Parallel Algorithms
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Lecture - 6

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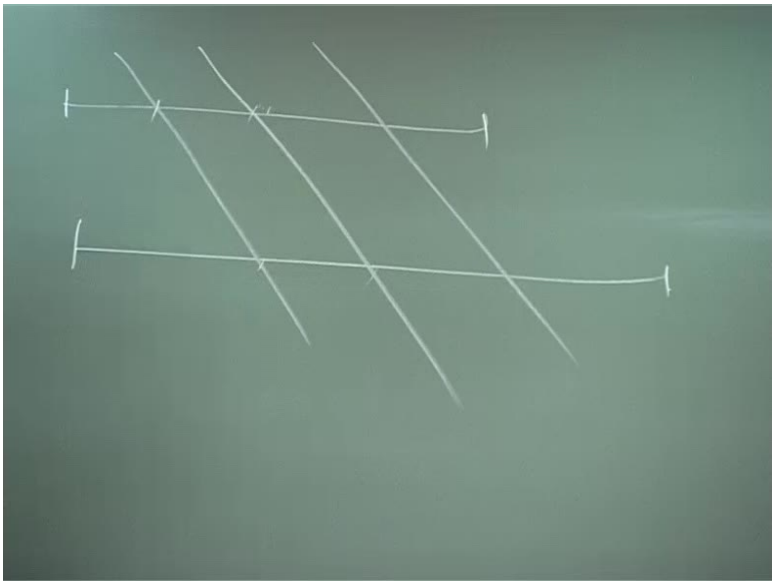

So, in the last class we were discussing about the medium pair algorithm, let in the medium pair idea is you have to sequence and you have to find out the medium pair. Such through that the devise exactly two equal half so, that next time this is done by one person. And next time I have applied two person two divide one person is to divide this part in two parts and this parts into two part and the size become same with condition is the size become same.

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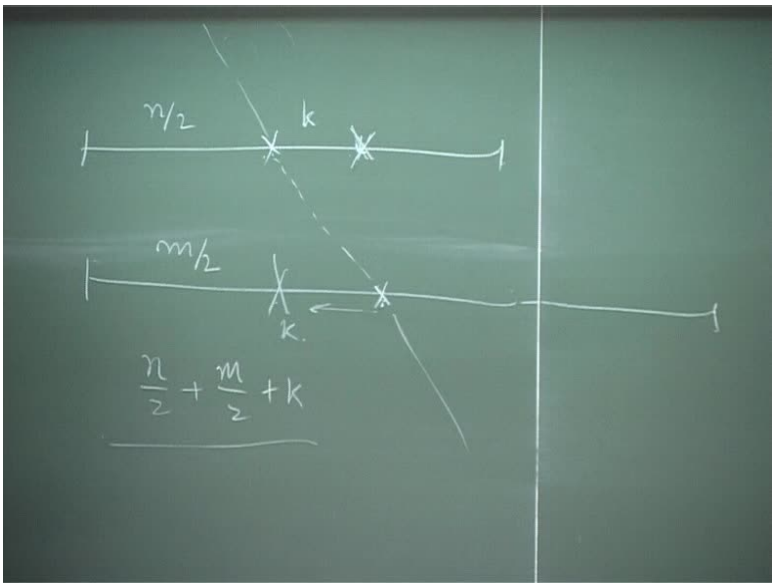

And all this elements will be smaller than these elements. This element would be smaller than this element now, I can apply n process two merge is sub part and they will take the same amount of time and then there is no need of concurrent read part. So, only if I do this partition that will take care so, the issue is how to divide or how to create the medium pair. Initially this process is like this initially, I will take the middle value so, I can assume that these two elements are the probable candidate for my median pair. Now if this is the case you also know this is two points because this is n by 2 this is m by 2.

So, take some of this then it becomes one of them will be n plus m divided by 2 that medium means that is the elements. There is a probable candidate but, as I told this cannot be a probable candidate for the median now, what it does? He checks is there any element in this zone which is larger this element over is there any element of this zone which is larger than this element.

Suppose, there exit one element here which is larger than this element. Suppose, what it means that all this element should be smaller than this element, there exit one element one element here which is, if I find that this is smaller than this element, then all this element should be smaller than this element. What it means suppose, this is k then n by 2 class m by 2 class k that many elements smaller than this element. So, this cannot be the median element. If you want to make this the median element you have to find out median element. Once this has to be shifted by smoothing has many house you have

shifted this side. So, you have to shifted by k house this side so, what happens that now probable candidates is that this and this are the member of the so, we if increase this side that should be decreased this side. So, that m plus n m by n m by 2 plus n by 2 that relationship is maintain.

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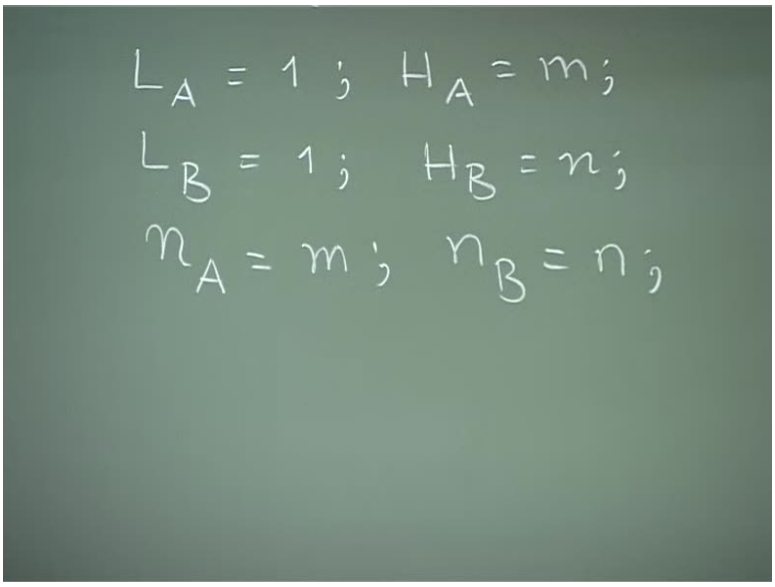

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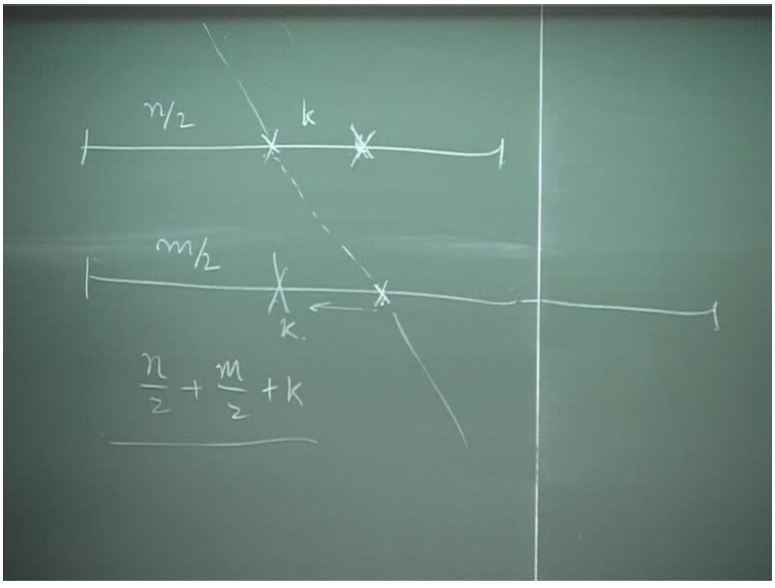

Now, since it is shorter order so, you can use the binary search to find it out. So, you have to find out the median pair suppose, you have a equals so, as usual you have low is one high is your m low in that sub b is 1 and height of b is your n than number of

elements in a is m number of elements in b is your n. So, in a o points to the total number of elements you are considering for finding the median para n a in this n a gives you the number of elements in the n a which you are considering for finding the median pair element. And n b gives to the total number of elements in the array b which are under consideration for median pair.

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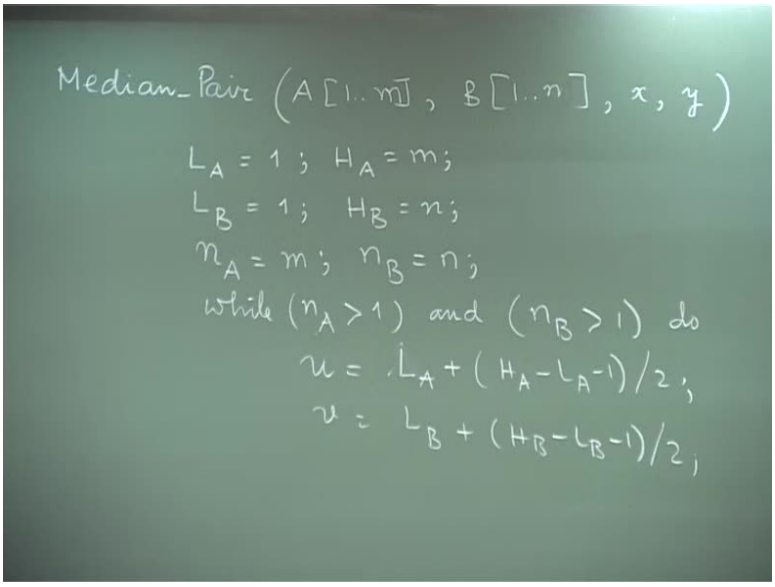

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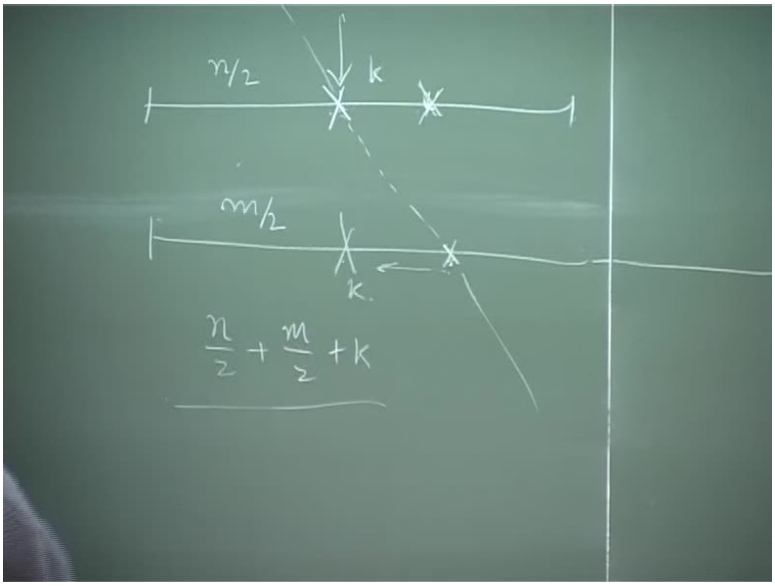

Now, this time doing this first time I am considering this one second time my searching zone may be this one or this side because the binary searching be halfing out the time.

You know there may be a terminal condition. What is that while n a is greater than 1 and n b is greater than 1. Do we have to find out the middle element middle? Middle element is low of a plus H a minus L a minus 1 divided by 2 v is a at this equation you will be able to discord some part. Suppose, I find out this element is smaller than this you have to shift this axis's searching that will be reduced.

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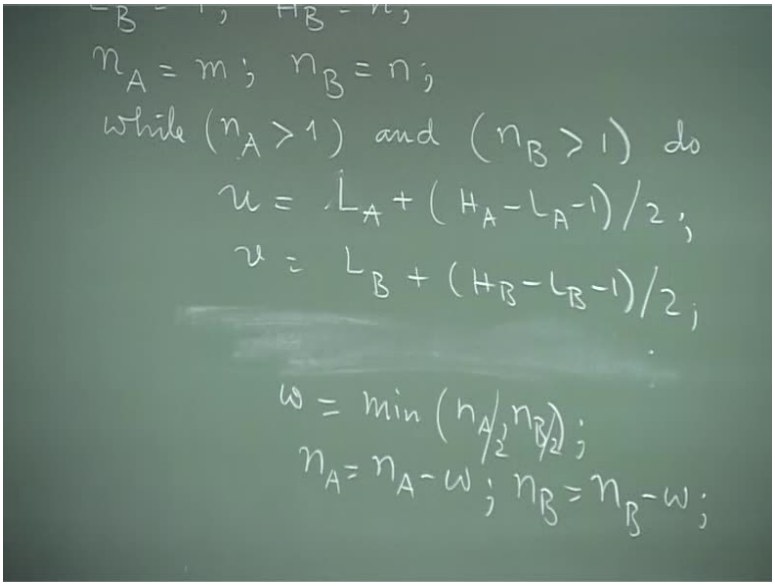

Now, as we do in the case of binary search, what you do in the binary search that you search here if it is larger than you. Go to this side and this part is reduce you have discarding same this would be here you are, you want to discards certain parts to find out the appropriate pair of elements So, new n a will be next time it will be half and new n b is also n b by half and your amount will be able to discard is minimum of n a n n b will discuss two example this one I think, I should not change n a n b now.

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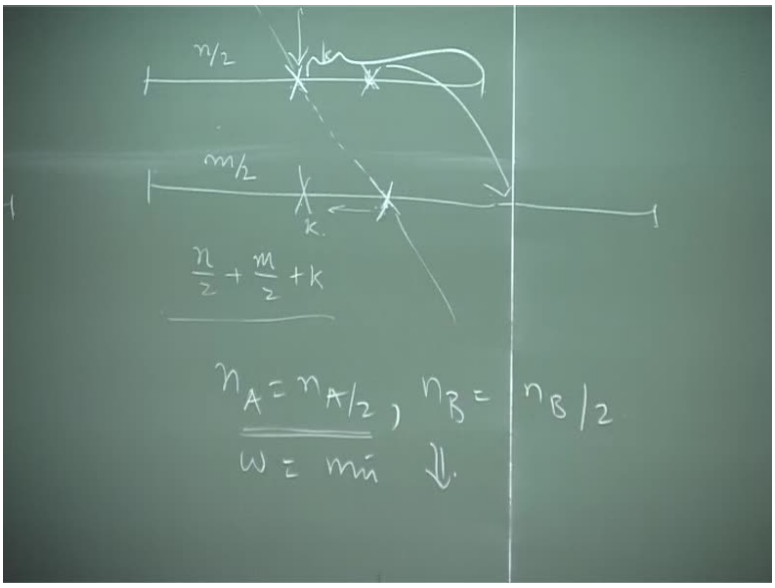

Then i may be little problem here you have put it by 2 by 2 and then you put n a is equal to n a minus w n b is equal 2 n b minus. Where there no guaranty otherwise difference is that see half if I write first n equals to n a by two parts. If I right n a is equal to n a by 2 and n b is equal to n b by 2 and w defined that minimum of this 2. What it and then n a size well be reduce by I have already reduce by size by half but, this side there is no guaranty I can reduce by half exactly. I can reduce maximum this side here.

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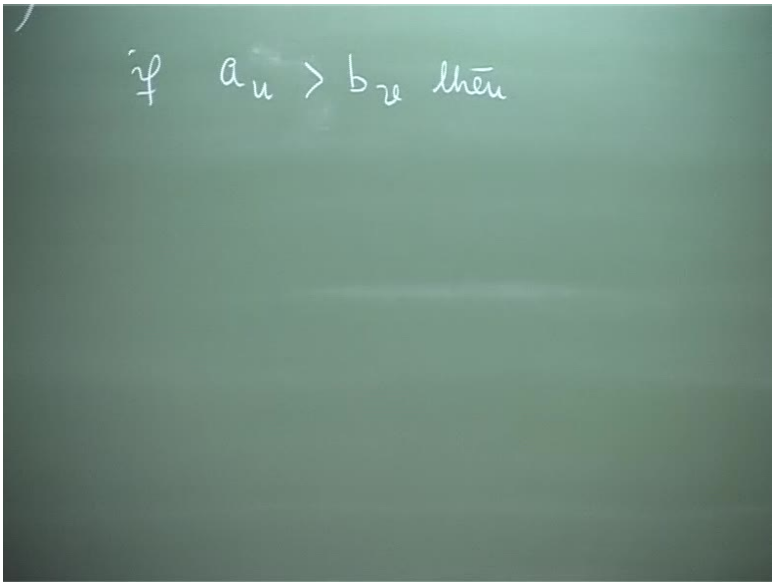

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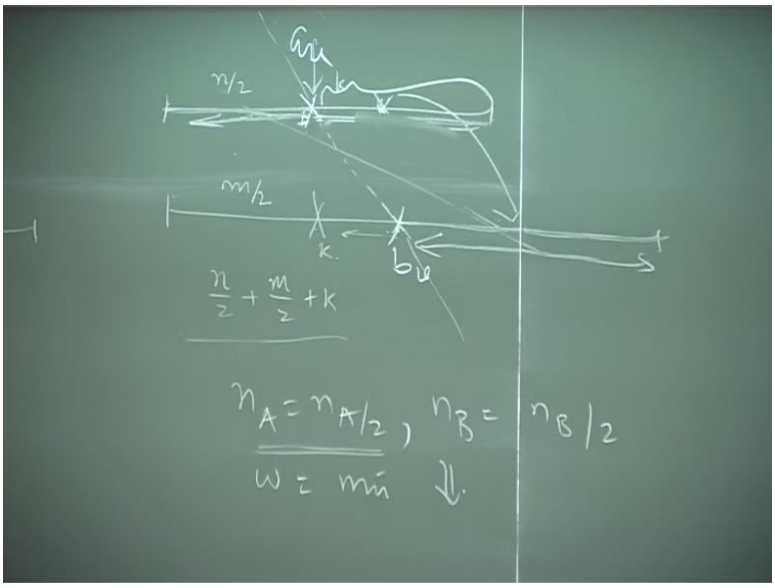

Not more than that because i have to get the guaranty because I have told that this is not a probable candidate. So, that guaranty also you have taken here. So, you got n a now if you find that a u is greater than b u than a u if, a u is greater than b v what it means that element I have to find out the element from this zone a u is a u greater so, I have to find out the element from this zone sorry element from this zone and this side is your b v so I have to find out this side a u if find greater than b v that means the median pair must be lining from a v here.

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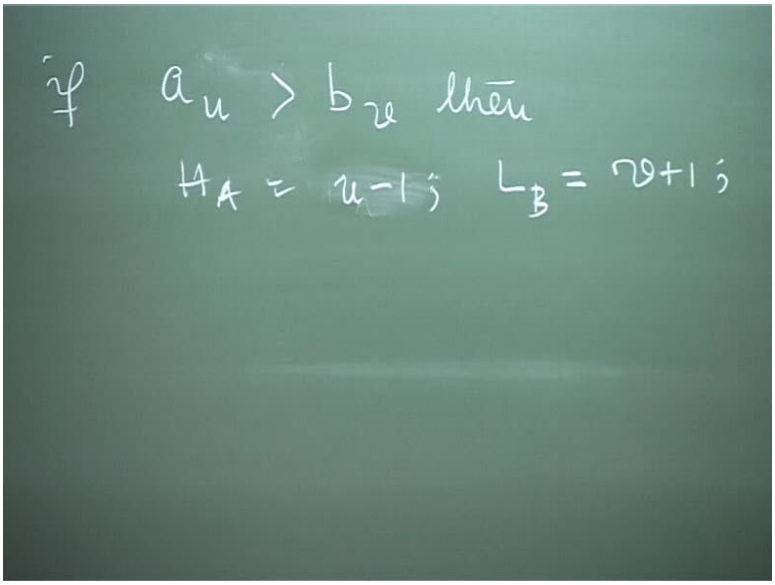

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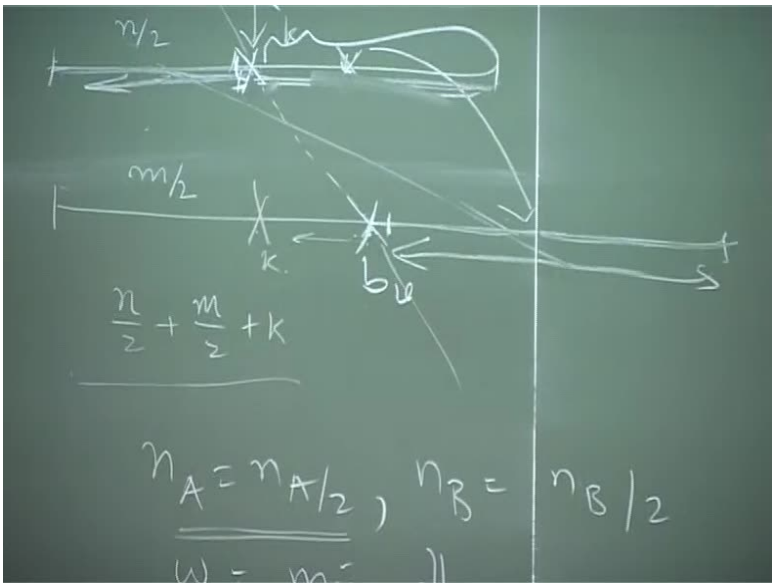

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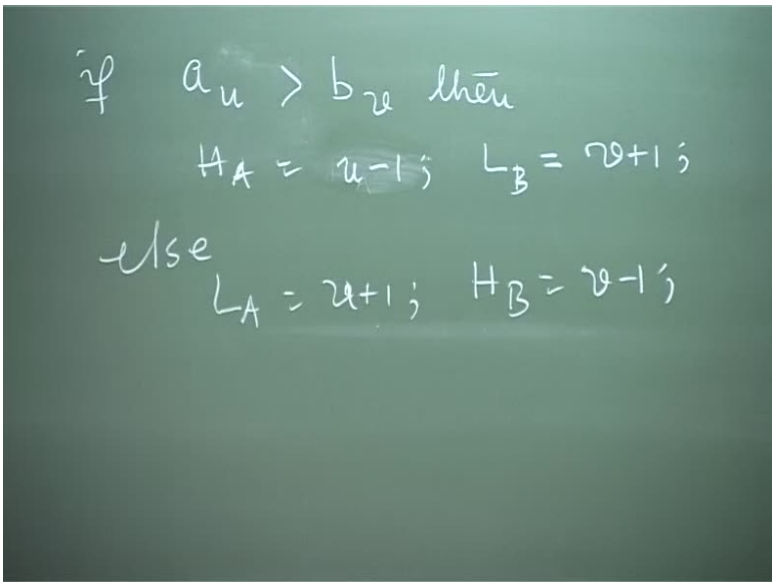

And from here so, that means in this zone means height should be high or high in that should be reduce. So, your h a becomes h a minus h a becomes u minus 1 and l b becomes v plus 1. So, h a becomes this 1 and will becomes this 1. Else is the reveres 1 l a becomes v plus 1 g b becomes nor be u plus 1 and this is v minus 1. h a becomes u minus 1 l b becomes v plus 1 l a becomes u plus 1 h b becomes v minus 1.

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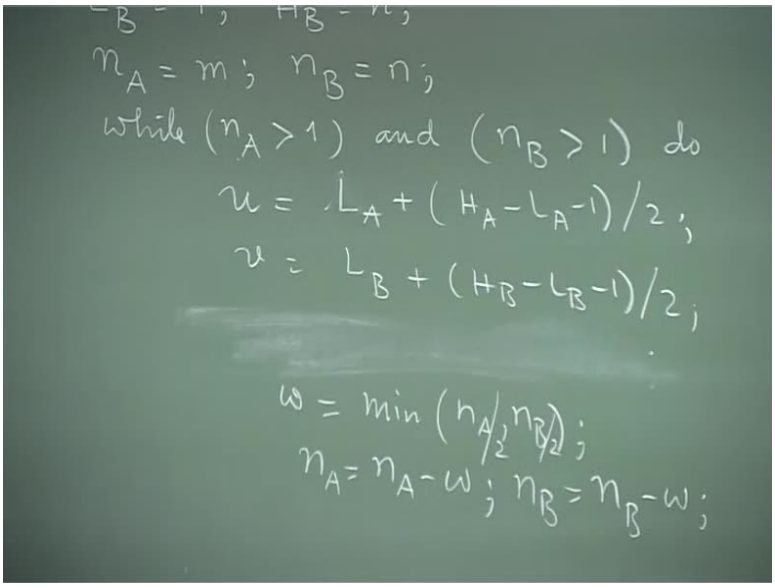

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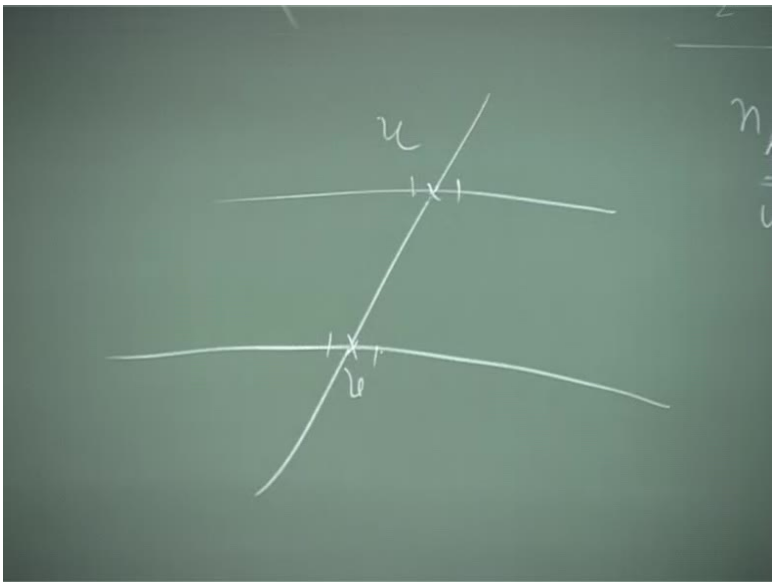

So, once this will be interested till there exit at list 2 elements in n a and n b so, when it is 1 of the median pairs you will be getting 1 you will be getting the u and v. So, if you get some u and some b this is u and this is b. But, you have to know that this may not give as you through an example you have seen this is the most probable candidate to be the median pair. So, what it has been observed because of that u minus 1 and v plus 1, that this are the three probable candidates can be the member of your median pair.

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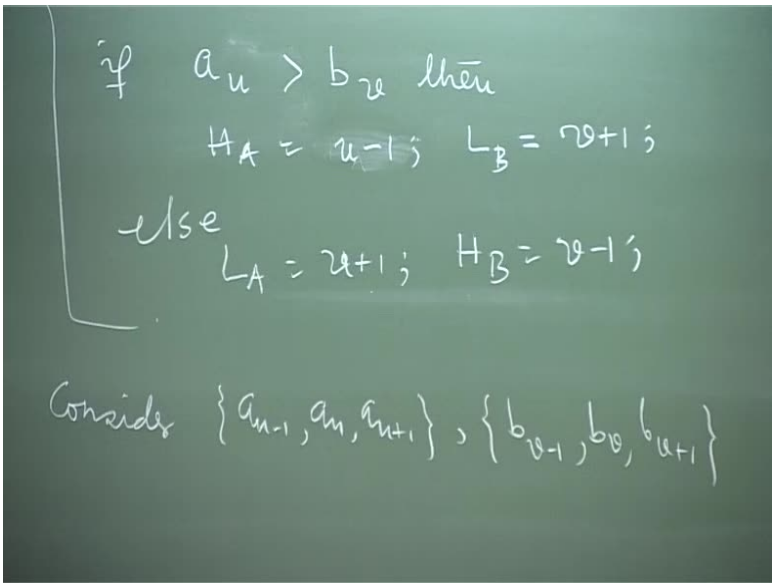

So, consider a u minus 1 a u and a u plus 1 and b b minus 1 b v and b v plus 1. So, how many pair will be getting with refers to this will get 3 you will get 9 and you select one which satisfy your definition of median pair that can be done in constant 10. Now, you know the definition of median pair so, nine element you select for this element whether he is the middle element or not all for this whether, he is the middle element or not that is the thing 9th element whatever you perform the rotate constant amount of time right so, you have form the nine element you have to get the 5th element. For example we have that summation of that means median.

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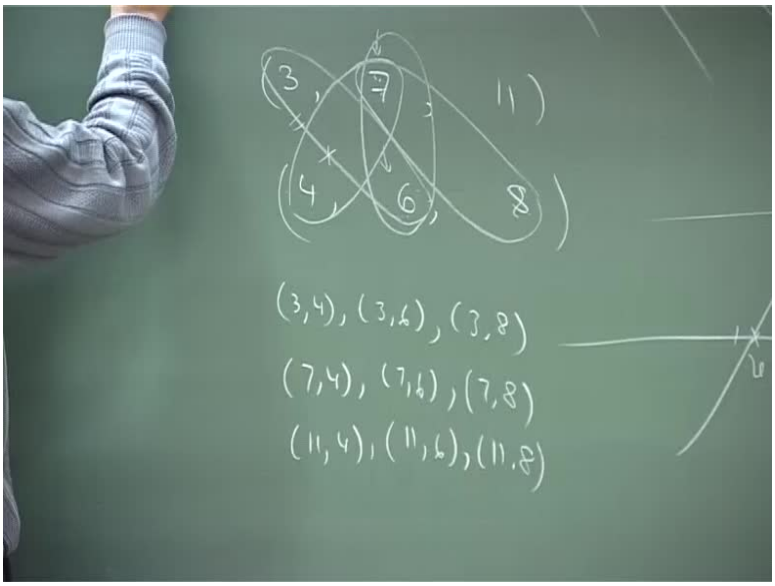

Let us think first this three elements 3 7 11 4 6 and 8. So, you get 3 4 3 6 3 8 3 4 7 6 7 8 11 4 11 6 11 8 this are the nine pairs should be get it. Now, which one is median pair for this 9 3 4 6 7 8 this so, 7 is your median element agreed. Yes 6 also possible middle elements so, one possible candidate is this one and another possible candidate is these one another possible is this one another candidates possible. This 1 6 8 6 11 cannot be a possible candidate.

No because 6 7 is line there right this are the 4 yes 7 4 is, If 3 6 also not possible candidate 3 6 is also not a possible candidate 4 7 6 7 7 8. All 7 4 7 and 6 4 7 7 4 what is the definition we may one of them is median and another element is.

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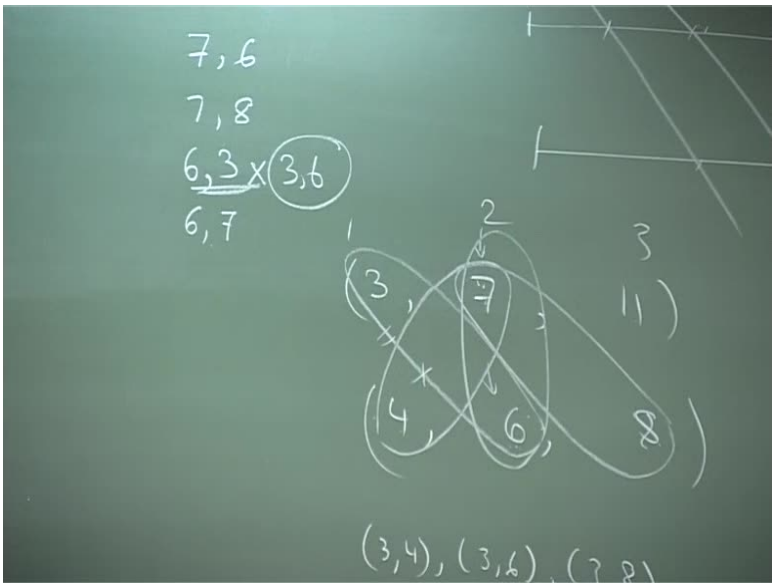

Yes other element is either smaller than this element or just higher than this element so, if 7 is median pair median element then all the element so, 6 is a probable candidate and 7 8 is probable candidate in the case 6 is a middle element. 6 3 is possible and 6 7 is possible so, not 7 4 from the definition now, we will be selecting the one whose some of the index is the smallest. So, here it is 1 2 3 so, this will be the 3 6 actually 3 6 is the your returned.

Sir why should be consider the element after larger than the minimum if you're taking the minimum some of the index? Yes. Element data's smaller than the median will have a smaller index and larger index. No see median pair definition first let, us some probable defined definition was made like that one of this two element will be the

median element the other element of the other sequence is that element we is the either the largest element smaller than the median element or the smallest element larger the median element. And if you get and more than 1 return than we select the one whose some of indices is minimum there is the definition depart right based on that only here finding this.

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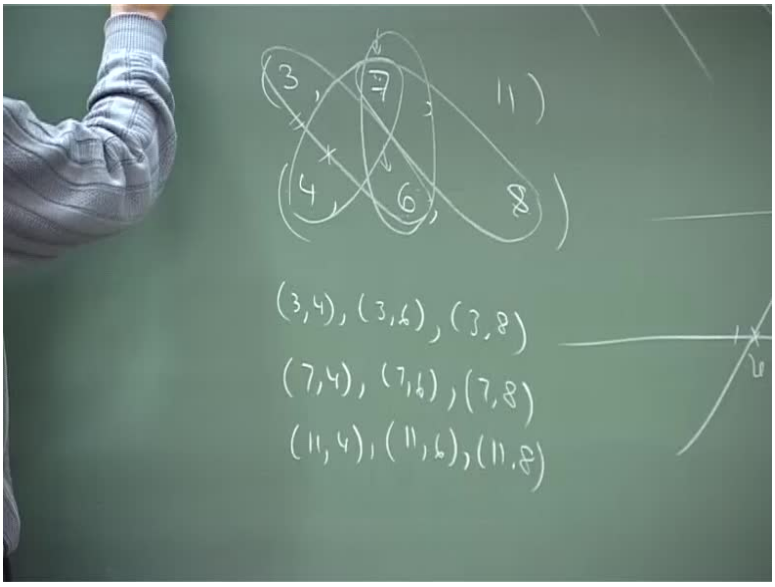

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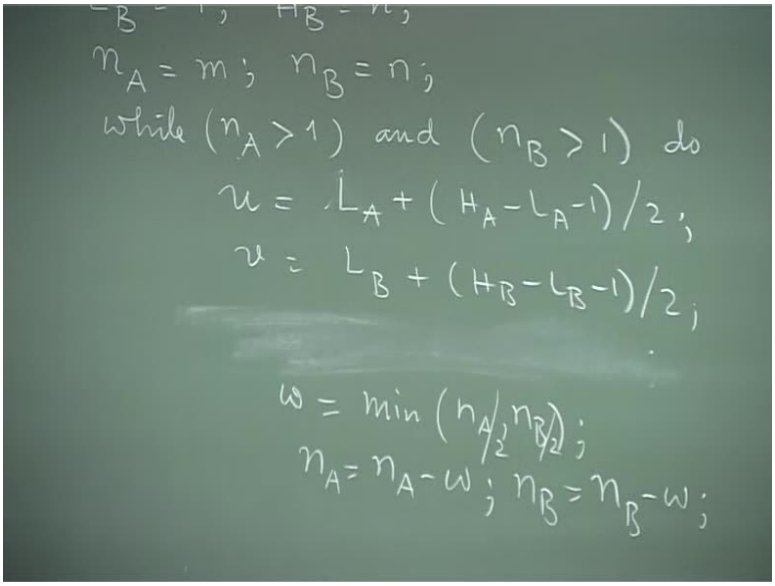

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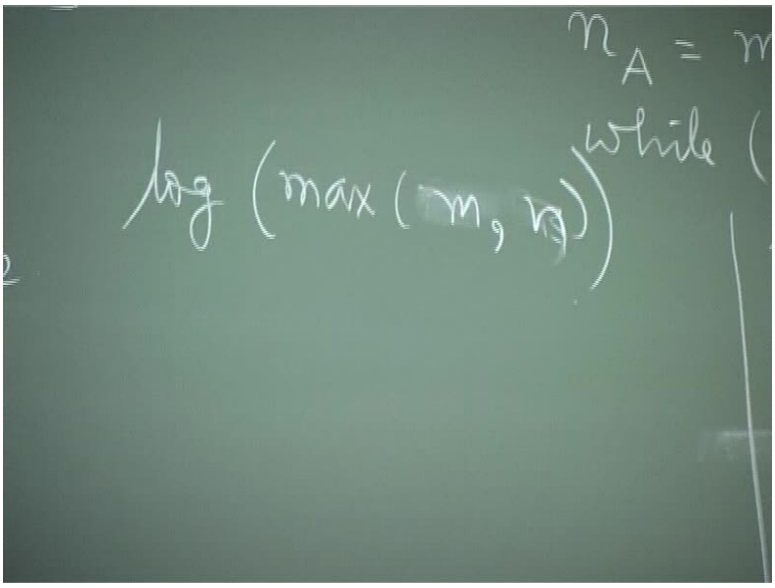

Now, this can be done in constant amount of time because my pear and manipulate all this thing agree and here theoretically, that every time we are dividing into half half half half in reality that is different but, theoretically that there of equal size and very time you are reducing by half. So, log after log nitration's i must be able to come here to six constant of time. So, complicity of this algorithm becomes longer or long of maximum of n a n b or n a n b means your n n m.

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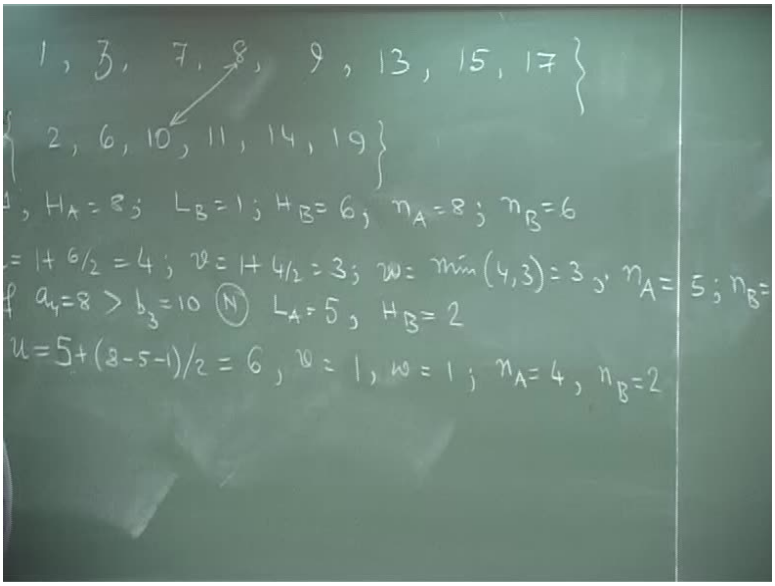

So, let us consider one example I think through that we will able to we so, regulate is 1 n

h a is your 8 initially, l b is 1 h b is your 6 n a is your 8 n b is 6 that this is the first nitration u is 1 plus six by 2 say this 4 v is equal to 1 plus 4 by 2 this w n a modified is 5 n b is 3. If a u a four which is a is greater than a 3 b 3, b 3 is what ten answer is no if it is no so, this is compact if it is no L a becomes u plus 1 minus.

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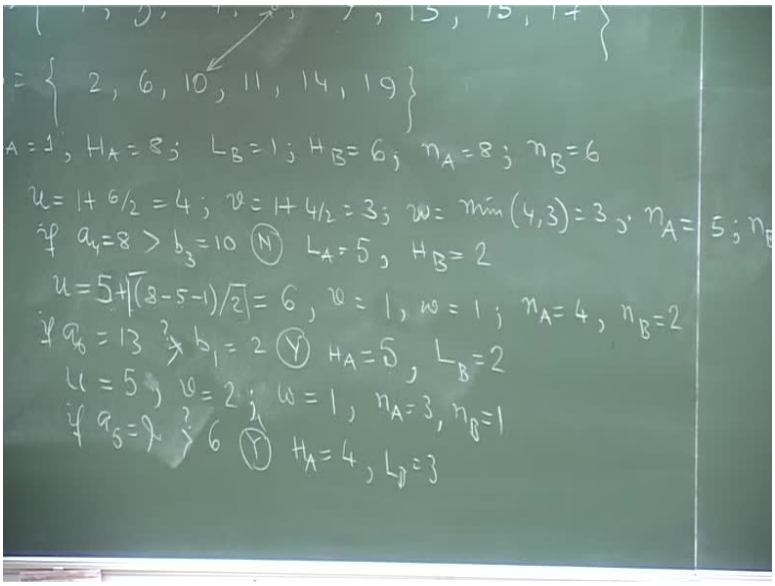

5 an h b is 2 now, u becomes 1 plus 8 minus 5 minus 1 so, divide by 2 if this l is 5 here 5 plus 1 it is 6 v is 1 plus 0 so, b i1 w is 1 so, n a is 4 n b is 1 than a 6 is 1 13 is it greater than it is less than or greater than b. What is b b 1 is 2 it is then h a h to be updated h a is 5 minus 5 and l b is 2. Now u is l a la still 5 plus 0, which is 5 5 minus 5 0 so, it is 0 you have take this index so, it is 5 u is 5 v is this is actually, v minus 5 minus 1. 5 minus 5 is 0 minus one by 2 is 0 to take the low index upper index on to be this, b is what? Now we have to take it 0 because we have in this a part of it say b is what now, b is l l b 2 plus 2 minus. So, this is also 0 so, this is 2 w is what n a. So, w is 1 n a is 3 n b is 1 now, if u 5 a 5 what 9 greater than equals to 6 answer is yes if yes than h a is 4 and l b is 3.

Student: Sir in the last second stage 5 w is 1 if 2 may com 5 com.

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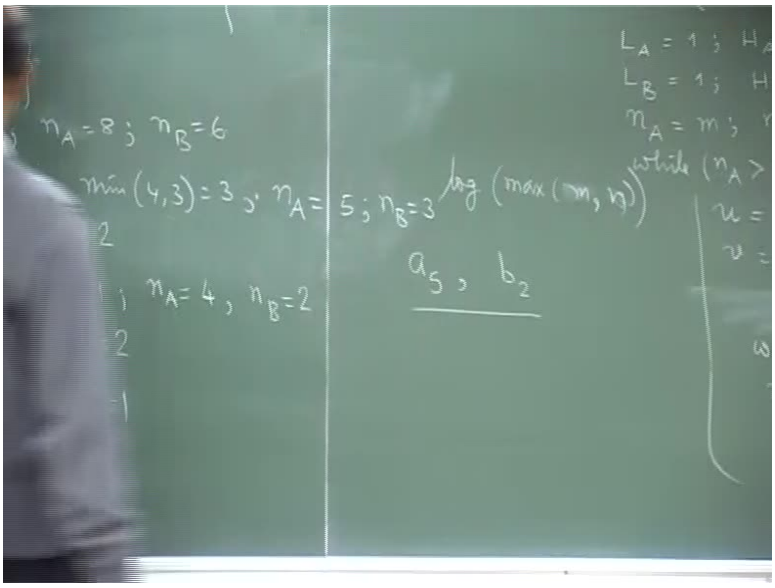

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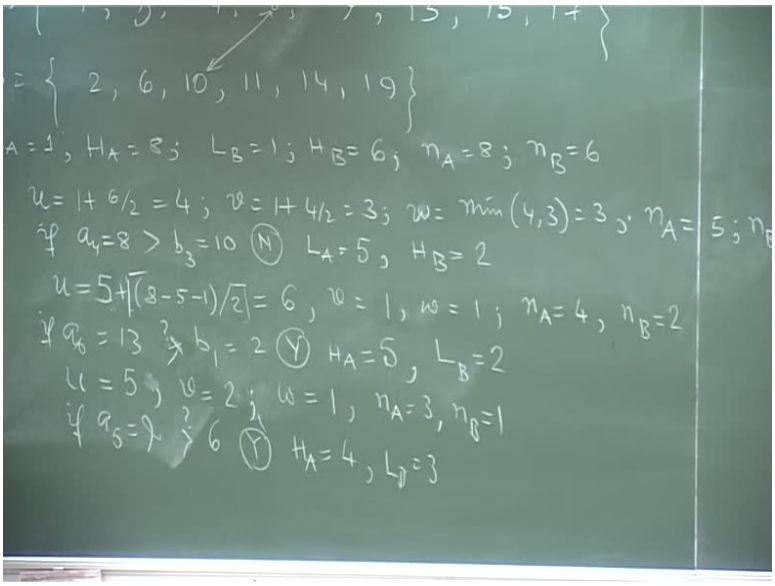

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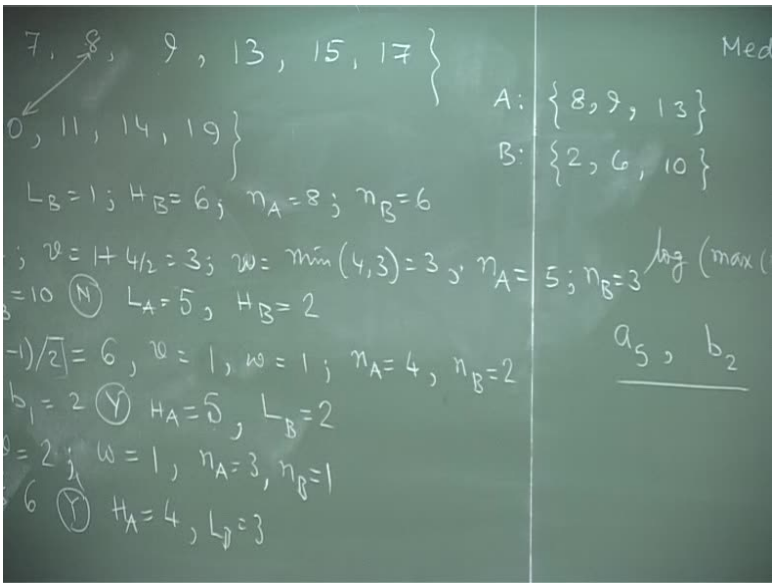

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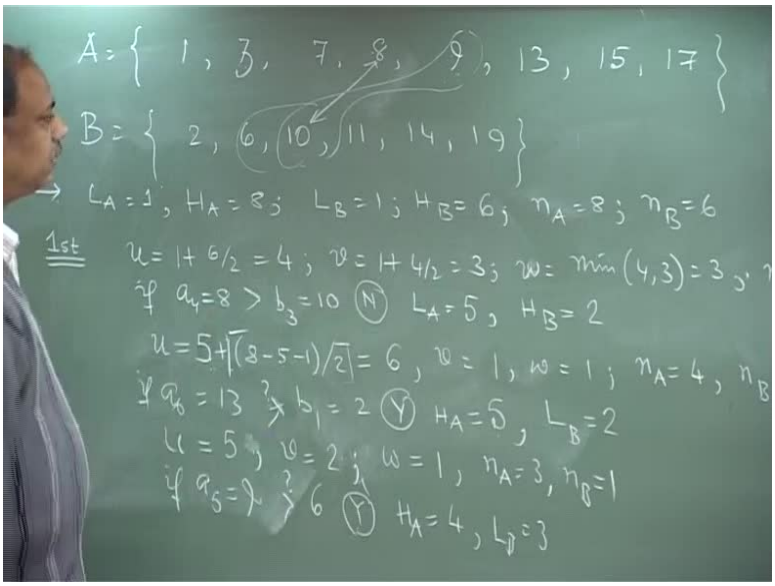

Minimum of n a by 2 n b by 2 so, here we are written well is a 5 and b 2 because now, n a has come n b is come greater than equals to 1 so, it will stop so, basically you get the sequence this is 1 2 3 4 5 so, 8 9 13 and 2 6 10 this are from the a sequence you get this from b sequence you get this and you have median pair this lines here. Let, us see 1 2 1 2 3 4 6 7 8 totally 14 elements, after find out the 7th element 12 3 4 5 6 7 so, this type of elements should be there

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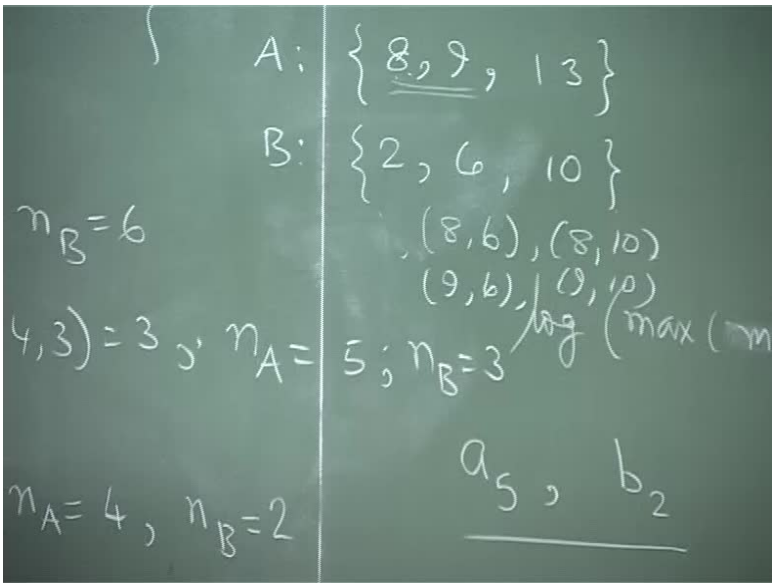

So, it will consider1 of the 2 6 yes 2 6 8 6 elements so, 3rd or 4th element by this or this median. So, this form either here it will be possible will be getting from there 8 and 9 are the median pair. Median element so, and it will be selected now if I find what are the probable candidate tell me from here median element one can be 8 if you 8 than it can be 2 8 or 8 6 if it is 9.

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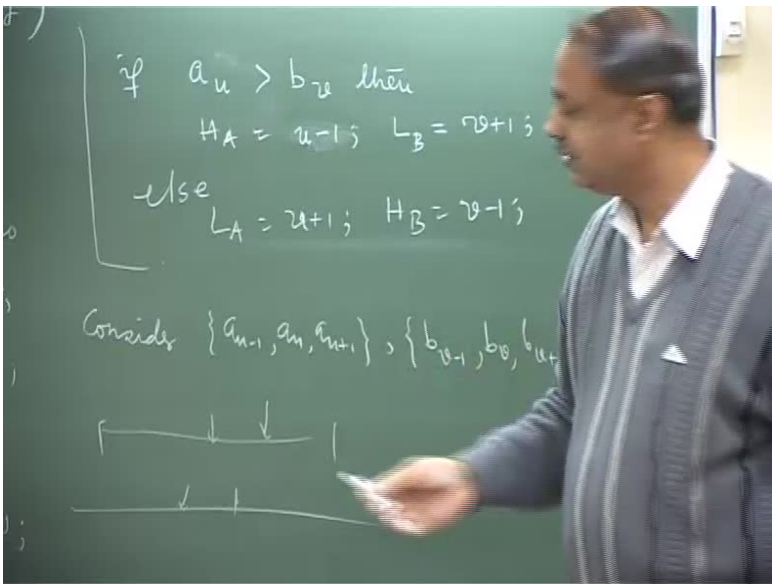

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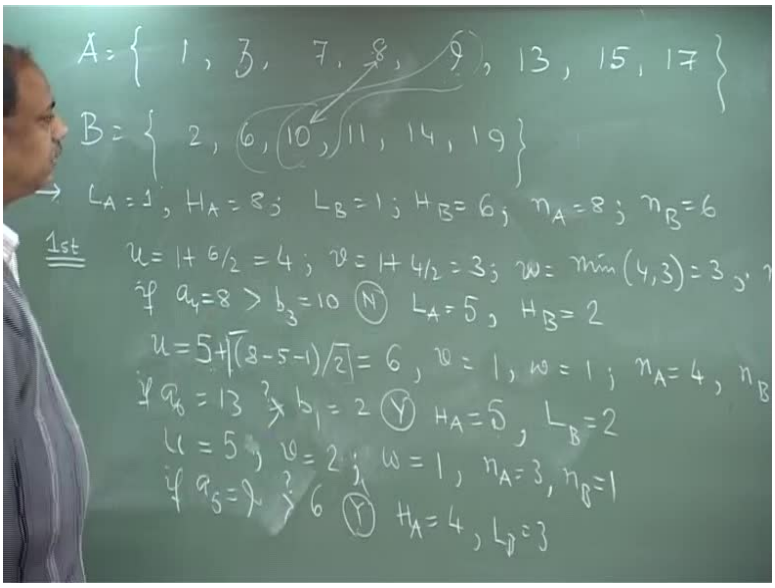

If it is 8 than how 2 you may 6 8 minimum element, yes 8 6 another one is 8 and 10 concurrent if is 9, 9 6 9 10 and the written will be 1 which is having the minimum index. Either this 1 8 6 it is the one of the element is the middle one so, there is the idea now this here right and what I here he use the idea of binary research 1 2 so, two arrays in such way that if this moves this side the other point moves in this side that is the only thing this is not a very big achievement but, we observe through that where is w problem can be solved there is the only thing. What is the array once I move the point from here to here this point will move from here to here so, that n by 2 half I to get a median pair. So, median element must be 3 so, that properties maintain so, once I know this median pair and you observe this median pair is a sequential algorithm.

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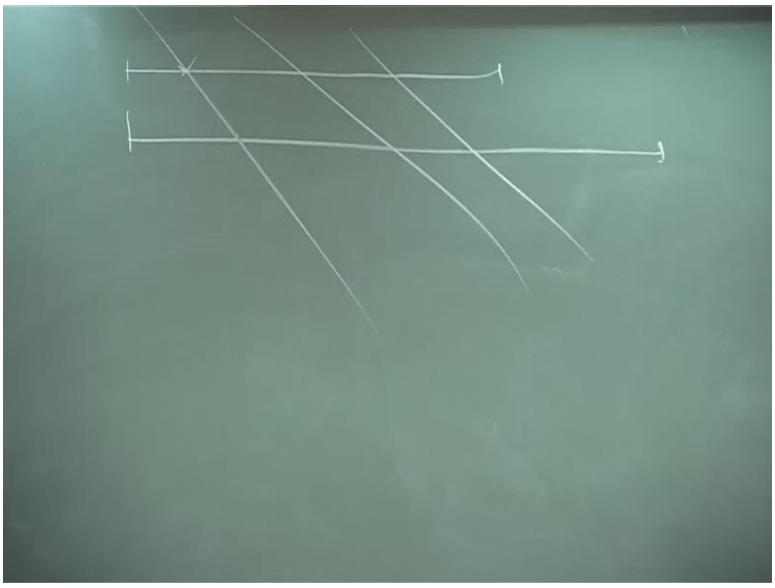

We got that you use the pair algorithm on this part. So, once I know this sequential algorithm for finding the median pair this can be use for here will be rotating. So, if the first iteration suppose, if we have two sequence in the first iteration I use one person to find the median pair in the second iteration I use 2 person to find out the median pair and 3rd you have use 4 person to find the median pairs and so on.

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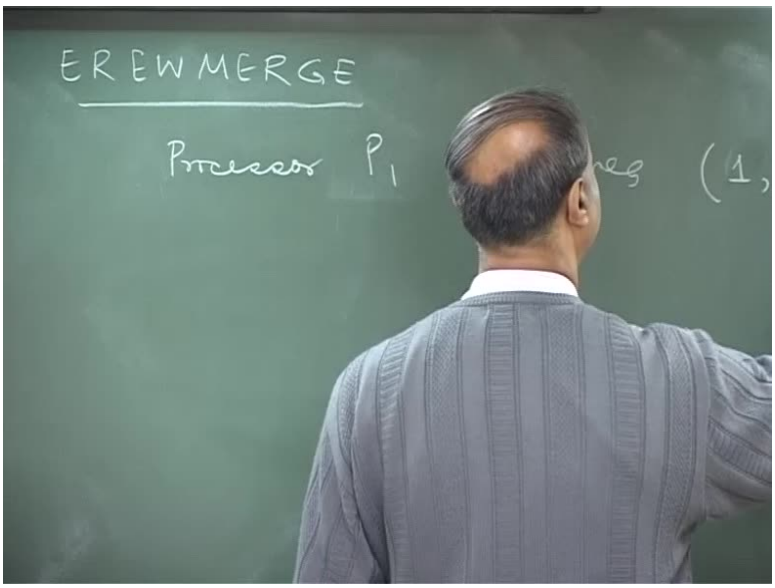

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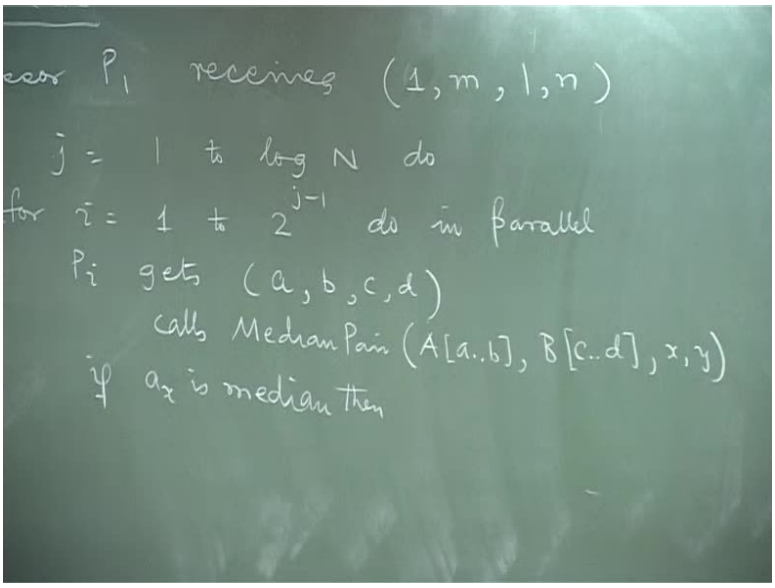

And we divide this pair such way that one element is the middle element the other element is selected in such way that all this elements are smaller than this elements. So, that the properties is maintain than this cluster is smaller than cluster on so, on it is so, e r e w merge e r e w merge processor p one receives. So, one m one n what it means that he grade the first index of a sequence last index of a sequence first index of b sequence last index of now, for j is equal to 1 2 log n is the total number of processor do for I is equal to 1 2 to the power two to the power j minus 1 l 4 l m.

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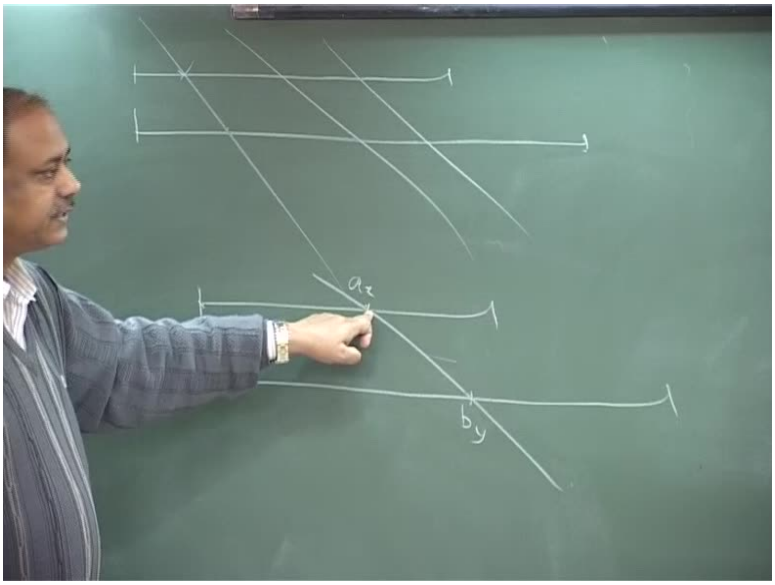

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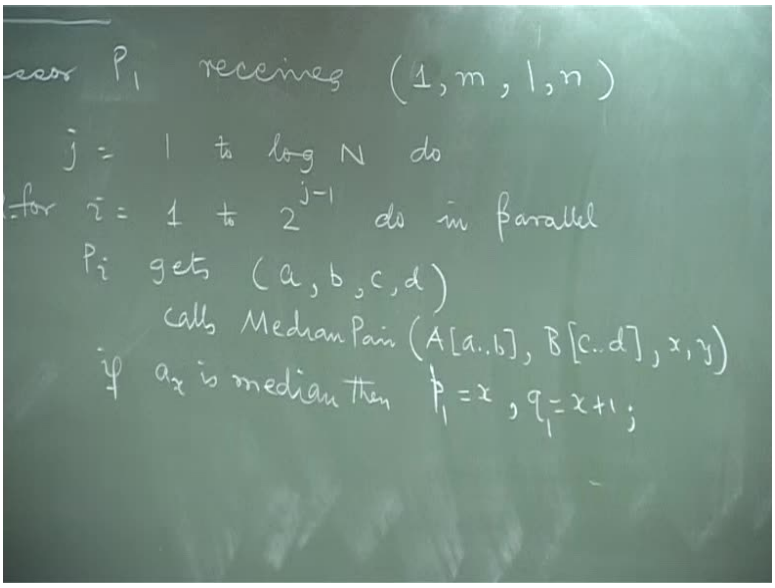

So, what I am telling first time one processor, I am accurating second time two person and so on p i get a b c d, a b c d means a is the first index b is the second index of the first sequence. First index and last index of second sequence p i calls median pair a now, if a x is median then if a x is median then I will be trans pitting from here to here and here to here to the next processor p 1 and here to here and here to here to the other processor so that next iteration he gets this bonds and he divides. So, from here to here this I have to said the index x. So, let us assume that p 1 is the starting index p 1 p 1 is x and q 1 is the starting index for the next processor which is x plus 1.

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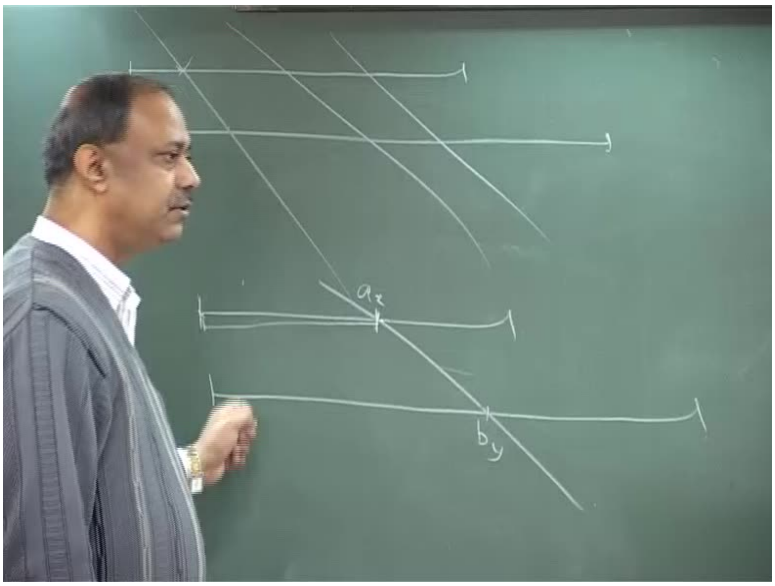

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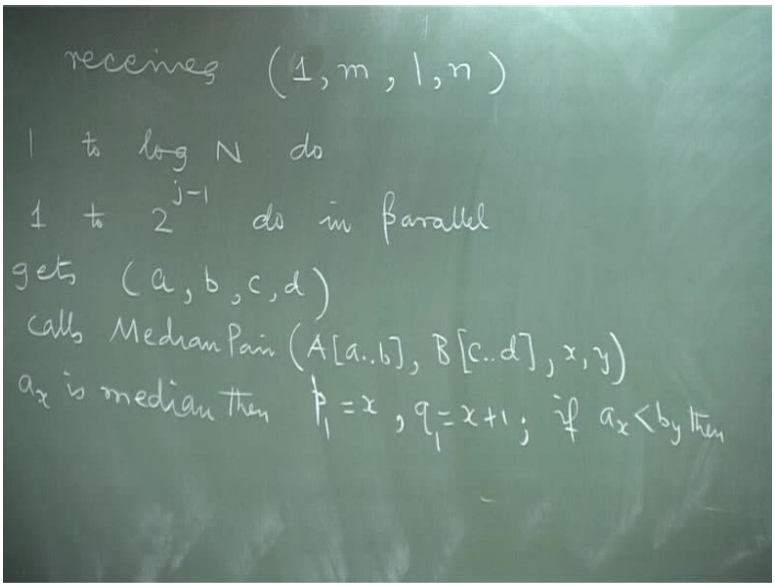

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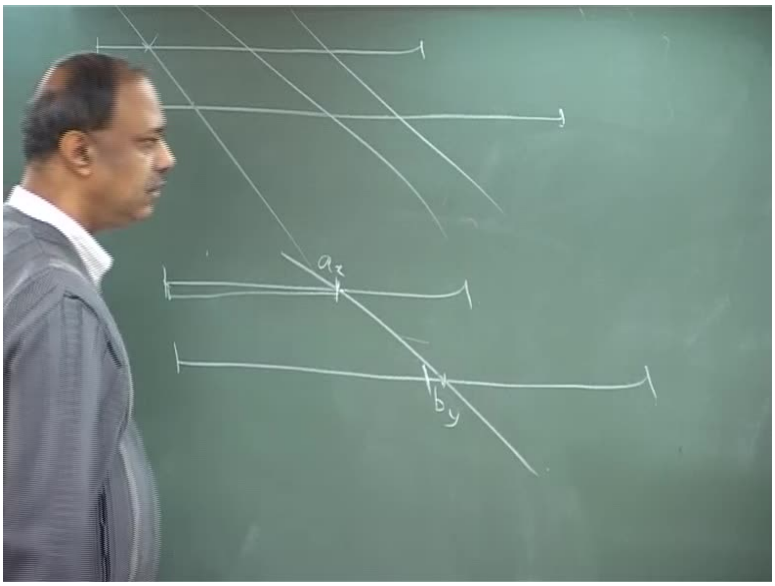

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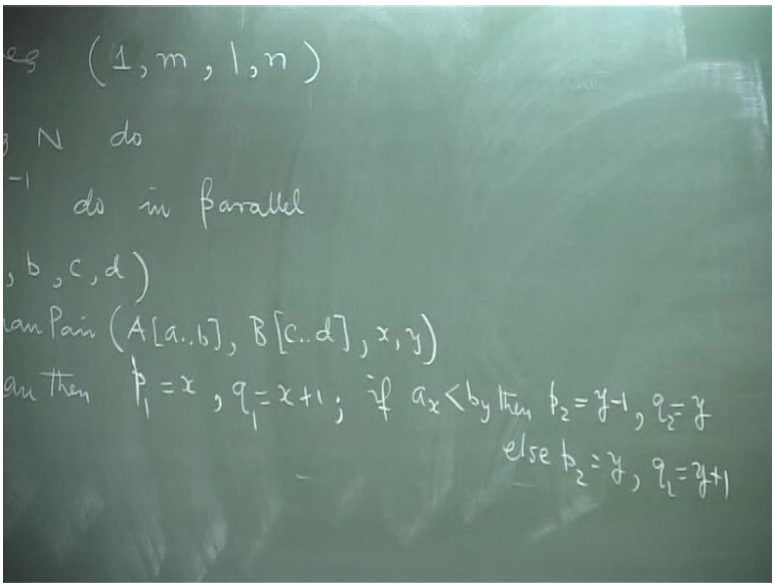

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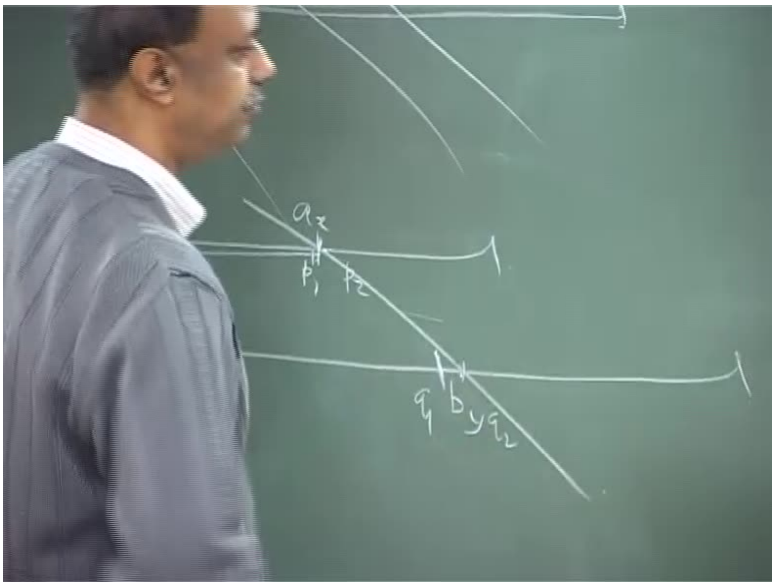

Up to this for the first processor and from here onwards to the next processor and here you have to find out now, if a x is less than b y if a x is less than b y then 1 b y is the larger than this that means this element will be smaller than a x this is the smallest element larger than this. So, up to this I have to send it to first processor so, p 2 is y minus 1 and q 2 is y else p 2 is y and q 2 is y plus 1 so, i got p 1 i got p 2 i got q 1 i got q 2.

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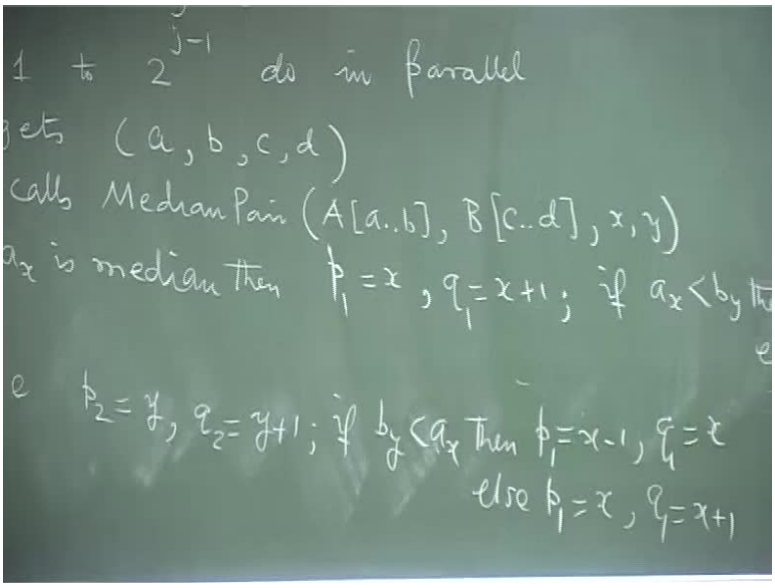

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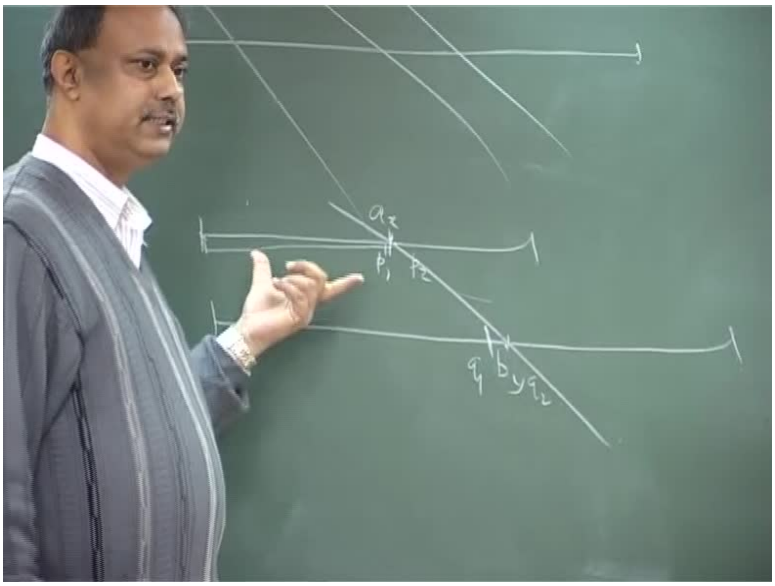

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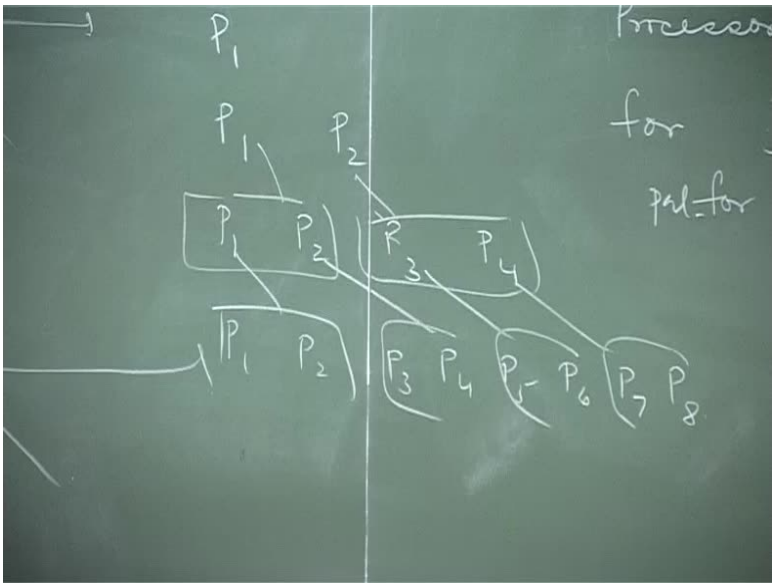

Else b y is the median. So, if b y is the median p 2 is your y q 2 is your y plus 1 if b y is greater than b y is less than x than p 1 is your x minus 1 q 1 is your x else p 1 is your x q 1 is your x plus 1. There is the p got it now next is i have to board cast it p i should communicate to the next processor p 1 should send the value 2 p 1 and should a this part and this part so, next iteration you observed so, first is p 1 does the work next 1 p 1 and p 2 than p 1 p 2 p 3 p 4 so, p 2 sends the information to this p 1 sends this information to this.

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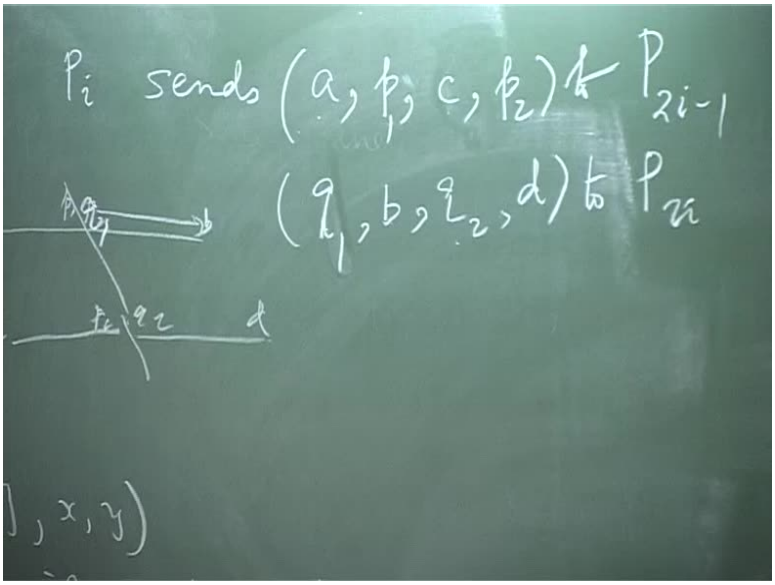

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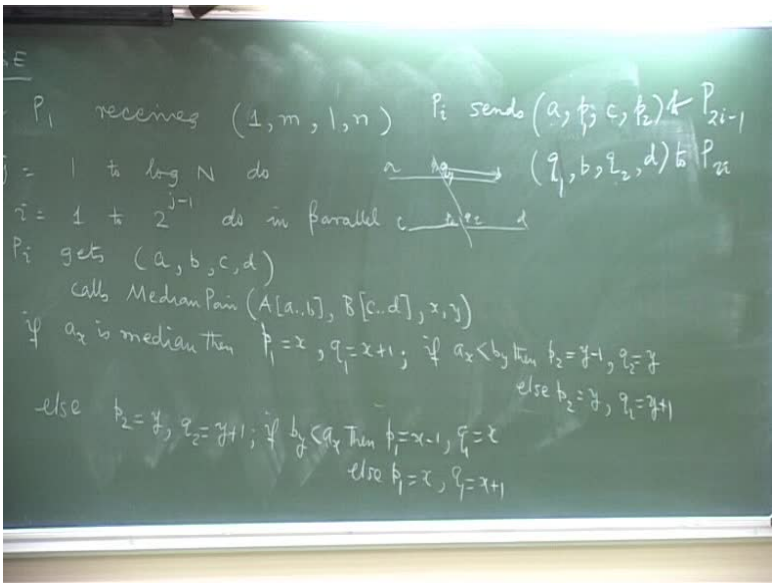

Next time p 1 p 2 p 3 p 4 p 5 p 6 p 7 p 8 8 processor. We have to transmit this that is the way you would broadcasting. So, if i in p 3 p 3 broadcast the direct to which p 5 and p 6 how to get that p i sends a coma p 1 than p 2 coma to the last index p 1 c is the last index defined. Which is p 2 an first index just immediate this is x plus 1 to the last index is b and because it was initially a b now will be send it a p 1 and p 2 and q 1 q 1 2 b l q 1 to b p 1 to b yes or no similarly, c p 2 and this side q to 2 should be q 2 to send 2 p 2 i minus 1 2 i so, p is send p i send to this next iteration p i p i is nothing but, the new p i gets this 4 parameters he compute this and so on.

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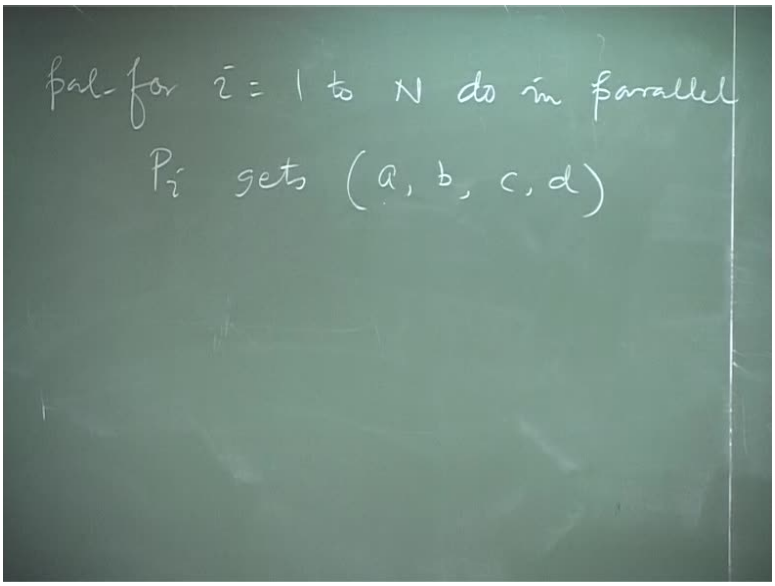

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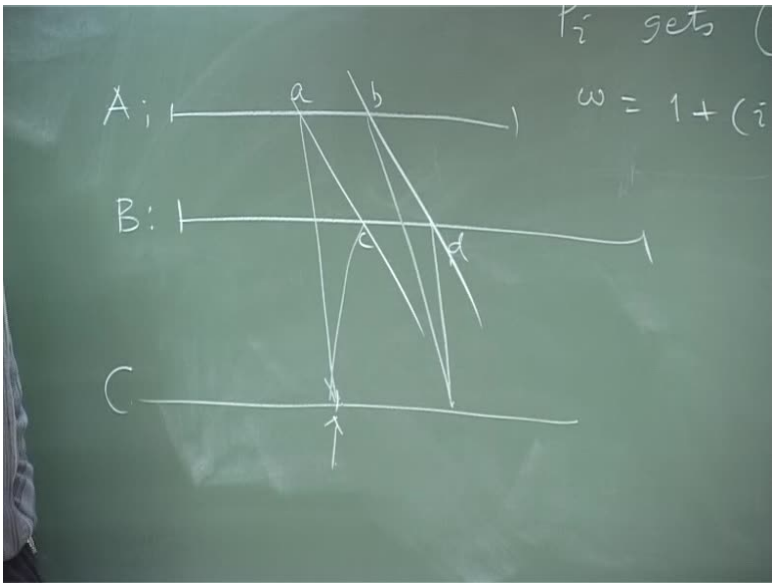

So after log iteration a b c d what is it is a starting in the p i, a b c d p i is got this part this is the sequence and this is the b sequence. Now, you have to merge it so, you need to compute in the c sequence. What is a starting index and the n in index yes or know now, how to get the starting index is every processor is doing now, you are dividing equal part every processor is having that much size. Because equal part so, there if i multiplied by n than i get m plus n total number of elements so every processor is m plus n divided by n block. So, i f processor starting index will be if it is w it is this is the starting index.

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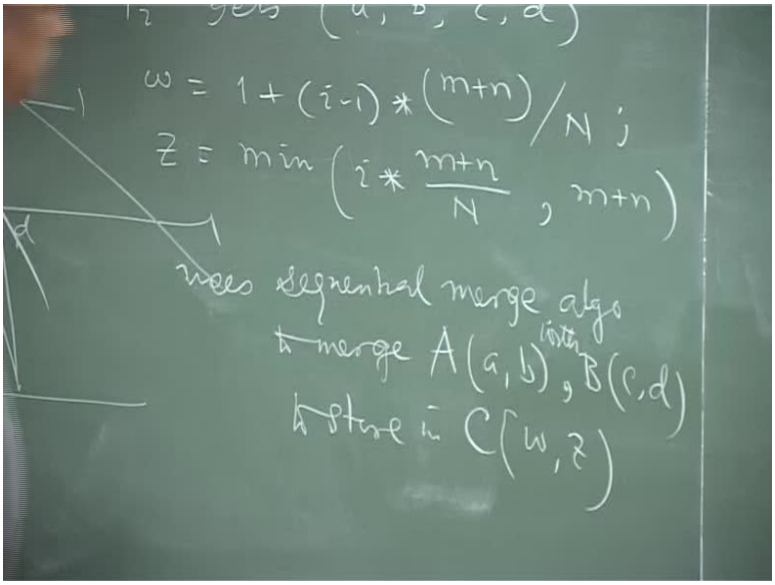

And what is your n index, n index is minimum of i times m plus n divided by n or m plus n because, last 1 last 1 may not be exactly r m plus n divided by n last 1 may not be exactly that is size. So, that will be little 1 2 be because you are dividing by n so, this i maximum index is m plus n so, p i merge sequence uses sequential merge algorithm to merge a a coma b b c coma d to store in c w coma z merge with and store it in c w coma z.

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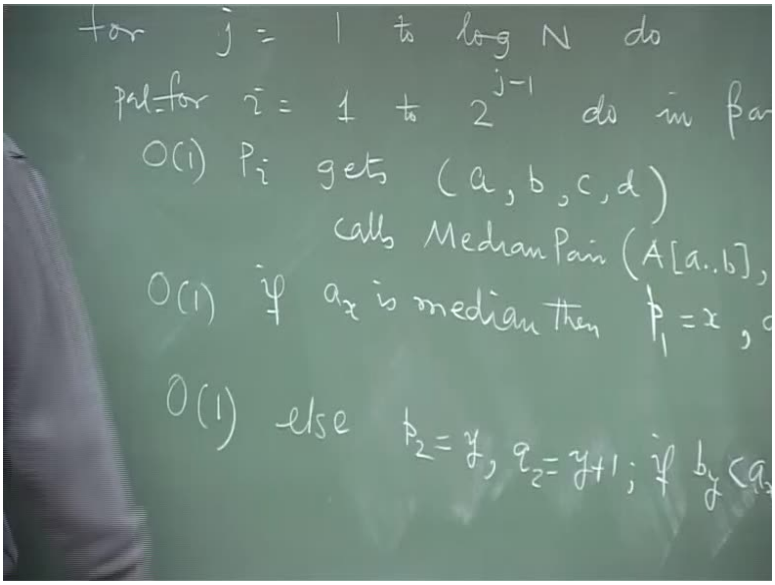

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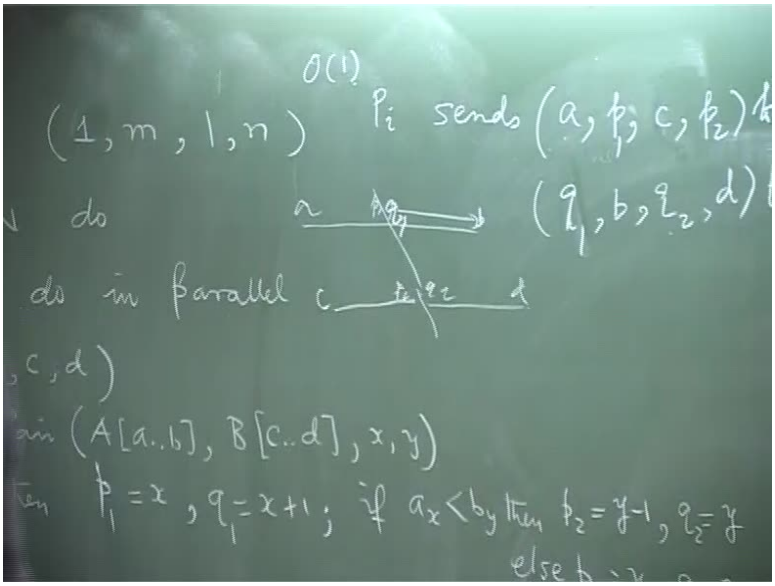

Now, if i have compute the time complexity this takes constant amount of time this take

constant amount of time this also takes constant amount of time now, what is the time needed here. What is the log algorithm so, at the ith at the j f s stage at the j f s stage? What is the sign 2 1 to log n th at the j fs stage at the first stage if the first it is the m plus n. So, time needs log of m plus n in the second stage m plus n divided by 2.

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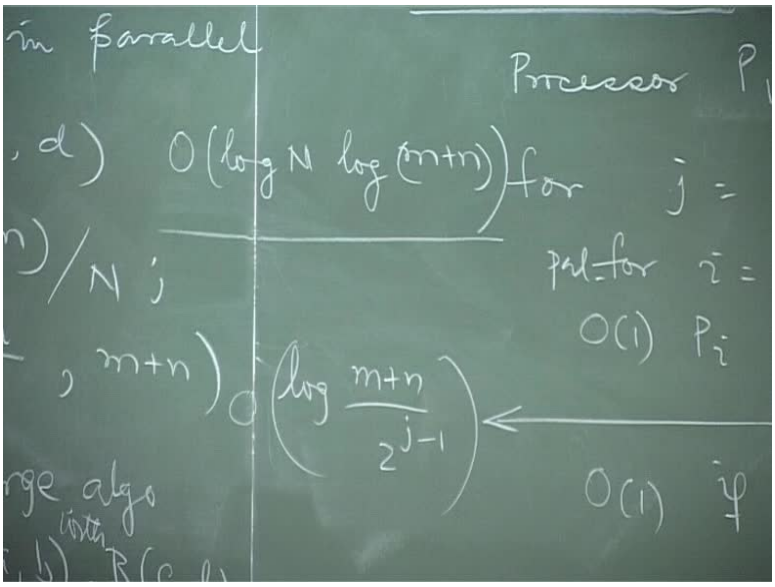

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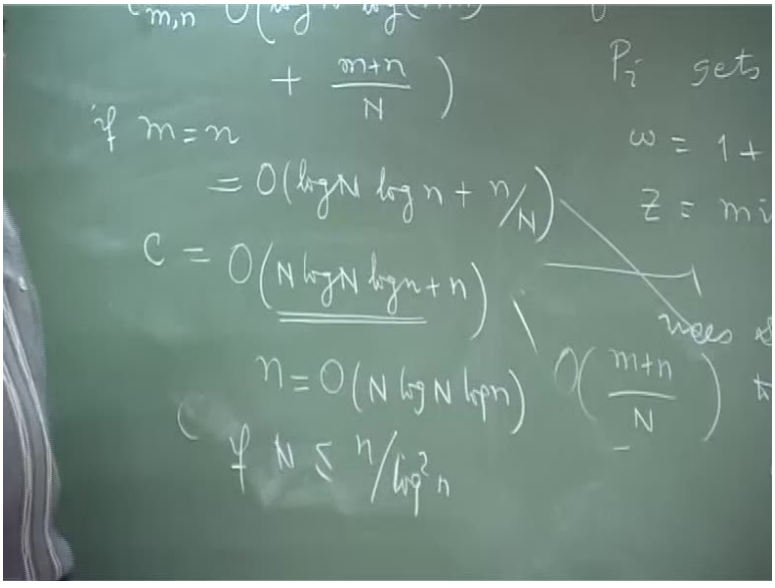

At the j fs stage it is log of m plus n divided by 2 to the power j minus 1 agreed. First stage it is m plus n second class there is a m plus n divided by 2 and so on agreed. So, this is your inside the loop but, summation if i take than it becomes what so, this

becomes whole algorithm becomes order log capital n log m plus n. What about this 1 sequential merge log it takes m plus n divided by n because if that is the maximum size your m plus n divided by n because if that is the maximum size you have agreed.

So, the total time complexity becomes t m is or not t m t m n because order log capital n log m plus n plus m plus n by n now, if m equals to n than this becomes order log n log small n plus n by n. So, the cost becomes n log n log n plus n so, it is cost optimal if small n is equal to order n log n log n. That is if n is less than equal to n by log square n that is if n less than equals to n by log square n. So, is cost optimal if is algorithm so, we have discussed the c r e w merge in algorithm c r e w merge in algorithm. So, in the next class to discuss out line short on c r c w and c r e r e w equals also c r c w.