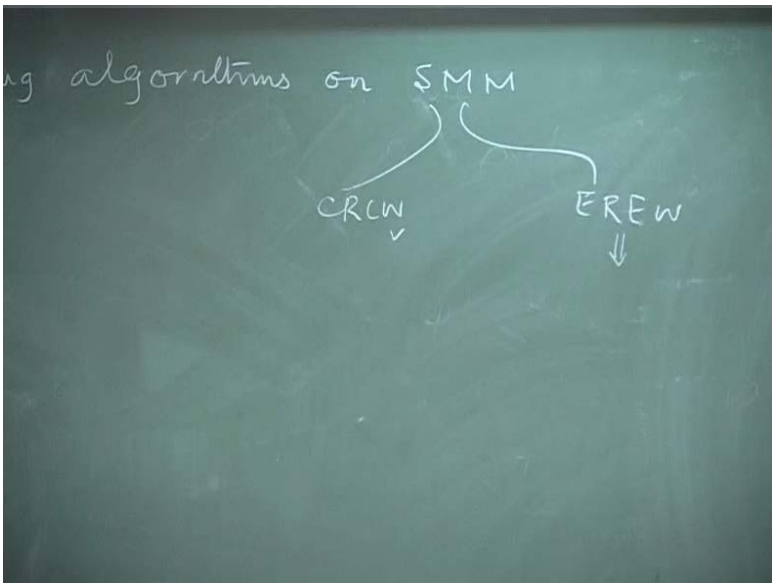
Parallel Algorithms
Prof. Phalguni Gupta
Department of Computer Science and Engineering
Indian Institute of Technology, Kanpur

Lecture - 12

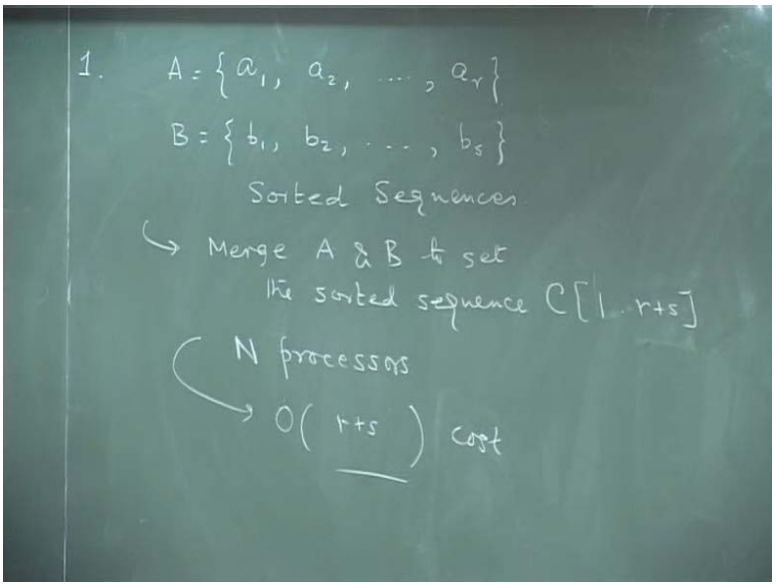
So, today before will be discuses about the sorting on shared memory model. We want to discuss, because one thing you what will you observe that all the algorithm so, what we covered they are based on the merging algorithm so, as bionic merge and all be when merge and other things. So, here on shared memory model bionic merge manner give you.

(Refer Slide Time 00:47)



We do not give the good results so we will be discussing merging algorithms on shared memory model. Now first we will see that CRCW model, and then we will see how to write an efficient algorithm on EREW models. So, if you can covered both this two algorithms today in word otherwise they… So, at least we will try to finish this one and some part of this. Now one issue has to be taken into account why on shared memory model the issue is nothing but the load balance in of its process, the process should be busy as spire it process are a otherwise we will have not be achieve that cast of quality.

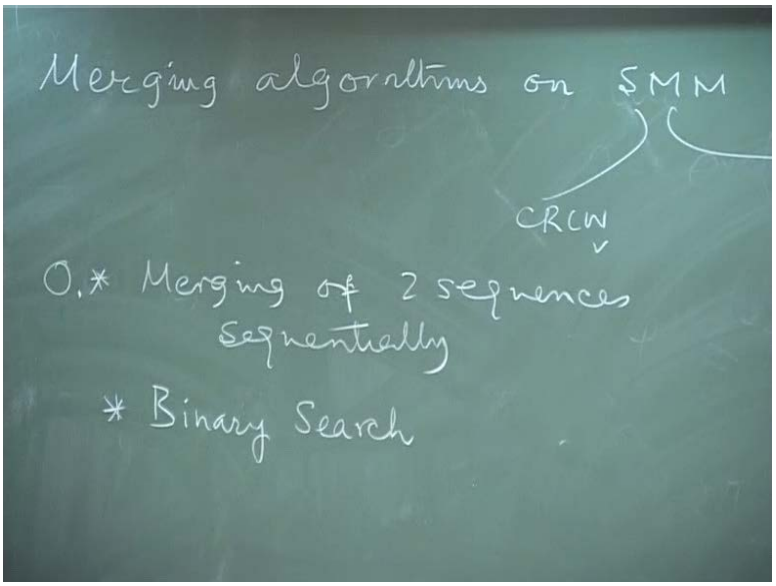
(Refer Slide Time 02:02)



Now, let the aecia of that there are 2 sequence 1 is sequence a 1 a 2 a r and un other the sequence is b b 1 b 2 b s 5 this 2 sequences are the sorted, they are sorted sequence see the problem is 2 merge a and b 2 get to sorted sequence c so size of c is 1 for pluses they had résumé that they are n processors and issued try to designed algorithm how is takes this first time like hear this time of cost not are time.

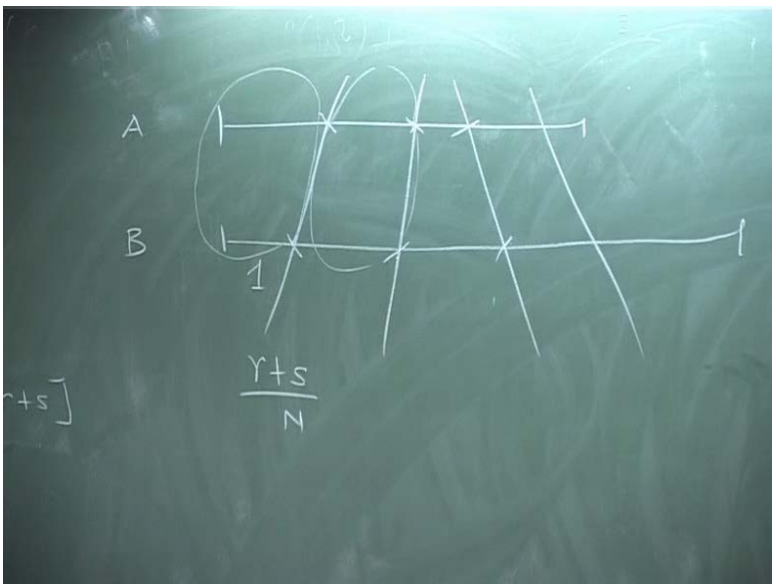
So, what we should try, here we should be designed in algorithms sauce that the cost of the algorithms is r pluses. So, that becomes are parts of humor so, this is the hard problem fix. Now let us to live the go back to the sequence of algorithm fast 2 first ward in that you now, the 2 merging of 2 sequences.

(Refer Slide Time 04:24)



Sequentially write this is note to you that you take on element of a sequence of other element is b sequence, upper it then the smaller 1 important it see and then you elevate of have select you lope the index of back seek well by 1 and then continue till all the elements of all sequence of tested and the meaning element are the success will be it 2 c sequence.

(Refer Slide Time 05:35)

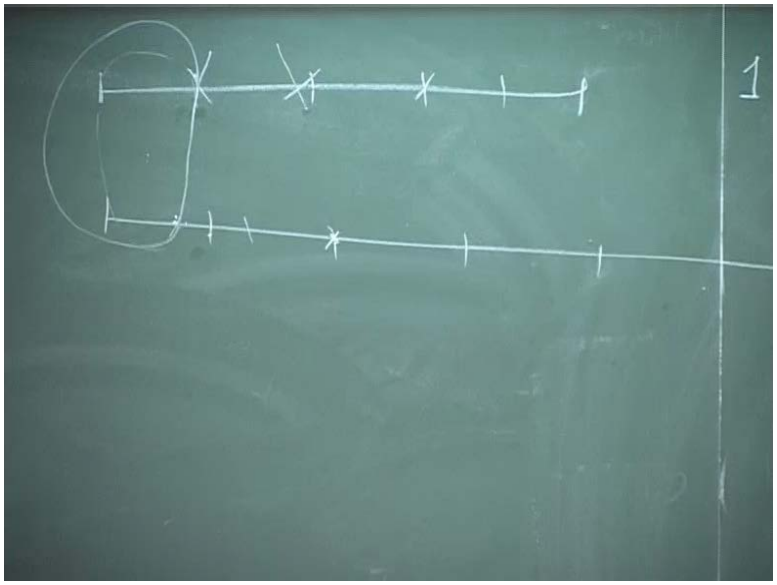


By this ways then, we all know the algorithms so, which is known as binary search write these algorithms in them this are the p requisite of design the new algorithms this is

known give you, we will take log on time now, what is the e is that you have the 2 sequences is a and b and in a put devise these a and b same end groups in to end groups such that that 1 process can by employed.

Two merge these to the other process employed to merge these 2 other employed merge these and so, all write and to see that the number of elements it is merge in, it is of the ordered it we do not. So, that these should be the order of put that they should be process is equally be 2 now, that means your main target is 2 of team these points so, it is now that to that.

(Refer Slide Time 07:15)



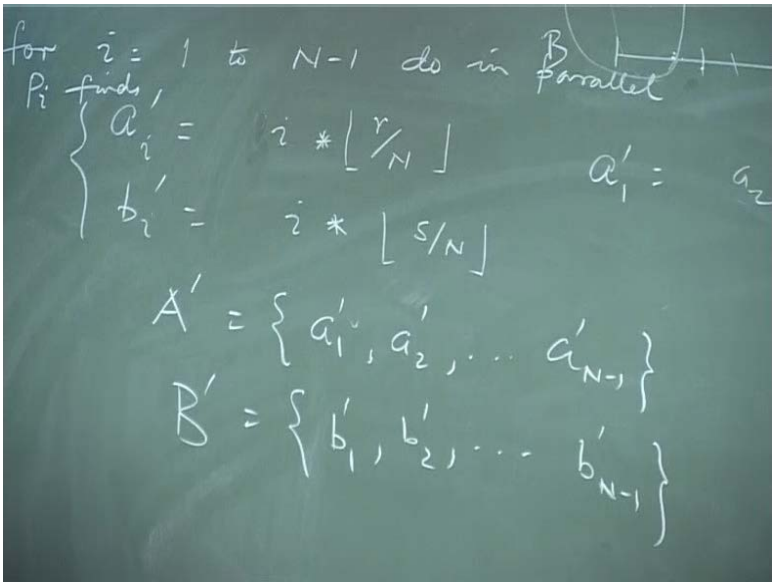
First what we are doing that the sequence a incept because it continued they are element switch off. We had lived they are elements you need to find out this point so, would be why devises this a in 2 n my n groups you devise and 2 gropes b in 2 and gropes and so, write and these and gropes but, suppose it devise exactly and gropes of equal size the problem we are facing key even have a merge that means would give you the sort this is acceptably because these element may be larger than these element some for so, exactly this will not help you.

So, what you to do first you assumed that these are estimate they let, us the estimate assumed to the likely. So, you kind dividing 2 n gropes n gropes means we need n minus 1 point 2 to get than gropes and these and minus 1 points now this pointes sub to be manipulated so, that if this is the position than that should be shifted by these are this

size. So, that this element became is the small of than these element but, these element is larger than this using the binary search that is the idea so, that the coasted which to be sorted are merge by the prosier i.

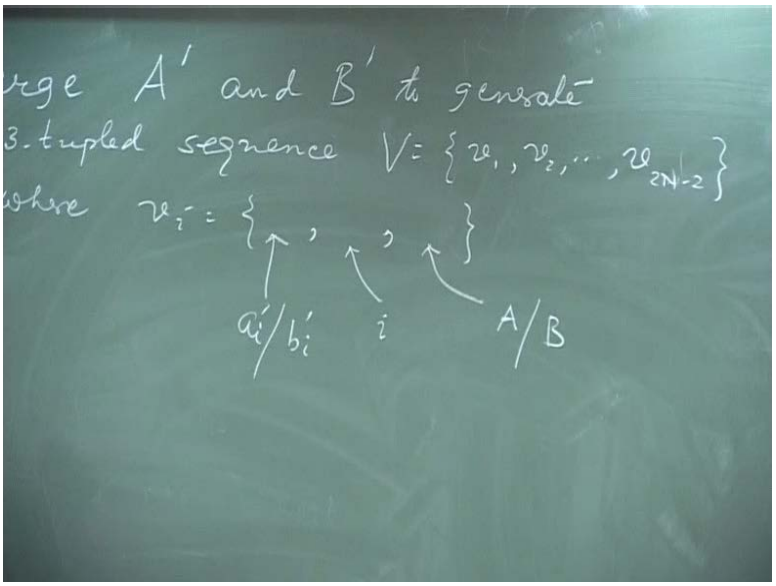
So, we have 2 divide in to end gropes that so, if I fix this 1 then other 1 I have to select from this groping such we would that this element e is smaller than this element and so, their I can get the clustered the reprocess can be employed to mark this 1. So, next element may be these all this some1 suppose I select this 1 so, I have to select and element from hear in such away that all this element are smaller than this 1 that is I d and then try to put the classes in sided the size becomes little large and process are reserve able are equal it is it does that.

(Refer Slide Time 09:10)



So, now do that first we innessiate that we divide this in 2 n gropes so, in words to get then gropes we need to get n minus 1 points like so, because first point is known if I now get this points to get in to immediate in minus 1 point so, I can write for i goes to 1 2 n minus 1 do in parallel a i dash is so, basically I get this is this element so, this is a or this is c or b so, a 1 dash is your this elements a 2 dash is the next elements and so on and this can be down cons entreat every prosier finds that well p 1 p 2 p 3 and p i finds p i finds so, every prosier finds 1 well which takes for number of time. Now, let this for 1 sequence is a dash which is a 1 dash a 2 dash a and minus 1 dash and b dash is another sequence b 1 dash b 2 dash b and minus 1 dash.

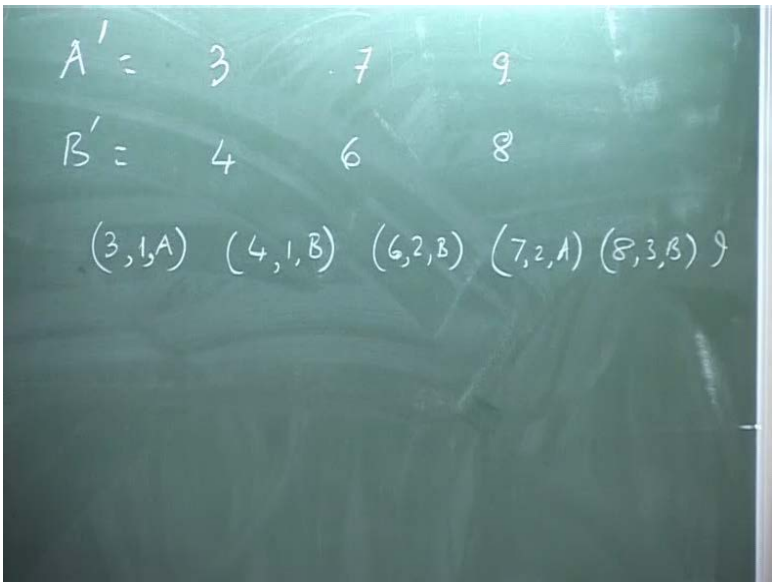
(Refer Slide Time 11:12)



So, now your merge in these 2 sequences a dash and b dash each is having size in minus 1. So, and merging that and we are generate in new sequence these b 1 b 2 and b 3 b 2 and minus 1 2 and minus 2 now, b i is 3 these now these 1 is the element. Elements means it may be a i dash or b i dash then these one give now i index and these 1 continent the information with that it is a sequence or it becomes 2 b.

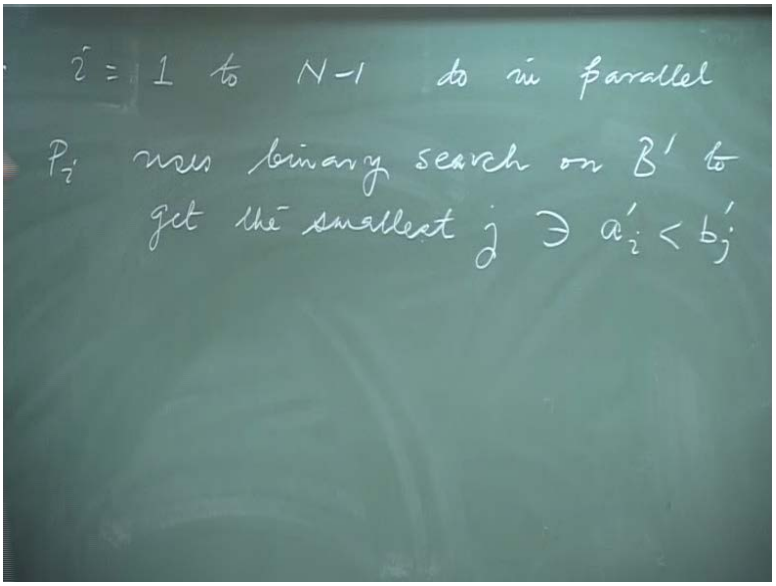
Sequence these this is this first 1 is the value of the element second 1 index of the element in a dash or b dash and this is third 1 is telling whether it is the family of a or it is the number of a big, this is the Last medium a and z last c say for example, I get 3 7 9 and this is you are a dash sequence this is b dash sequence 4 6 8.

(Refer Slide Time 13:07)

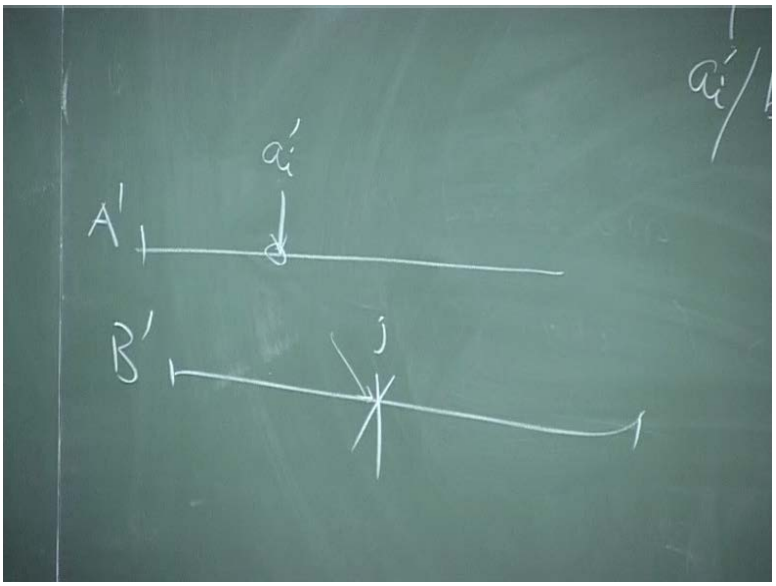


Now I want 2 merge these merger will give you 3 4 6 7 8 and 9 that now, these 3 will not be long to know whether in to gutted from first sequence or gutted from second sequence. So, I want to tried a 3 1 a 4 1 b 6 2 b 7 2 a 8 3 b and on so all so, that you by seen this element you can find out what is the index of that element and index of the element ultras files 2 a dash now, we dust i 5 know the a dash i now the sutras files 2 a.

(Refer Slide Time 14:35)

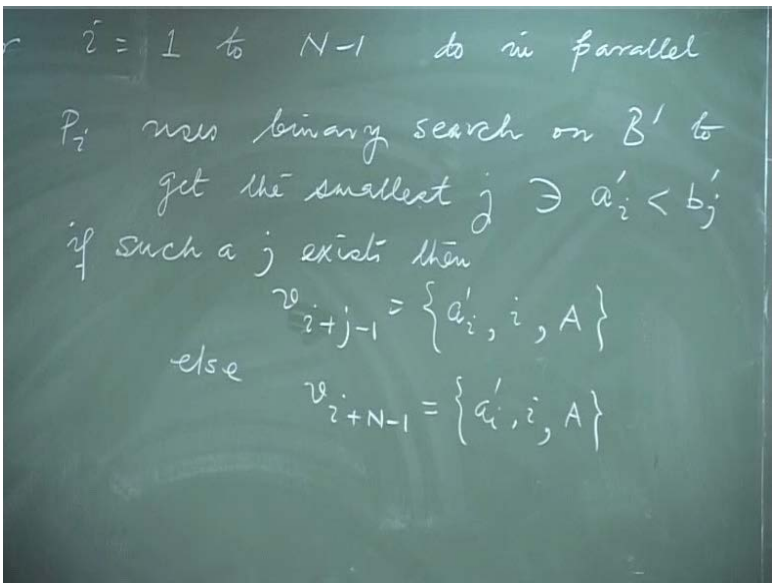


(Refer Slide Time 14:45)



Also by multiplying these factor now how 2 implement this for this is blue paint equation for i equals 2 1 2 n minus 1 to in parallel p i gets the smallest a i uses binary search on b that is to get the smallest j sues that.

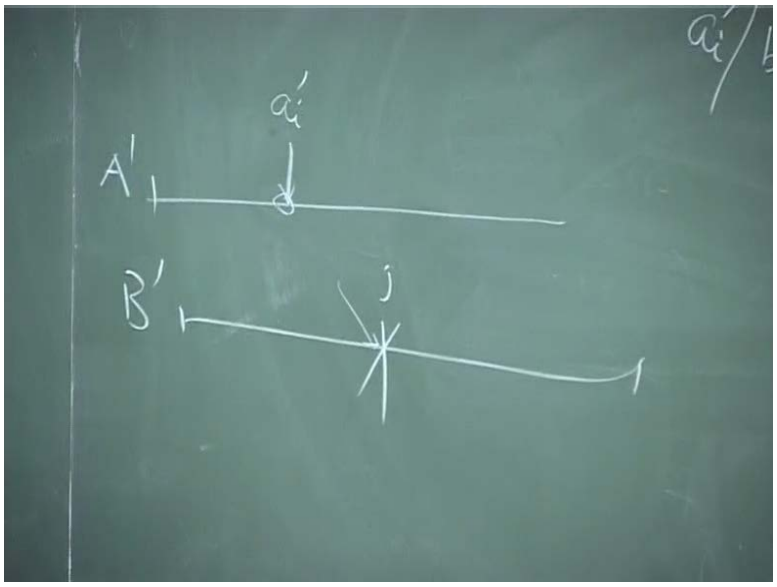
(Refer Slide Time 16:17)



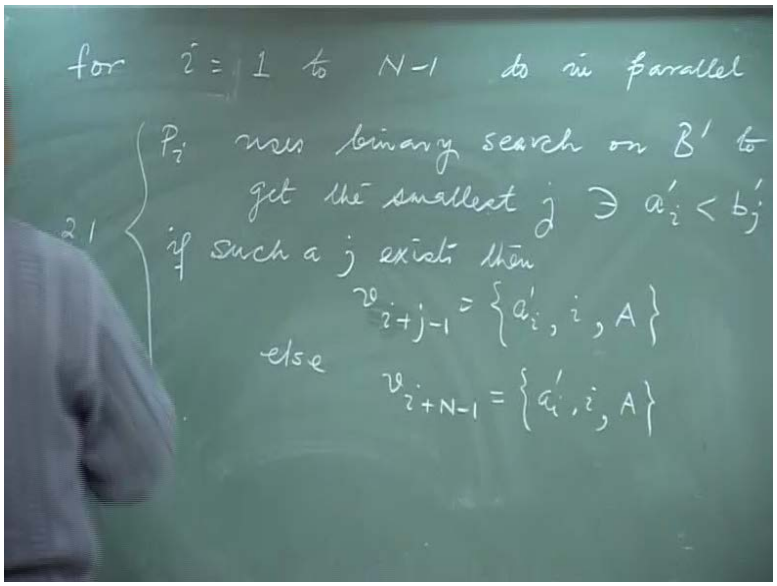
So, what happen that this is you are a dash sequence this is your b dash sequence now, this element is known to you that a i dash now your finding the smallest j smallest j using the this is sorted sequence because this is the so, you can do the by dent the search

and a i dash is les then this so, if such j east if such a j exist then b i plus j minus 1 these a i dash i a else b i n minus 1 is equal 2.

(Refer Slide Time 14:45)



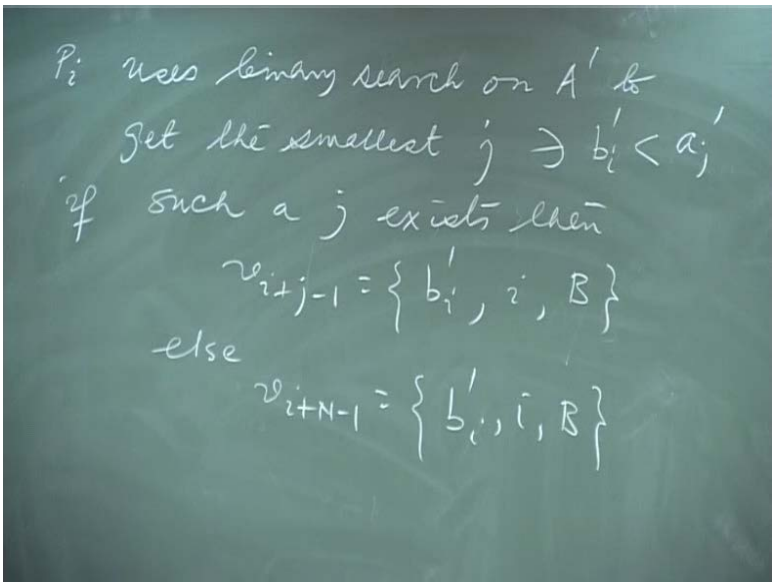
(Refer Slide Time 17:41)



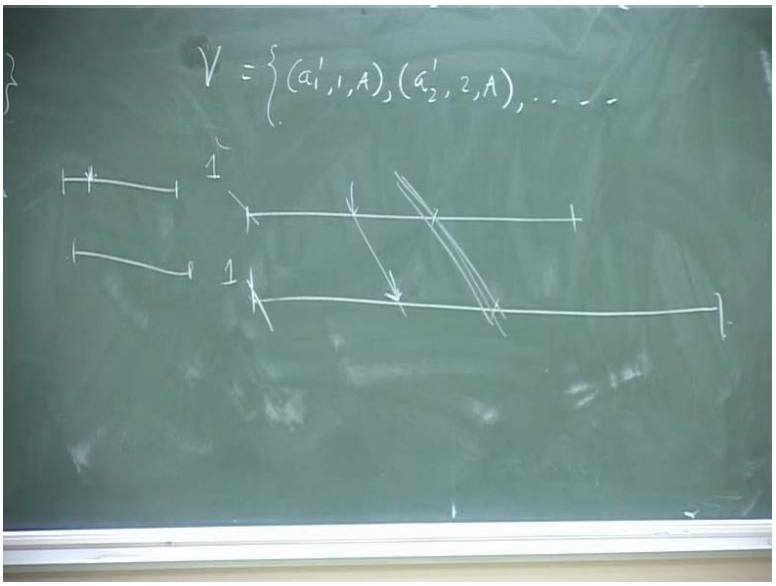
a i dash i a like these on occur if their dos to all these element of smaller than this that means n minus 1 elements plus i minus 1 that means element of smallest of then a i dos so, i so the position of they would be i plus and minus 1 i need is a simple binary search and then your insert in being so, this is the first part this is been in mute refers to a now a

doing to is refers 2 b so this is 1 2 point 1 size so, point 2 will be p i uses binary search on a dash.

(Refer Slide Time 17:55)



(Refer Slide Time 19:10)



To get the smallest j sues that b I dash is les then, a j dash if sues a j exist then b I plus j minus 1 is b i dash i b else b I plus n minus 1 is b I dash I b so, you got that merging of 2 sequence of a dash and b dash now, once you now these sequence which contents the information like that may be a 1 and 1 a then, we a 2 dash 2 a and like that but, you

cannot used this is for nation that is like that we what you have to know we are product patricianly.

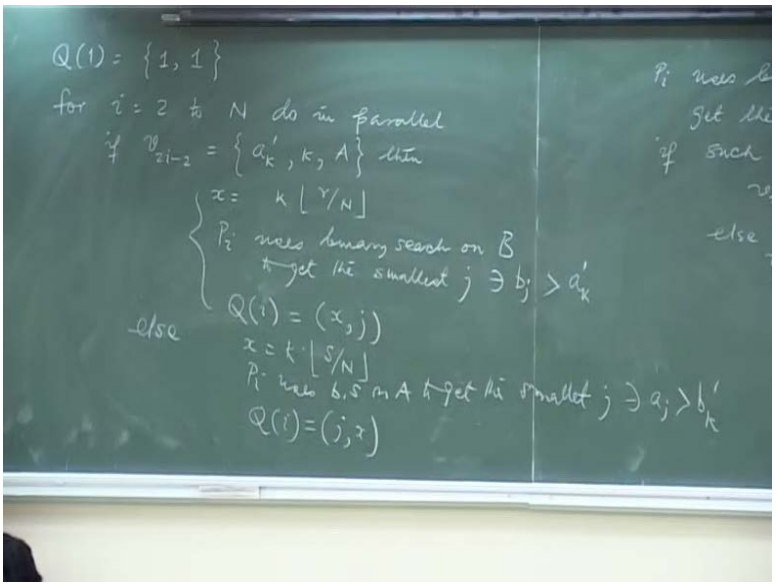
What is that participant and what is the end points that to be d1 now starting point you can get from hear n point of have to determined from the other ship wise that is how see what you are getting lap this is one virginal shift wise sequence from there you got this 2 sequence. Now this elements suppose you decide this elements now you tell me from the original basic sequence what should be the these elements so, the you can start your merging so is it so, we know this element here corresponding element can I thought here by multiplying or plus corresponding or by n times. Suppose it is the second element corresponding element is nothing but, to b y into here.

So, these elements known to you and using the binary sum of these elements would offer to this you will find transparent element here which is the once element larger than this element so, that you can start this part. These elements are castigation known of a not to get the starting element of the element of the you using finally, search on this element this part.

So, you will get the starting element similarly, you will be getting by another process part gating and in the part starting another a that is in the b so, you have mode up to this part element because this part indicate by another process now, you have total 2 n minus 2 elements in beat.

So, at 1 element is do to you starting element of process thing is 1, 1 this is the starting element of a starting element of this now, began and now to you there will be remaining of n minus 1 point so, that is perfect to do the budget.

(Refer Slide Time 22:10)



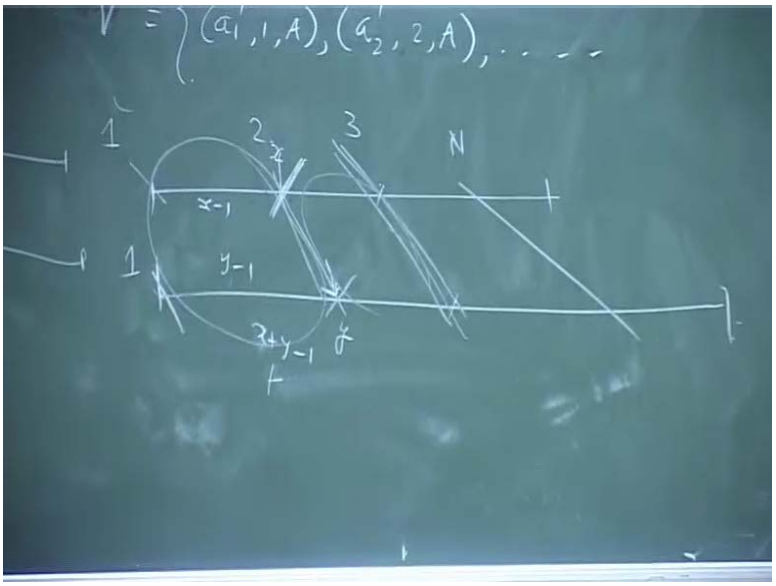
You know the starting elements its q, q is your starting element position so, this is known to you it is a 2 to 2 now you have find out the remaining n minus 1 points so, for i equals 2 to n do in parallel if v 2 i minus 2 every even index with v i, I am taking because there is 2 n minus 2 points i have get in n minus 1.

So, I have taking even index parallel to tell of if this is nothing but, a k dash k a than x is nothing but, k minus 1 it is this is clear that this x is nothing but, this correct thing that is cluster of a y r by n so, position of this will point k into r by n or if x equals to k equal by this k r by n.

So, that is the index you got for a c hence original x sequence and to get the elements, which is larger than x larger than ak did not get that you will be doing p i use as binary search on b sequence to get the smallest j such that b j is greater than a k dash b j is greater than a k dash.

So, in that case we can define q 2 q i q i by x comma j so, you observed that you got the starting index of x and starting index of b agree else you will be writing here x equals to else means what this is a k dash k b and p i uses binary search on aids to gets the smallest j such that a j is greater than b k dash and you will be finding q i equals to j x.

(Refer Slide Time 26:42)

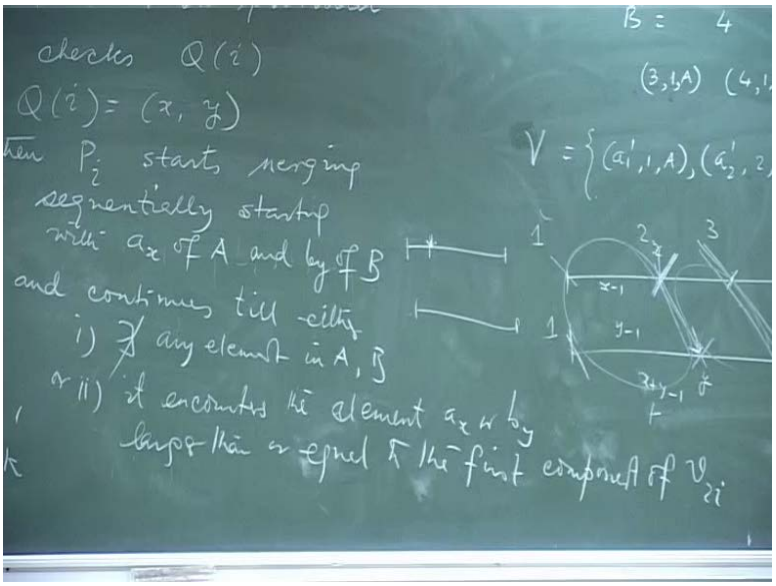


So, what did by the here found as the this element and this element would be gets similarly, this is p 2 that similarly, p 3 is obtain and similarly, n also p n also get agree now the you emphasize the this prescribes you want to process do the mode in of this e 2 merger this p and merger this and so on than you get the shortness.

So, if I know starting index at x and here y what did means if the index of a prospect of 2 is x and y what did means than exist these x plus y minus 2 elements x minus 1 element smaller than the x and y minus 1 elements smaller than this so, if i merge to the starting index of this see this should be the guarantee x minus 1 element this side this get will be the y minus element this side.

So, this starting index for this would be x plus y minus 1 over minus 1 over you will be storing it and how long you continue till either that element extracted the sequence or you encounter to this element of this elements. Because this account would be this part and taken impact of another process you could not do that.

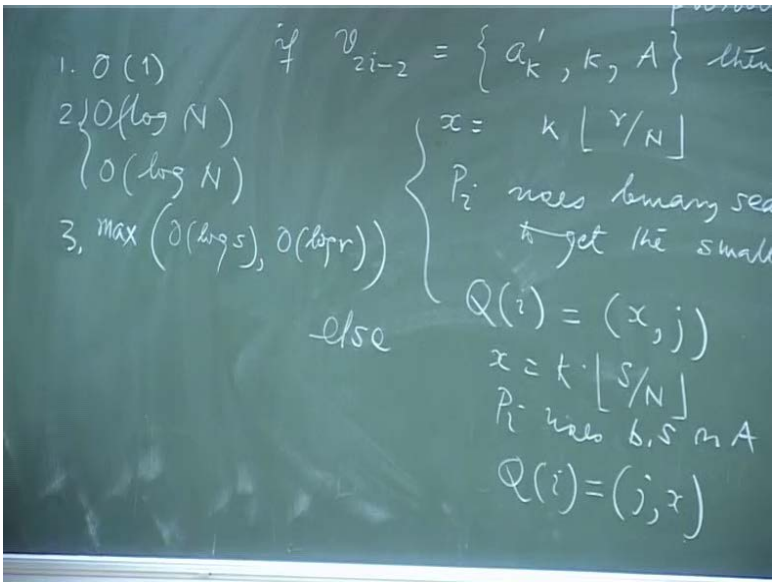
(Refer Slide Time 28:46)



So, that is the idea so, the next part is that how to merge it so, for that for i equals to 1 to n do in parallel pi checks q i if q i is x y than p i starts merging sequecially starting with a x of a and b y of b and continuous till, either they does not exist any element in a and b or it is encounters as it encounters us by with the first element.

So, it either that not exist on the element in or b for merging or it has it encounters and elements and x of b y. Which is larger than or equal to the first comp1nt of v 2 i and the v 2 i first comp1nt is 1 element which is member of this x and y of the another process so, what first can analyze than let us come first determine the time complexity and then will that would help of the firm.

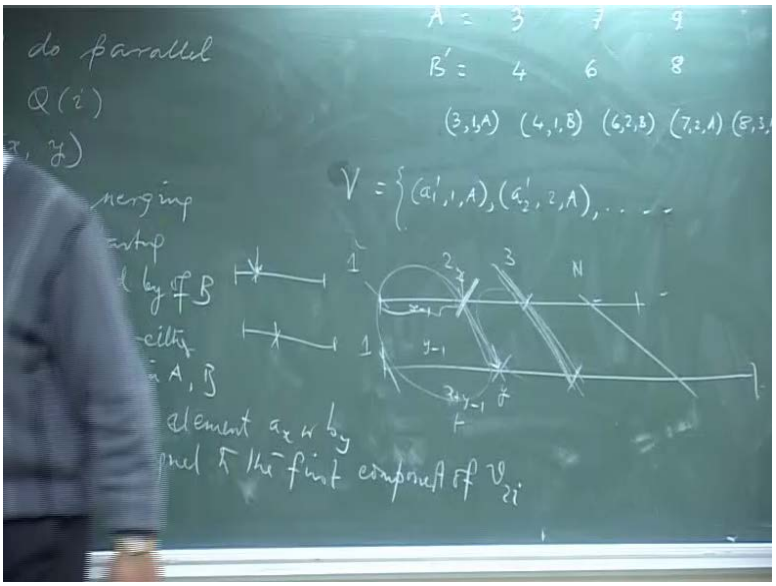
(Refer Slide Time 32:20)



For example are it was now first 1 is that in the step 1 what you are d1 are divided into n minus 1 row would takes order 1 ten that second step so, first step is are a 1 what is the second step that you are doing on a binary search or bitters and the size of the bitters is size of bitter is than so, log n type and again you are doing on at the sequence which is again log n time.

So, this is the step 2 now in the step 3 you are doing binary search on b this x constant time it will manage such of b and it takes you have to have s elements. So, logiest time if it is of the case you will be doing the binary search on which is log r time so, this is not known so you are going to select.

(Refer Slide Time 34:11)



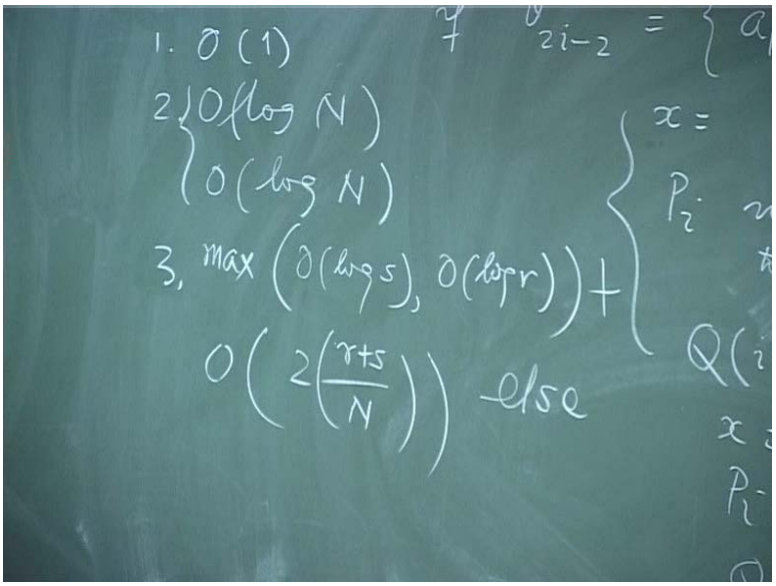
So, either this 1 of the so, larger 1 will be put here sum max maximum of order log s or log r now, come to this point this is the sequence merge you have but, you know the size of this now and you are looking for a every even index every even index v every even index v.

So, once I consider v means that there are 2 index as sleeping that next and next 1 to alpha every even index comp1nt time equator so, this is 1 v part this is another v part 2 to sub factor of a you get on a growth now, what happens to each group first than you multiply by 2 to get the maximum wall pet size that you have to get the size such you have object this size and then you obtain this size.

Now given the obtain this size what happens because what happen you will mapping this element on to this element on the original sequence. Now suppose this element is equal to this element suppose this element equal to 1 element here that means i malty on this play on this even there by r s elements would be smaller than the smaller equal to these elements.

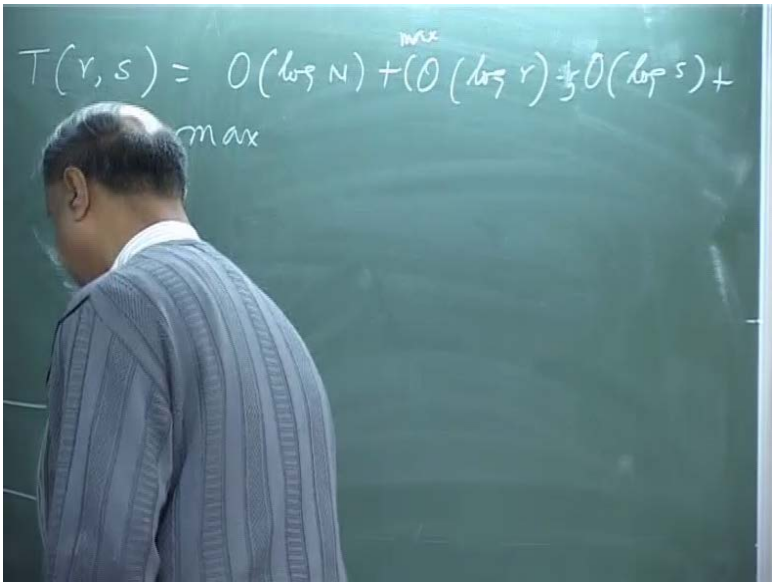
This point is present the r by s element because instead r by s element you have taken 1 element here or not here divided this r by n each of size and you have taken r by n element here now, if you find in the these element equal to 1 element here what does city implied on r by n element is would be smaller than this element.

(Refer Slide Time 36:37)

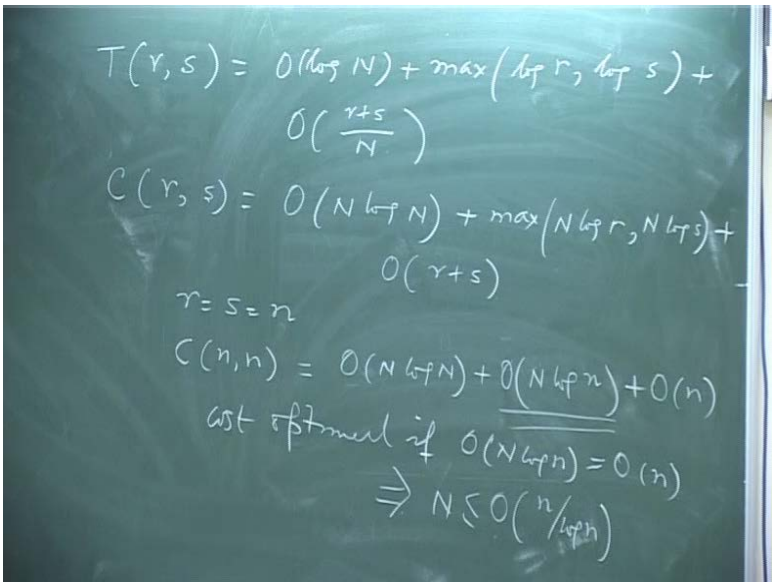


So, similarly, as the case by n element of this because this point is repeat by n representation so what r by r by plus is n because r of by n plus is by n similarly, r by n s by n because it is a 2 3 so, it is size is 2 times r plus s by n is the merging element total of number of element merge in the worst it is so, which cost is order this because this is sequential merge.

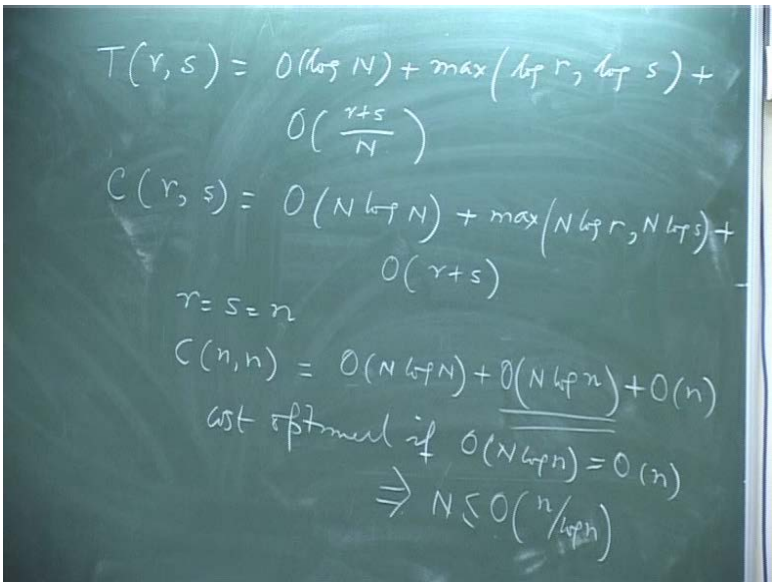
(Refer Slide Time 37:07)



(Refer Slide Time 37:45)



(Refer Slide Time 40:18)



So, if i add all this things the ant complexity term t r s equals to order log n plus order log r plus order log s plus maximum order log n plus maximum of log r log s order r plus s by n. So, this is the time complexity will get it and cost now, if r equals 2 s equals 2 n this cost becomes so, it is cost of optimal this is 1 because merging of tell 2 sequence of by than x order and time so this is cost of optimal if order n log n equals order n it implies n is greater than equals to there be 8 elements in sorted order 1 is 8 element, another take 8 elements 1 6 7 10 12 15 18 20 tell me 2 3 8 11 13 17 20 3 20 4 2.

So, let us show the 12 rope 8 sequences 2 and lets n equals to 1 so, initially you have a dash sequence and b dash sequence. What is your data sequence in aiders how many element is there 10 divided by 4 so, 2 first is 4th element what are write here 3 13 20 3 so, you multiply 2 to 2 not 13 11.

Now, you get next step is you compute v 3 index is 1 and it is belong into b 6 index is 1 belong into a 10 index is 2 belong into a 11 index is 2 belong into b 15 index that is 3 belong into a 17 index 3 belong into b this is your b then you have to q 1 comma 1.

So, first will checking the this 1 t 2 i minus 2 the prospect 1 is checking this 1 process 2 is checking the this 1 it is belong into s sequence and 1 so, what will write here 1 into r by n which is 2 agreed and now after multiplication you got 2 now will be checking that the number of elements.

The elements which is larger than smallest element larger than which is 8 position of 8 is 3 than the element simultaneously the process 3 will be looking for this 1. Which is 11 and 2 is 4 and the sequence a 1 2 3 4 5 in will be writing 5 and simultaneously this will comes sequence b.

So, it is 6 and 7 1 2 3 4 5 6 7 this is your q sequence the first process will be first process will searching to be 1 1 1 1 to 2 3 now, 2 3 will be 5 is here so, 5 4 2 3 q is this until above this and 2 3 2 3 is this column is starts up to 5 4 5 is here and 4 4 is 11 5 4 and then 5 4 goes up to 7 6 7 is here and 6 1 2 3 4 5 6.

So, this is the structure will be dividing this and the process you want does the merging of this 3 able merging of this merging of this in this step the obtaining few there in another is I will conclude with, what are the step I need actually this is completely based on sequential merge along with in every property doing as the equation merging than will because starting I know the inject and binary search.

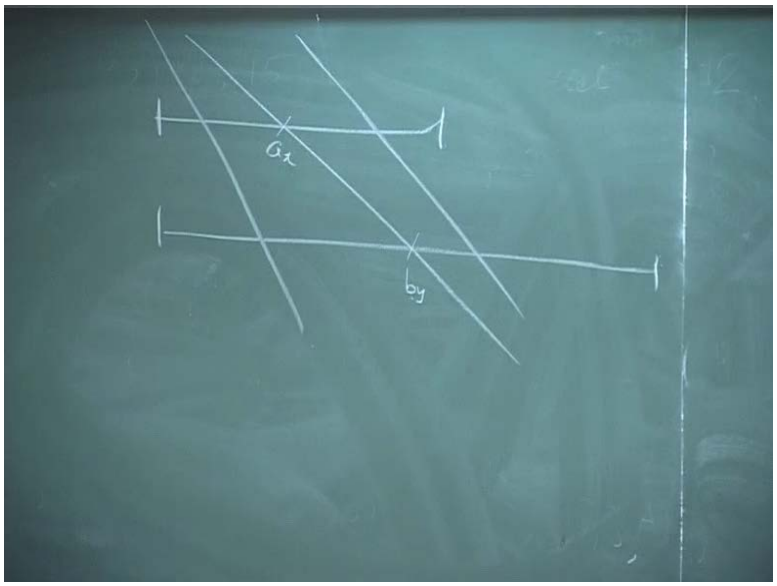
Now, that is the case of binary search every process is taking b sequence that is a sequence so, in transparently the concur will middle element will did by several process. So, due to that that will need concurrence did as through the subject question exclusively the conceal you say the bring d1 life part bring that independent in distinct idea.

So, exclusive it is the concur is due to the binary search now, if you get read of the concur read for binary search then it can be improvement as on a r w order now as you known that concur you get operate this, which further that they log n and emphasize time.

So, the time complex would increase by log n factor here so, multiply by log n factor so, then it will be decrease like you not able to change the cost of parallel algorithm. Now what problem you observe would you want into divide it is sequence into such way that estimates size of the subsequence it is order r by s plus divided by n.

So, that every process equally busy to give by the sorted sequence. What inability observe in the example that is last k f c in than that number is not exactly r by s r plus divided it is not more than that agree so, what as to be taken another because you are taken r by n or s by n s is divided by stand 4.

(Refer Slide Time 52:09)

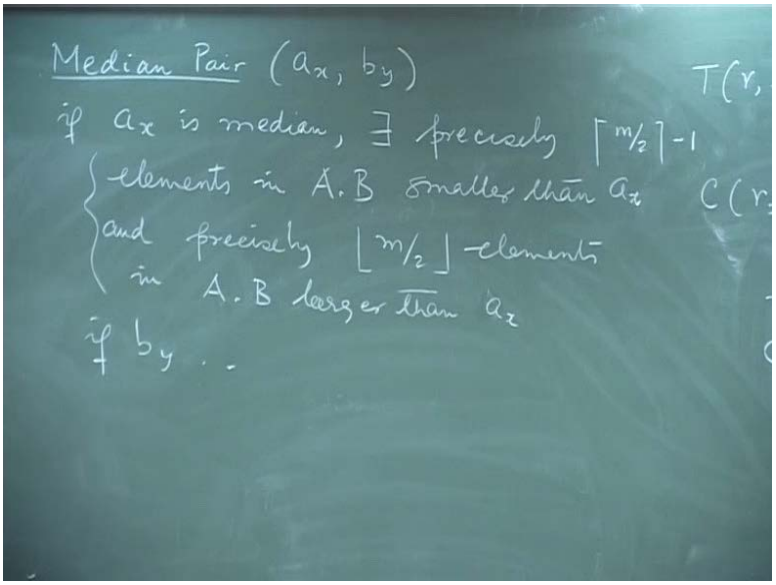


So, some important point and you have to take 2 every time so, what are the equivalent enter to this probably you may would to dictionary more efficient and more require but, in the case of in the case of EREW module this may be little difficulty little difficult to that type of manipulation. So, the idea in EREW module is little different what we are doing a planning to do is that we want to divide into 2 equal parts.

Now, how to divide into 2 equal parts that if I can find out the median the median of this 2 sequence. So, suppose this is a median of these 2 sequence means there will be 1 element here, there will be another element here so, this is a x and this is b y by some method i divide into the 2 part so, that a x b y 1 of them is median and the other 1 is the neighboring element of the median may be the smallest element larger than this or largest element smaller than this.

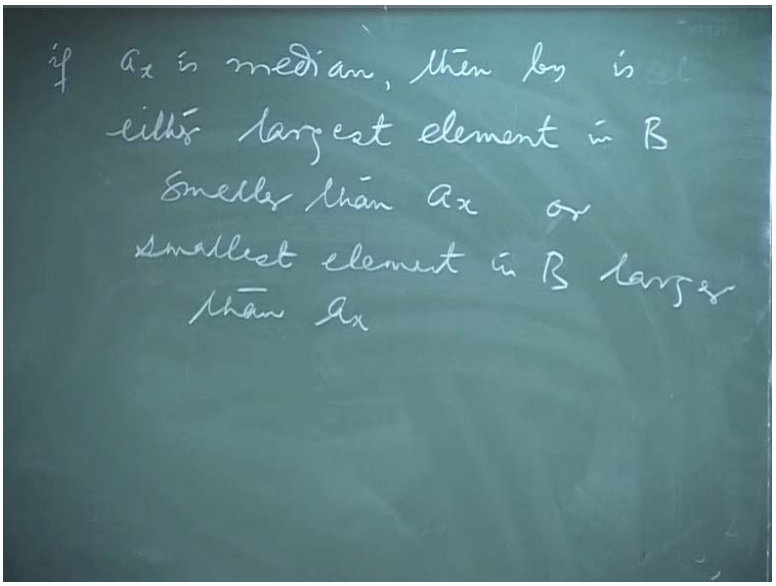
Next you divide this into 4 parts using the 2 process next time. using 4th process this is 1 block another block you find the middle of this and so on in that case you observe after log in steps you are dividing this whole sequence into n equals sub groups or not say you so, you will be getting the anything equals the sub groups and sequenceutialy you merge it so, to do that first you have to define median pair.

(Refer Slide Time 54:00)

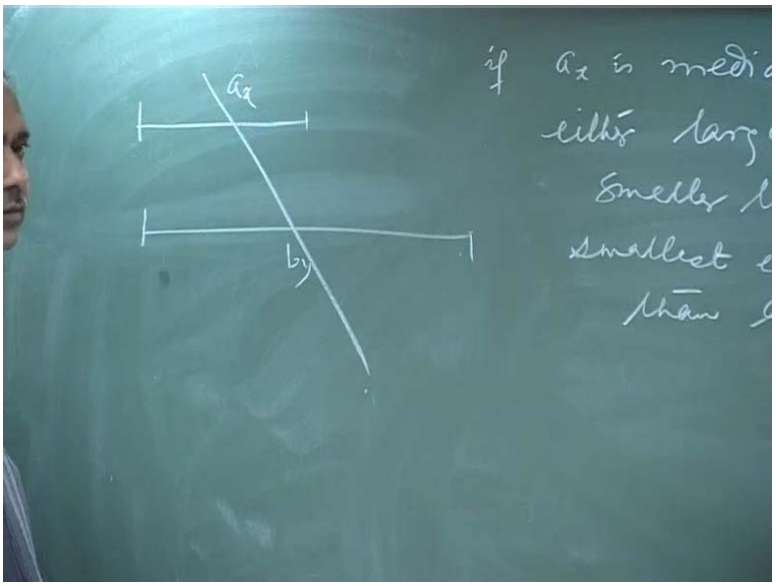


Because exactly finding median say if I have the 2 sequence a and b and you want to find exactly a median it take it will take lot of time because there is a question of merging and then find the middle element and so, on before doing there we are finding a median pair, a x b y and if a x is median it means their exist precisely or exactly if a x is median their exist precisely m by 2 minus 1 elements in a dot b smaller than a x and precisely m by 2 elements a w larger than similarly, with the reverse 1 if b y is to be there b y is next.

(Refer Slide Time 56:27)

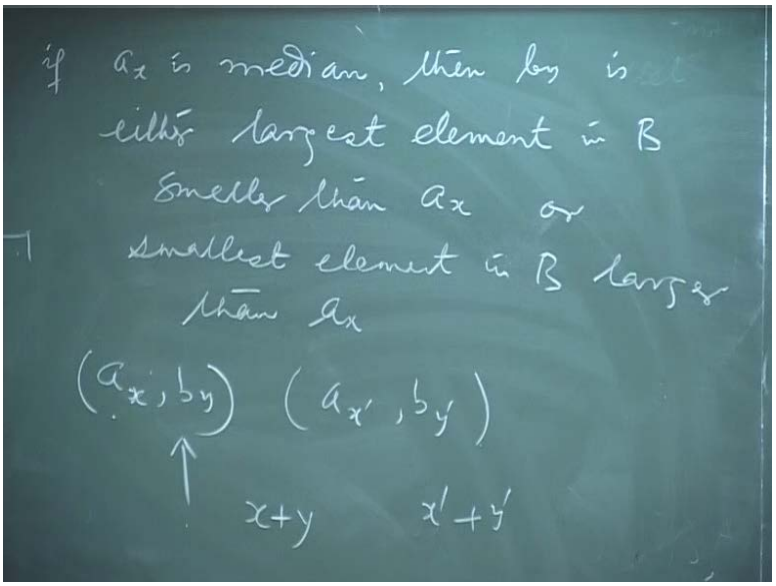


(Refer Slide Time 57:09)



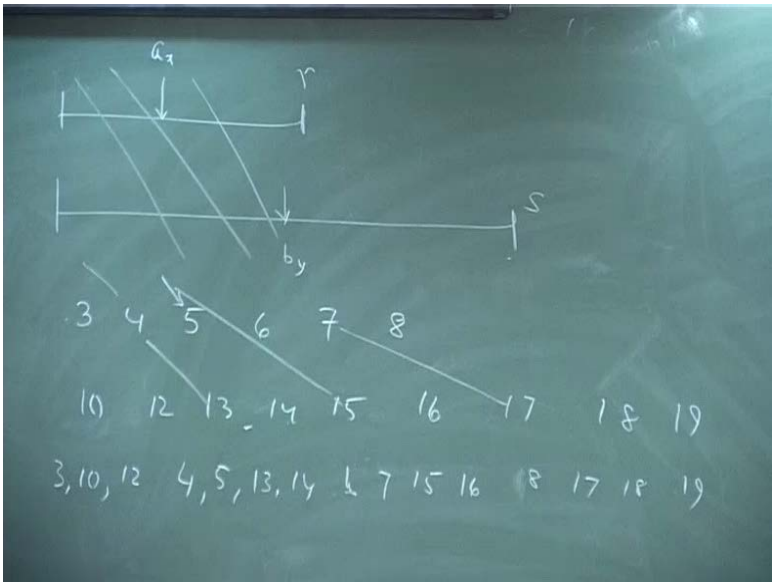
Now what is your b y if a x is median in the median pair what is the role of b y what is b y so, b y if a x is median then b y is the largest element in b smaller than a x or the smallest element in b larger than if a x is the median than b y is so, suppose this so, if a x and b y now the b y is either just larger than a x or just smaller than a x.

(Refer Slide Time 57:33)



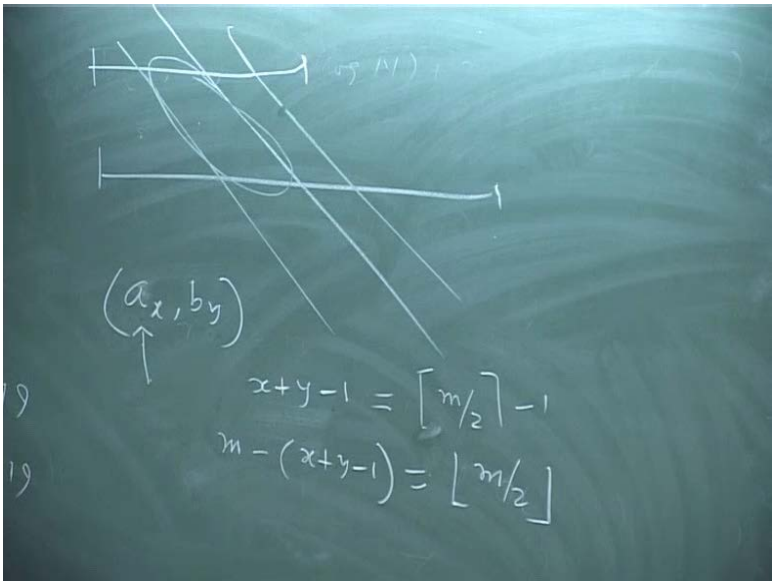
Now that means so, happen that in your programmed we will be here able to will be able to produce the 2 median pair 1 is a x 1 is b y and other 1 is a x dash b y dash if it is a case then we have to select 1 of them, 1 strategy we have to follow that you will be selecting the 1 whose indices is smaller than the other 1. Some of the index is smaller than the sum of the other than that will be your median because you cannot handle 2 median pairs for your 1.

(Refer Slide Time 58:38)



If your algorithm generates the more than 1 median pair than you select the 1 some of the indices is the can you fine tell me yes, i have 1 sorted sequence we have another sorted sequence this size is r this size is s now, tell me tell me understand this median element got this 1 and median this you got this 1 than how tell me little give me I got the initially a x b y than you want to merge this 2 dividing into you want to divide this 1 than this 1 like this that now, the scenery is if you do this way why it will be a problem let, us consider 1 example 3 4 5 6 7 8 and here 10 11 12 13 14 15 16 17 18 19 now tell me yours studies.

(Refer Slide Time 61:54)



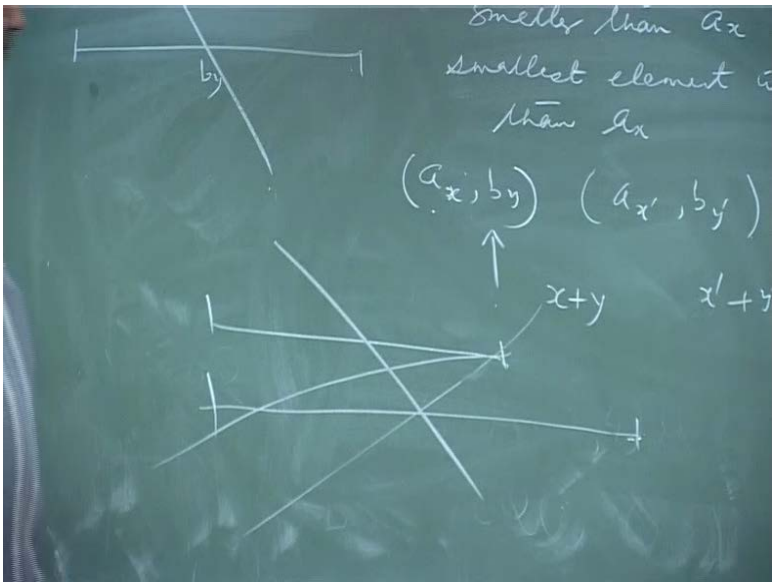
So, you want the median suppose this is the median and 1 2 3 4 5 6 7 8 9 so, fifth 1 is your median so, first you divided this 1 than second this and suppose 13 you divided similarly, you have divided 7 and 17 now, you want to merge this so you get 3 10 12 you get 4 5 13 14 6 7 15 16 may be 8 17 18 19 then so, we the idea is not only to divide this than the retards with this in such a way that all this elements will larger then, this elements that is why we looking for median like all this element not only equal parts but, only this element is larger than this element and so on.

But there is idea of b t n p l clear if you have a x b y is a median n p r it is median n p r and let, us assume that this is your median suppose a x is median then can i drop this conclusion and x plus y minus 1 is precisely x is median and x plus y minus 1 m by 2 and

because x is the index y is the index for the b that tells you that many elements either smaller than this.

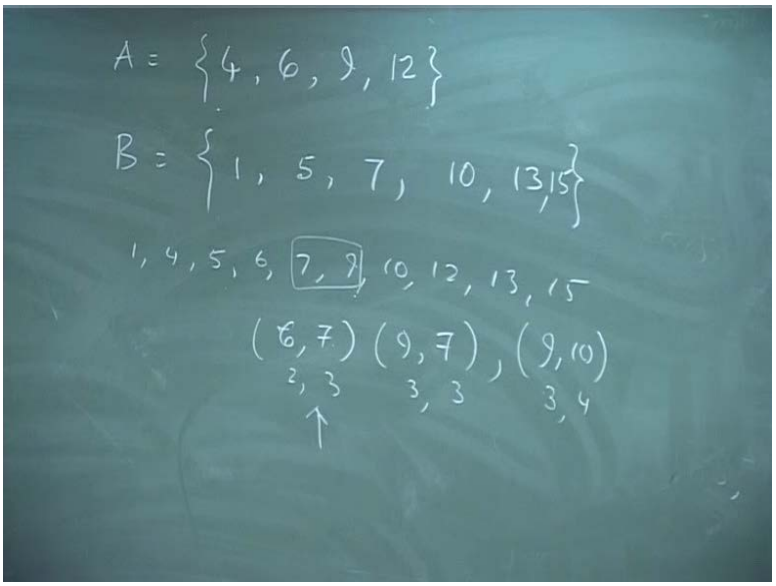
So, this is the what precise the x minus 1 x minus 2 or m minus whatever what is m m is the number of elements number of r précis so, m minus x plus 1 y minus 1 is m by 2 right that is the precisely that elements from the other side by would be larger than this elements. Either elements faller than this that elements larger than this these are the just from definition I am not that anything additional.

(Refer Slide Time 63:23)



Similarly, the case will be why b y is the medium along things I understand your problem ax is the element and by is an element in b and 1 of the miss media other 1 is the either largest element 1 of the than what is monastery larger than b y. It can be this 1 can be this 1 not this 1 if this is here than this be here and so on what if but, this cannot be media cannot be median if it is median analyze to your analytical activity but, lot of good property this exist can began here.

(Refer Slide Time 64:54)



So, you can try and think by this can be use beyond from the point to do something else can be fall elements from bored elements 4 6 9 12 and this side 6 element from 1 5 7 10 13 common elements here 15 which is the began here first 1 is 7 if so 7 is possible 6 7 than 9 7 9 10. So, this of the 3 main elements can be possible did for your median emphasize. So, you will be the selecting 1 so, this is index 2 3 index is 3 3 and this index is 3 4 you will be selecting the 1 which was the minimum 1 so, see 7 will get the began than with the other stepping 7 is the began.