

Biometrics
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Lecture No. # 02

So, in the last class, what I gave, I told you that, what is the aim of this course? What we want to do throughout this period? And also we told that different types of biometrics system, we like to discuss. And it is already told that we need to know little bit about the image processing. And I know I understand that you do not have the background of image process, but again I am not covering going to cover the total image processing thing. Whatever I need throughout the, my biometric system that part only I will be covering. You know otherwise itself is a course I do not want to cover all these. Now I mention that what is image.

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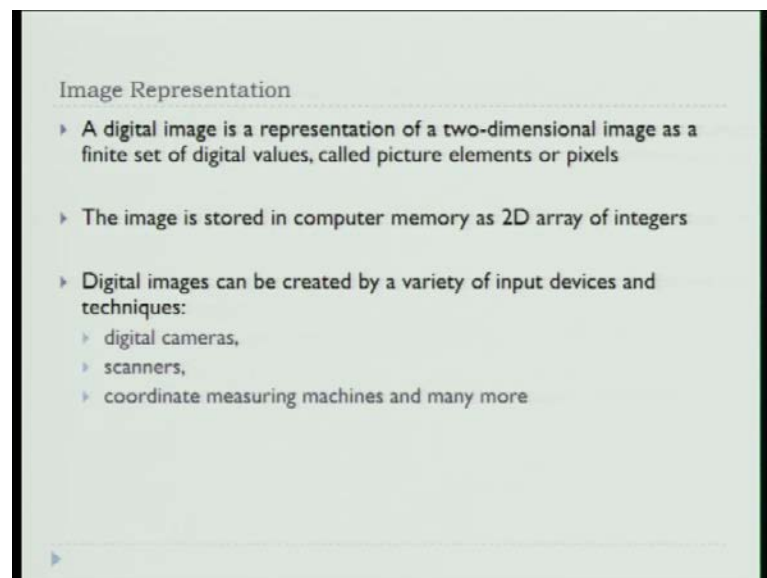


Image is representing a certain picture or certain photographs and once you think about the picture or photographs or something like that it automatically comes to your mind it is a two-dimensional pictures. Now, the two-dimensional picture means what? It is a two-dimensional array and each once it is an array, what it means that it gives you m cross n elements. Now each element represent to some part of your picture. Now, if I tell

that it is 100 cross 100 images and it is representing that 1 kilometer by 1 kilometer area then obviously, one picture represents to 10 meter by 10 meter or something like that.

So, that 10 meter is a big number even though you feel that the 10 meter is very small, but 10 meter per square 10 meter by hundred square meter area is representing. Now one thing you remember that, if I go for the satellite image or remote sensing satellite image it is coming from the safe top of say, 250 kilometer and it is giving the image of say, 100 kilometer by 100 kilometer. So, one pixel is representing something, it is assume that it is it obvious the normal distribution or you know if there exist a greenery it will be suddenly the greenery will not be restored. So, greenery is a certain area you will find green.

And then again there is a transition with the green and urban area; again you will find the certain area is urban and so on. Now if I think that no 1 picture represents to 100 kilometer by 100 kilometer; obviously, you will not see anything, because it is a green plus urban everything would be in one picture. So, this image we stored it in the two-dimensional array in the case of finger print it is a very small area, but I am telling that it is 500 PPI, 500 PPI means 500 pixels per inch, 500 dots per inch, sometimes we tell DPI sometimes we tell PPI dot per inch.

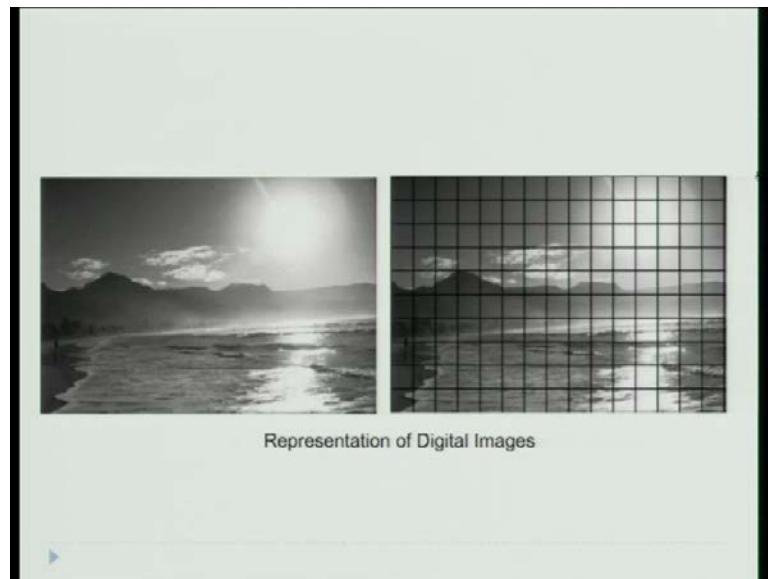
So, it is a big number. So, full resolution is very high you can see every beans every minor point or in your fingerprint image, but there exists the thousand PPI fingerprints scanner also. So, it gives a bigger image and every small things also you can see. Now, how can you collect this image? One way is that through camera, this is simple way you take the camera and then you have the different types of scanners; flat bed scanners are there where you scan it. And that third one is the coordinate measuring unit. Have you seen tech plot type things? Have you seen those stuff that in the civil engineering department people what they do they want to draw the maps digitized the map.

What they do they have a pen, digitized pen it puts on the map and automatically the coordinates are getting stored and obviously, there exist certain error because human being whenever you are fixing a dot or using the digitized pen it may not be exactly on the, but error would be very minor. But this is another way you can digitize your whole map. So, one is digital cameras and that is you know, if you go to the market to purchase

a mobile phone see have you recently visited any mobile shop to purchase a mobile phone?

You have visited what they tell about you? Mobile phones. See they will not talk about the phone characteristics they will talk about the camera characteristics all those it has or music that you know this much capability it has, you can store this, not they would not talk about the phone they talk about the camera the 5 megapixel, 10 megapixel camera. Now, why they are telling about this one because they want to make use this system for multipurpose and better the number of megapixel you will get the better is the quality of the image. Because more dots you are putting in one small area.

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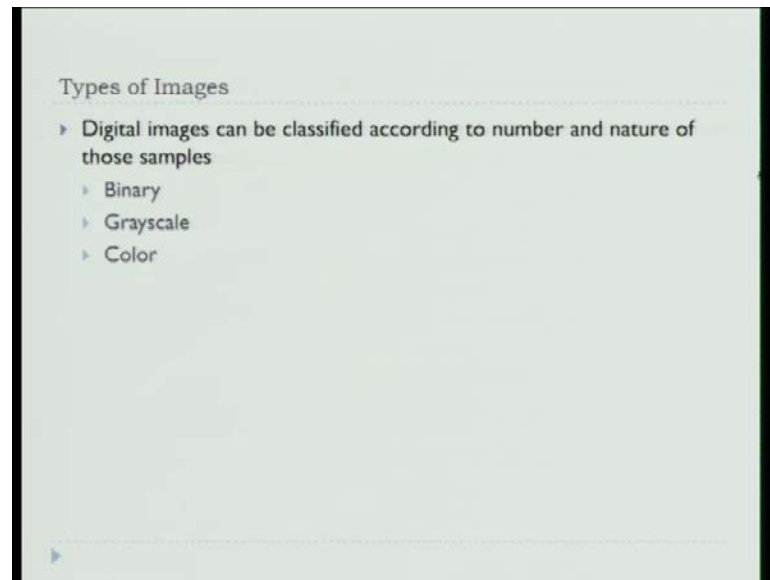


So, this is an image and this image as I told you, but do not think that that much size pixel will be given, this pixel size will be very small the remaining area, but I have for visual point of view. I am dividing into the grid grids and assume that this is 1 coma 1 this is 1 coma 2 and so on. Now how this image being read? See generally image you are reading from this one way could be like this the raster scan made method right this way.

One could have thought no I will read from this way another, but another way one can think about doing this way, but generally we read from the this corner and we read from the left to right. Now, how to read the image? There is standard image for those who know that open CV, their straightway daily routines are available or mat lab image routine. So, we are not going to discuss how to read an image, that part you read yourself

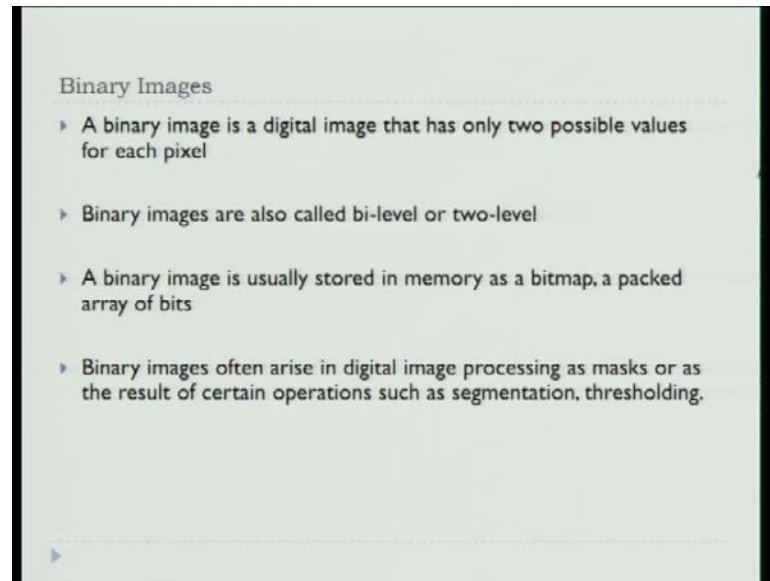
that open CV you can take the help of open CV or you can take help of mat lab. For us it is a matrix A or matrix I where I_{ij} is telling about or pointing to the pixel at the i eth row at the j eth column. What if I tell I_{ij} , this is the i eth row j eth column this element I am looking for.

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Now types of images, there are generally three types of images, one is known as binary image another one is grayscale image and finally color image and most of us are habituated with the color images. Yes or no? But why color image? color image gives you the better soothing effect, but or you can visualize the texture or color complexion part more better way, but our most of our algorithms will find that we ignore the color. we are depended on the other two methods, the other two image representation. Because that color you know today you are coming with the normal color and tomorrow you can put lot much that talcum powder or another things the color will be different. So, the color will not play major role in determine or using the system for identifications.

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Now, what do we know binary image? Name itself is telling you that it has the two numbers that each pixels intensity, intensively value that as I told you in the image this is a $I \times m$ or $m \times n$ and I, j eth element is represented by I, j eth that contains one value that is either 0 or 1. That is why it is known as binary image and sometimes we tell is bi-level or two-level images. Since it contains only the value 0 and 1 or each pixel represents 0 or 1 so I can have eight intensity values in one byte or you can you take the help of bitmap to represent the whole image. Once I have the bitmap or once I have 8 pixel values in one byte so I can use the bit operations to perform any operation on the image. Now where are you going to use this binary image? First thing is that you remembered that binary image is useful while we use as a mask.

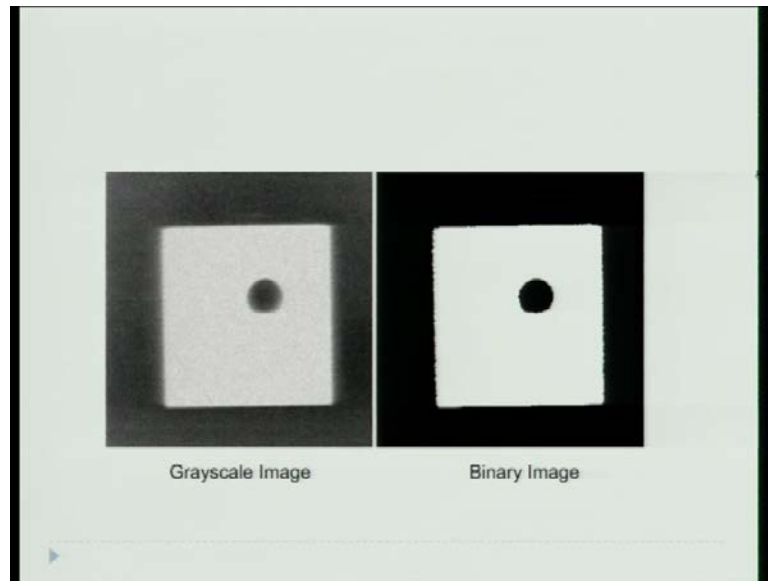
Mask means the small image, I want to move throughout the whole image will see when we will be performing our certain operations, we will be using that as a mask. Sometimes binary image comes out as an output of the certain operation on the image. For example, that suppose I want to see only the edge of image what is a edge of an image?

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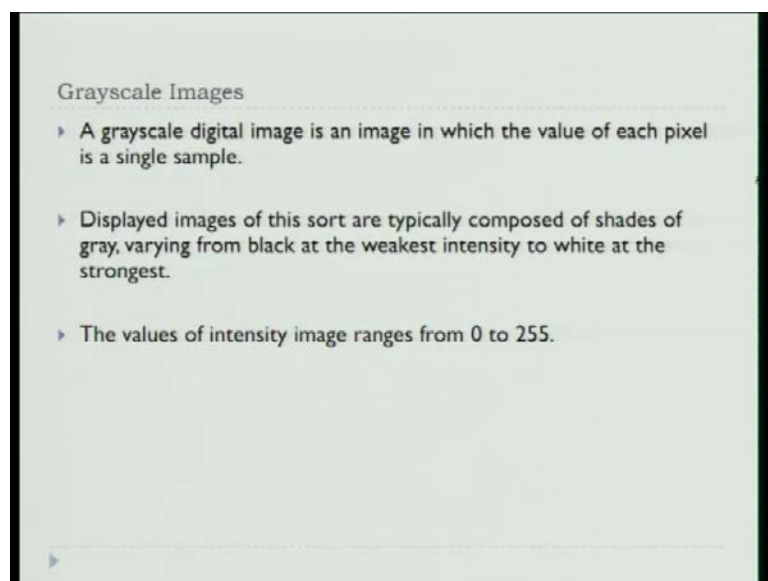
Suppose I have a house here and here is a path and this I need I do not have to see the color of this or the texture of this I want to see that boundary of this or the boundary of this path or boundary of the railway tracks and so on or the river. So, only the I need to know the line to see that is useful. So, you need these are all zeroes and this I put one then I can see this is a white 0 and 1 itself sufficient to represent your edge. Similarly, for our case also what happens that I have say, I got my hand like this and I want to segment **I want to segment each fingers I want to segment** each fingers out of this. So, for to segment it I need only this area I do not need in to see the inside of the things I need to see this part I need to see this part and so on.

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So, to segment something the binary image is very useful, because you know if you perform the operation on other image, it will take huge amount of time and it may create lot of problem. See for example, this is your binary image, you see the sharp difference between the 0 value and 1 value pixels, whether it is the grayscale image here the values are not 0 they are having certain other intensity values that is why it is you know shade is different. So, is it ok? So, you know now what binary image is and you have just heard the term grayscale image.

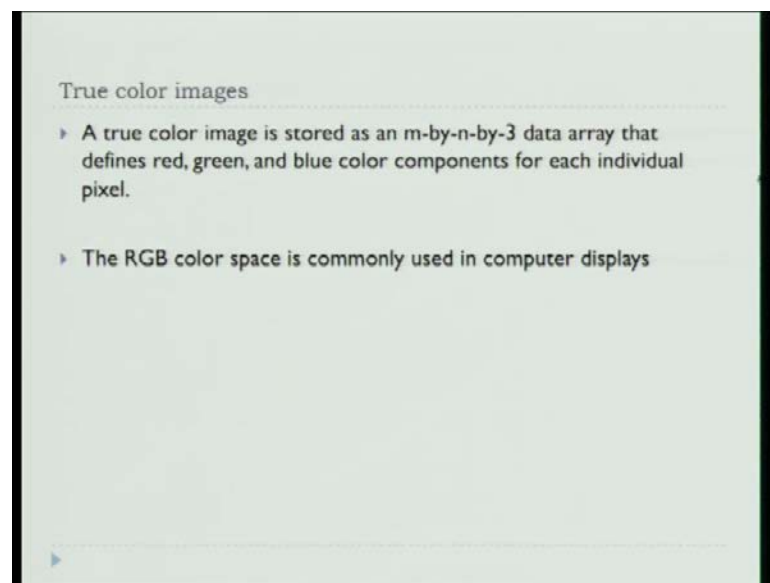
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What does it mean in the case of grayscale image each pixel is represented by eight bits; that means a byte is allocated for a pixel in a grayscale image and it is of course, that remember one thing that intensity value of each pixel is a positive integers, the intensity cannot be negative. So, the number; obviously, lying between 0 to 255 because 8 bit and each bit will give you some information about the intensity. So, value 0 is that it is very dark and value 255 tells you value 255 tells you that it is white.

And in between the values that is telling you the texture shades of your images say, if you go back to this image this is a grayscale image. Now here the value is 255 or nearby values exactly may not be 255. This area you may get it is near 0 and here you are getting some shades. So, these values this is near 0 these values are need not be 0 these are may be 55, 60 like that values. You understood? So, in the grayscale image there the intensive value should be lying between 0 and 255. So, this is an example of grayscale image.

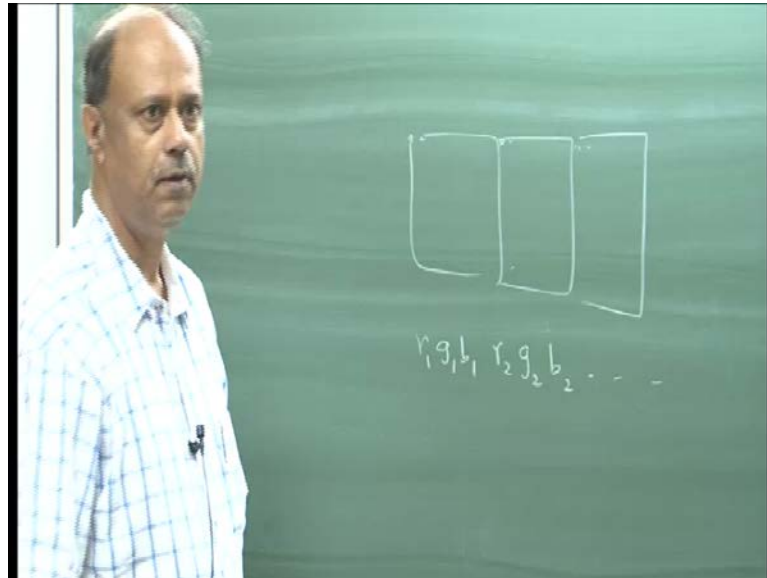
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Now, obviously, the another term is coming the color image, the color image is composed of the three components one is red, green and blue. We tell RGB and each of them is having the same as gray scale values lying between 0 and 255 and if I combine all these three factors then you can see visualize the color effect, one of them will not give you the color effect. Now how to represent because there are 3.1 is R another one is

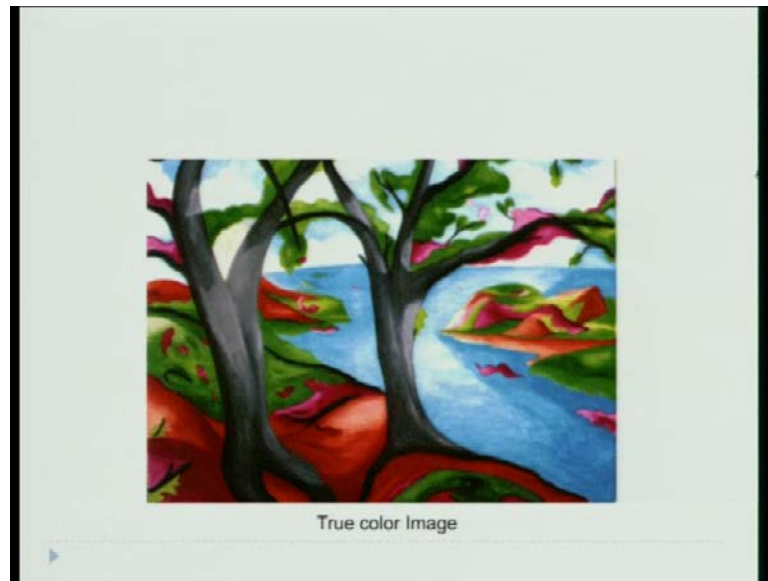
G and another B. How are we going to represent it? There are two ways you can represent, one is that whole R part is there then G part then B part.

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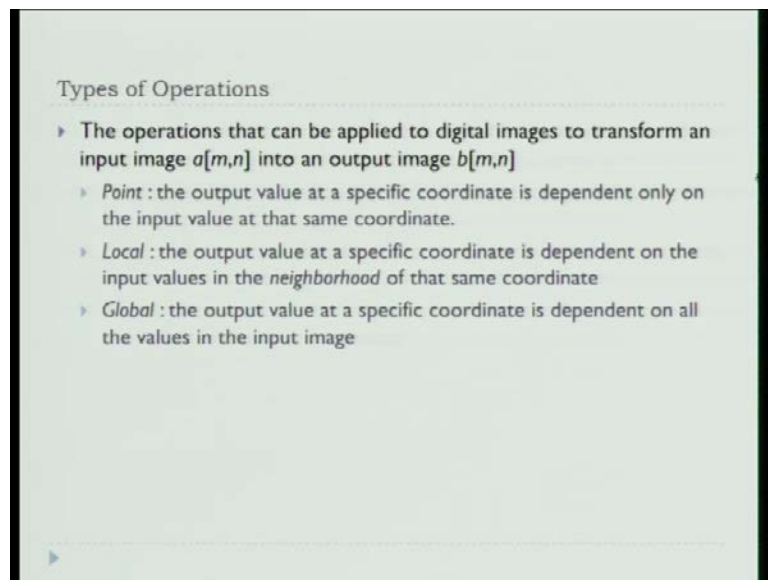
So, r image g image r image then followed by g image followed by b image then when you want to combine that you take this pixel this pixel and this pixel show it display it then take this pixel and this pixel and this pixel display it and so on, this is one format. Another format is that I have RGB, RGB and so on. So, R component of the first pixel G component of the second first pixel and B component of first pixel they are kept together. That means three bytes I am keeping for the for each pixel. So, that is why it is written m cos n cos three and this is mainly used for your you know color display.

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So, this is a pseudo color basically that is artificially on the same is the I have put some color and to show you the pseudo color.

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Now, once I have an image, you need to perform some operation on the image otherwise what for you are representing the image. So, for example, **for example**, that satellite sent you a data you got an image and that image satellite is what satellite is moving.

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And you here is a satellite and its and it gives you this is a path of the satellite and it gives the picture of this path whole path from here to here. Now, handling such a whole path at any will be very difficult. So, what we do we divide into the frames. So, these frames has to be taken into account to analyze. Now you observe that it contains several problems, first part is that once system is giving you the data it has different types of distortion.

So, you need this for example, the distortion is that when is giving the setting the data earth is moving or earth curvature is there. So, all those are satellite behavior is an issue, all those because you should not think that satellite will be lying moving like this only it may move like you know it has lot of all those factor should be coming. So, those corrections are required to make. So, that you visualize the image. It may so happen suppose of my satellite is a CCD based CCD camera it has say, you have say, 2048 say, CCD lights there and one light is not functioning well it is not giving the it is not making it on and off.

So, what you will get? You will get one dark line, because this is always will not give you any image because if that light is not on. So, if there is a line here dark line, you will not be able to sell your image nobody will purchase your image if you tell that my satellite is not giving you one giving you the whole image. So, you need to do certain operations or it may so happen that because of some reasons that some noise enters into

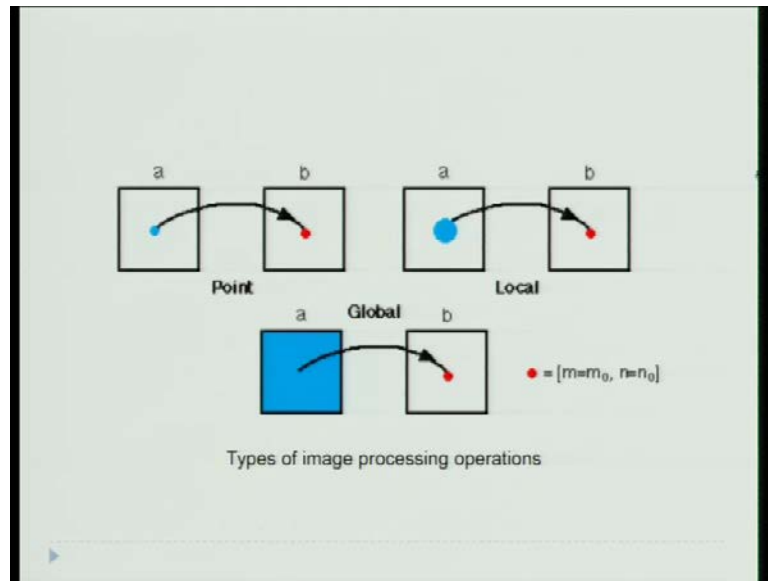
each image. That noise will be there because for any image you cannot think that you will have that an image without any noise, but that noise should be as minimum as possible so that you get the soothing effect people will purchase your system otherwise if it is a full of noise nobody will take your image.

So, this after getting an image you need to perform some transformations to transform the original image to the output image and that is coming under these operations. So, one thing is that there are point operations what it means that I have an image **I have an image** and each point I have to change by some value. Each point means each picture representing the certain position the image and it contains some intensity value, but you need to modify this intensity value. So, this is a point operations.

Say for example, what happens that even though I am thinking that in a grayscale image the intensity values will be lying between 0 and 255, but in reality it may so happen that you are getting the data lying between 60 and 120. Because you know there is no guarantee the system will give you the intensity value 0 to 250, you would be getting say, 60 to 252 120. Now you want to improve your contrast you want to show that no it is covering the whole range. So, you need to do some operations on each pixels that is your point operations.

Then it is coming local this operation is different from the point in the respect in this **in this** following respect to compute or to estimate this pixels intensity or to obtain some operation performance on this or intensity on this I will take the help of some neighboring pixels. I will take the help of some neighboring pixels to obtain some intensity value of pixel. So, that is your local operations finally, the global operations the global operation means the whole image I will be studying to draw a some conclusion. So, for example, I want to know **I want to know** that what the intensity range? So, I will read the whole image and then I will tell that minimum is 60 and maximum is 120. So, this is a global operation.

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So, these are the three types of operations will be performing and since we are working each pixel mainly we are working on the each pixel. So, our work will be mainly upon these two parameters, because generally this is only to draw a some conclusion to tell about the image behavior, image quality or something what is the contrast of the image what that type of things we need we perform this type of operations.

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- ### Types of neighborhoods
- ▶ Neighborhood operations play a key role in modern digital image processing
 - ▶ The neighborhoods are basically of two types
 - ▶ 4-connected
 - ▶ 8-connected
 - ▶ Two types of sampling schemes are
 - ▶ Rectangular sampling - In most cases, images are sampled by laying a rectangular grid over an image
 - ▶ Hexagonal sampling - An alternative sampling scheme

Now, once we think about the local parameters that local operations so the concept is coming the neighborhood. What do you mean by the local that who are the people who are neighbor of a pixel.

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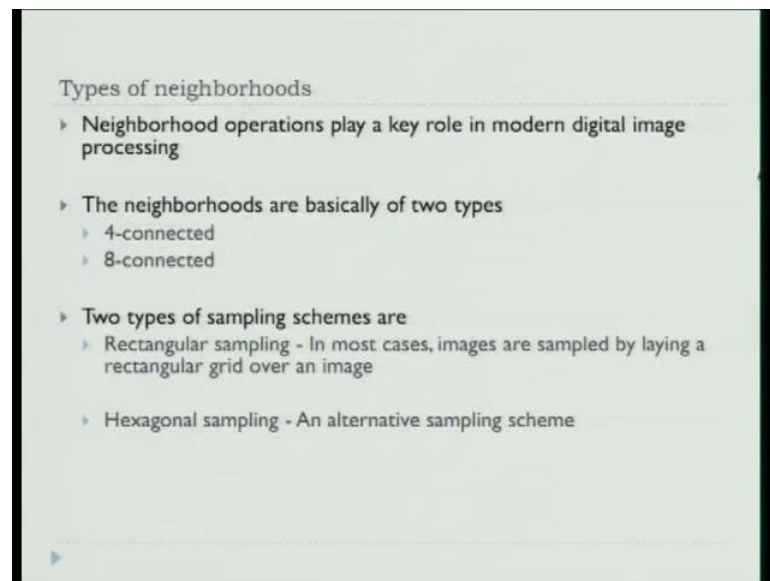
So, you have say, for this image for this for this image, suppose this is any pixel suppose this is a pixel and who are its neighbors and one neighbor could be this four things they are known as four connected neighbors. Now another one is that 8 connected these eight are neighbor of this. See you can think those I want to consider another level to have the neighbors more you are increasing the size more you are making the size, complexity will be increasing and you will not gain much. The two estimate this pixel, if I consider these 8 neighbors what it will give that behavior of 8 neighbors you are taking to consideration to estimate the intensity value of the center pixel, that is the idea.

That if I consider the 4 neighbors to estimate this, I take the behavior of this four neighbors I take the behavior of this behavior means I take the intensity information of this four neighbors to estimate the intensity of the central pressure. Is it clear? Why it is so? So, say for example, as I told you the remotely sense image I always feel that the behavior of the intensity levels of the neighboring pixels will be maintained. What it means say for example, here this is a green shade is there, if you consider the neighboring pixel that also will be the green shade, if there is a major change in your path say this is your path, there is a major change between these two then it will not be

one pixel it will not be it will be certainly it will be not there because it will be you know soothing impact will be there.

That sudden change will not be there slowly it will be changed. So, that behavior of the neighboring pixels you know will help you to estimate the pixel value of this one. So, one way say for example, I have a one CCD light is off, now this is I have to estimate this value what I will take I will take the behavior of the two neighboring pixels and I put a average value of this here. So, the dark becomes the behavior of these two have been taken into account and I get the soothing effect. Now, if it is a sharp change here you will not get that sharp change because you have taken the help of the four neighbors or eight neighbors and it to estimate the there will not be sharp change there will be some soothing things. that is the only possibility of loss of some information.

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So, this is a 8-connected and 4-connected neighborhoods. The next one is that this will be very useful for estimating something. Estimating the or output image say what happens that, you have got a satellite image here, now this image has full of distortion and you have corrected them. So, once you correct them, so this point I will we I will be discussing when we will be discussing the geometric transformation will find that this point will be this is your input image and this is your output image. Now this is a grids of the image now this corner points once you do the corrections it need not be this point. It

will may be here this point need not be here this may be here this point need not be here this may be here and so on.

But see this coordinate has been say one comma one has been moved to say may be 1.5 and 1.7 may be it need not be the integer, but these grids are integer. So, you have to get the or intensity value of this integer points of the output image from the input image original input image. You understood? So, you need to do some operations to get the estimated value of the output image on the integer grids not on the real grid **real grids** you will **you will** not be able to visualize the thing, because this points are integers is it ok? So, that is your re-sampling we will be discussing when we will be covering the geometric transformation.

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Operations on Gray Scale Images
> Clearing an image
  for (j = 0; j <= n; j++)
    for (i = 0; i <= n; i++)
      I[i][j] = c;

> Copying an image
  for (j = 0; j <= n; j++)
    for (i = 0; i <= n; i++)
      I[i][j] = J[i][j];

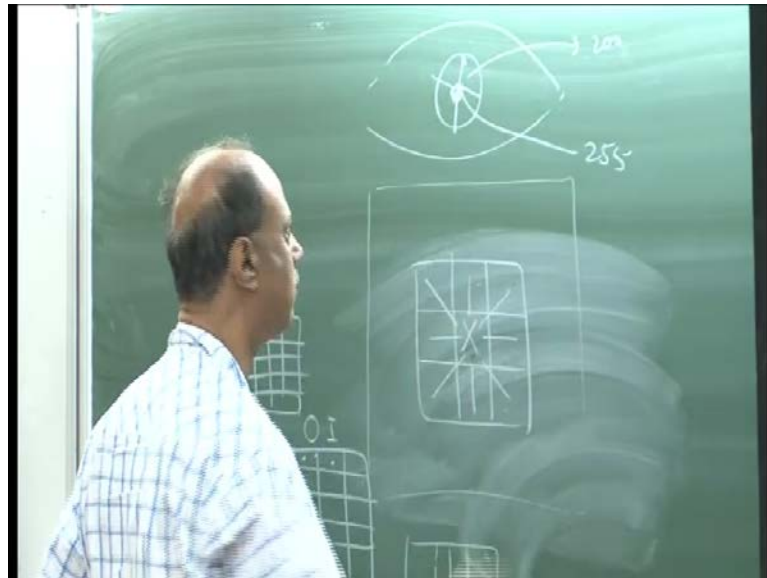
> Inverting an image
  for (j = 0; j <= n; j++)
    for (i = 0; i <= n; i++)
      I[i][j] = 255 - I[i][j];
```

So, this is your neighbors, now that operations the this is not actually a world global; this is a point operations on the whole image point operations on grayscale image, first one is clearing an image this is the you know initializations I want to initialize the image by a white intensity value white means 255. So, for each for each pixel at the I am putting a 255. So, this image becomes snow type things or it contains the white values. I want to copy an image from j to i. So, because here I am assuming that you have already read the image.

Next one is, I want to involved an image what does it mean that I have the grayscale image, I want the in negation of that because your negative image is very sometimes

very useful. So, what we do we subtract each element by from 255 that will give you the negative of your image, the negative of your image sometimes useful for example, I want to see whether there exist some dots or not right.

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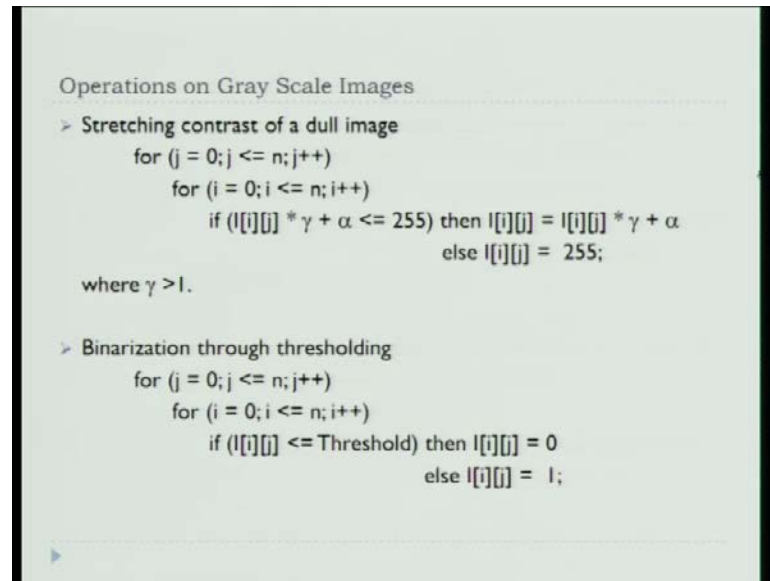
Say for example, on your iris **iris** you have the black and inside there is a small area another dark, very dark if you see that in your iris image this is your eyes and this is a dark area and inside that there is a very small dark area. In that I want to get that small dark area if I do the negate of this 255 minus this value this will become very white. This small area will become this will become almost 255 and rest this may be 200 and so on. So, this area you can visualize that this is your pupil area. So, I need I need to get **I need to get** this part. So, if I know this area, I know the centers I can find out the through circle circular expanding I can obtain the outer range of the outer part of your black area. So, the inverting an image is a very powerful operation for our case.

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Operations on Gray Scale Images
> Column-wise Shifting an image
  for (j = 0; j <= n - c; j++)
    for (i = 0; i <= n; i++)
      I[i][j] = J[i][j+c];
> Row-wise Shifting an image
  for (j = 0; j <= n; j++)
    for (i = 0; i <= n - r; i++)
      I[i][j] = J[i+r][j];
> Increase of brightness by constant factor  $\alpha$ 
  for (j = 0; j <= n; j++)
    for (i = 0; i <= n; i++)
      if (I[i][j] +  $\alpha$  <= 255) then I[i][j] = I[i][j] +  $\alpha$ 
      else I[i][j] = 255;
```

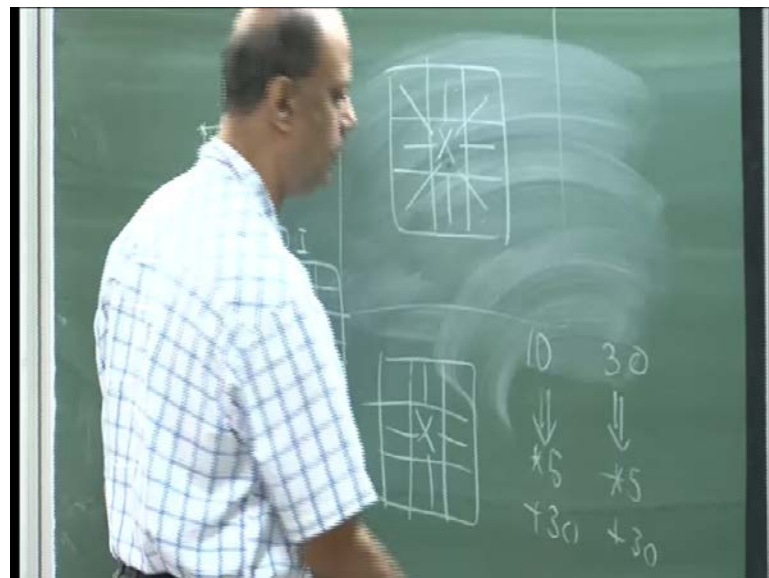
Sometimes you need to shift the image towards right towards left and so on. So, I am shifting the image towards your left. So, similarly I am shifting the image up and now next one is that I want to increase the brightness of an image. How to increase the brightness? So, simple operation is that I just add some values on each image component each pixel. Say, I have the intensity value x , I had alpha with that and you will be getting. So, what I am doing the intensity value which was lying between 60 and 120 and I am adding now 30. So, intensity level will be range will be 90 to 150, just I have added. now while you are doing this operation you must ensure that it must be less than 255. If it is more than 255 you are in problem, then it will take some negative value to be overflow.

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So, you have to ensure this part. That no I do not want to increase the intensity level by 30 what I am doing that, I want to stretch it I want to expand in different ways that so that **so that** not only the shifting it is also stretching all the spans. So, I am multiplying by a multiplier factor and plus 30 movement alpha one this value should be less than 255.

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So, if the number was like intensity value was lying between say, 10 and 30 now what I am doing I am not only multiplying by say 5 plus say 30. So, what happens now the range is coming 80 and 180. So, instead of 1 to 30 I am now stretching it only thing that

while stretching this you must ensure that this is less than equals to 255. Now here we observed that I have told that gamma is greater than 1. See why gamma is greater than 1? I am taking that suppose gamma is less than 1, then you is reducing you will not get more information you will be getting the less information. So, that is why I am assuming that gamma is greater than 1 and gamma should not be negative because our aim is to our image is always containing the positive integer values. So, we are taking care that one.

Then given a grayscale image how to **how to** get the binary image. Now you have to; obviously, you have an intensity value lying between 0 and 255 and you have to convert into 0 and 1 image. So, some part will be 0 some values will be converted to 1 now this some part some or some intensity will be converted into 0 some will be converted to 1 that should be dependent on some threshold value, that these elements if it is less than this threshold value you assume that this is 0 otherwise it is 1. How to determine that threshold that will discuss later on. So, say if image value is less than equal to threshold value put it 0 otherwise you put it one. So, you get a binary image now this sometimes we need to do the of the inversion of the binary image.

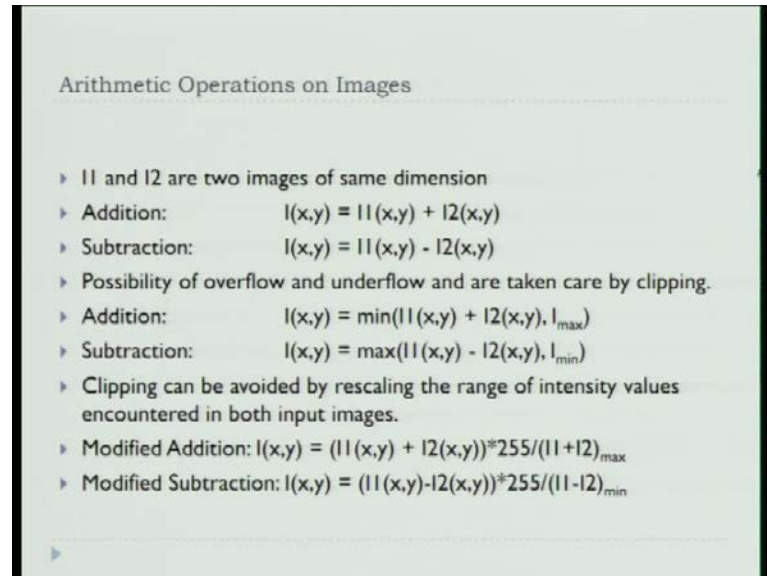
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Operations on Gray Scale Images
> Inverting after Binarization through thresholding
  for (j = 0; j <= n; j++)
    for (i = 0; i <= n; i++)
      if (I[i][j] <= Threshold) then I[i][j] = 1
      else I[i][j] = 0;
> To display the resultant binary image in a form as close as possible
  to the original image, we can have
  for (j = 0; j <= n; j++)
    for (i = 0; i <= n; i++)
      if (I[i][j] = 255 * (1 - I[i][j]));
```

So, this is a simple one i equal to 0 less than equal to 1 is 0 and the other 1 is 0. Now can I **can I** get back from binary image to grayscale image it is impossible, because you have we have already to their holding you have made them in such way that if 0 is less than

threshold value it is 0 otherwise so, but you can you can visualize you can increase the intensity level one is 0 another one is 255 based on this statements.

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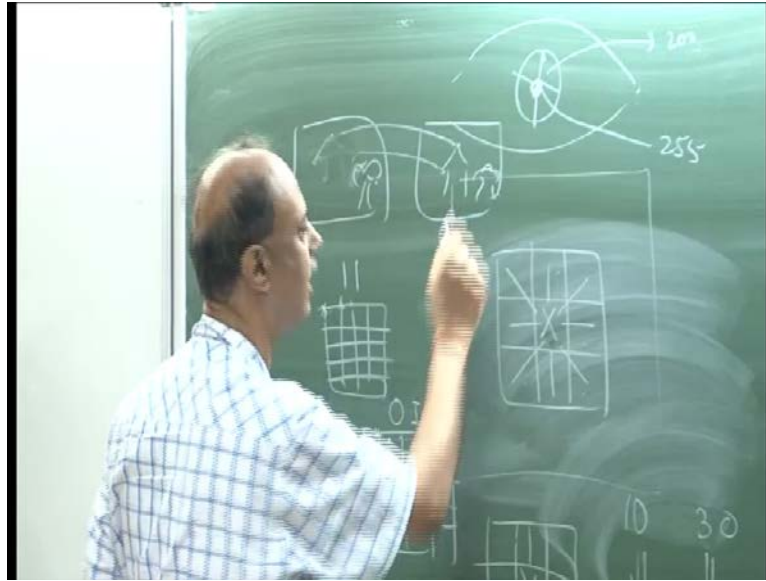


Arithmetic Operations on Images

- › I1 and I2 are two images of same dimension
- › Addition: $I(x,y) = I1(x,y) + I2(x,y)$
- › Subtraction: $I(x,y) = I1(x,y) - I2(x,y)$
- › Possibility of overflow and underflow and are taken care by clipping.
- › Addition: $I(x,y) = \min(I1(x,y) + I2(x,y), I_{max})$
- › Subtraction: $I(x,y) = \max(I1(x,y) - I2(x,y), I_{min})$
- › Clipping can be avoided by rescaling the range of intensity values encountered in both input images.
- › Modified Addition: $I(x,y) = (I1(x,y) + I2(x,y)) * 255 / (I1 + I2)_{max}$
- › Modified Subtraction: $I(x,y) = (I1(x,y) - I2(x,y)) * 255 / (I1 - I2)_{min}$

But here it would be either black or white 0 or 255. Now, arithmetic operations on the images suppose I want to add the two images, first thing you remember these two images must have the same dimension. Second thing that once you are adding I assume **I assume** they are representing the same objects, if they are not same objects does not have much meaning to add the two images. Now second thing is that it is so it is assume that there are of same size then they are representing the same objects. Third part is that they are registered what it means that a registered means,

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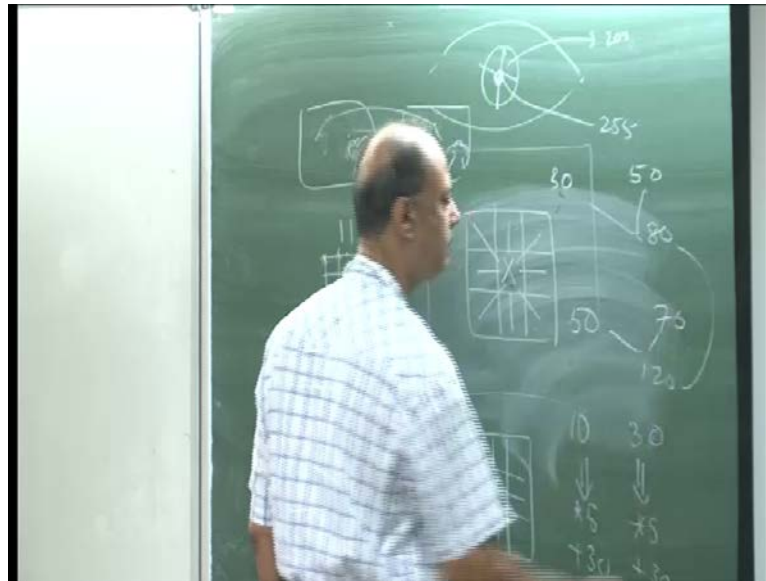


that suppose I have in my image I have a house here, as this image also I have a house. So, these pixel positions these pixel positions these pixel positions this should be same. If they are not registered by adding something you may not get something get a proper image thing one way could be sometimes, what we do I artificially I want to put here a tree I want to put a tree here. So, one image is having the house, another image is having the tree if I add I get I get a tree plus house this type of activity also there people wants to fake something. So, anyone is to add some generally we use we assume that there are two images same size they are registered, but for other purpose also people are using for addition operations.

So, addition operation is $I(x, y)$ equals to $I_1(x, y)$ plus $I_2(x, y)$ similarly it is the case in the subtraction. The subtraction is useful when I want to compare that whether these two images compared between the two images I want to match the two images. So, I will subtract and I will see whether difference is 0 or not. So, this is the addition and subtraction, but you observe if I add this two there is a possibility the total intensity value may exceeds 255. So, you have to control because you cannot **you cannot** add the two pixels, just like that because it is there is a possibility it exceeds 255. So, one way is that to control that to maintain their than to control this overflow underflow problem. So, you will be putting the $\min(I_1(x, y) + I_2(x, y), 255)$ that is what it means or 255.

So, if it is more than 255 then it is 255. Similarly, in the case of subtractions, it is a maximum difference between these 2 and 0. So, if it is coming less than 0, I will forcibly making it 0. Now, once you do it that basically you are adding the two positive integers. So, intensity level will be shifting from you know from one large value to the very large value. Say, you have 30 and you have 50 if I add then the value will be 80.

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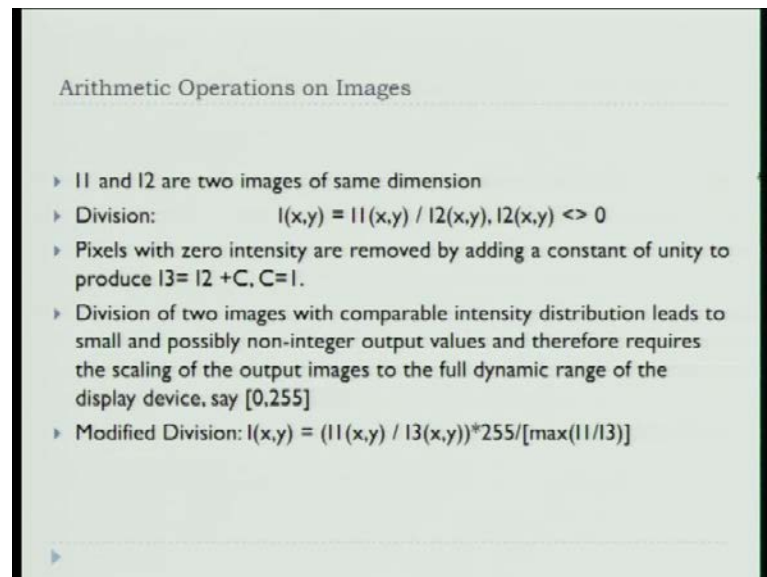
So, if my intensity value initially minimum was 30, here and on another in this 50 the minimum value will be 80, it is shifting to the larger side and that is again intensity range of intensity level will be you know higher side, but it will not be stretched one. So, in order to do that in order to cover the whole image whole intensity level what you did this to add multiple by the range I want 255 minus 0. 255 is the maximum intensity value I_1 and 0 is the minimum intensity value I_2 . So, I_1 plus I_2 this divided by the image max values, first image max value and first and second image values adding after the whatever the maximum intensity value. So, see I have 30 plus 50 it is 80 another one may be 50 plus 70 120. So, maximum of these values will be taken into account for division.

Now, once you are dividing remember the value may not be integer. So, you have to take the you know your nearest integer value. So, the division is very carefully you have to handle. Similarly subtraction so you have subtract the value multiplied 255 divided by again I_1 minus I_2 minimum. So, two image intensity level what is the minimum after

subtraction, what is the minimum that will be taken into account to divide. Not the I 1 minimum and I 2 minimum it is I 1 minus I 2 whatever minimum value that will be taken into account.

Now, here also there is a possibility of negation, because subtraction is there. So, if it comes to negative value you forcibly make it 0. So, subtraction is very powerful for us because sometimes we want to see whether the two images, if I convert into same intensity level whether after subtraction I get zero or not, in reality you may not agreed, but this is a useful parameter.

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Arithmetic Operations on Images

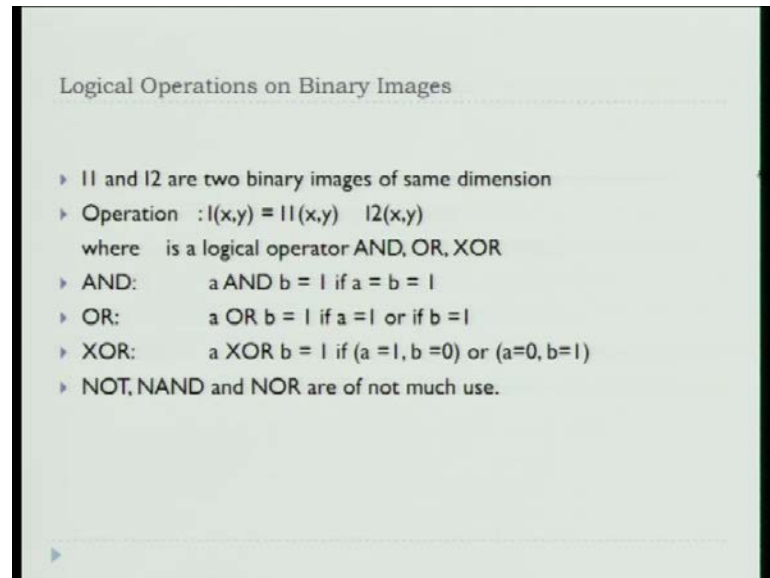
- ▶ I1 and I2 are two images of same dimension
- ▶ Division: $I(x,y) = I1(x,y) / I2(x,y), I2(x,y) \neq 0$
- ▶ Pixels with zero intensity are removed by adding a constant of unity to produce $I3 = I2 + C, C=1$.
- ▶ Division of two images with comparable intensity distribution leads to small and possibly non-integer output values and therefore requires the scaling of the output images to the full dynamic range of the display device, say [0,255]
- ▶ Modified Division: $I(x,y) = (I1(x,y) / I3(x,y)) * 255 / [\max(I1/I3)]$

Next one is, divisions and the two image I 1 and I 2 of same dimension and they represent the same objects then I 1 (x, y) is equal to I 1 (x, y) divided by I 2 (x, y). I 2 (x, y) must be greater than 0. But there is no guarantee that it will be greater than 0, there may be a black spot is there. So, 0 divisions will create a problem. So, what we did we add some constant to ensure that it is not zero to **ensure that it is not 0**. Now, once you divide it, something that number will become very small there is a possibility because you are dividing x by y and y if it is a very large number and x is corresponding x is very small number then number will become very small.

So, what we did we need to stretch it. So, we multiply by some big number say 255 to get the number again in that range. So, you need to do some manipulation here. So, what you have to do you have to first divide by this, now you get the min and max and the

same stretching algorithm you use it to stretch the number lying between 0 and 255. Is it possible? Is it clear? Yes or no? Clear? So, this is first I_1 divided by I_3 into multiple 255 and divided by I_1 by I_3 . So, this will give the range lying between 0 and 255.

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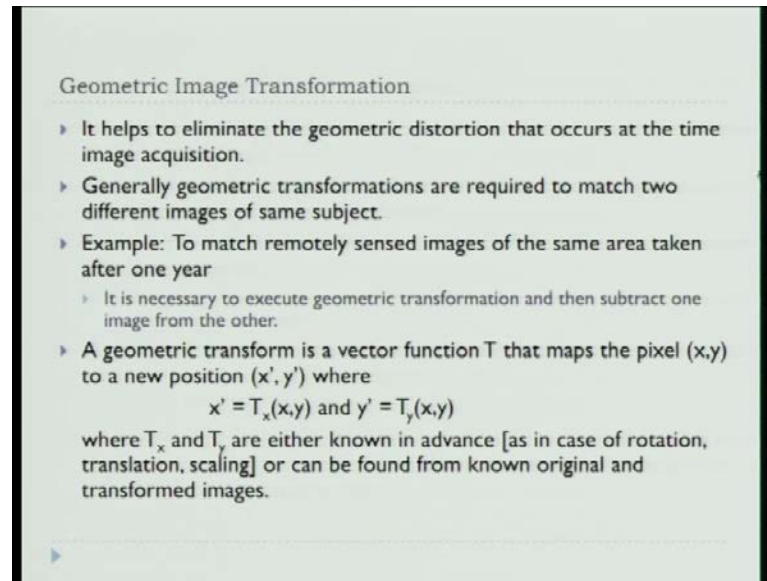


Logical Operations on Binary Images

- ▶ I_1 and I_2 are two binary images of same dimension
- ▶ Operation : $I(x,y) = I_1(x,y) \ I_2(x,y)$
where $\$ is a logical operator AND, OR, XOR
- ▶ AND: $a \text{ AND } b = 1$ if $a = b = 1$
- ▶ OR: $a \text{ OR } b = 1$ if $a = 1$ or if $b = 1$
- ▶ XOR: $a \text{ XOR } b = 1$ if $(a = 1, b = 0)$ or $(a = 0, b = 1)$
- ▶ NOT, NAND and NOR are of not much use.

Sometimes you do the logical operations on the image, generally this is useful only for the binary image. So, binary image $I_1(x, y)$ and $I_2(x, y)$ and this is a logical operator and these are AND, OR, XOR. So, this one that I_1 by I_3 I got the max of this, this is the max of this. So, we will go to the next slide. So, then a and b this is the OR operator XOR, but this are not that you see negation of a binary image. We have already discussed how to do it other part is not that important.

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Geometric Image Transformation

- › It helps to eliminate the geometric distortion that occurs at the time image acquisition.
- › Generally geometric transformations are required to match two different images of same subject.
- › Example: To match remotely sensed images of the same area taken after one year
 - › It is necessary to execute geometric transformation and then subtract one image from the other.
- › A geometric transform is a vector function T that maps the pixel (x,y) to a new position (x', y') where
$$x' = T_x(x,y) \text{ and } y' = T_y(x,y)$$
where T_x and T_y are either known in advance [as in case of rotation, translation, scaling] or can be found from known original and transformed images.

Now is coming the geometric transformation. So, as I told you that there are several in any image you will find that there are many reasons to get the geometrically distorted image now you have to correct that, otherwise your image will not be useful at all. So, this distortion occurs at the time of data acquisitions. For example, as I told you one case that one CCD light is not on, similarly, when you are collecting the photographs or face photographs there may be full of light so the total illuminated images. So, it will not be useful or there is a blurring effect or the image is rotated right because it is not essential that, I will get always the frontal and straight photographs may be rotated by some angle or it may be so happened that it is transcendent or it is scaled

Say, I have and one image of your photograph which is I have taken the data from one meter distance and another photograph is from 10 meter distance. So, the 10 meter distance photograph will very small. So, I need to scale it in such way that, I can compare these two images. So, scale is an important parameter. So geometric transformation are equate to match the two different images of the same subject to match remotely sensed. I told you this issue the same area when after one year what you have to do. First you have to transform it and then you compare it. So, the geometric transform is the vector function T that maps the pixel (x, y) of the original image to x dash coma y dash of the output image.

So, x' is represented by $T(x, y)$, T_x is your some transformation with respect to x and y' because $T(x, y)$ where T_y is some transformation along the y directions. So, these transformations are either known in advance what it means in the case of as I told you that scale things I know what is your distance of the camera and to. So, I need to scale it or I know that it is rotated by θ angle I need to correct it and so on. So, this is known in advance or I know the original image and the transform image and I have to correct it. So, this T_x and T_y will be either known in advance or can be found from the original image and the transform image. One is original and another one is your output image. So, based on that you have to obtained what is your T_x and T_y .

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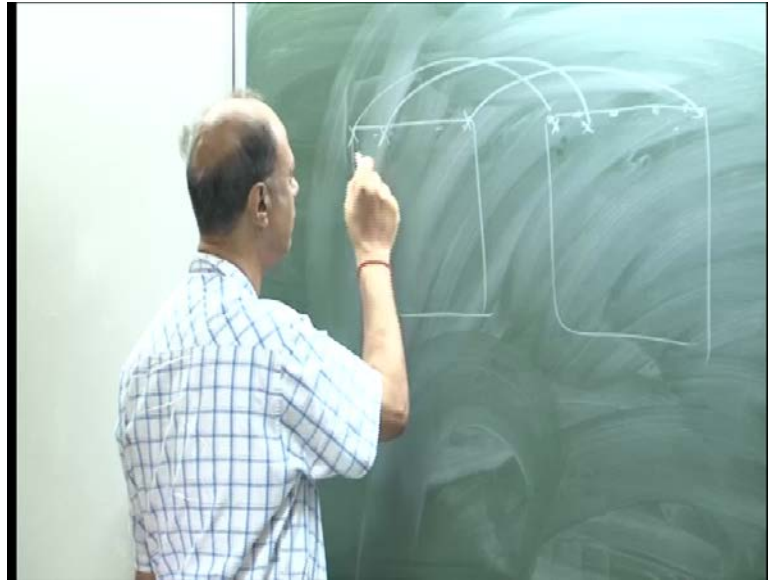
Geometric Image Transformation

- ▶ A geometric transform consists of two basic steps.
 - ▶ Pixel coordinate transformation which maps the coordinates of input image pixel to the point in the output image.
 - ▶ Finding the point in the digital raster which matches the transformed point and finding its brightness value using interpolation of the brightness of several points in the neighbourhood.
- ▶ The geometric transform generally is approximated by a polynomial equation

$$x' = \sum_{r=0, m} \sum_{k=0, m-r} a_{rk} x^r y^k \quad y' = \sum_{r=0, m} \sum_{k=0, m-r} b_{rk} x^r y^k$$
- ▶ This transform is linear wrt a_{rk} and b_{rk} and so if pairs of corresponding points (x, y) and (x', y') are known, it is possible to determine a_{rk} and b_{rk} by solving a set of linear equations.

So, it has the two, the geometric transformation consist of two steps. First one is that given x, y of the original image I have to get that x', y' of the output image r . So, that is first. Once I know the x', y' which need not be which not be the integer coordinates of the output image, I have to estimate now the grey values of the output image or intensity value of the output image.

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What I have to do, for each pixel values I have to get the output image pixel value. So, and so on now this estimated way x dash, y dash they are not integer. First problem, then once you know this values I have to get the intensity value of this output image. So, these are the two steps first is that for each image for each pixel point you have to get the estimated pixel coordinate of the output in the output image and then you have to get the intensity value of this. Agreed? So, intensity means the brightness values, now as I told you **as I told you** that there is a **there is a**, this intensity values these are coordinate values are dependent on these values and the some transformations. So, the transformation I am considering it is a polynomial degree and polynomial of degree and.

So, x dash is equals to $a r k x r y k$ and it is a since it is a x direction and y directions. So, it is a I have written x to the power r , y to the power k and similarly y , y dash equals to this. So, this $a r k b r k$ are the coefficients this $a r k b r k$ we have to obtained. You have to agreed in such way that you can estimate x dash and y dash. Agreed? So, these are linear equations. So, you can easily or system of linear equation.

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Geometric Image Transformation

- ▶ In case the geometric transform does not change rapidly depending on position in the image, lower order approximating polynomials of degree 2 or 3 are used needing at least 6 to 10 pairs of corresponding points.
- ▶ The corresponding points should be distributed in the image in such a way that can express the geometric transformation- usually they are spread uniformly.
- ▶ If it is approximated by a bilinear transform, then 4 pairs of corresponding points are sufficient to find the transformation coefficients
$$x' = a_0 + a_1 x + a_2 y + a_3 xy$$
$$y' = b_0 + b_1 x + b_2 y + b_3 xy$$
- ▶ Simpler one is Affine transformation for which one needs 3 pairs of corresponding points
$$x' = a_0 + a_1 x + a_2 y$$
$$y' = b_0 + b_1 x + b_2 y$$

This transformation includes typical geometric transformations such as rotation, translation, scaling and skewing

So, you can obtain these things. Now, if you see the original image and output image there are not much changes symmetry is maintaining so, that means, you do not need to fit a polynomial of larger degree right there is small polynomial degree is self sufficient. So, degree two or degree three is sufficient to estimate your x' and y' . If it is degree two or three I need basically I need to estimate this I need basically 2 or 3 or 4 6 or 10 corresponding points in the output image. So, if I have to make the bilinear transformation the x' comes $a_0 + a_1 x + a_2 y + a_3 xy$ and y' comes $b_0 + b_1 x + b_2 y + b_3 xy$ this is a bilinear equation simple formula. So, you have the four coefficients four coefficients, you have to estimate now the two estimate the four coefficients you need the four coordinates of the output image

If I know the four coordinates of the output image, if I know the coordinate of this input image I can estimate the coordinate, I can estimate the value of a_0 a_1 a_2 and a_3 along with this x' y' and so on. Am I right? See for this points, if I map to this point this point like the four coordinates, if I have corresponding mapping points, I can estimate a_0 a_1 a_2 and a_3 similarly b_0 b_1 b_2 b_3 there is a bilinear transformation.

If I use the and affine transformation this is your $a_0 + a_1 x + a_2 y$ and similarly y' equals to $b_0 + b_1 x + b_2 y$ what it means that you need to get the three pairs of **three pairs of** data to estimate a_0 a_1 a_2 and b_0 b_1 and b_2 . Now, this affine

transformations is very useful to make the corrections with respect to rotations translations scaling and skewing So, this is clear bilinear these are transformations we required to estimate, now one thing you remember that these transformations is uniformed say, in a one image for one part is rotated and other part will not be rotated that is not there. So, if I have rotated it, so it is across the image same fact will be there.

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Geometric Image Transformation

- ▶ A geometric transform applied to whole image may change the coordinate system and a Jacobian J provides information about how the coordinate system changes

$$J = \left| \frac{\partial(x', y')}{\partial(x, y)} \right| = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{vmatrix}$$

- ▶ If the transformation is singular (no inverse) then $J = 0$
- ▶ If the area of the image is invariant under the transformation then $J = 1$
- ▶ The Jacobian for the bilinear transform is

$$J = a_1 b_2 - a_2 b_1 + (a_1 b_3 - a_3 b_1)x + (a_3 b_2 - a_2 b_3)y$$
- ▶ The Jacobian for the Affine transform is

$$J = a_1 b_2 - a_2 b_1$$

Now, the term Jacobean we want to introduce the geometric transform applied to whole image will change the coordinate system because; obviously, you have the original image this one and this is the output image once you transform it. So, the coordinate system would be changed. And Jacobean is the parameter by which you can tell how the coordinate system changes. So, that is important. So, this is nothing, but J is nothing but determinate with respect to that whatever transformation you have taken is partial derivatives. Now, if there is no change there is a singular no inverse then J will become 0. If J is equals to one it indicates that it is invariant under the transformation.

It is invariant to say it is invariant to rotation that, J should be one that is important that to prove that it is invariant under certain of transformation, you have to tell, you have to show that J is equal to 1. Now, the I have already shown the formula of bilinear and if I take the derivative and then the determinant, you will find for the bilinear transformation Jacobean when the J equals to this while for the affine transformation you will get a $1 \ b \ 2$ minus $a \ 2 \ b \ 1$.

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Geometric Image Transformation

- > Some important geometric Transformations are
 - > Rotation by angle θ about the origin
 $x' = x \cos \theta + y \sin \theta$; $y' = -x \sin \theta + y \cos \theta$; $J = 1$
 - > Change of Scale a in the x axis and b in the y axis
 $x' = a x$; $y' = b y$; $J = ab$
 - > Translation
 $x' = x + c$; $y' = y + d$; $J = 1$
 - > Skewing by angle θ
 $x' = x + y \tan \theta$; $y' = y$; $J = 1$
Skewing is the act of making a rectangular image slanted. That is to make it fit into a parallelogram instead of a rectangle.

So, the some well known transformation geometric transformations, first one is that, I want to rotate the image by an angle theta. So, x dash equals to x cos theta plus y sin theta and y dash equals to minus x sin theta plus y cos theta. And if you see the determinant of the value of J becomes 1. Change in the scale x dash, I want to increase the scale a x and y dash equals to b y and if you see the determinant mean J it will becomes a b. Translation x dash equals to x plus c and y dash equals to y plus d and here also you will find J equals to 1 and skewing by an angle theta is x dash equal to x plus tenth theta y dash equals to y and J will become 1.