C-Based VLSI Design

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Module - 04
C-Based VLSI Design: Scheduling
Lecture - 12
Heuristic Scheduling: Forced-directed Scheduling

Welcome students, in today's class, we are going to learn about Force-directed Scheduling. So, this is one of the Heuristic Scheduling.

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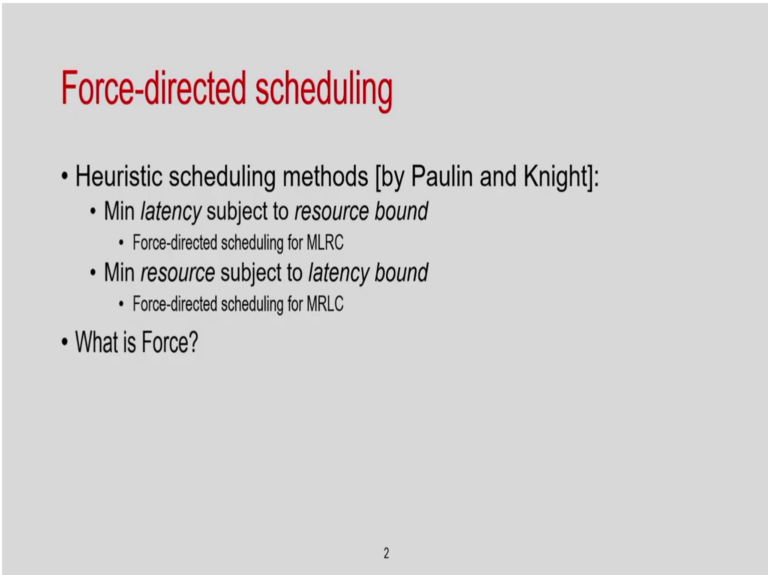

So, as we have learnt in the previous classes that, the scheduling problem is in general a NP complete problem. So, we usually go for a heuristic base solution, where we have an option to prioritize the operations right. So, there will be a priority among the operations and based on the priority, we are going to select operations in a particular time step.

So, and also we have discussed that list scheduling algorithm in the previous classes, where we have having one such priority we have understand, which is basically distance of the node from the sink node. So, the distance is in generic graph when there is may be multiple paths, I am going to consider the distance of the longest path. And we have seen that, we have can have both MLRC and MRLC problem formulation using list based scheduling.

So, in going in that directions, now we are going to think about another heuristic base solution is called force directed scheduling; here the priority function is different, which is basically force. So, that we are going to learn today. But this was proposed by Paulin and Knight way I mean maybe 20 years back and both this mean latency and this MLRC and MRLC can be formulate using this heuristic which is force. So, let us try to understand, what is force?

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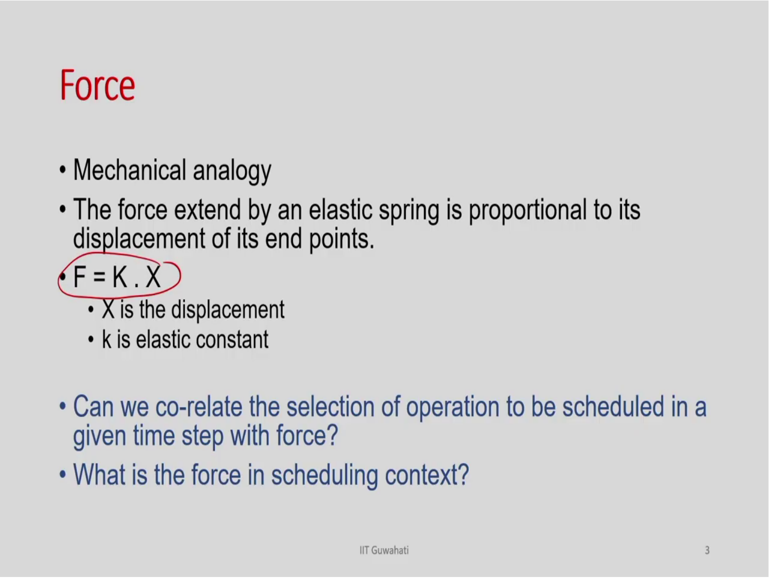

So, it is a basically mechanical analogy that you have a say elastic spring and you try to spread it. So, you try to extend it. So, once you extend it, what is the total force is generated? It is the displacement of two end points, whatever the displacement happened into some elastic constant, that is what is the overall force.

So, basically in this particular work, they try to correlate the scheduling problem with this force, this elastic force. So, let us try to understand how we can correlate little bit. Now, we will discuss this in subsequent part, but let us try to understand.

So, as we mentioned that we, what is the problem for us? We have given operations and we are just considering whether I am going to schedule this operation v i in time step l or not. So, that is the decision I am making.

So, now, think about this. So, what is going to happen that, if I going to schedule this operation v i at time step l; so you what is going to happen? So, it will increase some kind of concurrency in that particular time step; because if this particular operation is type K, so the I need at least one more of resource in that particular time step l.

So, it is basically increase the concurrency in time step l. So, that something is the you can think about the concurrency is the force in this context. So, it is actually increasing the forced.

It is not only that, you basically since this operation could have been scheduled within some time frame; that we are going to you know that, this is basically TLP versus TALAP to TSAP. So, that is the range where this operation is schedulable and if you schedule a particular time step in that range the other part it is not going to schedule. So, that means the concurrency is reduced for those state.

It is not only that, you think about that now you basically schedule this operation in time step l and it has some subsequent operation, it has some predecessor operations.

Now, the kind of operations the predecessor and successor operations, their schedulability also they might be impacted. Because for example, say one operation if you schedule in time step l and this time the next successor operation could have been also times schedule is time step l, because this operation could have been scale, if it is the previous operation is scheduling l minus 1.

So, now since I have scheduled this operation in time step l, I actually have an impact on the next operation that now this operation cannot be schedule in time step l, it has to be scheduled in time step l plus 1. So, it is concurrency for time step l is also gets impacted.

So, these are the kind of force it can generate once you schedule an operation in a time step. And if we just calculate this force formally, mathematically and I can actually utilize this force during my scheduling. So, that something is important.

So, let us try to understand how to calculate this force and then we will see how I can utilize this force in developing both MLRC and MRLC algorithms.

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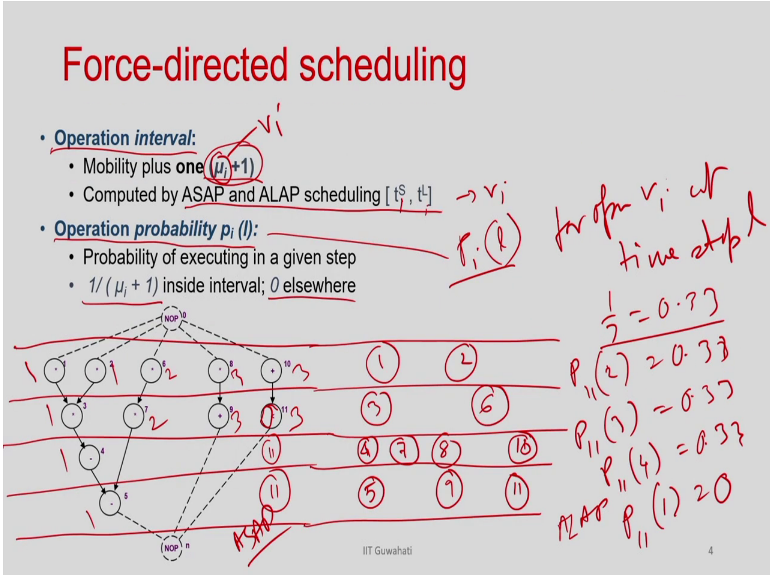

So, let us move on. So, first we will understand the operation interval, which is already we are aware of that; we know that given a latency bound lambda, I know the ASAP tells me the earliest possible time where the operation can be scheduled and the ALAP which is as last as late as possible scheduling algorithm will tell us that, what is the latest possible time step where this operation can schedule.

And if I say this the gap is mu for the operation i mu i; so mu i is for operation v I, then. So, the mobility is basically mu i plus 1. So, why? Because say one operation can schedule it 1 or 2. So, its ALAP is 1, a sorry ASAP is time step 1, ALAP is 2; so the difference is 2 minus 1 is 1, but it its mobility is 2 so that 2 minus 1 plus 1. So that is why this is mu i plus 1.

So, let us take this example the ASAP solution is this, we all know that. So, this is the ASAP schedule, this is the earliest possible time step; now you think about the ALAP schedule. So, let me just try to you, assume that all operations are single cycle and the lambda is 4, that the given time is 4. So, for ALAP I am going to put the ALAP quickly, which we already know. So, this is this 5, 9 and 11 can schedule here; 4, 8 and 10 will schedule here.

So, this is 4 and then this 4, 8 and 10 can schedule here and then sorry this 4, 7 also. So, 7 can also schedule here. So, this is this can schedule here and then 3 and 6 is schedule here ALAP and 1 and 2 is schedule here; this is the ALAP schedule and this is ASAP. So, we can find out the mobility for each operations, which is basically we can actually identify from the scheduling time t S to t L, which is the t S is the ASAP time and t L is the ALAP time and if I just put i here, this is for v i.

So, what is the mobility for this mobility plus 1 the operation interval for operation 1; it is basically 1, because it is schedule in the same time step, for this also 1, 3 is 1, 4 is 1, this also 1, because it is can only schedule in one time step. So, for this it can its schedule here 2 and this 4 here. So, 4 minus 2 it is 3, 4 minus 2 is 2 plus 1. So, this is also 3, this is also 3, this is also 3, because it is 1, 2, 3; 3 minus 1 is 2 plus 1 and this is 2, this is 2.

This is what the operation interval for these nodes. Now, try to understand that operation probability. So, let us try to understand this term, which is called p i of l; so that means this is for operation v i at time step l. So, basically the probability of scheduling operation v i at time step l. So, that something is given by p i l, very important to understand this term notation.

So, now you think about this operation say 11. So, 11 can schedule in time step 2 and it can schedule in time step 3 and it can schedule in time step 4 also; because it in ASAP its schedule in time step 2, ALAP schedule in time step 4. So, there is a possibility that this is the operation interval and it can schedule in any time step.

So, what is the probability of scheduling in time step 2? It is 1 by 3. So, this is basically 0.33; so that means p 11, operation 11 in time step 2 equal to 0.33. Similarly, probability of operation 11 in time step 3 also 0.33, operation 11 in time step 3 sorry, time step 4 again it is 0.33. And it cannot schedule in time step 1; so probability of operation 11 in time step 1 is 0.

So, this is how I can identify the probability of each operation in each time step; basically in the range of t S to t L, in that range it will be 1 by mu i plus 1, this is what I have done here and other cases it is 0. So, this is how I can do it. So, for each operation, each time step I can find out the probability value, understood. So, this something is clear.

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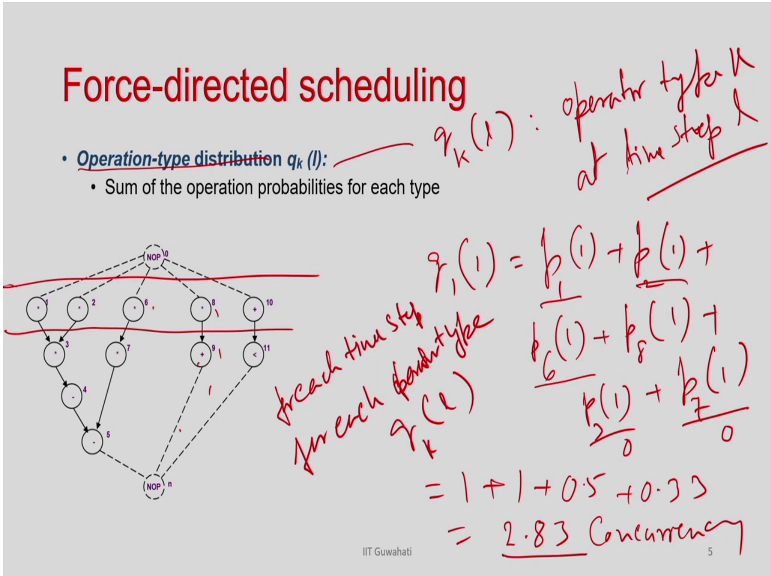

Now, if I just do this operation type distribution. So, now, say we I have different type of operator and say this is that q k l; that means it is for operator type k at time step, time step l. So, this q l k is saying that is, I already identified that what are the probability of each operation in each time step.

Now, some of the operation is multiplier and some of the operation is ALU type. So, it can have different type as well. So, now, I can actually can sum up the all the probability of that operation type k in time step l, then that will give me the q l k. So, for example, say I am try to find out the for time step 1 and for operator types say multiplier which is say 1.

So, q 1 in time step 1. So, what is the probability of operation 1 going to happen here? Because this is only can happen in time step 1, that we have already understand.

So, this is 1. So, basically it is probability of operation 1 in time step 1 plus probability of operation 2 in time step 1; because I have to take all the multiplier type operation. So, then probability of operation 6 in multiplier 1, then probability of 8 in time step 1. I should write for all multiplier; so there are two more multipliers are there, probability of 3 in operator time step 1 and probability of operation 3 in time step 1.

Out of that, 3 cannot schedule in time step 1, 7 cannot schedule in time step 1. So, this is 0 and this is 0. So, I can write that. So, p 1 is 1, p 2 1 is 1; because this operation can only schedule in time step, there is no probability it can schedule in others time step, because the ASAP and ALAP time step is same. So, that is the there is no kind of mobility for this operation, that we have already discussed many times.

So, now, this is 1, this is 1. So, this p 6 is how much? It can schedule here or here, because this can stretch down. So, probability here is 0.5, half 1 by 2; 7 is cannot schedule here, because 7 is already discussed that 3 and 7 is 0. So, and then probability of 8 is something, because it is 1 by 3; because this can schedule here, here and here, because 8 can come here also. So, because operation 9 can schedule here; so 8 can schedule here, here or here. So, there are three options. So, 1 by 3 which is basically 0.33.

So, what I understood? So, that operation type distribution or multiplier in time step 1 is 2.83. So, this basically gives you the concurrency; because this says that, we need kind of at most 2.83 number of multiplier which is kind of 3 multipliers in time step 1. So, it is roughly very abstract level.

So, this we can actually calculate for each time step for each type of operation. So, for each time step for each operator type. So, I can I have to calculate this q k l; because l will vary from 1 to n and k will vary for all type of 1 to k. So, this we can calculate and we have the operation type distribution. And this is kind of giving me the kind of concurrency of that type of operator in a time step. And that I have to do it for multiplier and adder or adder is basically ALU here. So, I have to identify this.

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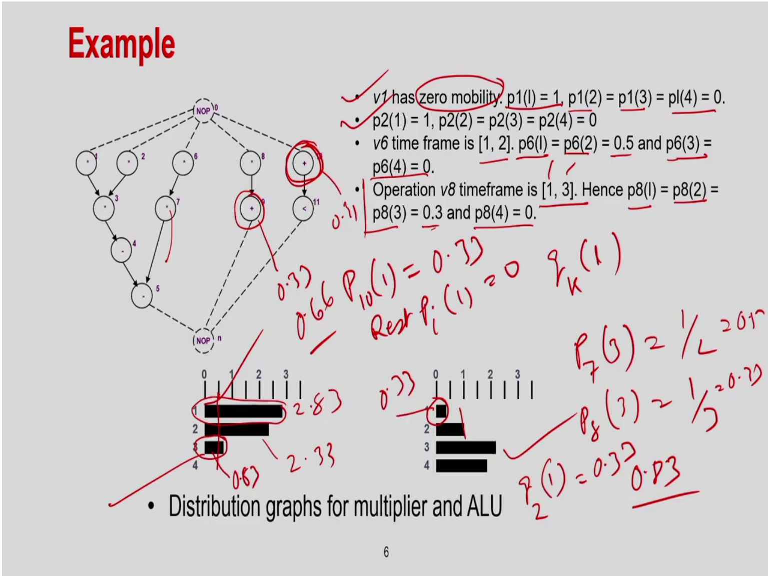

So, let me take this complete this example, because we already identified the probability of each operations in each time step. So, for example, if you take this operation v8, which can whose time frame is 1 to 3; this is the ASAP time and this is ALAP time, hence it is basically 1 by 3. So, it can schedule in time step 1, 2 or 3, probability is 0.3 and p 4 is 0. If you take v 6, which time frame is 1, 2. So, the probability of operation 6 in time step 1 or time step 2 is 0.5 and operation 6 in time step 3 and 4 it is 0, because it cannot schedule in time step 3 and 4.

And this I have already understood that p 1 is 1 and p 1 1 is 1 and p 1 2, p 3, p 1 3, p 4 is 0, because it cannot schedule in other time step, because it has a zero mobility, it has to schedule in time step 1; similarly for p 2. So, this way you can actually identify all the p value for each node in each time step. So, those is calculated and then I have, I told you that we can actually identify the q k l, which is the operator type distribution. So, it is just the summing of all the probability in each time step.

And I already calculate this for multiplied in time step 1, which is very 2.83; if you just do the same thing in time step 2, you get 2.33. And it is if you do it in time step 3; so time step 3 is basically it is I can calculate. So, for say in time step 3, so 3 cannot schedule here, only 7 can schedule here.

So, which is basically, in time step 3, 7 can schedule so this is basically p 7 in time step 3 which is 1 by 2 and 6 cannot schedule, 8 can schedule, because this 9 can schedule in time step 4. So, 8 can schedule, so p 8 of time step 3 is 1 by 3. So, which is basically 0.5, 0.33, this will be 0.83. So, this is basically you can see, this is 0.83. So, I can actual and for 4 it is 0, because none of the multiplier can schedule in time step 4.

Similarly, I can actually do for addition, because say addition this operation can schedule here, here or here. So, only this operation can schedule in time step 1. So, that is why it is basically 0.33, because p 10 of 1 is 0.33 and rest are 0; rest p i of 1 is 0. So, these are all for all operations are 0; so that means this is q 2 operators type 2 in time step 1 is 0.33.

I hope you understand this, because this is just say the what are the operations can be possible to schedule in time step 1 of type k. So, this is 0.33 that we can understand; in time step 2, this operation can schedule and this operation also can schedule. So, this is basically, this is again 0.33; because it can schedule in any time step and this is also 0.33, it is basically 0.66, that is what I got here.

So, this way I can actually calculate the value for this q i k operate pi operator type distribution, this operation type distribution for each type of operation in time step and this is given by this; you can try yourself also.

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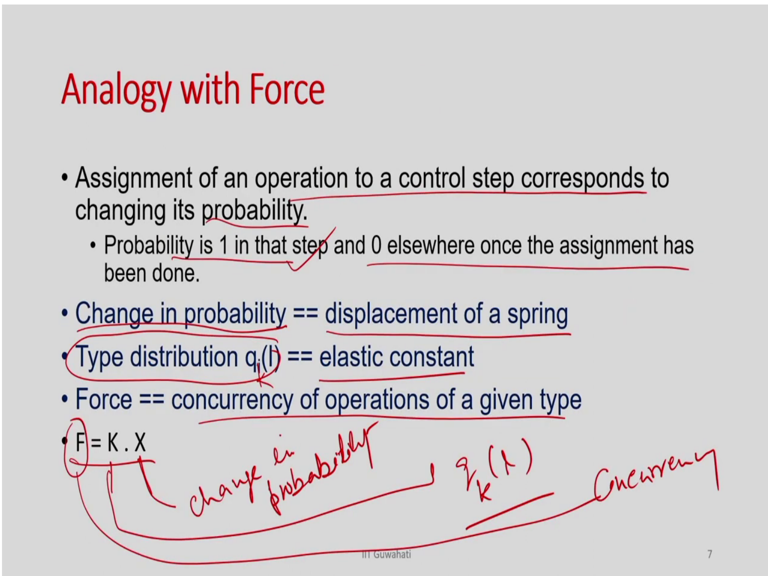

So, basically what I have learnt so far that, we can actually identify the operations type distribution. We can actually identify the maximum kind of possibility of concurrency possible in each time step, using the value that we have already calculated.

So, now what is the analogy with the force? Because we are actually try to develop a force directed scheduling in this class. So now, you remember that force was K into X; K was some elastic constant is the displacement value. So now, in our context of scheduling what is X, what is K and what is F let us try to understand that.

So, basically as we have understood that, because of the mobility an operation has a probability equal probability of executing the in the time frame. For example, I have seen that v 10 can schedule in 2, 3 or I said 1, 2 or 3. So, it has probability 0.33 in this all time step equal probability.

Now, let us suppose that you already schedule this operation in time step 1. So, earlier it was 0.33 in time step 1 and 0.33 in time step 2, 0.33 in time step 3; now we schedule it time step 1. So, what is going to happen? It probability become 1 in time step 1, probability become 0 in time step 2, probability become 3 in time step 0, probability become 0 in time step 3 also. So, that is the change in probability. So that is, what is the displacement in context of scheduling.

So, it has equal probability in 3 time step, I schedule in a specific time step; so probability will increase in that particular time step and probability becomes 0 in other time step. So that is, what is the elasticity here. So, that the kind of sorry displacement here. So, assignment of operation to a control style corresponding to changing the probability. So, probability become 1's, where it get schedule and 0 for other places.

So, change in probability is the displacement of the spring. So, this is my change in probability. Now, what is k? Because you already have calculated sum this operator type distribution, so that is something you already identified this q i l, q k l for each type.

So, that mean it is already, because of the uniform distributions of the operations; it is sorry according to their operation distribution, I have already calculated the type distribution. And which it was saying that in a time step what was the kind of concurrency you already can achieve, at max can achieve.

Now, what is going to happen? Because you now move this operation here and there, so it will impact that value. So, here that this of type distribution q k l is the elastic constant. So, this is basically the type distribution, q k l. So, that is the type distribution.

So, I am going to multiply the change in probability with the this type distribution; because that is going to be get impacted and that is the constant value we have already obtained by distributing that operation equal probably in all the time step, where it can a schedule and then, so basically that become my elastic constant in this equation.

So, I am just trying to correlate the value that I have already calculated with this terminology. And what is the force? I have already talked to that force is nothing but the concurrency. So, the force is if you have a value high; that means a higher concurrency. So, for example, here it is 2.83 have a higher concurrency here, 0.83 here the less concurrency, it is very obvious. So, concurrency defines the force. So, this is basically concurrency in our context.

So, in scheduling constant, F is the concurrency, the number of operation can be parallelly executed in a time step, so that is my F. K is the type distribution; type distribution was basically you understand, that is basically the kind of operations at max what is the kind of maximum concurrency can be achieved in a time step, and that is what we have calculated.

And this X is the displacement in mechanical context and here it is the change in priority, because of scheduling operation in a particular time step.

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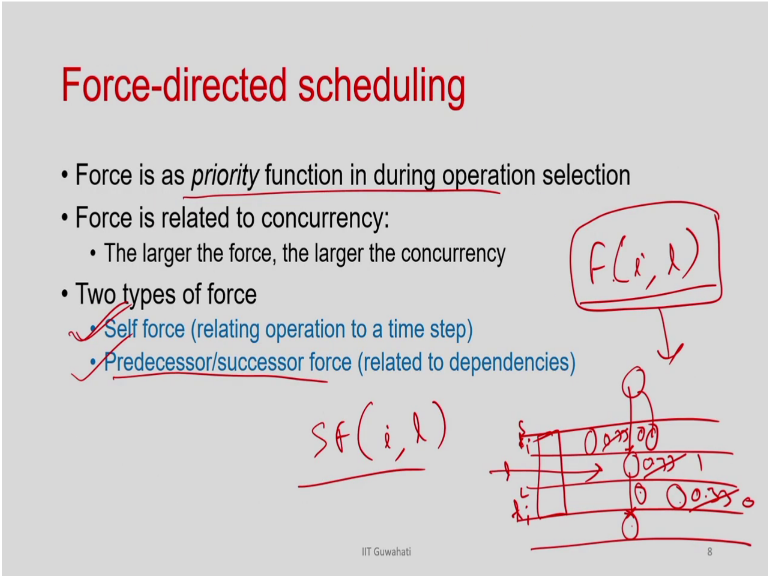

So, this we have understood. Now let us try to develop the algorithm. So, what I am going to do is that, we are going to calculate the force for each operation, in each time step. We are going to calculate that force and I am going to use that force as the priority function. So, if the force is high that means it will actually increase the concurrency in a particular time step and if the force is low, that means in a particular time step, it will decrease the concurrency if I schedule that operation in time step.

You can actually think about the force of operation i or say you can write this way that force of operation i in time step l. Because we are talking about whether I am going to schedule this operation v i at time step l or not. So, I am going to if; so what I am going to do?

If I decided to schedule this operation i in time step l, what is my total force. So, if the force is very high; that means it is it actually increase the concurrency in time step l; if I schedule this operation v i in time step l and if the force is negative or less, that means it will decrease the concurrency in the time step.

That is the kind of analogy or the priority function that I am going to consider and using this priority I am going to schedule this. So, I am going to come to this actual algorithm later part of this discussion. So, let us identify what is this force now. So, far I have not talked about the force. We just calculate these things; I know that the force is concurrency, but how to calculate? So, how to calculate from this q i k and this p i whatever the value I have already obtained so far how can I calculate this force? Let us try to understand.

So, there are two type of forces, one is basically the self-force and another is the predecessor, successor force. So, the self-force is something if we mention that so self-force, you can think about this operation v i in time step l. So, you have this say this is your time step l, and this operation i could have been scheduled here, here or here any places.

So, it has the say the time frame was 3; the t S, this is t S, i and this is t L i. Now, if you schedule this operation here, and so you can understand if I assume it is 3. So, earlier the probability was 0.33, it was 0.33 here and 0.33 here. Now, you try to schedule this operation here. So, here probability become 0, here probability become 1, here probability become 0, that is the idea.

So, now, if we just do this. So, this is basically the displacement. So, earlier it was 3.33, 0.33, 0.33 now it is 0, 1 and 0. So, this is the kind of displacement I can if I think about the mechanical analogy; and then if we do this what is the impact, that is given by the self-force.

And now if you just schedule this operation here and there is a successor operation of this node; now this successor operation if I was decided to schedule this here, it cannot schedule here. But if I schedule this operation v i here, the successor can schedule here as well. So, the impact; so once you actually calculate, I mean decided the time step for operation i, it will impact the time step for the successor or predecessor node as well, because there may be some predecessor node also, that will also can now flexible it can come here.

So, that is something we have to calculate and which is given by the predecessor and successor force. So, let us try to understand how to calculate the self-force and predecessor force.

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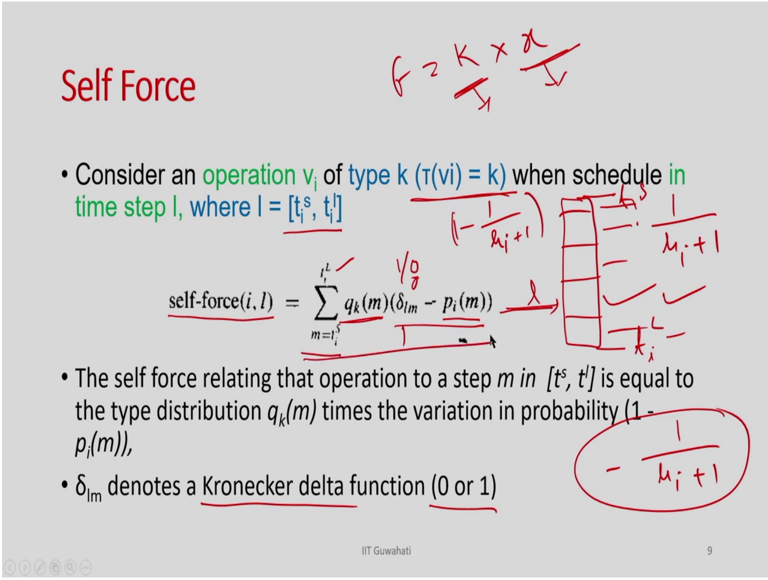

So, if you I mean always recall that your force is basically K into x, x is the displacement and k is the elastic constant. And we already know that this is my type distribution and this is the displacement now. So, let us try to find out this value now.

So, now, as I mentioned that I want to calculate the self-force for operation v i in time step l. So, basically the operations could have been scheduled anywhere between t to s, that I have already discussed; that it was basically the time frame, this is my time frame t i S and this is the time frame t i L and this it can schedule anywhere. And I try to schedule this operation and this is my time step l.

So, what is going to happen? So, I am going to, because it could have been; so basically it the probability of this operations was 1 by mu plus 1 in every time step, where it could have been scheduled and that was given by your p m. So, p i of in time step l, p i l.

So, now I am going to calculate this self-force for all nodes; because these are the all time step it is going to be impacted as I shown in the previous slide that; if I schedule it here, so it will become 0.33 to 1, and it is become 0.33 to 0.

So, I have to calculate the force for all time step. That is why I am doing a summation from t i S to t i L basically in this time frame range. What I am going to do? I am going to multiply this elasticity constant with the displacement. So, what I am going to do? This is my, that q k m; it is the type distribution, operator type distribution in time step m of type k. I am assuming this the type of operation v i is k, this is the operation type k. And I am going to the multiply with the change in probability.

So, here this delta l m is call Kronecker delta function, which is can take value 0 or 1. So, it is basically you can understand that if this particular; so earlier the probability is equal in all time step, it is basically 1 by mu i plus 1. So, it is 1 by 3 or 1 by 2, 1 by 4 that we have understood; if there are range is 4 it is 1 by 4 and so on.

So, earlier the probability was 1 by mu i plus 1. Now, if I schedule this operation in a particular time step m; then this will become 1. So, it will increase the probability; earlier it is 1 by 3, now it is become 1, so it is basically change in probabilities 1 by 1 minus, it is the positive increment. So, 1 by mu i plus 1.

So, then I am going to put 1 here and for all other state while it does not schedule it becomes 0. So, earlier it was 1 by mu i plus 1; it will decrease by this number. So, it is decrease by this number. So, I am going to put 0 here. So, that this become minus 1 by mu plus 1, understood.

So, you can understand clearly that this is the self-force; because it was having equal priority in this all time frame. I am calculating that if I try to schedule this operation in time step l, what is the overall impact it will impact in all those time frames, concurrency in all those time frames, it will increase the concurrency in this time step and it is going to decrease the time concurrency in this time step. I am going to sum all this impact.

So, whether it is impact, it is impact positively here, it will impact negatively here. But I will take this summation of all this impact and then that summation is my self-force, I think it is understood.

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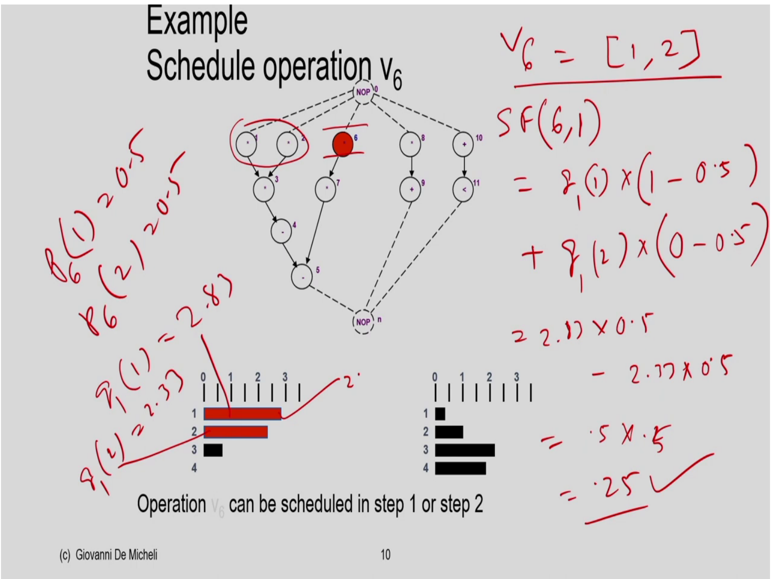

So, let me take the example and explain. So, say for example, this operation 6. So, operation 6 cell it can schedule in time step 1 and 2; because you know that the time frame is 6 is time frame is 1 to 2, operation v6 it can schedule in time step 1 or 2.

So, now let us. So, I also know that the this is a multiplier type operation, the type is multiplier and this is the q i that q 1 of 1 is 2.83, this value and these value is q 1 of 2 operator type 1 in time step 2, which is 2.33. And you know that this p 1 6, sorry this is in other way. So, that p 6 operation 1 in time step 1 is 0.5 and p 6 2 is 0.5, it can schedule in this time step.

So, if I decide to schedule this operation in time step 1. So, that means I am not scheduling it in time step 2. So, what is my total self-force? So, that means self-force of operation 6 in time step 1. Which will be this q 1 1 into; because I have scheduling it here, so I am going to put 1 here into 0.5 plus q 1 2, that means operator type 1 in time step 2 into, this is 0 now because I am not going to schedule in time step 2 minus 0.5.

So, what I did, I just put this value into this equation. So, if I just do this and I can calculate. So, it is basically 2.83 into 0.5 minus 2.33 into 0.5. And I will get some value, it will be a positive curve value, I think it will become, it is basically 0.5 into 0.5. So, it is basically 0.25. So, this is a positive value.

So, basically the conclusion is that, if I although this operation 6 could have been schedule in time step 1 or 2; what we what we will get is, basically if I schedule it in time step 1, it will increase the overall concurrency. Because you understand that it will, because it was already 2 these two operations are scheduled here; if I put it here, so it will increase the concurrency in time step 1, but overall concurrency will be increased. So, this is how I am going to calculate the self-force.

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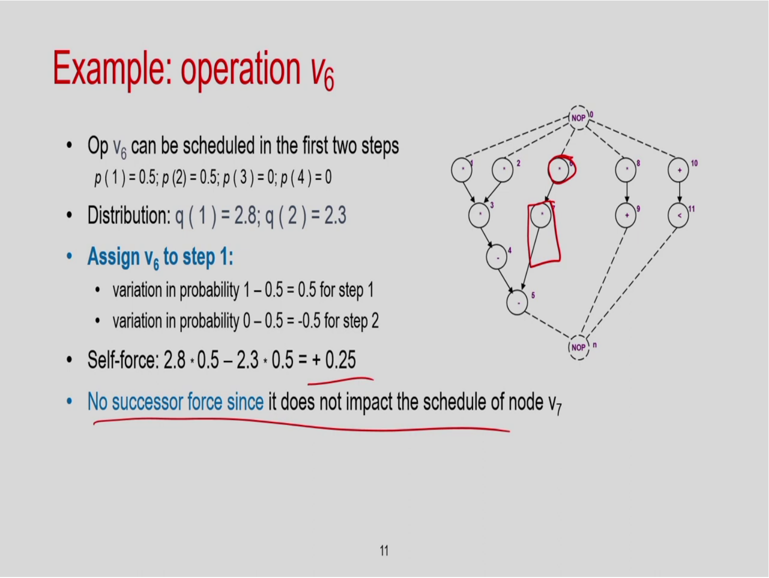

So, this example was already here. So, I have already calculated. So, the value was 0.25. So, this is understood.

And here because if we schedule this operation in time step 6 so it does not impact the scheduling of node 7, because 7 could have been scheduled here or here it does not impacted. So, there is no successor force here, because it would not impact the schedule of operation 7 or any predecessor or successor node. So, there is no successor force.

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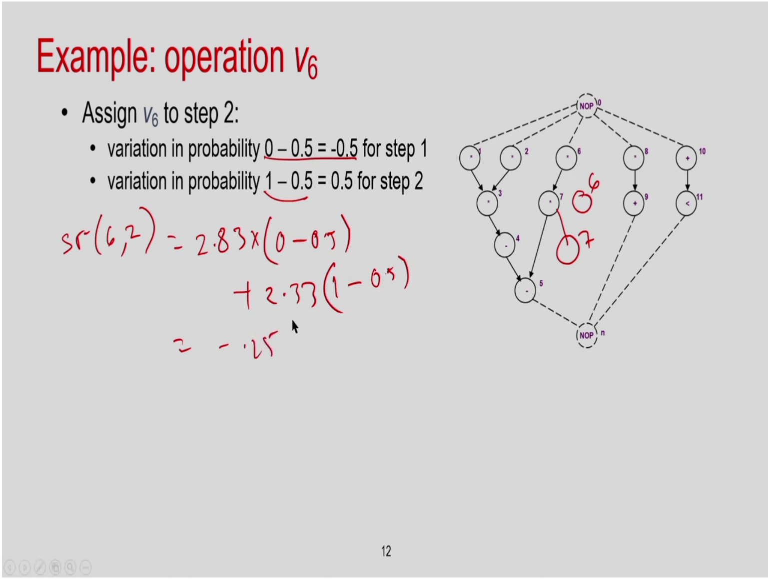

So, now let us understand what is; now if you try to schedule the operation in time step 2. So, if you try to schedule this operation in time step 2 operation 6 in time step 2; what will happen? So, this will come here and 7 will come here. So, this is 6 and this is 7. So, that means let us try to calculate the self-force first.

So, change in probability in time step 1 is negative, because earlier it was. So, if I just write this self-force of operation 6 in time step 2, which is basically 2.83, which is the q 1 1 into 0 minus 0.5, because it is not scheduled here.

So, this is minus 0.5 and plus 2.33; here it is now scheduled, so earlier it was 0.5, so 1 minus 1 is see, if it is scheduled here, it will become one that Kronecker delta function minus 0.5. So, if you just do this, you will get minus 0.5. So, that is, that I do this, so I will get minus 2.5. So, that something is understood.

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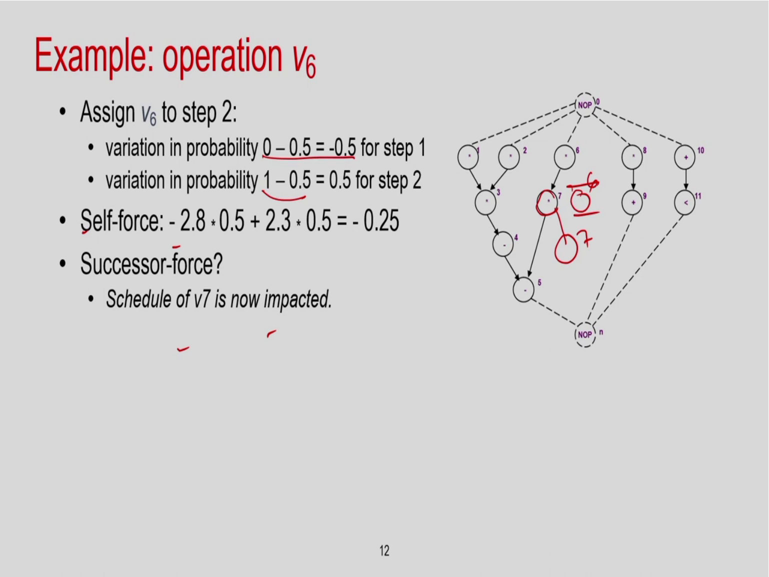

But you can understand now that, if I schedule this operation 7 a 6 in time step 2, now operation 7 is now impacted. So, now operation cannot be scheduled in time step 2, because now because this operation is depending on 6 and it is going to execute here.

So, earlier the operation in general if the 7 has a possibility of scheduling in 2 time step, now it is 1 time step, so it is get impacted and it will create some successor force. So, now, I will see how we can calculate the impact of 7 of scheduling of operation 6 in time step 2.

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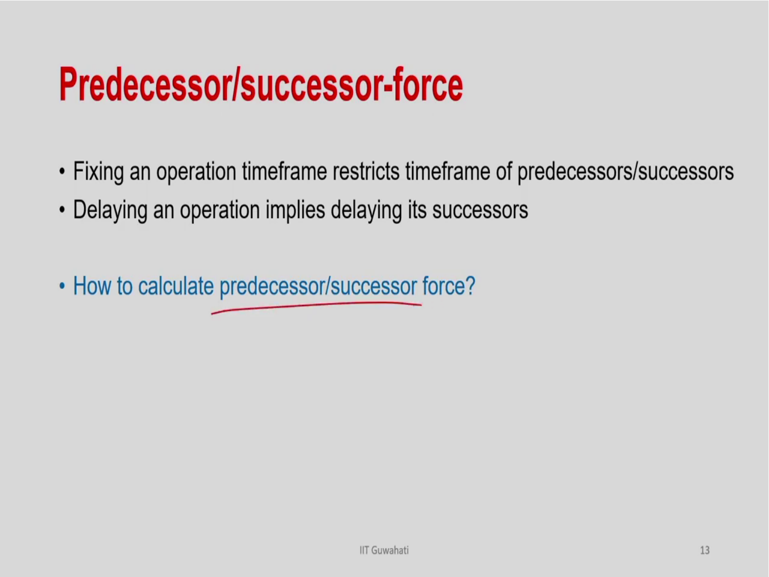

So, how to calculate that? Something let us try to understand.

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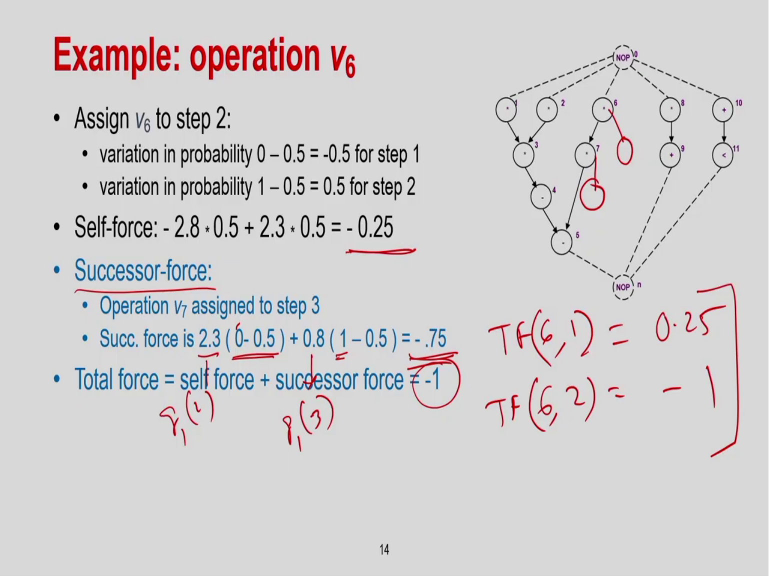

So, we have already understood that the self-force is minus 2.5, if I schedule this operation here. And what is the impact of 7? So, 7 now cannot schedule in time step 2 that is the bottom line. So, what we can do now? So, successor force is that; so this point 2.3 is the q operation in time step 2, the type distribution of operator time in time step 2.

So, now this 7 cannot schedule in time step 1, the time step 2. So, that is why it is 0. Now, but it can schedule in time, it must schedule in time step 3, because it is going to schedule here now. And the type distribution that q 1 3 is 0.8, that we have already calculated.

So, this is the self-force; if I just you calculate this equation you will get point minus 0.75. So, that overall force is what the self-force plus successor force, which is minus 1.

So, what you understood; if I calculate, I mean if I schedule this operation in time step 6, then my total so, S F total force of operation 6 in time step 1 is 0.25; because it does not have any successor force, because it does not impact 7. But the total force if I have schedule the operation 2, in time step 2, it is minus 1, because it is minus 2.5 plus minus 0.75, it is minus 1.

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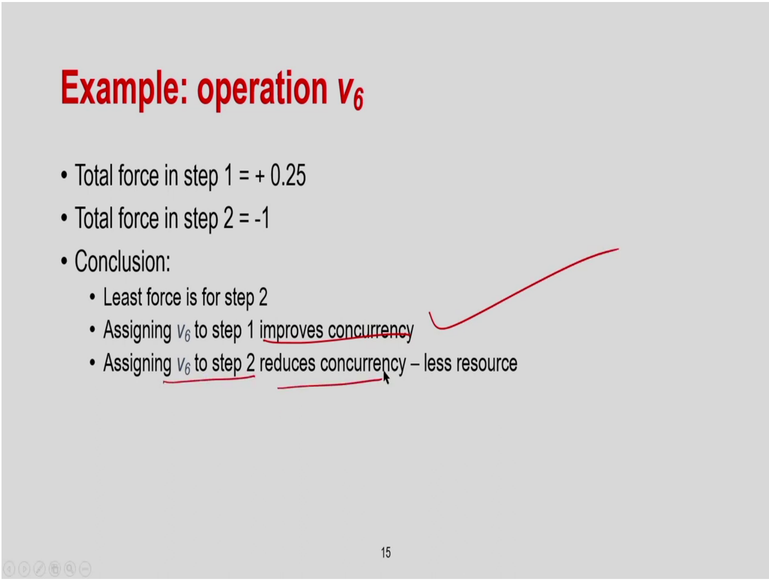

So, what does it mean that, if I schedule this operation in time step 1; it will improve the concurrency and if I schedule this operation in 2, it is reduce the concurrency. That is what we can understand from this total force. And that is, what is the impact of certain operation to be scheduled in certain time step.

So, let us now try to understand how we can actually calculate the successor force for all cases. Because here you can understand that it is not, whenever you try to schedule an operation in a particular time step, it is going to impact all the successors and all the predecessors. So, I have to calculate that things for all successor and all predecessor and I have to sum it up with the successor the self-force.

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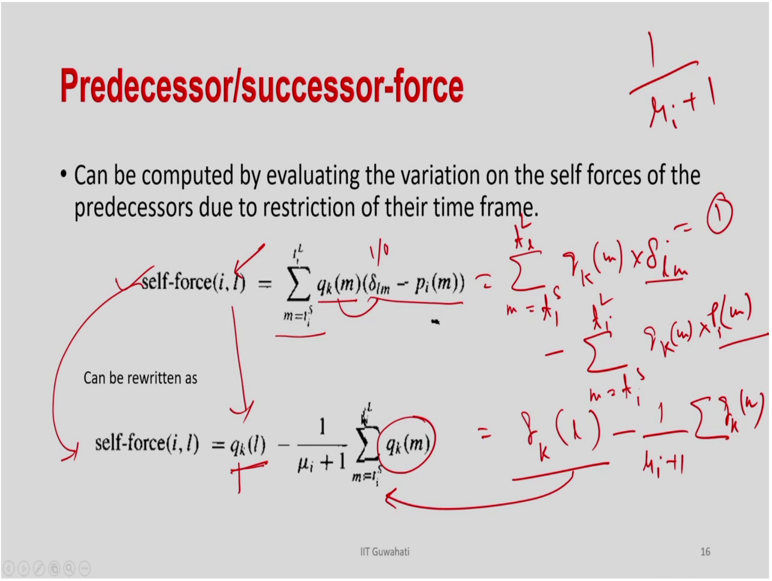

And how I can calculate, let us try to understand. So, what I am going to do is, this the self-force; we already know that self-force is basically the summation from t i S to t i L, where I am going to multiple this type distribution with the change in probability and the probability will be either 1 or 0 minus the actual probability. So, that I have already understood.

So, what I can do? I can rewrite this equation as this, let us see how. So, if I just do this. So, it is basically summation of m equal to t i S to t i L, q k m into delta l m minus summation of m t i S to t i L. So, I am going to multiply these two and these two. So, q k m into p i m.

So, what is going to happen? So, if we know that delta m is 1 for 1, where if it is for L, where it is getting actually getting scheduled. And for all other cases, so I am actually talking about the time step l. So, delta i m is true for only this in time step l and all cases it is 0. So, I can if I just multiply this. So, it is basically turned out to be that q k l; because in l time step, it will become 1 and other time it is 0. So, the summation will be 0.

So, this is nothing but q k l. So, that is what I got here; minus and you know this p i m is 1 by mu i plus 1 because it is the probability in every time frame. So, if I just rewrite this term now it is basically 1 by mu i plus 1, then summation into q k m. So, that is what I got this equation. So, what I can do? I can rewrite my self-force by this equation; let us try to understand this.

So, if you can understand here that this is basically the type distribution in time step l for type k, minus probability into the type distribution in other time step. This is what the self-force.

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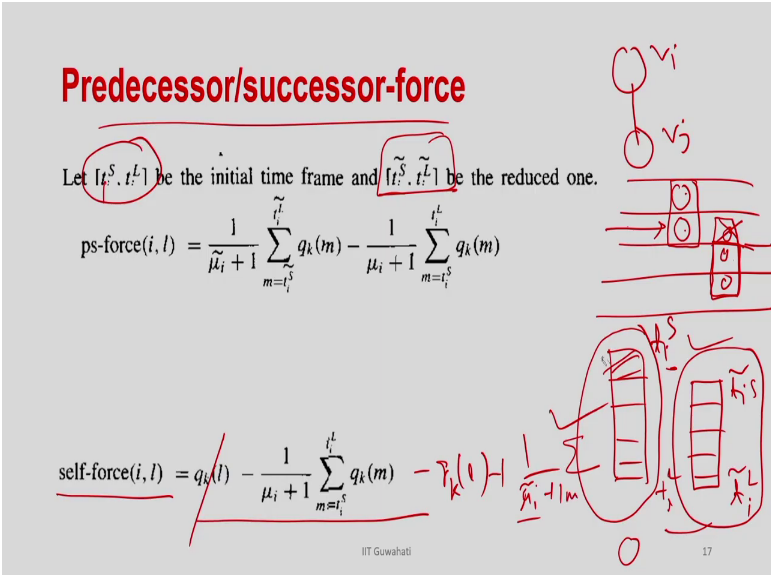

So now, let us try to understand, what is the predecessor or successor force? So, initially as I mentioned, suppose I have this operation v i can schedule here and here and the successor node, so there is an edge from this to this. So, this is my v i and this is say v j. And say this time frame of v j is this v i and time frame of v j is this right; because so this is my for v j. Because if I schedule this operation here, I can schedule it here; but if I schedule it here, I cannot schedule it here, but I can schedule it here. So, this is the time frame.

So now, if you schedule this operation v i here; it is impacting here, it will not going to schedule it here, now the revised time frame become this. So, that is what is given by this.

So, initially the time frame for a node is t i S to t i L. So, it was from t i L and this is t i S; this is the range where you can schedule. Now, because I schedule the previous operation, in some steps probably I cannot say I can schedule it here say, now this is actually modified to this. This is my t i L tilde and this is my t i S tilde.

And similarly it can get impacted if there is a successor node is impacted. So, this t L can also get impacted. So, what is going to calculate is that, the self-force the that predecessor or successor force is that; what was the actual self-force when the range was t S to t L and what is the range when I actually terminated to t i S tilde to t i L bar. So, this is the revise frame.

So, if I just subtract this revised one so or say change in basically this minus this. So, if I just revise this, so earlier it was this value, it was this value and now this is value, this is change is this. So, this minus this. So, if I just put this in this equation t i L and t i S and then next time you put t i L 1, t y 1. So, then if you do a minus this, this two term will be. So, I will just write down here. So, this is for t i L and minus q k l. And so, this will become plus now, because there is a bracket here.

So, this will become mu i plus 1 and then this is 1 by mu plus 1, sorry 1 by mu i plus 1. So, here now the mu also changes right, this will become mu bar.

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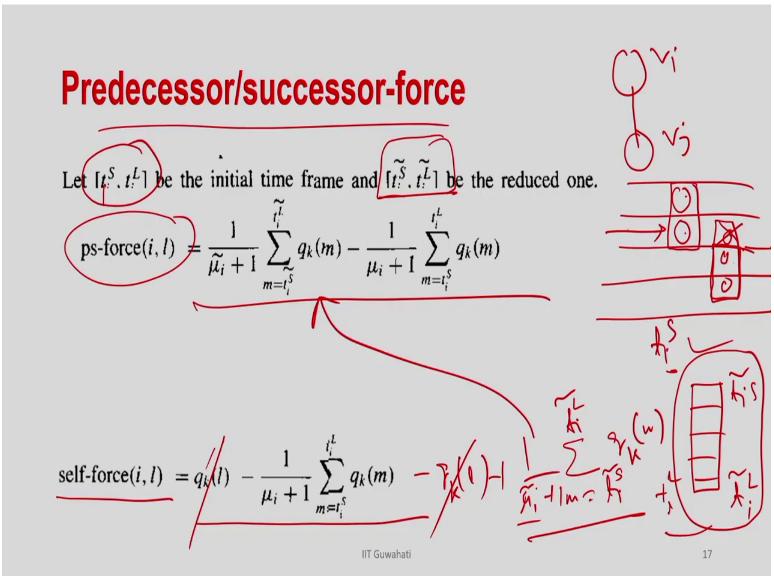

And then this become m equal to t i S tilde to t i L tilde; because this is the revised one. If I just do this, what will be going to happen? This and this will be separate out. So, this minus this. So, this is what I got here.

So, basically once I schedule a operation in say it is some node in say time step l; it will impact the time frame of the successor node as well as the predecessor node. So, for all successor or predecessor what I am going to do; I will find calculate the self-force when it was could have been done in any time step and it is in the revised time frame. So, this is the revised time frame the tilde and their difference is the predecessor or successor force.

So, basically I am now impacting the overall scalability of the predecessor node. Similarly I can also do the for successor node. So, this I can calculate.

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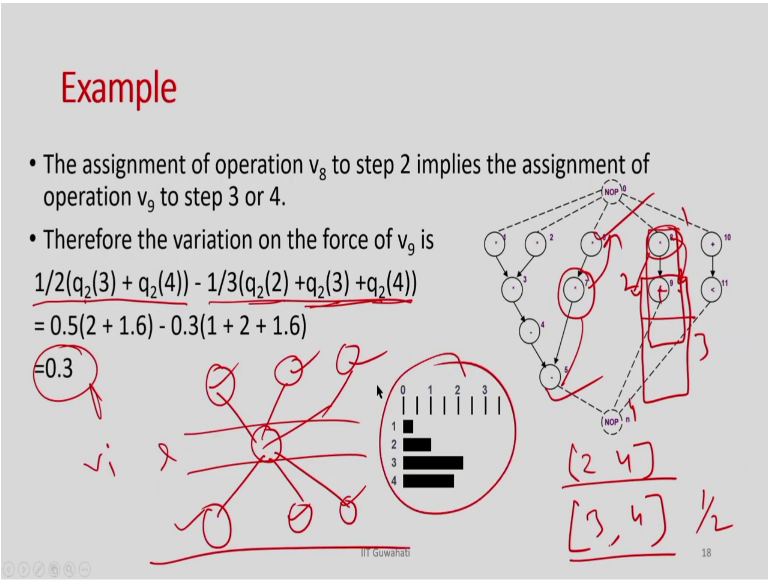

So, let me just give an example say, suppose I schedule this v8 in time step 2. So, you understand that v8 time frame was 1, 2, 3. So, 1 to 3. And v9 was the time frame is 1, 2 to 4; because it can schedule in time step 2, 3 and 4. Since I schedule this v8 in time step 2, now v9 can only schedule from 3 to 4. So, it is revised time frame become. So, it was earlier 2 to 4, now it will become 3 to 4.

What I can do? I can just find out what is the self-force once it is 2 to 4 and what is the force when a self-force when I calculate, it is basically 3 to 4. And I can just make the separate out the difference and that is the change in force. So, that is what I am done.

So, for v9. So, this is the revised force 1 by 2 because it is now it can only schedule in 3 or 4. So, there is a probability is 1 by 2 into q 2 3; because it is on time step 3 or 4 of type 2, because this is an operator type plus, this is operator type plus and it is q 2 4, 3 or 4. And earlier it could have been scheduling 2, 3 or 4. So, it is q 2 2 q 2 3 and q 2 4, 1 by that.

So, if I just do this, I will get 0.3. So, what does it mean? So, that if I schedule this operation 8 in time step 2; that predecessor force due to this schedule is 0.3, because this is can only impact this node. So, if I now think about this node. So, it will impact this node and this node. So, you have to calculate the self this predecessor or successor force for this node as well as this node and then you have to sum it up with the self-force for v7.

I hope you understand this. So, what I have to do it? I have to do it for whenever I schedule a operation v i in time step l, v i in time step l it will impact. So, suppose this is impacted. So, there are same, there may be 2, 3 predecessors of this may be 2, 3 successors. So, I have to calculate this force that predecessor or successor force for all these six nodes. All the nodes that can be impacted by the schedule of this node.

And I have to add that particular force with the self-force of this node and then only I can actually identify the total force. And if the total force is high; that means it will if I schedule this operation v i in time step l, it will increase the concurrency and if it is low, that mean it will decrease the concurrency. I think you, it is clear to you now.

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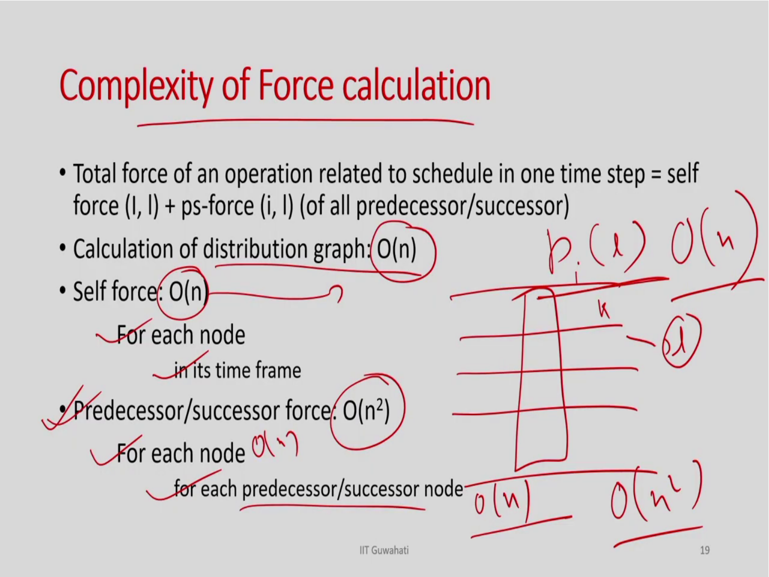

And now I will talk about the little bit of complexities use here. So, what is the overall complexity? So, what is the calculation of the distribution graph? So, distribution graph is this. So, for each node, what is the value for this? So, this can be calculated. So, I have to identify the priority of each node i in each time step. So, which is basically of order of n; if I assume there are n nodes, n into l. And since this l is basically is a time frame which is fixed, I can actually assume this is order of n.

And then I have to sum it up. So, I have to just sum it up this for each type k, in each time step l. So, this is basically the calculation of the distribution graph will be order of n. And what about the self-force? Self-force is order of n, because it does not impacted the other predecessor or successor node. So, if I just I have to just; how do I have calculated the; for all the time frame? I have to just multiply the type distribution with the probability.

So, this will be order of n, because you have to calculate for each node; for each node for each, it is time frame and time frame is a very fixed constant value. But this predecessor or successor for node is an order of n square, because this is has to done for all node.

So, for each successor or predecessor node. So, this is order of n, this is order of n; because whenever I take a node, I have to check what how many nodes are the predecessor and how many node are successor and for each of them I have to calculate this.

So, for every node, I have to search the whole graph again to identify the predecessor and successor node and then you have to calculate their force, so it will be basically order of n square. So, this is the high level, no need to think too much about this; but this is a bit basically complex to calculate this.

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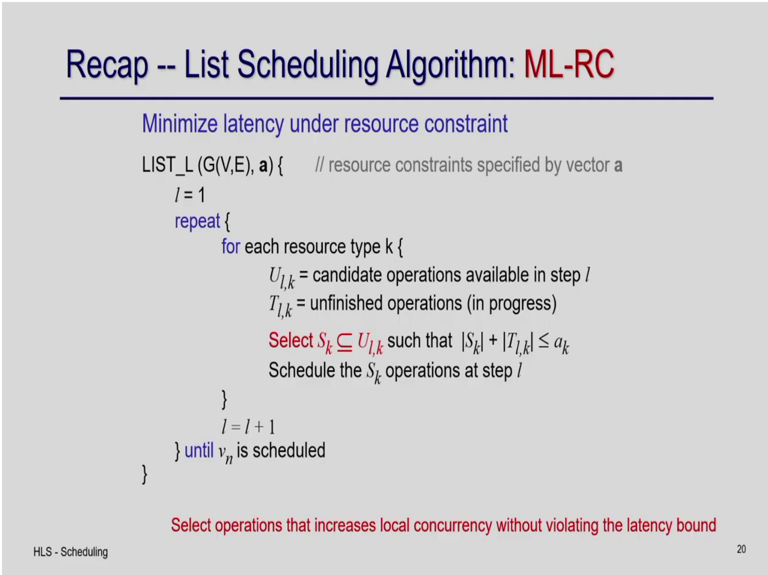

So, far we have. So now, if you just to recap, we have understood what is the force and what is the how it going to be calculated. So, in conclusion we have understood what is force and how to calculate force for each node. And in summary the force is basically is the kind of concurrency for the, it is a concurrency of the operations when I am going to schedule an operation v i in a particular time step and how it is going to impact the concurrency in the subsequent nodes.

So, that is something we have calculated. And in the next class, I am going to see how I can utilize this force in developing the scheduling algorithm both MLRC and MRLC.

Thank you.