Advanced Graph Theory
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Lecture – 13
Connectivity and Paths: Cuts and Connectivity

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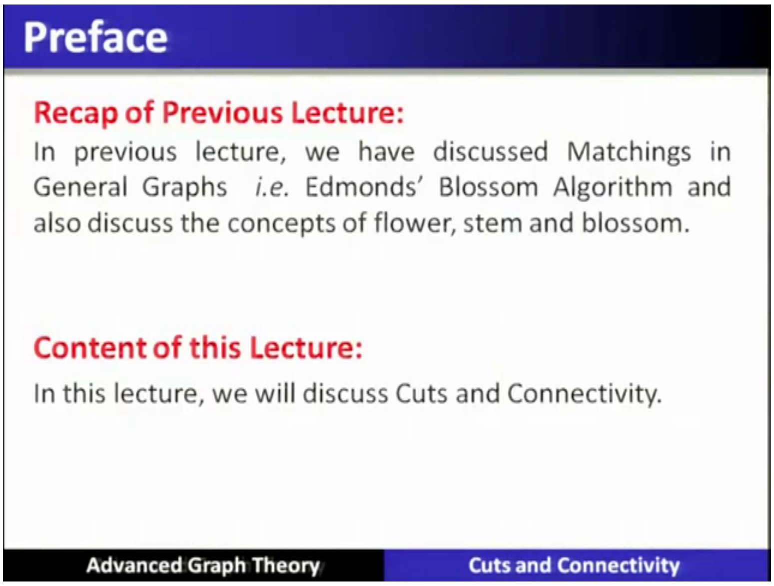

Cuts and Connectivity recap of previous lecture, we have discussed matchings in general graph, and we have also seen the corresponding algorithm that is known as Edmonds blossom algorithm. We have also covered the concepts of flower stem, and blossom in that reference content of this lecture.

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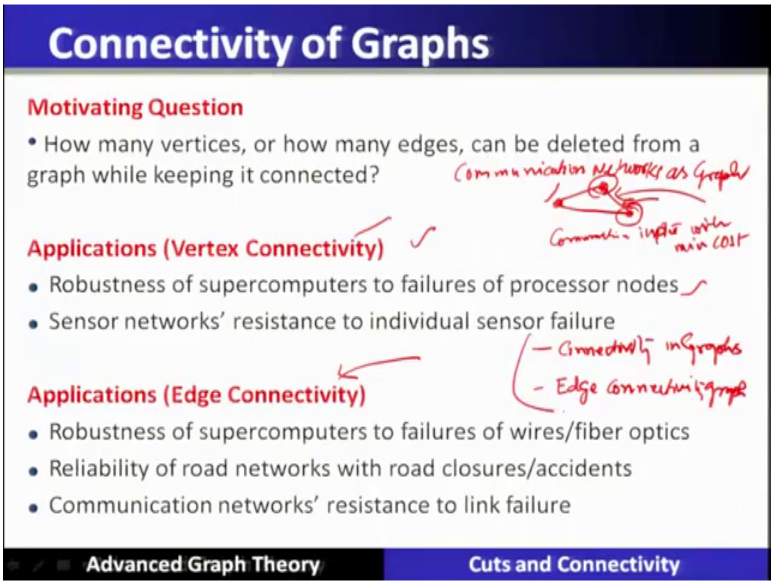

We will discuss cuts and connectivity. So, when we talk about the connectivity of a graph, then the motivating question will make more understanding about the connectivity the question is about how many vertices, or edges can be deleted from a graph while keeping it is connected.

So, the application for this scenario is the communication networks. So, communication networks as the graph if we visualize, then it comprises of the nodes which represents, the communication devices, and these particular communication devices they are connected through the channels, or through the edges, there is a possibility of failures of these vertices or, also there is a failure of the channels, or a links may go down, because of noise are some other disruptions. So, the idea is how to communicate how this network will communicate inspite of the failures of vertices, and the edges with the minimum cost, because when you talk about the edges that is fiber optic cabling.

So, if we want to develop this kind of network, then the cables are to be laid. So, what will be the minimum cost of getting this particular type of network to be connected in spite of these failures? So, there are 2 different aspects I told you when the vertex or basically the nodes, or the communicating nodes are basically if they fail then how we can basically, still make the network communicating in spite of their failures. So, that particular problem setting is called the vertex connectivity. So, the robustness of that vertex connectivity issues, we are going to see that will arise in many applications.

Similarly, if the edges are failing that is the link is down, then how we can ensure that still some links are down then the communicating entities, that is the transmitters should be able to communicate or exchange the messages or the data, in spite of the failures, then those conditions in the network requires some more edges to be put in the network, and that will require an extra cost. So, what will be that minimum number of such additions in the network, which can tolerate some of these failures on the edges so, that the communicating node, can still communicate in spite of these failures.

So, both the problem settings when we are looking about the vertex connectivity so, simply it is called as a connectivity, in the graphs and if we consider the edge connectivity. So, it is called the edge connectivity, in the graphs both these problem settings, we are now going to cover up.

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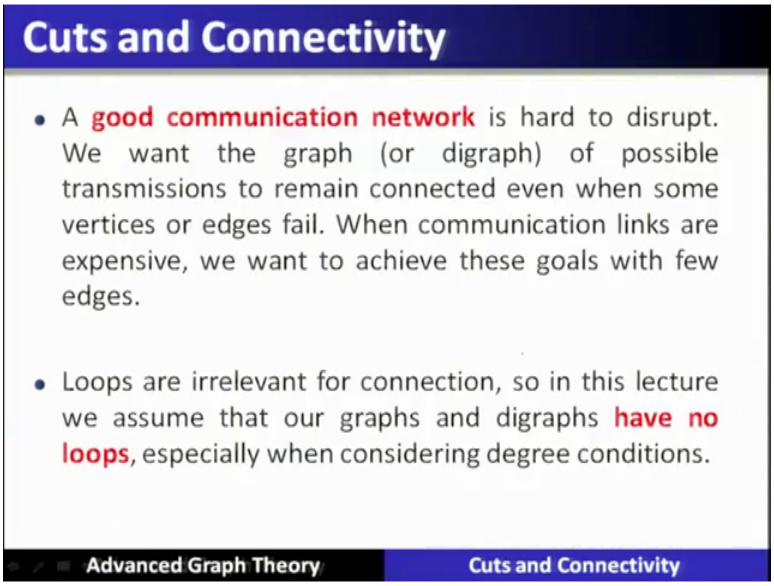

So, a good communication network is hard to disrupt, and we want the graph of possible transmissions to remain connected even when some vertices or the edges fails. So, when the communication links are expensive we want to achieve these goals with a few edges.

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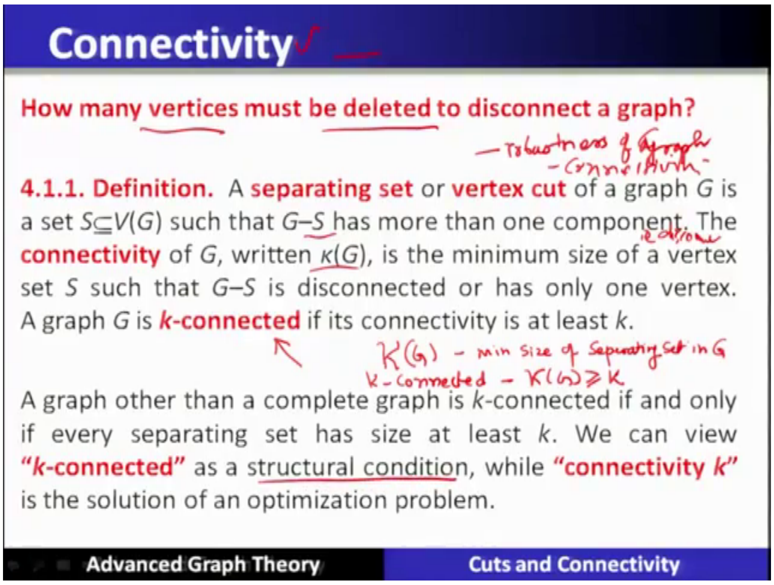

Connectivity now how many vertices must be deleted to make your network disconnected. So, again I am repeating how many vertices must be deleted to disconnect a network so; that means robustness of the graph, we are referring to and that is nothing, but the connectivity. So, the connectivity deals with how many vertices are required to make the graph disconnected. So, when you say disconnected they will not be communicate each other, or they will not exchange in the data when they are down.

So, definitions a separating set or a vertex cut of a graph is a subset of vertices of a graph such that the graph, without this separating set S will have more than 1 component; that means, the graph will be disconnected. If S set of vertices are not present in the graph hence the connectivity of a graph is represented by k of G is nothing, but the minimum size of the separating set in a graph. So, a graph is called k connected if it is connectivity is at least k.

So, for k connected graph, the connectivity that is k of G is at least k. Now when we talk about after having the background about the connectivity, then in the next lecture we will talk about the k connectedness of a graph which is more subtle in the discussion. So, it requires a clear understanding of this particular lecture is the background. So, a graph other than the complete graph is k connected, if and only if every separating set has the size, at least k. So, we can view the k connected as by structural condition, and the connectivity k is an optimization problem.

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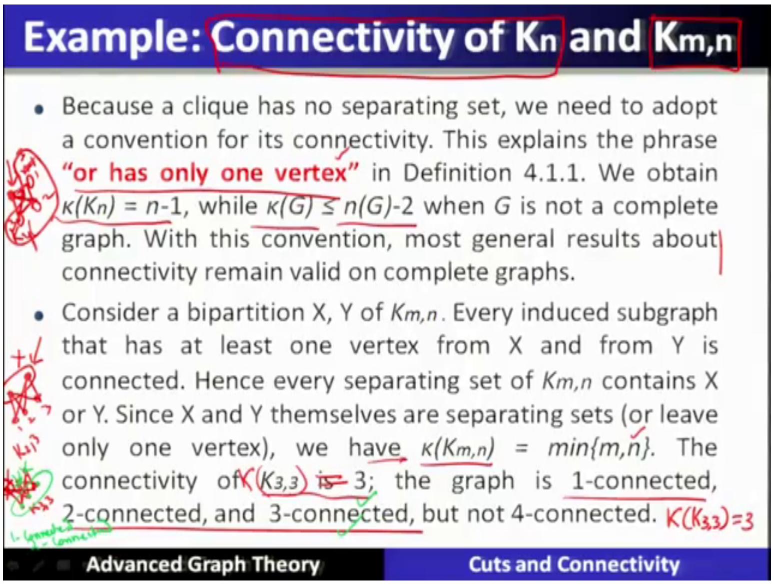

Now we will discuss the connectivity of a complete graph K n, and also the complete bipartite graph K m, and before we go ahead let us revisit the definition of a connectivity once again. So, the connectivity of a graph is written by k of G is the minimum size of the vertex set, such that g minus S is disconnected or has 1 vertex, this is an addition which we have to see. So, either the graph is disconnected or one vertex remains, let us see where these conditions are useful. Now since we know that the clique let us draw a clique this is k 4.

Now, every vertex of a clique is basically having a connections with all the other vertices; that means, this vertex is connected to other n minus 1 vertices if this particular or you can say it is connected to other 3 vertices 1 2 and 3 you know that the clique is very tightly coupled. So, if we remove all these 3 vertices, then this particular vertex will remain in isolation, hence we have to include in the definition of a connectivity that either the separating set, will disconnect the graph or 1 vertex remains this particular the inclusion of only 1 vertex remains, in the connectivity is to also include the complete graphs.

So, again I am reading it because the cliques has no separating set, we need to adopt a convention of it is connectivity, this explains the phrase or a has only 1 vertex in the definition. So, we obtain the connectivity of a complete graph n kappa n is equal to n minus 1, while for other graphs other graphs means which is not a complete graph, then this kappa G will be at most n minus 2. So, with this convention most general graphs remain valid even you include the complete graphs.

Now, talk about the connectivity of a bipartite graph, let us say this is K 2 3 every vertex is connected to all other vertices on the other part I would set. Now if we remove this is a partide set if we remove them, your graph will be in a disconnected situation where 1 2 3 it will be disconnected into 3 components, minimum of 2 is required to make this particular graph disconnected. So, minimum of m and n whatever is the minimum that particular will become the connectivity of the complete bipartite graph.

So, the connectivity of K 3 3 here, both m and n they are same. So, connectivity is K 3. So, k of K 3 3 is equal to 3 here in this particular case using this particular formula. Now a graph is 1 connected 2 connected, and 3 connected what do you mean by this. So, when k of K 3 3 is 3 that is the connectivity of this particular graph is 3, what will happen. So, let us draw a 3 3. So, this k is K 3 3. Now if you remove 1 vertex what will happen this graph is connected only this vertex is removed the remaining graph is connected.

So, it is one connected, if you remove 2 vertices still that graph is connected the rest of the component is connected. So, it is 2 connected, but the moment 3 vertices are removed this particular graph will be disconnected into 3 components. So, hence the connectivity of this graph is 3 so; that means, if 3 vertices will disconnect this particular graph, hence the definition of connectivity is followed. So, when you say that a graph is to be connected it means that it is one connected, it is 2 connected; that means, if you remove one vertex it will be still connected, if 2 vertices are removed it is still connected, but if 3 vertices are removed we disconnected.

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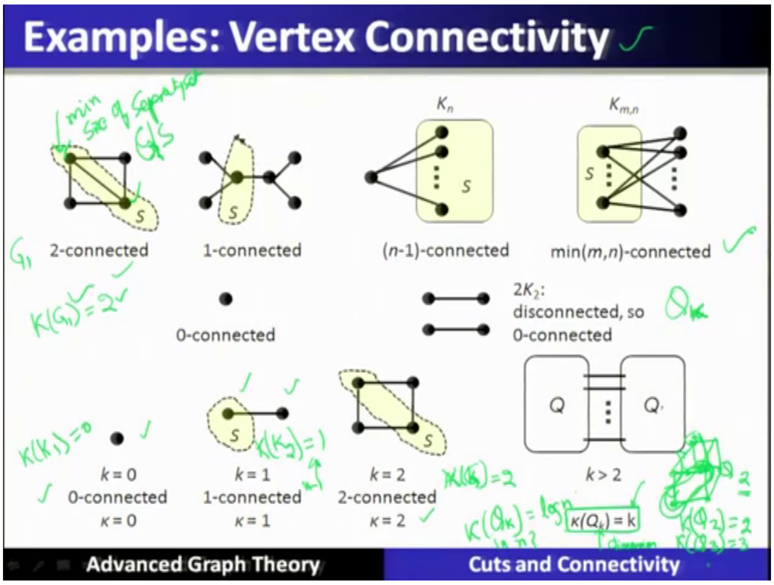

So, we cannot say that it is 4 connected so here the example. So, this particular graph is 2 connected; that means, if these 2 vertices are removed, then it will break into 2 components the graph will be disconnected. So, it is 2 connected or the connectivity of this particular graph, let us say it is G 1 will be 2 that is the minimum size of separating set, separating set is the set of nodes if they are not present in the graph. So, G minus S it will make the graph disconnected. So, the size is the minimum size of that separating set.

Now, there is a convention which says that a single vertex. So, this particular vertex what is the connectivity of these particular vertex. The connectivity of this particular vertex is 0 this we assume. Similarly when k it is 1 connected; that means, if 1 vertex is removed. So, this is called k 2 graph. So, the connect k of K 2 is equal to 1 why because if you remove it will basically give a particular node, that is you know that it will be under that definition of a complete graph that is n minus 1. So, k 2 is a complete graph of 2 vertices. So, here it will be 2 minus 1 that is 1. This you know already the connectivity kappa of this particular graph will be 2 why because it is a minimum size of this separating set.

So, we are looking up all this connectivity part, and this we have already seen about the complete bipartite graph connectivity. Now important thing is about the hypercube of n dimension, if the hypercube is it is represented as Q k, the hypercube is basically having 2 dimension. So, this is Q 2. So, meaning to say that these particular neighbors, if you remove then it will be disconnected, here the size of the neighborhood is that particular dimension, it is Q 2 2 dimensional things. So, it becomes 2 here in this case for Q 2 k of is equal to 2. Now the moment it becomes a 3 dimensional 3 dimensional means 2 Q 2 is being basically added to make it as 3 dimensional, and hence the number of neighbors of a particular node will increase to 3 that is 3 dimensions.

So, all 3 if basically are removed then only the, this particular vertex will be disconnected, hence kappa of Q 3 will become 3. So, if you generalize. So, for a hypercube of k dimensions, the connectivity will be that k itself. Now if you want to find out the connectivity of Q k in terms of n. What will be that value? So, that will be the log of n then in that case that is the property of the hypercube.

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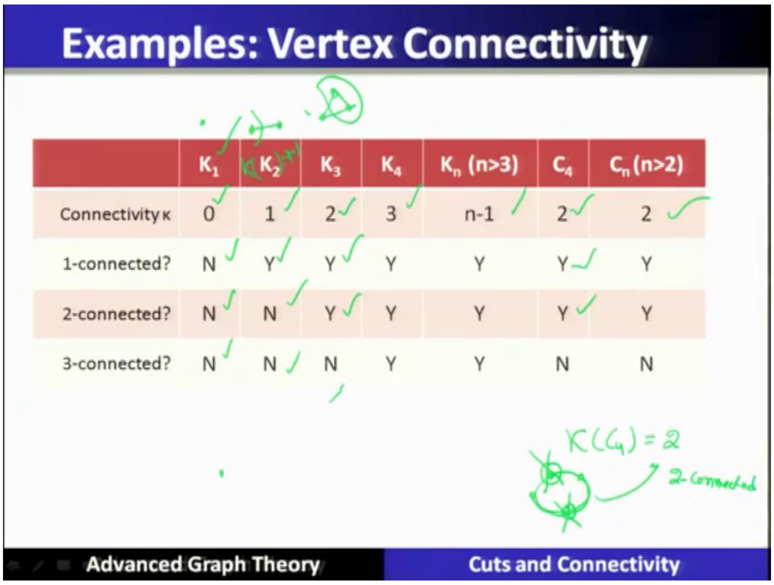

So, this we have already stated, but let us summarize it. So, if K 1 K 1 means single node that we have already told that it is having a connectivity 0, it is neither 1 connected not 2 not 3, if K 2 K 2 means it is a complete graph of 2 nodes, you know that it is n minus 1 connectivity.

So, here n minus 1 will be applied hence, it is 1 connected; that means, if you remove 1 vertex it will be remaining in isolated vertex, and that is the definition of the complete graph connectivity. It is not 2 connected neither it is 3 connected the complete graph of 3 vertices will have n minus 1 that is 2 3 minus 1 that is 2. So, it is 1 connected because the connectivity is 2, it is 2 connected why because if 2 vertices are removed, then only single vertex remains, but it is not 3 connected K 4.

Similarly, it is n minus 1 and K n becomes n minus 1 C 4 will be having the connectivity 2. So, it is one connected and again it is 2 connected C n similarly it is having that connectivity of 2 so; that means, it will it is a cycle, and if you remove one node still the remaining part of the cycle will be connected. So, if 2 nodes are removed then only it will be disconnected hence it is 2 connected, but not 3 connected.

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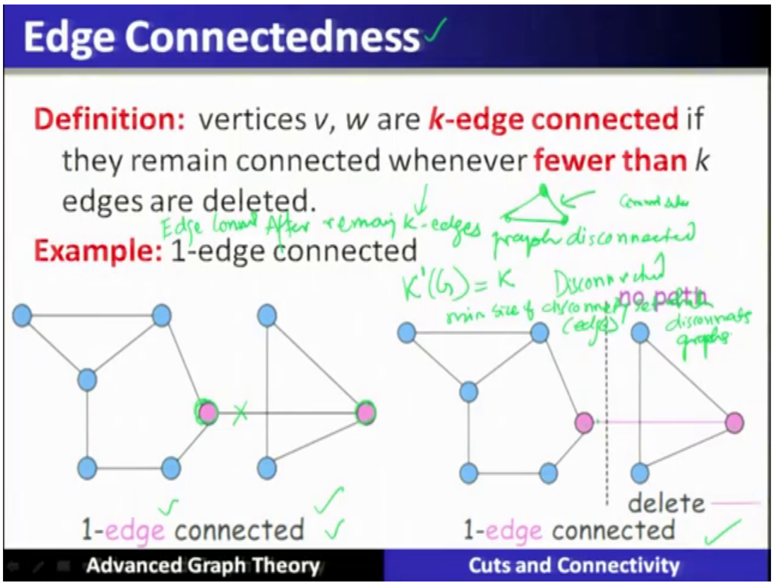

Now, we will talk about the edge connectedness, vertices u and v are edge connected if they remain connected whenever a fewer than k edges are deleted, then the edge connectivity is basically k. So, take this particular example.

Now, before we go ahead in the example we are referring to the connectedness with reference to the edges; that means, we are talking about the graph representing the communication network these links may fail. So, what is the property of a connectedness that if one link is failed whether the transmitters will be able to communicate the data with each other that is basically the motivation for edge connectivity, and we have to look separately this particular aspect.

So, here this particular graph is 1 connected why because if we see these pair of vertices which are shown in a different colors is connected by an edge, if we remove this particular edge this particular a graph will not have any path connecting these vertices hence, they will be disconnected. Hence the 1 edge will disconnect the graph hence it is 1 connected, this graph is 1connected. So, the definitions vertices u and v are k edge connected if they remain connected whenever fewer than k edges are deleted, or if you if k edges are removed then the graph becomes disconnected; that means, after removing k edges the graph becomes disconnected, and that is called edge connectivity.

So, what is that minimum number of edges which will disconnect the graph and that is represented by kappa prime G that is nothing, but some value K. So, that minimum size of disconnecting set disconnecting set means it is set of edges which disconnects the graph into components is called edge connectivity. So, this example will give you about 2 edge connectedness.

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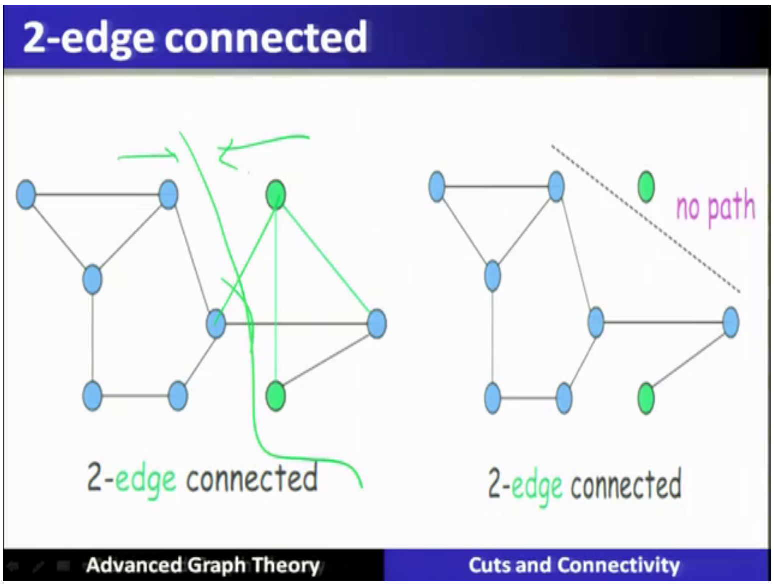

So, this particular this example basically shows it is a 2 edge connected; that means, if you remove these 2 edges. So, the graph will become disconnected; that means, this both the ends will not be able to communicate with each other hence it is 2 edge connected.

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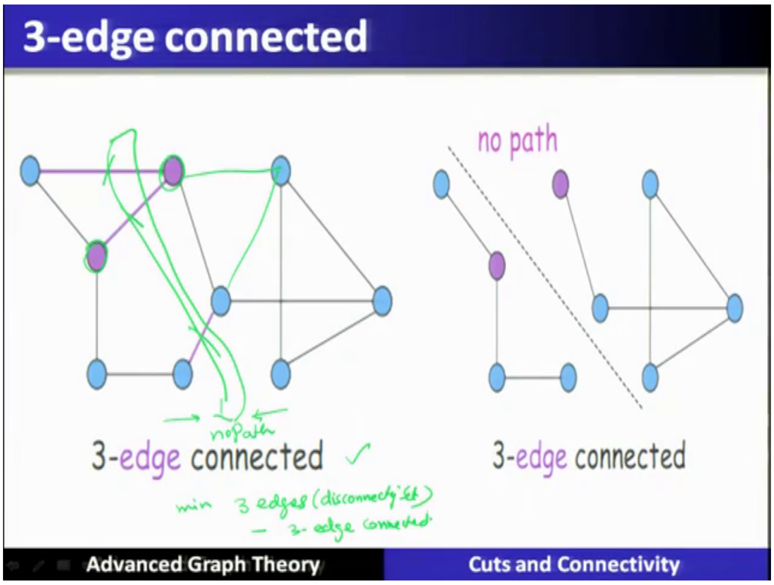

Now, we are looking up the example of a 3 edge connectivity, now this particular graph is 3 edge connected let us consider these 2 vertices which are shown in a different colors, if 2 edges if we remove this particular graph is still connected as a 1 unit or is it is a (Refer Time: 23:28), but if you remove this vertex this edge also. So, these 3 edges if they are removed, then this will disconnect into 2 parts 2 components where the vertices on this side cannot communicate with each other, there will not be any path no path which will link these 2 sides.

So, hence this is called 3 edge connectivity why because 3 minimum 3 edges which are called disconnecting set is required to disconnect the graph, hence it is 3 edge connected.

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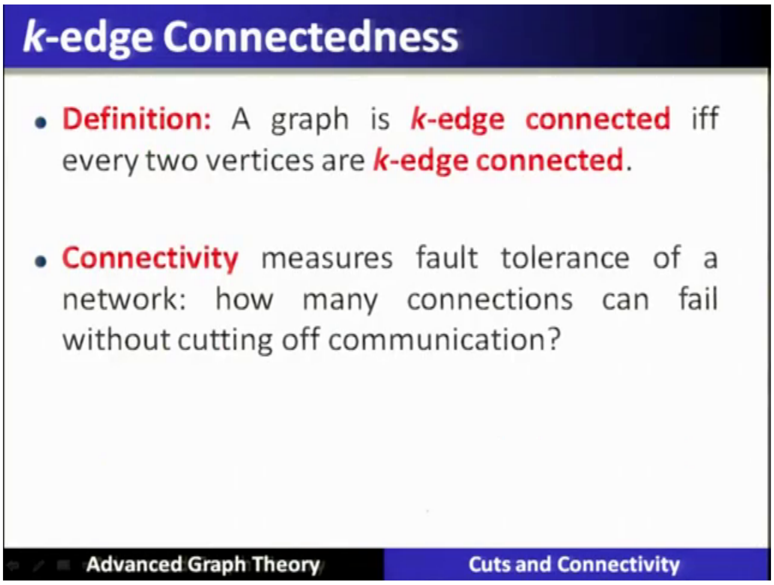

Similarly, k edge connected that is the graph is k edge connected, if and only if every 2 vertices are k edge connected so; that means, we are talking about for the entire graph, earlier in the picture we have seen 2 vertices, and these 2 vertices are edge connected. So, that particular minimum value of k in which that particular entire graph can be disconnected hence that becomes the entire graph property.

So, connectivity measures the fault tolerance of a network, how many connections can fail without cutting off the communication network.

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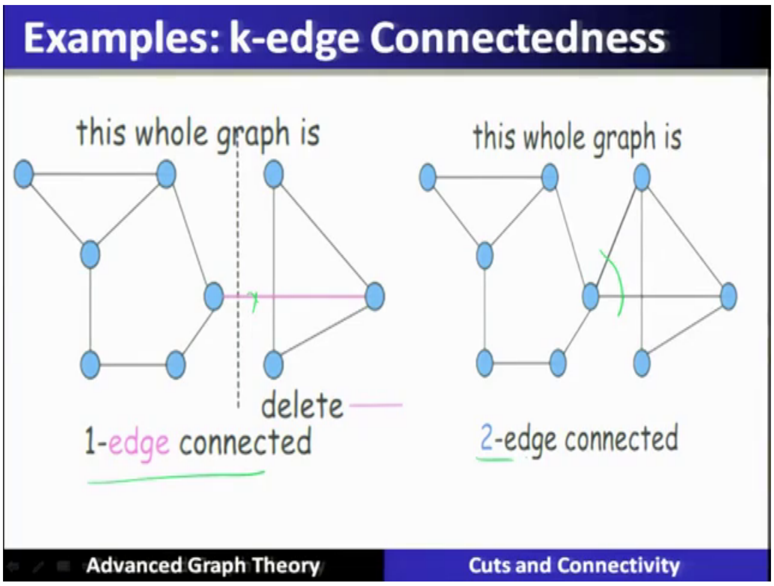

So, here we will talk about the entire graph. So, these particular again repeating the same thing that is this edges remove the entire graph will disconnect, hence this graph is one connected, if we modify this graph these 2 edges are removed minimum hence it is this particular graph is 2 edge connected.

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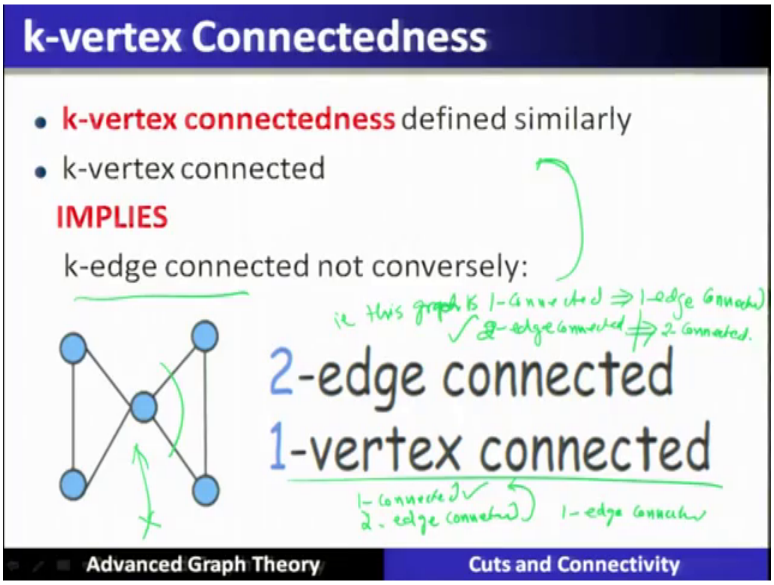

Now k vertex connectivity. So, k vertices k vertex connectivity means that the separating set is of size k to disconnect the graph. So, if a graph is k connected that will imply that it is also k edge connected, but not vice versa that we will see take this particular example that this particular graph is 1 connected, that means if we remove this particular vertex this graph will be disconnected, but as far as edge connectivity is concerned. So, edge connectivity will require 2 edges to be removed that is disconnecting set is of size 2 hence it is 2 edge connected graph.

So, if it is 1 vertex connected; that means, it is also 1 edge connected, but not vice versa; that means, it is 2 edge connected; that means, does not means that it is 2 vertex connected.

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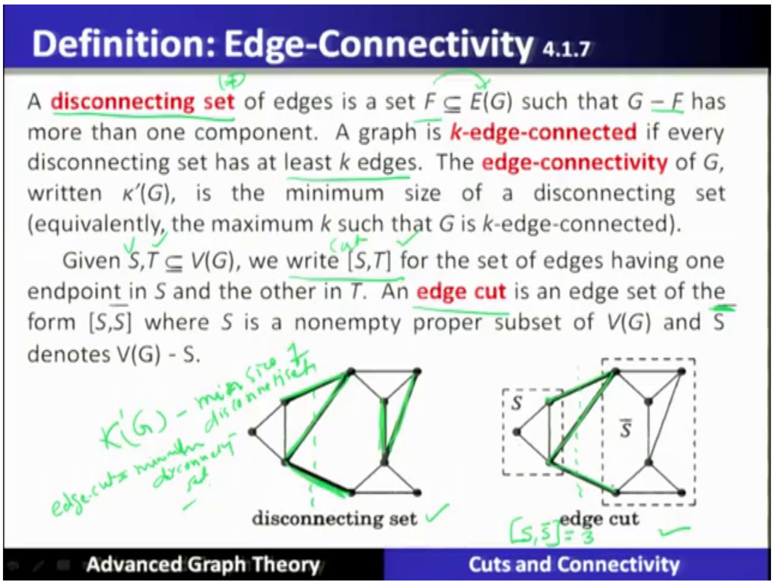

So, let us go in more detail about the edge connectivity or disconnecting set of edges is a set F of the edges of the graph such that the graph without this set of edges F has more than 1 components; that means, the graph disconnects, if this F set of edges are taken out from the graph, hence this particular set F is called a disconnecting set I have referenced this in the previous slide.

Now, with this definition that is our disconnecting set definition, now we will be able to define the edge connectivity. A graph is k edge connected if every disconnecting set has at least k; that means, this becomes a property of the entire graph that means, all among all the disconnecting sets the set of minimum size if it is k, then that becomes the k edge connected graph. So, again I am repeating a graph is k edge connected, if every disconnecting set has at least k edges the edge connectivity of G is written by kappa prime G is the minimum size of disconnecting set. So, given S,T a subset of the vertices of a graph G we write down S, T cut for a set of edges having 1 endpoint in S, and the other endpoint in T.

So, an edge cut is an edge cut is an edge set of the form comma S bar or S complement where S is the non empty proper subset of V, and S prime or S bar or S complement denotes V minus S take this particular example. So, this example shows the disconnecting set shown in dark edges; that means, if they are removed the graph will be disconnected, on the other side we are showing you the edge cut. So, edge cut is defined in S and S complements is nothing, but these set of edges which crosses from S to S bar. So, here you can see that the there are 3 edges which cross from S to S bar. So, this will form the edge cut. So, here the size is 3.

On the other hand if you see here besides these 3 some more edges are also there in the disconnecting set. So, every disconnecting set is not an edge cut, and edge cut is a disconnecting set or you can say a minimum edge cut is nothing, but the minimum minimal minimum disconnecting set, we will take this reference in this particular discussion.

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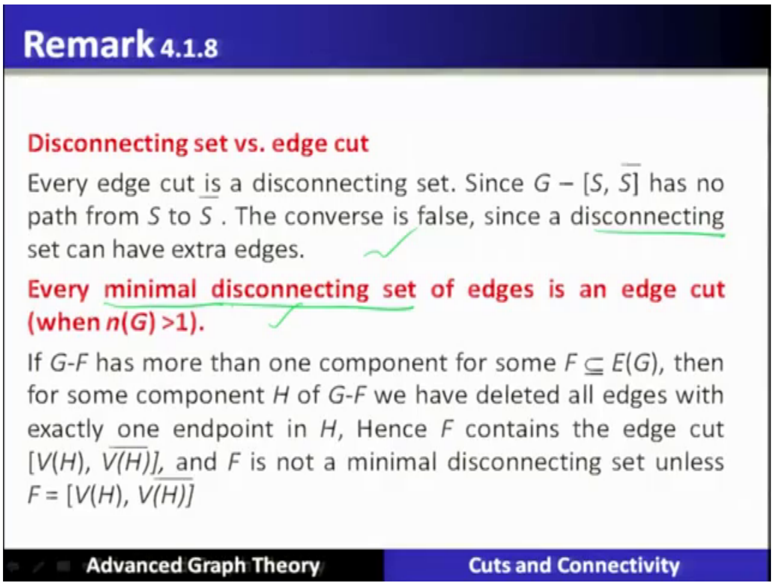

So, whatever I explained is written over here a disconnecting set versus edge cut, every edge cut is a disconnecting set. Since G minus S comma S bar has no path from S to S bar hence the converse is false, since disconnecting set can have an extra edges that I have explained.

Now, every minimal disconnecting set of edges is a edge cut. So, not entire disconnecting set can be same as the edge cut, but minimal disconnecting set is equal to the edge cut even G minus F has then for some component H in G minus F, we have deleted all the edges with exactly 1 endpoint in H, hence F contains the edge cut V H bar, and F is not a minimum disconnecting set unless F is an edge cut connectivity and minimum degree for simple graphs.

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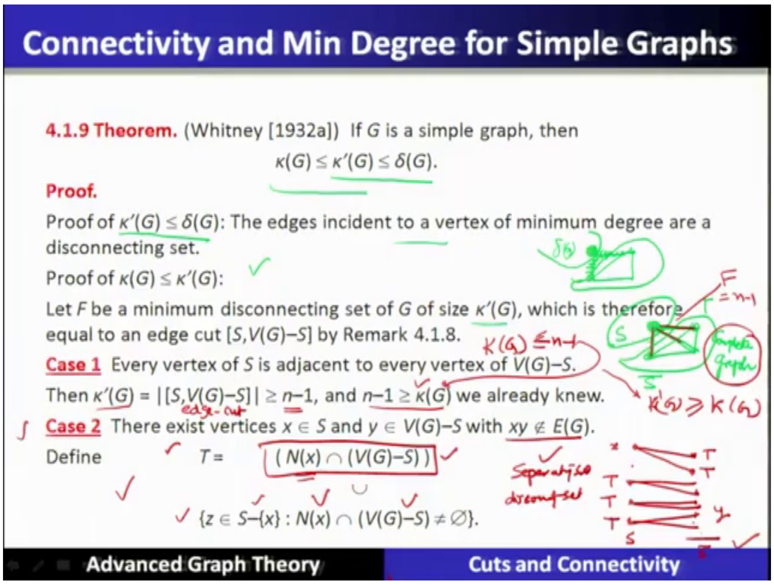

Let us see the theorem given by Whitney in 1932 if G is a simple graph, then kappa G is less than or equal to kappa prime G which is also less than equal to little delta G that is the minimum degree of a graph.

Let us see the proof. So, we will see this particular proof in 2 parts, the first part quite straightforward. So, here kappa prime G is referring to the edge connectivity, and that is upper bound by the little delta of G little delta is the degree of that minimum degree. So, the so the edges which are incident to the vertices of the minimum degree are the disconnecting set for example, this is the edge which is having the minimum degree of a graph. So, this edge and this edge if they are removed, then this particular node will be disconnected with the other part of this particular graph.

Hence this part of the theorem so, the edges which are incident to a vertex with a minimum degree are the disconnecting sets, now another part of the proof we will see that is the kappa G is less than or equal to kappa prime G. Let F be the minimum disconnecting set of G of size kappa prime G, which is therefore equal to the edge cut. Now if every vertex of S is adjacent to every vertex of V minus S, then kappa prime G is equal to S and S bar, and that particular size will be greater than or equal to n minus 1 and n minus 1 is greater than kappa G we already know.

So, this becomes the case one when every vertex of S is adjacent to every other vertex of V minus S, let us take this particular example. So, this particular vertex is adjacent to all other vertices. So, this becomes a complete graph situation. Now we are constructing edge cut. So, this becomes S this becomes S prime, and these sort of edges are crossing S to S bar hence it becomes F. So, F is nothing but the size of F is nothing, but how many edges will be going in the complete graph and minus 1. So, kappa prime G is equal to that particular edge cut, and the size of the edge cut in the case of a complete graph is basically greater than or equal to n minus 1.

We already know that for a complete graph kappa G is less than or equal to n minus 1. So, for n minus 1 we plug in that particular value, hence this particular kappa prime G is greater than or equal to kappa G, in this particular case when the graph is a complete graph. Suppose the graph is not a complete graph, then another case 2 will be applicable let us understand, if every vertex is not adjacent to all other vertices let us say x and y there are 2 vertices, x and y they are not having a direct edge, but since the graph is connected. So, it will form 2 sets let us say and in 1 side it is having x on the other side it is having y and there is a path which will connect x to y, but there is no direct edge.

Hence this scenario is different from the case 1. Now we will construct the separating set, and the disconnecting set the minimal disconnecting set out of this particular graph. So, let us to consider the separating set. Let us call it as T so T will comprise of all the nodes which are lying on the other side that is on the S prime side this is S. So, the neighbors of x let us say there are 2 edges which are the neighbors of x, and also intersection S prime is which are lying here on the other side, let us call them as T this is 1 part, another part of the T, T will have this portion union this portion.

So, this portion I have told you that all the neighbors of x which are lying on S bar that will be designated as T, there is another component of T which says that the vertices on S which is not x other than x all the vertices let us say this vertex, and this vertex and this they have the neighbors on S prime. So, neighbors of x here which are lying on the other side will comprise all such vertices. So, there may be more than one neighbors like this. So, we will consider those particular nodes as T that is all who are having neighbors on the other side why these particular edges are important, why these nodes are important, because the path from x to y will contain all these T S.

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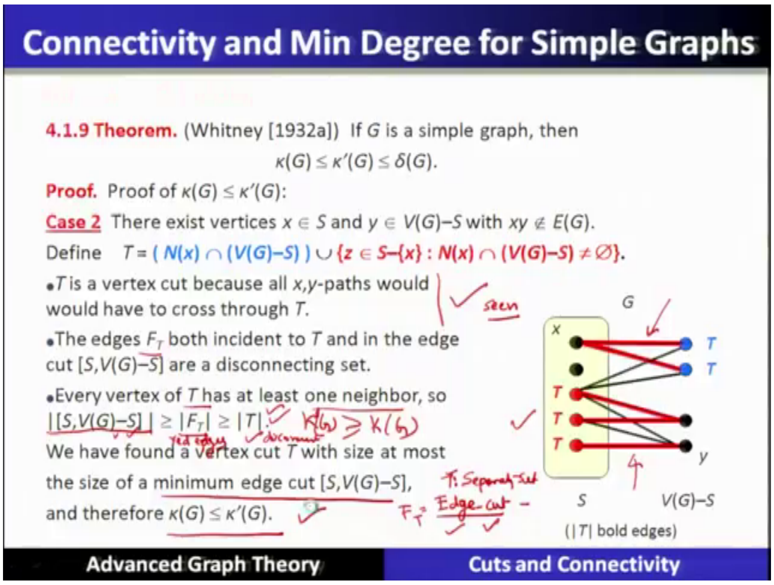

Let us go ahead and see more details. So, we have reached up to this point, now these vertices T we have included. So, we have basically formed our separating set that is nothing, but T we have identified a T as a separating set. So, we says that if all T S are removed, then this graph will be disconnected; that means, S x and y they will not have any path. So, T is the vertex cut, because all x y is would have to cross through these T S. So, hence this we have seen. Now we will form an edge cut. So, edge cut means these sort of edges which if they are removed then this will disconnect. So, those that kind of those edges are called edge cut.

So, we are now constructing the edge cut out of this particular graph. So, let F T the edges F T both incident to T, and in the edge cut are basically the disconnecting sets. So, we will be finding the edge cut out of that. So, every vertex of T how we are constructing every vertex of T has at least one neighbor. So, what we will do for x both these edges we will select, but for other than x all other nodes, we will pick one of these edges out of many we are picking only 1 edge, hence we are forming an edge cut separating set will be having a bigger size because other edges are also there.

If all these edges are included that will be separating set that also will disconnect, but we are forming an F T. So, let us see this particular F T, this separating set which we consider all the edges with are crossing from S to S bar is separating set is; obviously, greater than F T, the set of edges which we have selected, which are shown as red color, and the size of F T is basically greater than T, because T is the disconnecting set thus we have found out a vertex cut C with the size at most the minimum size of that particular edge cut.

Hence you know that T is nothing, but kappa of G which is less than or equal to F T or F T is basically kappa prime G. Hence this particular theorem is proved for both the cases when the graph is a complete graph or when the graph is not a complete graph, then we have constructed both these conditions that is you have constructed a separating set, and also we have constructed an edge cut, and through their comparison, we have seen that kappa G is less than or equal to kappa prime G.

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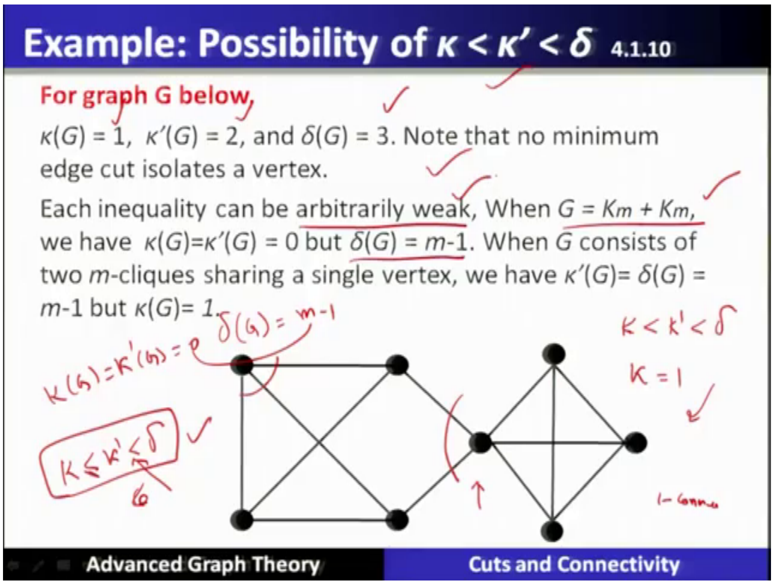

Now, these particular inequalities is basically quite varying and let us see that this there is a possibility which says that kappa is less than kappa prime is less than delta G, in this particular scenario it will happen, in this particular figure, if you see this is basically a vertex cut. So, if this particular vertex is remove this will be disconnected. So, it is 1 connected or the kappa is equal to 1. Similarly as far as these 2 edges if they are removed, then the graph will become disconnected hence kappa prime G is 2 and the minimum degree of this particular graph is 3.

So, if you compare them then kappa is less than kappa prime is less than delta. So, that inequality here equal is not there, since no minimum edge cut isolates of so, basically this we have checked. Now the inequality can be arbitrarily weak when the graph is Km plus Km. So, this particular graph is a disconnected graph where the minimum degree is little delta of G is m minus 1. So, if graph is disconnected then kappa G, and kappa prime G value will be equal to 0, but little delta of G is equal to m minus 1.

So, there is a gap between 0 and m minus 1 although this particular inequality holds, but this particular gap is quite big, and that is why it is arbitrarily weak.

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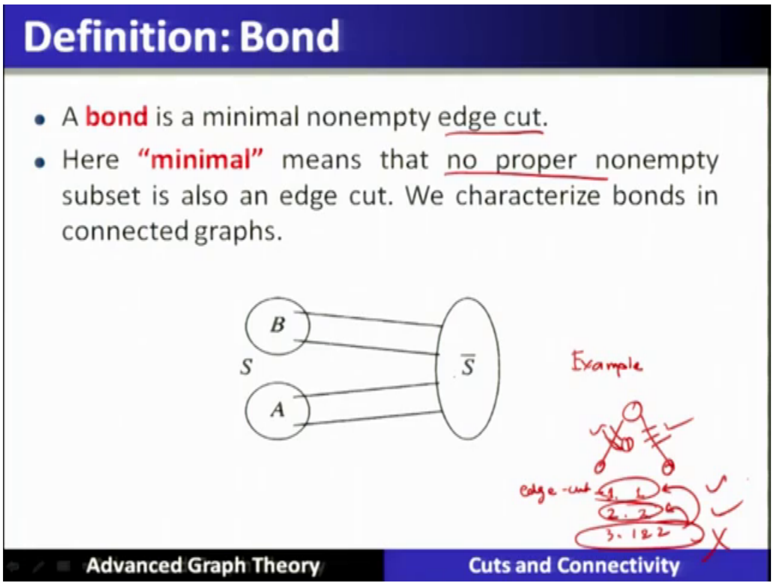

Let us see the next definition of a bond; bond is a minimal non empty edge cut, here minimal means no proper non empty subset is also an edge cut we characterize the bond in a connected graph. So, example this particular graph has 2 edges. So, this edge is an edge cut. So, edge cut there are 3 different edge cuts. So, this is 1 this is another edge cut third edge cut will include both of them, 1 and 2.

So, this is not a bond why because this will include the other smaller edge cuts within it. So, hence the bond is the minimal edge cut, and minimal means it is not a proper subset no proper subset is also an edge cut so; that means, if this is there then the proper subset is this 1, and this one they are also edge cut hence this is not a bond, but they are the other bonds this is shown here, in this particular example and this is the proof.

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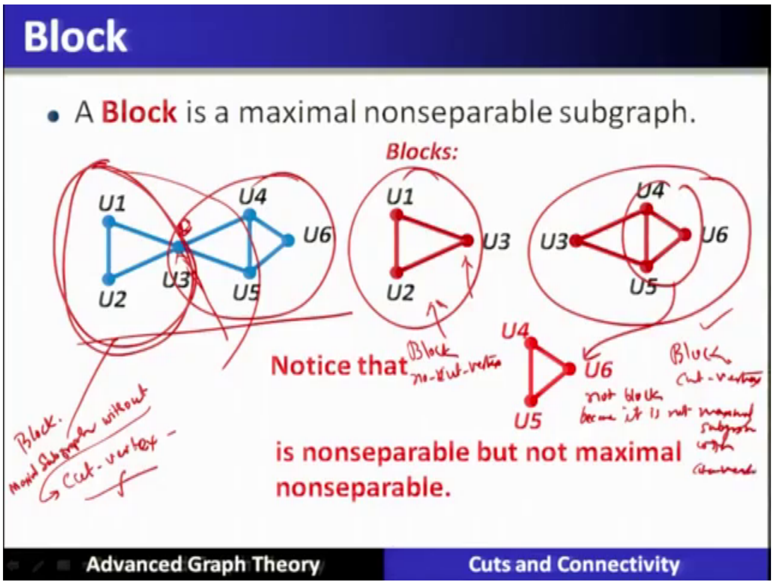

Let us see the block; block is the maximal non separable subgraph. So, non separable in the sense the cut vertex cut vertex can separate the graph. So, that particular maximal subgraph without so it is not a subgraph, but it is a maximal subgraph without any cut edge cut vertex is called a block.

So, take this particular graph. So, this graph if we take this part this is called sub graph, and is a maximal why because if we grow further; that means, up to this point then this will become an edge cut. So, without this up to this point they will not be a vertex cut. So, this is the maximal sub graph you might have notice that the vertex cut vertex of a graph is also a part of this sub graph, but that does not means in this particular portion a which is called a block there is no vertex cut there is no cut vertex.

Similarly, we can see this bigger another maximal subgraph of this particular graph G this also is a block why because it does not have any cut vertex; however, a smaller portion is not a block, because it is not maximal it is not a maximal subgraph without cut vertex.

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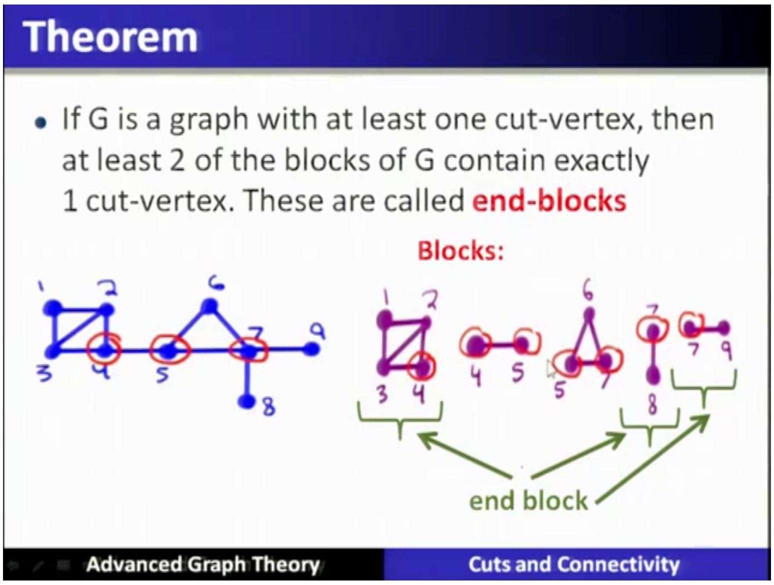

So, there is a theorem which says that if G is a graph with at least 1 cut vertex, then at least 2 of the blocks of G contains exactly 1 cut vertex, and these are called end blocks. So, here we can see that this particular block, and these blocks they have only 1 cut vertex. So, there are 2 exactly blocks having this particular property and they are called end blocks, others are other blocks are having more than 1 cut vertices.

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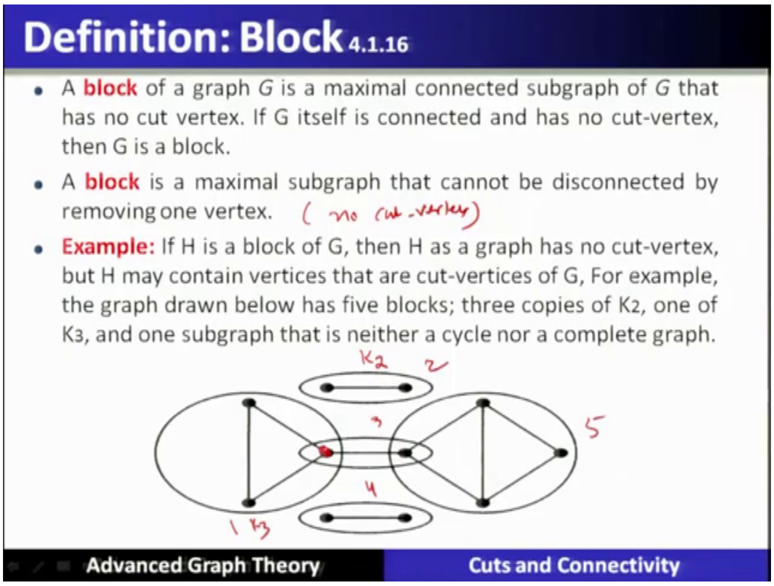

Let us see the definition of a block then a block of a graph G is a maximal connected subgraph of G that has no cut vertex, if G itself is connected and has no cut vertex, then the entire graph is a block a block is a maximal subgraph that cannot be disconnected by removing 1 vertex why because it is not having a cut vertex, example if H is a block of G, then H as a graph has no cut vertex, but H may contain vertices that are cut vertices of G that I have all already explained for example, the graph drawn here has 5 blocks 1 2 3 4 5 blocks, 3 copies of k 2 all 3 are k 2, 1 k 3 and 1 subgraph that is neither cycle nor a complete graph.

So, these vertices are cut vertex, but they are the part of cut vertex of a graph, but that is present in the block that is not a cut vertex of a block, but cut vertex of a graph can be present in the in the block.

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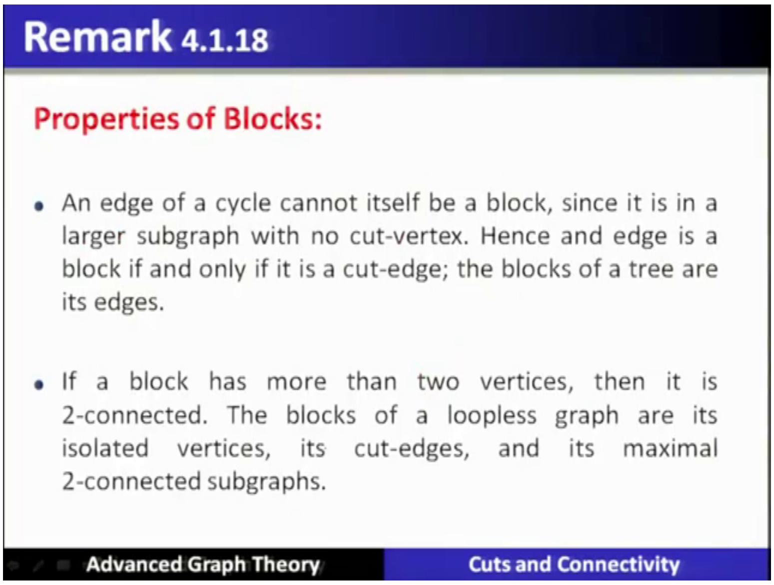

Properties of a block and edge of a cycle cannot itself be a block since it is in a larger subgraph with no cut vertex, hence edge is a block if and only if it is a cut edge the blocks of a tree are it is edges, if a block has more than 2 vertices then it is 2 connected the blocks of a loopless graph or it is isolated vertices, proposition 2 blocks in a graph share at most 1 vertex.

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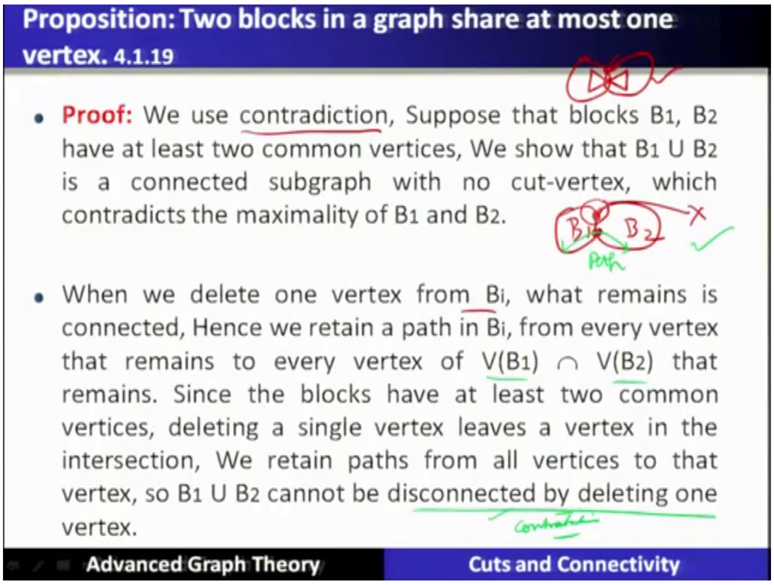

Let us see the intuition and then we will see the proof. So, intuition is says that let us see that this is 1 block and this is another block, 2 blocks in a graph share at most 1 vertex and that will be the cut vertex, and that is why these cut vertex is shared with these 2 blocks. So, it is at most 1 such vertex which they are sharing. So, that is the intuition which we have explained let us see the proof for it, and proof we are basically saying that by using contradiction. Suppose that B 1 and B 2, they have at least 2 common vertices not 1, but 2 common vertices in which they will basically share.

So, we will show that B 1 union B 2 is connected graph with no cut vertex which contradicts the maximality of B 1 and B 2. So, when we delete 1 vertex from B 1 what remains is the connected, hence we retain a path in B 1, we retain a path from every vertex that remains to every vertex of B 1 union B 2 that remains. Since the blocks have at least 2 common vertices deleting a single vertex leaves the vertex in the intersection. So, we retain the path from all the vertices. So, that the vertex B 1 union B 2 cannot be disconnected by deleting a vertex hence the maximality of particular block is basically violated, and this is the contradiction.

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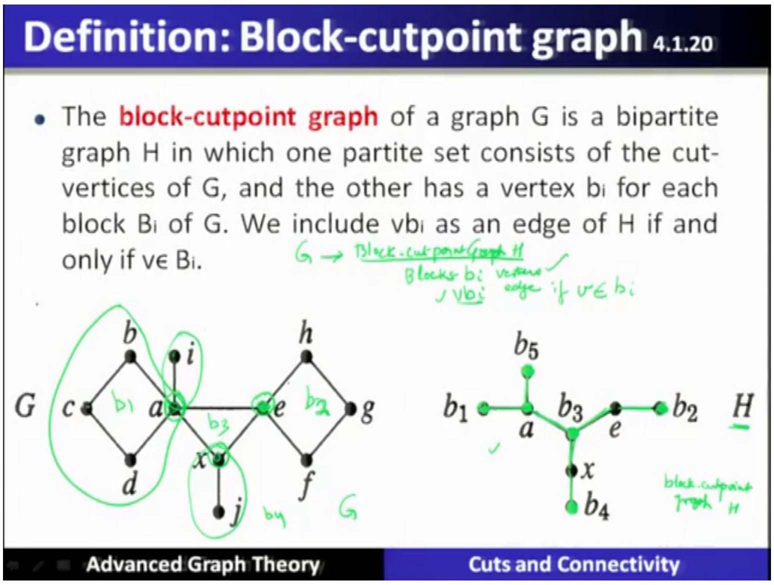

Hence there cannot be 2 different vertices which are which are common in 2 different blocks, block cutpoint graph a block cutpoint graph of a graph G is a bipartite graph edge in which 1 part I would set consists of the cut vertices of G the other has the vertex vertex bi for each block capital bi of G. So, we include v Bi as an edge of H if and only if v is in Bi. Let us see how to construct the block cut point graph, and we call it as a H, and we are constructing from a given graph G. So, it is a bipartite graph here we say that the blocks are represented by the vertices Bi blocks are represented by a vertices, Bi and we also include an edge v Bi, if and only if this v belong to this vertex v belong to that particular block i.

Let us see this is the block and this is the cut vertex let us first mark all the cut vertices. So, there are 3 cut vertices. So, for every cut vertex we write down we place an edge with that particular block, let us say this is b 1 block, this is b 2 block, this is b 3 block, this is b 4 block. So, for every block a vertex is placed let us say this is b 1 this is b 2 this is b 3, and this is b 4. Now the cut vertex belonging to block a we are placing an edge that vbi. So, here it is vb ab 1 this edge will be placed here ab 1.

Similarly, there is a block b 4 and b 5, this is also a block. Similarly let us say that b 4 is represented as a vertex, and this a is a part of this vertex a is a part of this particular block that is b 5. So, there will be an edge between a and b 5. So, these edges we have worked out. Now this b 3 block will have the cut vertex a so a and b 3, will have an edge similarly b 3 is having a cut vertex x also. So x and b 3 will have an edge, now this x is a cut vertex of this particular block b 4 so, x and b 4 will have an edge.

Now, another thing is b 3 is having an a cut vertex e. So, b 3 and e will have an edge. Now this is b 2, b 2 having a cut vertex e. So, b 2 and e will have an edge. So, this will form a block cut vertex cut point graph.

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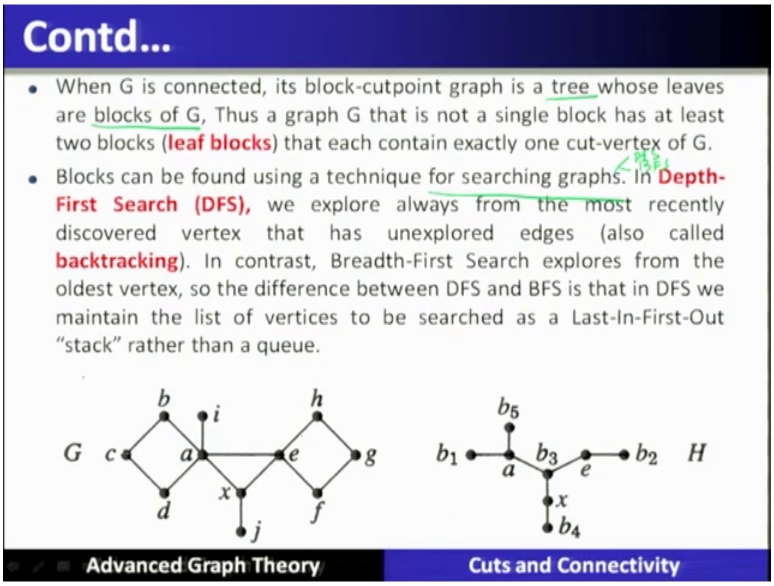

And we call it as the H and this graph is constructed out of a graph G now when g is connected it is block cut point graph is a tree that we have seen, and whose leaves are nothing, but they are the blocks of G thus a graph G that is not a single block has at least 2 blocks that is the leaf blocks that each contain exactly 1 cut vertex of G.

So, blocks can be found using the techniques for searching the graphs. So, 2 techniques we know one is called DFS to search a graph the other is called BFS. So, in DFS we explore always the most recently discovered vertex that has unexplored edges also called backtracking. So, in the contrast breadth first search explores from the oldest vertex. So, the difference between DFS and BFS is that DFS we maintain a list of vertices to be searched as a list and last in first out step. So, let us see how the DFS can be utilized to construct this particular block cut point graph, in this particular example.

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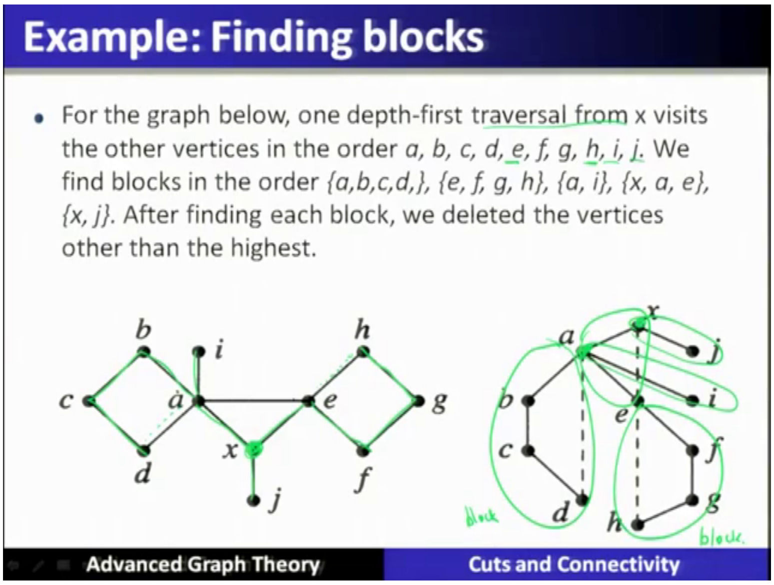

In this example, we will do the DFS traversal from vertex x here starting from here. So, x visits the other vertices in the order it will visit a then b c, then d then it will go to a a is already visited.

So, it will backtrack and then it will backtrack, and visit e from e it will go to f g h, and then it will reach to e e is already visited. So, it will backtrack after h it will backtrack, and now we will go to i from i it will backtrack, and then it will visit j. So, if you see the points where it backtracks. So, that becomes a block. So, in here also in backtracks. So, this becomes a block. Similarly this particular place it backtracks from e to x so a x e will also become a block.

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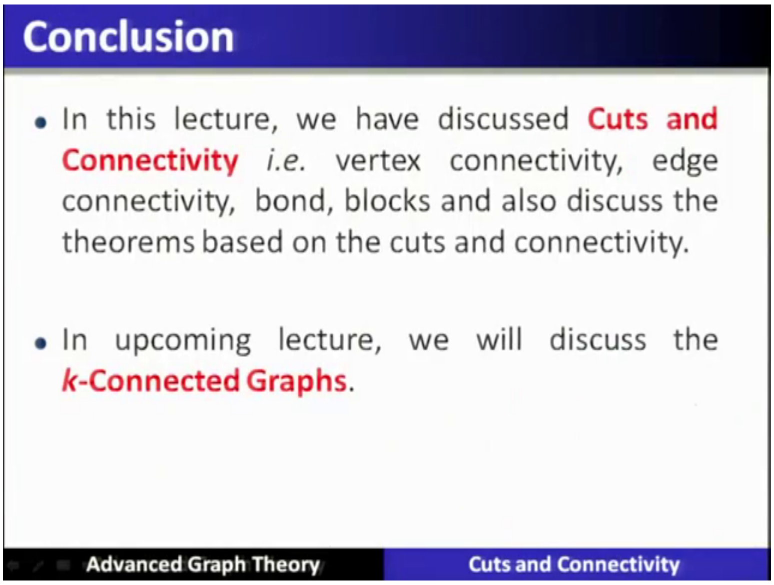

So, after visiting j it backtracks. So, this also will become a block, from a it back from I it backtracks to a it will become a block, and the point where it backtracks they becomes the cut vertices. In this lecture we have discussed a cuts and connectivity that is vertex connectivity, edge connectivity, bond blocks and also discuss the theorems that based on cuts and connectivity, in the upcoming lecture we will discuss the generalization of this connectivity that is the k connected graphs, which will require these understanding why because k connected property is settle.

Thank you.