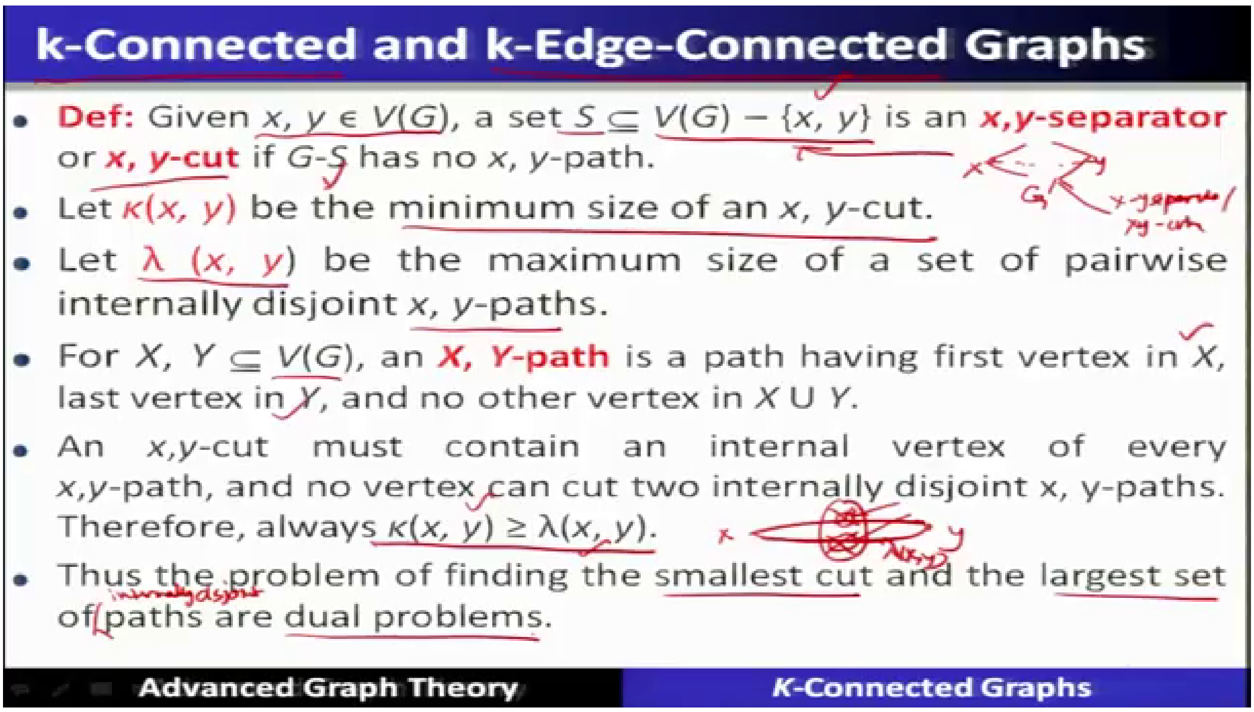

Now, we will go ahead about 2 connected sorry k connected graphs, and k edge connected graphs. So, we started with 2 connected graphs now we generalize the connectivity up to k that is it can be more than 2 also. So, k connected graphs, and k edge connected graphs. So, there are 2 different type of connectivity we are talking about when we say k connected graph is a k vertex connected, and k edge connected graphs.

So, let us see the few definitions. So, given x y pair of voltages in the graph G, the set S which is a subset of vertices minus x y is then x y separator or a x y cut if G minus S or G without S has no x y path take this example. So, x and y they are set of vertices which are there in G, they passes through a set of vertices called S, where S is without x y a subset of vertices which can be there. Now this is called x y separator or x y cut, if we remove [noise] x y so if we remove S from the graph. So, x and y will have no path to connect x y. So, x y becomes disconnected if S will be not present in the graph hence this is called the x y separator or x y cut.

[vocalized-noise] So, let kappa x y be the minimum size of this particular x y cut, [vocalized-noise] and lambda x y be the maximum size of [noise] the set of pair wise internally disjoint x y paths for x y a subset of vertex set of the graph G, and x y path is a path having the first vertex in [noise] the vertex set x. and the last vertex is in y, [vocalized-noise] and no other vertex in x union y exists, and x y cut must contain an internal vertex of every x y path, and no vertex can cut 2 internally disjoint x y paths therefore, always [noise] kappa x y [noise] is at least lambda of x y.

Again I am repeating. [vocalized-noise] So, far internally for two internally disjoint x y path, if we take out an vertex from this path do not disconnect y because it has internally disjoint path. Another vertex also [noise] if it is remove [vocalized-noise] together they will disconnect the graph, hence if let us say the graph has lambda different [noise] x y paths, then taking out vertex from each path will form [noise] the x y separator or x y cut hence the minimum size of x y cut must be at least the maximum number of internally disjoint paths between x and y, that is the problem of finding a smallest cut, and the largest set of internally disjoint paths are the dual problems that we are going to encounter here in k connected graphs.

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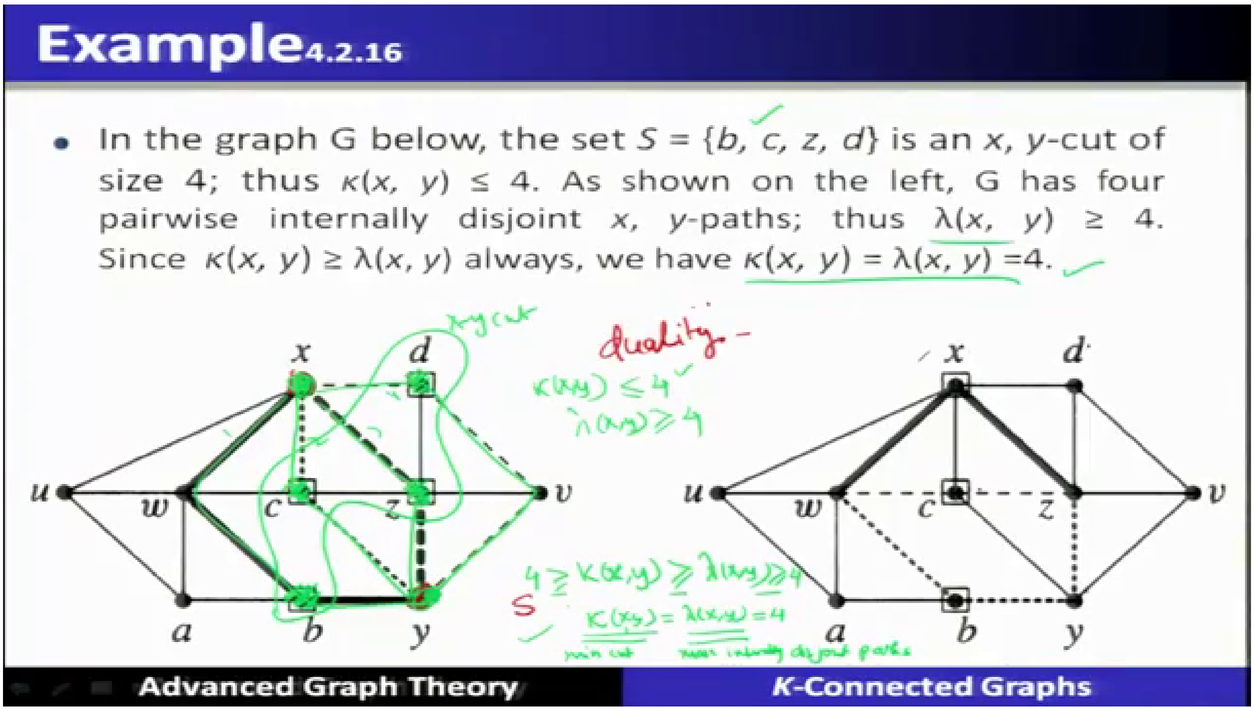

So, to illustrate through an example this particular concept of duality, [noise] we will see this particular example here, the separator for x y cut, this is x, and this is y. [vocalized-noise] We have to identify a separator, we will use a green ink for that separator vertices. [vocalized-noise] Now this S comprises of b is plugged out from this internally disjoint path, [noise] then C which is plugged out from this internally disjoint paths between x and y, then z which is plugged out from this internally disjoint paths [noise] between x and y, and d this plugged out from this internally disjoint path from x and y.

[vocalized-noise] So, four vertices which will form the separator [noise] or x y cut [noise] is being picked up [noise] 1 from every vertex, hence the kappa that is minimum of minimum size of x y [noise] cut is at most 4. [noise] Now we have also seen that these particular four vertices separator of size 4, we have taken out from 4 different pair of internally disjoint x y paths, 1 2 3 4 hence this particular lambda [noise] x y [noise] which is the maximum x y disjoint paths [noise] is basically at least 4. [noise]

Since we know that from the previous discussion [noise] kappa x y is basically at least lambda of x y, and kappa x y is basically 4, and lambda x y is at least [noise] four hence by taking up these all inequalities, we can conclude that kappa of x y [noise] is equal to lambda of x y, [noise] and that is equal to 4 [noise] in this particular example that is what is the duality [noise] that is [noise] the minimum [noise] cut [noise] is equal to the [noise] maximum number of [noise] internally disjoint paths between a pair of vertices x and y. [vocalized-noise]

So, we are solving this local problem between the pair of vertex vertices which is x and y what about other pair of vertices, the same [vocalized-noise] particular concept, we will check and find out [vocalized-noise] this particular inequality for other pair of vertices.

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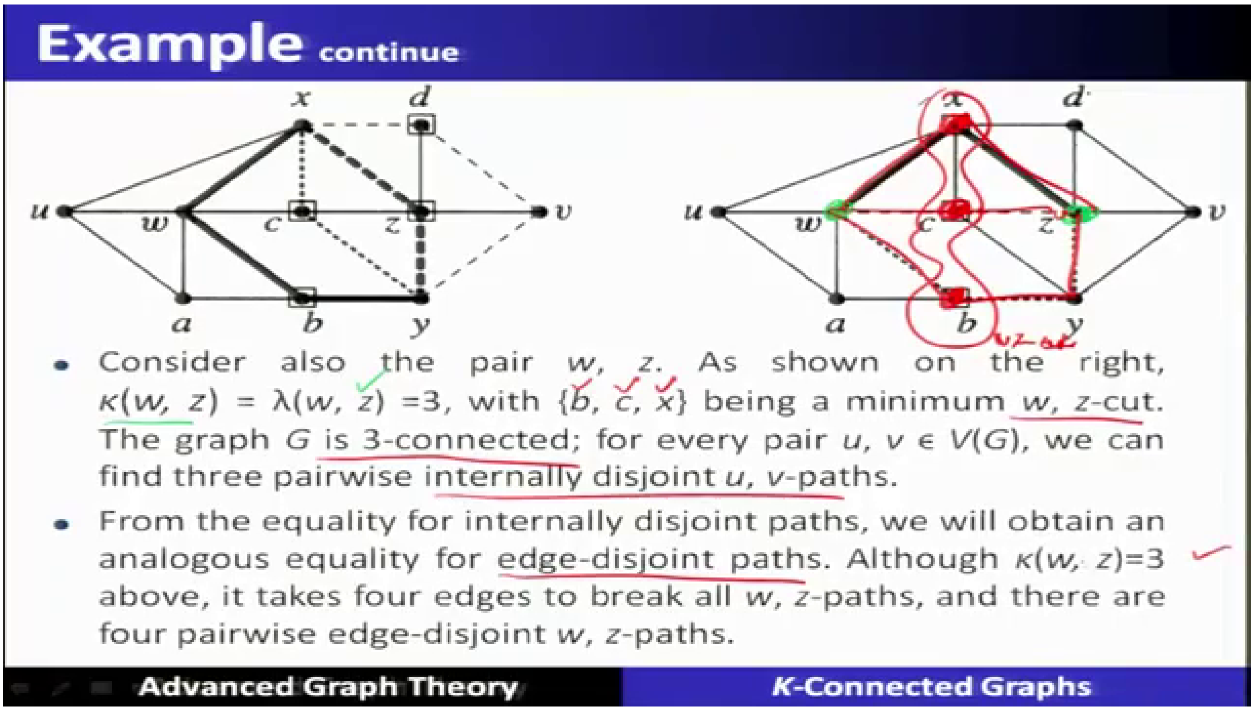

Now, we will consider another pair that is now the pair is w and z. So, the kappa w z [vocalized-noise] so kappa means this is ones vertex this is another vertex, this is another vertex. So, three vertices if we plug out this will disconnect w and z, hence kappa is equal to the to the lambda w z lambda means each vertex is taken from each vertex is taken from internally disjoint paths, [noise] w and sorry w and z [noise] this is one path, then this is one path, w and z w and z this is another path. So, let us again see that. [noise]

So, this is w and this is z [noise] let us see whether it has internally disjoint paths are not. [noise] So, between w and z this is one path, and this particular vertex we have included in the separator [vocalized-noise] between w and z there will be another path, and this particular vertex x we have included in the separator [vocalized-noise] between w and z there is another path which is going y of y and b is another included in the separator.

[vocalized-noise] So, if you remove them from the graph [noise] this is w z separator cut, [noise] if we remove it then w and z will be disconnected, [noise] hence the minimum size of w z here is 3, [noise] and the and the graph G is basically having for every pair of vertices, there are 3 internally disjoint u v paths.

We have seen we can [vocalized-noise] obtain analogous equality for edge disjoint paths also, and [noise] there we can see that although the kappa w z is equal to 3, [noise] it will take four edges to break all w z paths, [noise] and there are four pair wise edge disjoint w z paths. [noise] So, although there are 3 kappa value is 3, but when we talk about the edge disjoint it requires 4 let us see [noise] where are those 4. [noise]

So, this is w z [noise] if you want to disconnect through the edges so; that means, if we plug this, [noise] 1 2 3 4 then w and z will be disconnected, [noise] and the size 1 2 3 4, hence [vocalized-noise] it takes 4 particular edges to break w z paths, [noise] and there are four pair wise edge disjoint w z paths. [noise] So, this is one w z path, this is another w z path, this is another w z path, and this particular edge if we take. So, edge disjoint w z path we have also obtained.

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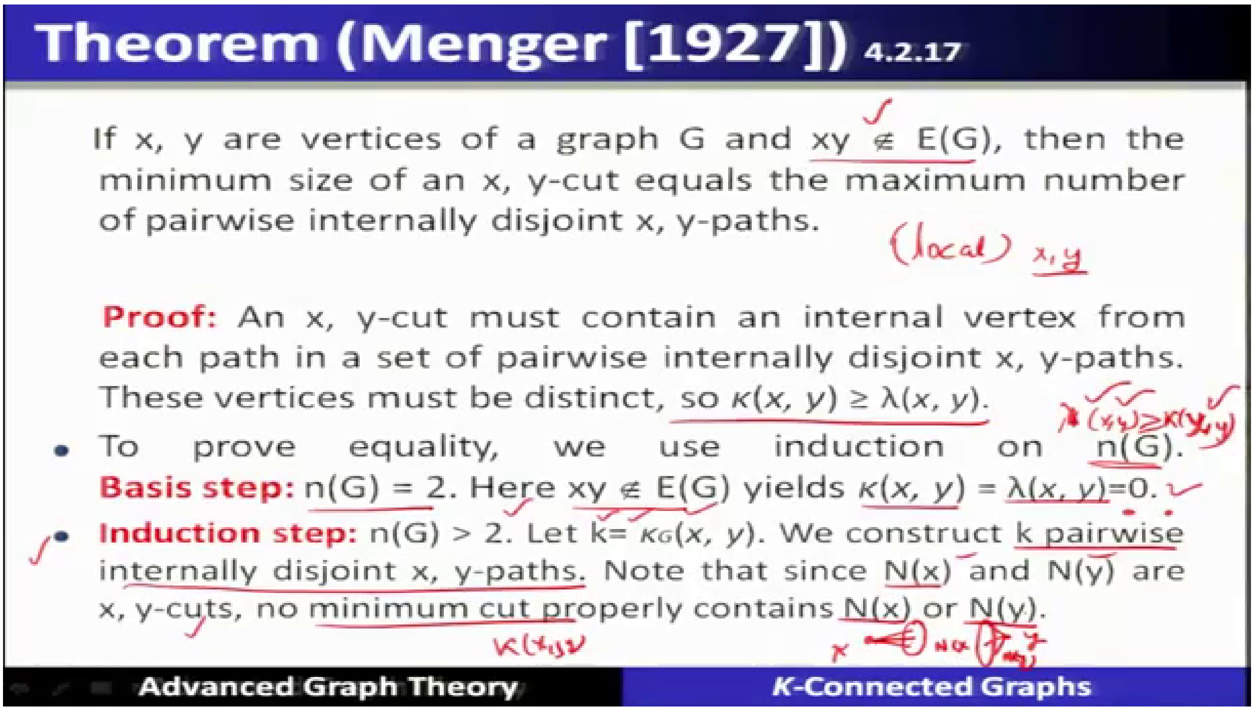

[vocalized-noise] now with this local phenomena between x and y vertices, now we see that the Menger theorem which is given in 1927, if x y are the vertices of the graph G, and x y is not having an edge in G, then the minimum size of x y cut equals [noise] the maximum number of pair wise internally disjoint x y paths. [vocalized-noise] So, this is the local [noise] theorem that is we this particular Menger theorem is now being stated between [noise] a particular pair of x y vertices, but for the entire graph for every pair of vertices, [noise] this has to be satisfied and only this particular condition for Menger theorem, [noise] in globally applicable for a graph that we will see at the end.

So, let us see the proof of ah this Menger theorem [vocalized-noise] for the proof let us assume an x y cut, and this particular x y cut must contain the internal vertices from each path in the set of pair wise internally disjoint x y paths that, we have already seen these vertices must be distinct. So, this inequality we have already have also seen that kappa x y is at least lambda x y by because, [vocalized-noise] each vertex is picked out from internally disjoint path which is lambda x y number of such paths are there. So, hence the minimum size of the cut is at least equal to the number of internally disjoint x y paths that is lambda value.

Now, to prove the equality so to prove the equality; that means, we have to we have to prove that lambda x y is also at least kappa x y. So, we have to basically show that there are lambda x y paths are there so, to prove the equality we use the induction on the number of nodes on in the graph, let us assume that the number of nodes is equal to 2, and also the condition of the theorem says that x and y should not have an a direct edge. So if there are only 2 nodes, and there is no edge what will happen, then the connectivity between x and y is the 0 and there are no internally disjoint paths, hence the basic step is proved. Now let us go to the induction in step when the number of nodes is greater than 2.

Now, here let us assume a value k which is nothing, but the size of x y cut in the graph G. And now we construct k different pair wise internally disjoint x y paths to show that lambda is at least kappa x y, and kappa x y is equal to k. So, k different internally disjoint x y path we have to construct, and hence to prove this particular theorem [vocalized-noise] note that since neighbor of x this is x. So so this is the neighborhood of x and if this is y so, they are exists a neighborhood of point since neighborhood of x, and neighborhood of i are x y cuts; that means, if we remove they remove all the vertices which are there in the neighborhood of x it will disconnect x and y. Similarly if we remove all the vertices of the neighborhood of y and also x and y will be disconnected.

[vocalized-noise] also no minimum cut, but we are looking for a minimum cut that is kappa x y, no minimum cut properly contains N x that is the neighbor of x and neighbor of y that we know means.

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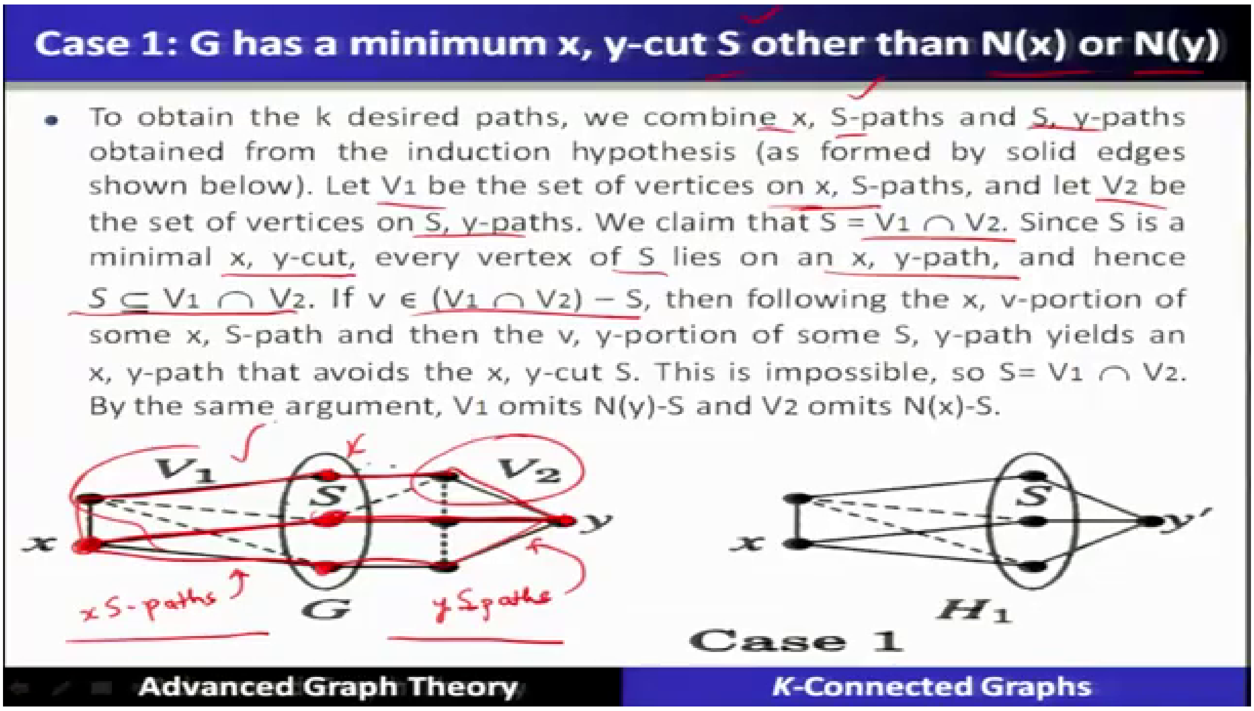

We have to form ah minimum cut. So, case 1 we have to see when G has a minimum x y cut S other than the neighborhood N x neighborhood of x or neighborhood of y. So, to obtain the k desired path we combine x S paths and S y path. So, S is you know that a separator or x y cut. So, we combine x S path and x y paths obtained from the induction hypothesis. So, that we can see so, this is x this is S all the vertices, [noise] and now we we form x S paths. Similarly for y [noise] and S is edges will form y S paths this is x S [noise] paths shown by red lines, [noise] this is y S [noise] paths shown again here on the right side. So, by induction hypothesis we have obtained these particular desired paths.

Now, let V 1 be the set of vertices on x, on on x S paths let us see that these set of vertices will be the V 1 and V 2 be the vertices on S y paths. Now we claim that S that separator is equal to V 1 intersection V 2 since S is the minimal x y cut. So, every vertex of S lies on x y path, [vocalized-noise] and hence S is a subset of V 1 intersection V 2 [vocalized-noise] my V is an element of V 1 intersection V 2 minus S, then following the x v portion of some x S path, and then v y [noise] portion of some S y path yield, the x y path that avoids x y cut S this is impossible.

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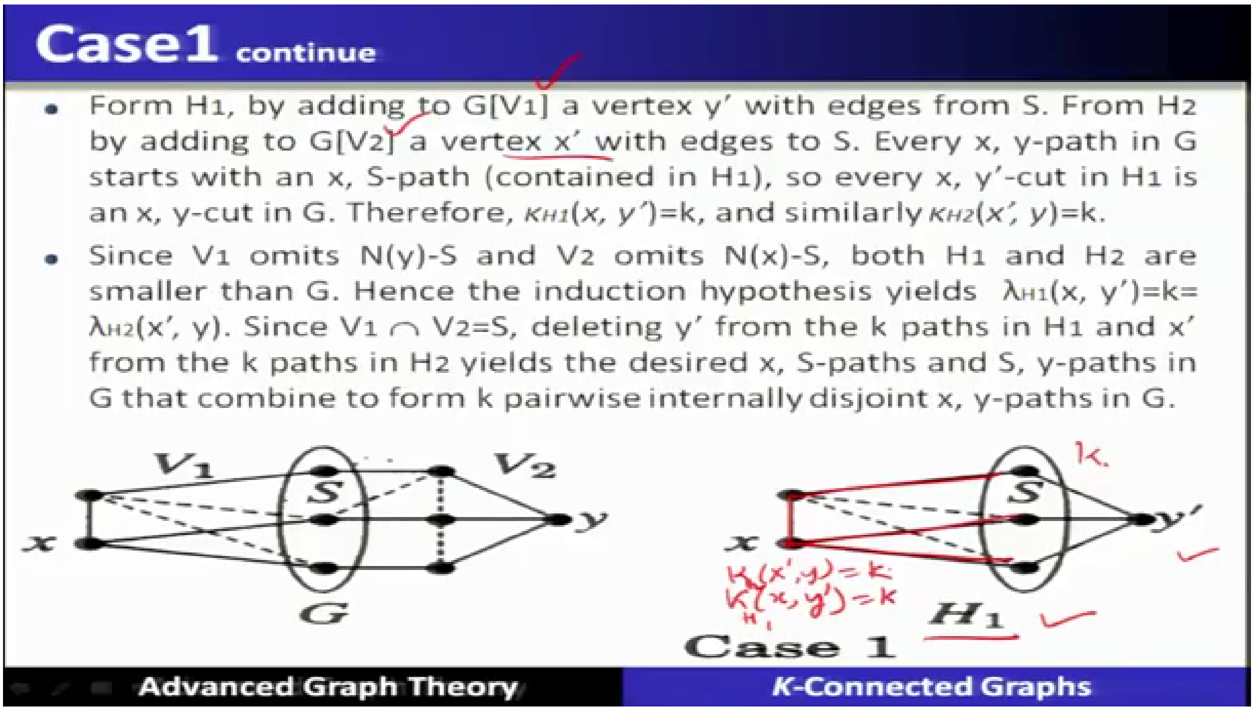

So, S is equal to V 1 intersection V 2 by the same argument V 1 omits N y neighbor of y minus S and V 2 omits neighbor of x minus S. So, from H 1 by adding to the graph which is induced by V 1 a vertex y prime with the edges from S from H 2 here, this is S 2 so, from H 2 by adding the vertices to to the induced sub graph of V 2 a vertex x prime that we will see in the next slide with the edges to S, every xy path in G which is starts from within x as path contained in H 1. So, every x [noise] y prime cut in H 1 is an x y cut in G therefore, kappa of x [noise] y prime is equal to k.

Similarly, kappa of H 2 is x prime and y [noise] that is equal to the k. [noise] Since V 1 omits neighbor of y minus S, and V 2 omits neighbor of x minus S both H 1 and S 2 are smaller than G hence the induction hypothesis yields that lambda H 1 x comma y prime is equal to the k is equal to the lambda H 2 x prime y.

Hence since V 1 intersection V 2 is equal to S. So, deleting y prime from k paths in H 1 and x prime from k path in edge to yields the desired x S paths, and S y paths in G that combine to form k pairwise internally disjoint x y path in G.

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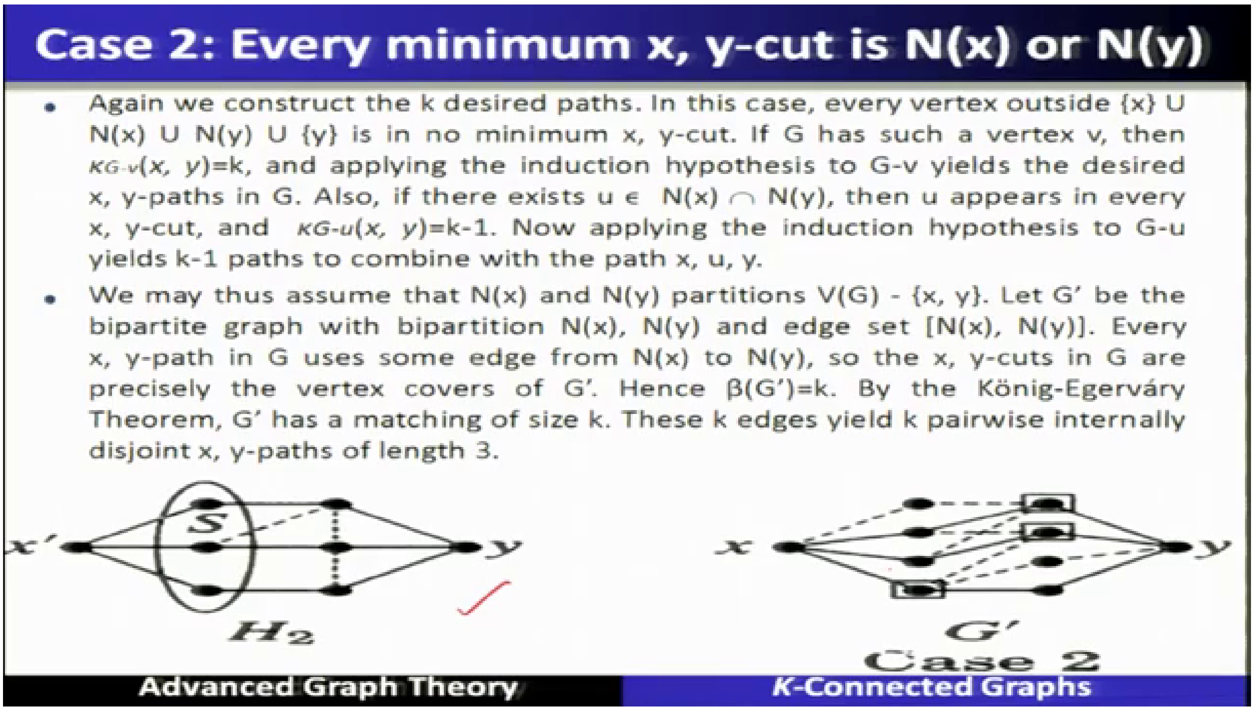

Now, case 2 is also very similar case to says that every minimum x y cut is either the neighbor x N x or N y. So, the same thing also is applicable in this particular case, hence we have constructed pairwise internally disjoint paths that is of size lambda [noise] hence it proves the theorem.

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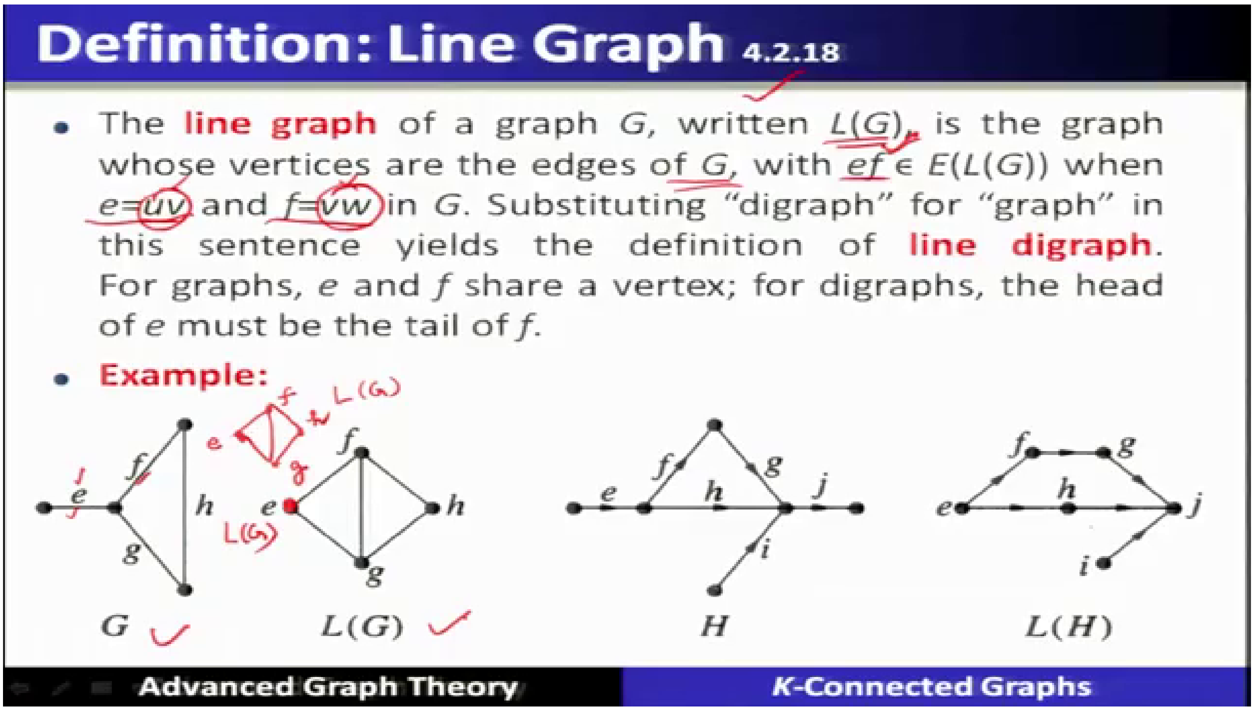

Now, we will see a definition why because [noise] this vertex version of Menger theorem we have seen now, we have to prove the edge version of the Mengers theorem for that we have to see this particular special kind of graph which is called the line graph, we are using the line graph for it. [vocalized-noise] So, a line graph of the graph G is [noise] denoted by L G is a graph whose vertices are the edges of G.

So, again let us see the definition line graph of G is written as L G, which is also a graph whose vertices are the edges of the original graph G, and the edges of line graph is ef, when e is basically an edge u v in original graph and f is again an edge v w in the graph; that means, these 2 edges are touching at v then it will form an edge in the line graph let us take this particular example. [vocalized-noise]

Let us see that this is a graph you want to construct a line graph of this particular graph according to this particular definition. [vocalized-noise] So, line graph is a is a graph whose vertices are the edges of G. So, this particular edge is E. So, this becomes a vertex this is f. So, let us see that this is an edge so it becomes a vertex called E, this is another edge in a graph so, here it will become a vertex, and here this is the H edge called H. So, this also will become a vertex, and G will also becomes a vertex of a graph. So, there are four vertices, because 1 2 3 4 different edges are there, in the line graph 4 different vertices will be there. [vocalized-noise]

Now, edges of this line graph e f when an edge 2 edges of the main graph the joints, they will form an edge e f in the main graph. So, e f these this edge and this edge will join at this end so it will form an edge e f. Similarly f and H they are joining in the vertex. So, f h will be an edge, similarly g h will be an edge then e g will be an edge also f and g they are touching so, f and g also will be edge.

So, this will be a line graph of the graph G, the line graph of a graph is the graph whose where is the vertices of a line graph or the edges of the original graph, and the edges of the line graph is when the edges of the original graph are meeting then it will form an edge in the line graph. So, if I will graph is given we can construct the line graph of a graph now let us see the theorem.

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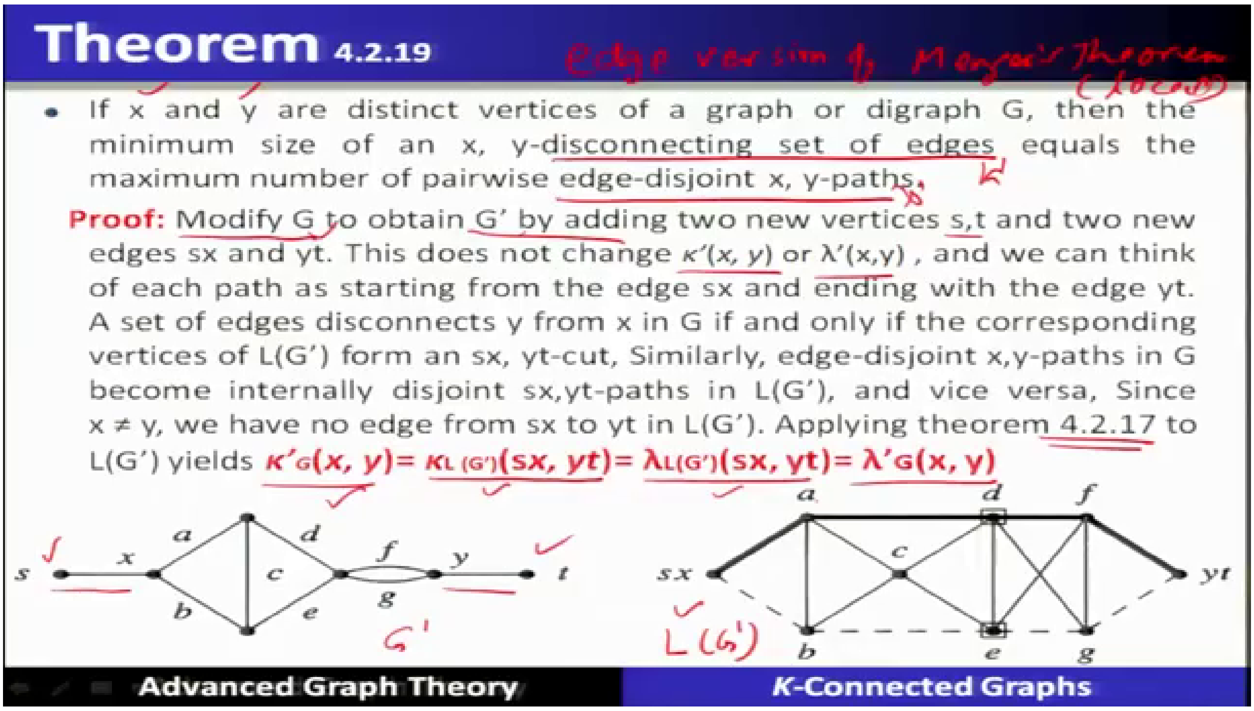

So, if x and y are distinct vertices of a graph or a [noise] graph G, then the minimum size of x y this connecting set of the edges here, we are we have changed the terminology here we are calling it as disconnecting set of edges, [vocalized-noise] in the previous theorem we have seen the separating set of vertices. [vocalized-noise] So, the minimum size of an x y disconnecting set of edges equal to the maximum number of pair wise edge disjoint x y paths, they are we were talking about vertex disjoint paths. Hence this is the edge version of the Menger theorem, [noise] this is a local local means for x y only we are considering one pair of vertices.

Let us see the proof we are given a graph G we will modify to d prime by adding 2 more new vertices s and t [noise] this s and t, they are rated up, and two new edges s x and y t they are added. So, this is called a G prime. So, this G prime does not change kappa prime and lambda prime. So, this is kappa prime, and this is lambda prime. So, this is not going to change in the original graph, and we can think of each path as starting from this particular edge s x, and ending with this particular edge y t.

So, a set of edges disconnects y from x in G if and only if the corresponding vertices in L G prime, we have to obtain a line graph of G prime. So, if the set of edges which disconnects y from x in G this corresponds to or this is equivalent to saying that a the vertices of L G that is the line graph form an s x and y t cut. Similarly the edge disjoint x y paths in G becomes internally disjoint s x and y t paths in L G prime.

So, edge disjoint here in this particular graph will become vertex disjoint in the line graph and vice versa. So, having done this, now we will apply the previous theorem that is the vertex version of Menger theorem 4.2.17 to this line graph L G prime, and this will yield as kappa prime. [vocalized-noise] So, this particular on the line graph if we apply this particular kappa prime kappa L G prime, s x and y t minimum cut is equal to lambda of L G prime s x and y t, this is the internally disjoint pairs between s x and y t.

[vocalized-noise] And we know that this particular kappa of L G prime this is equal to the kappa prime of the graph G. So, kappa prime that is equal to kappa prime x y, [vocalized-noise] similarly lambda L G prime is equal to the lambda prime in the graph G. So, the line graph will convert it to the line graph of the graph will convert the problem. So, that the vertex version can be applied and that is equivalent to the edge version solution.

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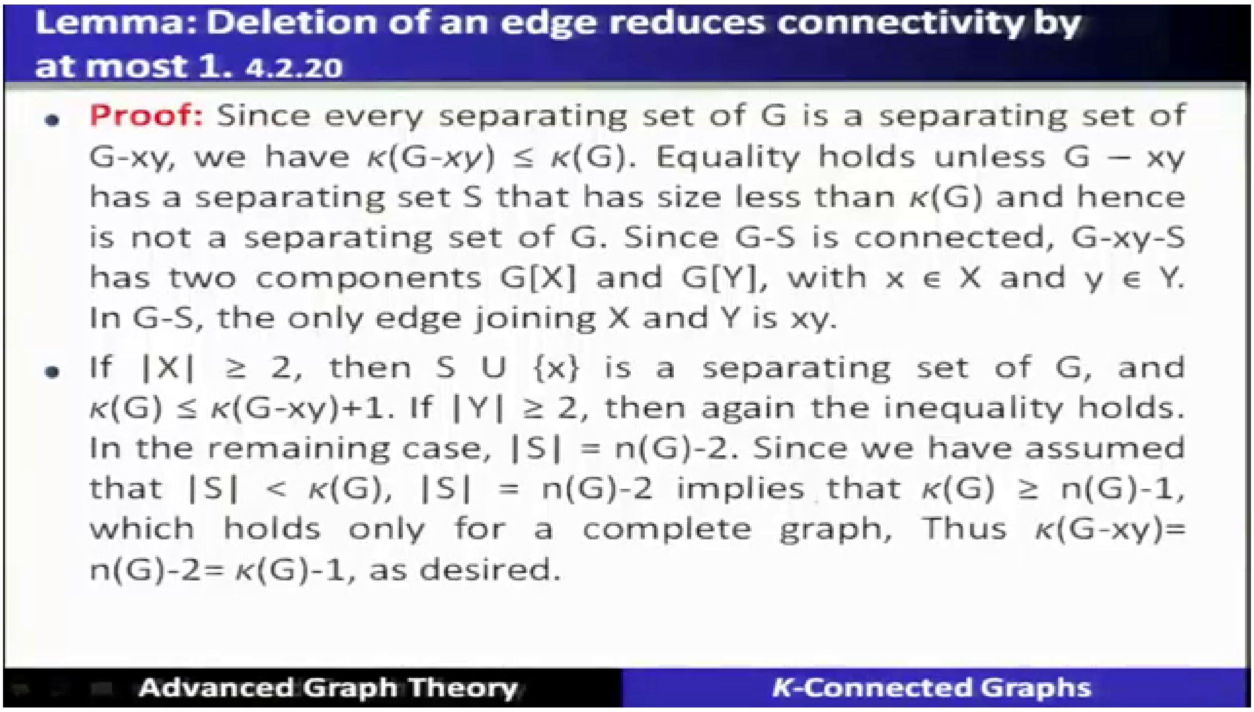

[vocalized-noise] So, division of n H will reduce the connectivity by at most 1. So, let us use this particular lemma in the next theorem, and this is the global version of the theorem. So, the theorem says that the connectivity of G equals the maximum k such that lambda x y is at least k for all x y [noise] pair of vertices. So, the edge connectivity of G equals the maximum k such that lambda prime x y is at least k for all x y set of pairs.

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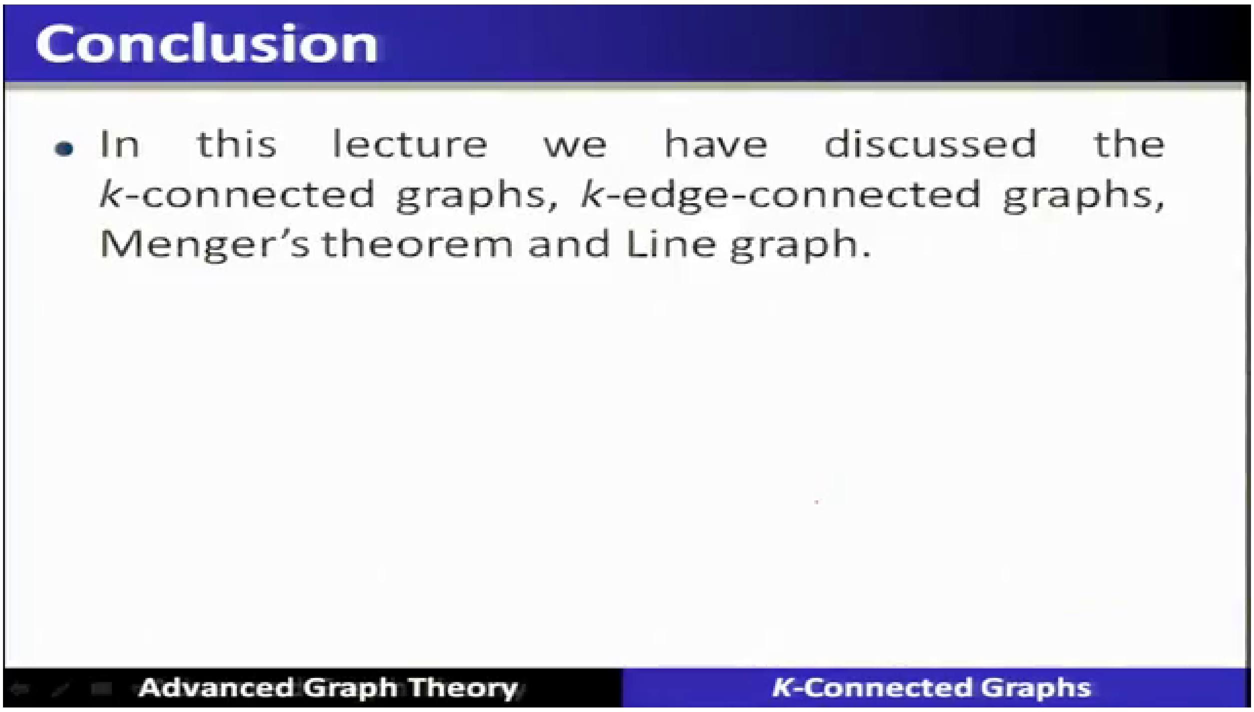

So, conclusion in this lecture we have discussed the k connected graphs, k edge connected graphs Menger theorem and line graph.

Thank you. [noise]