Discrete Mathematics
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Lec 08
Set Theory

Welcome to the NPTEL book on discrete mathematics. This is the first lecture on set theory. Set theory deals with sets.

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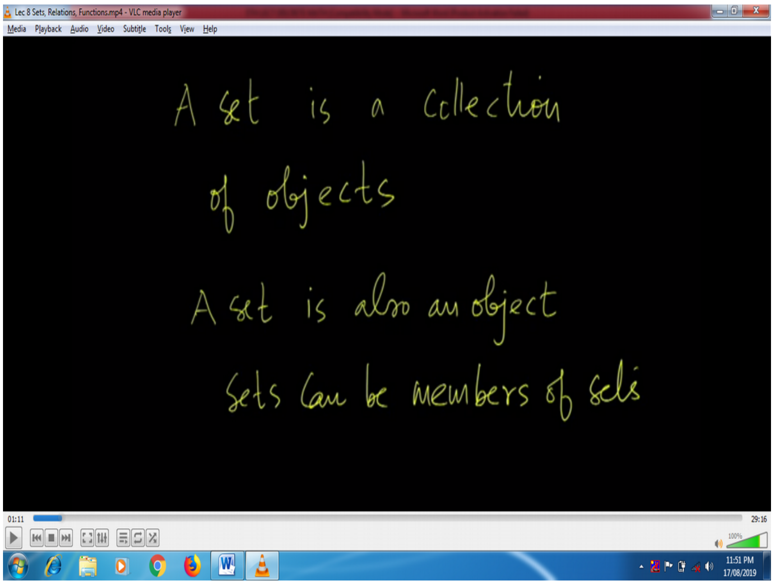

A set is collection of objects. In that sense a set is also an object. Therefore, a set can be a member of another set.

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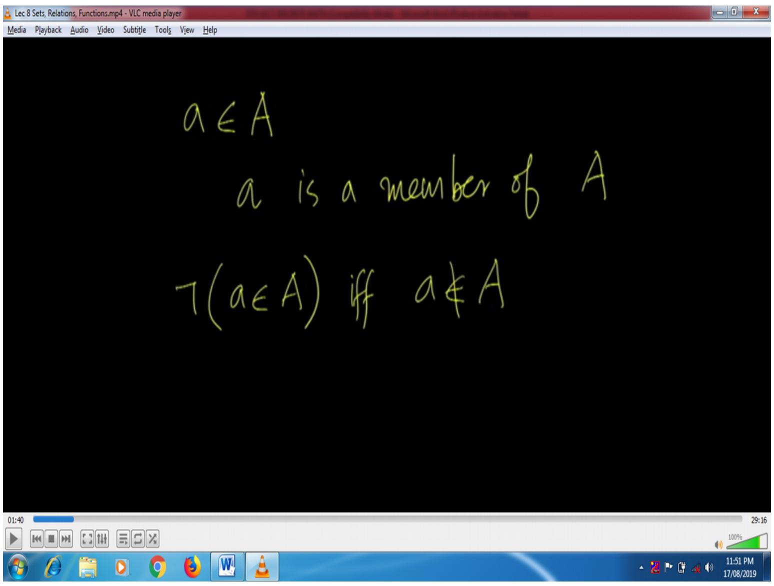

When we write a belongs to A what we mean is that small a is a member of set A. The negation of the statement which says that A is not member of a is written like this.

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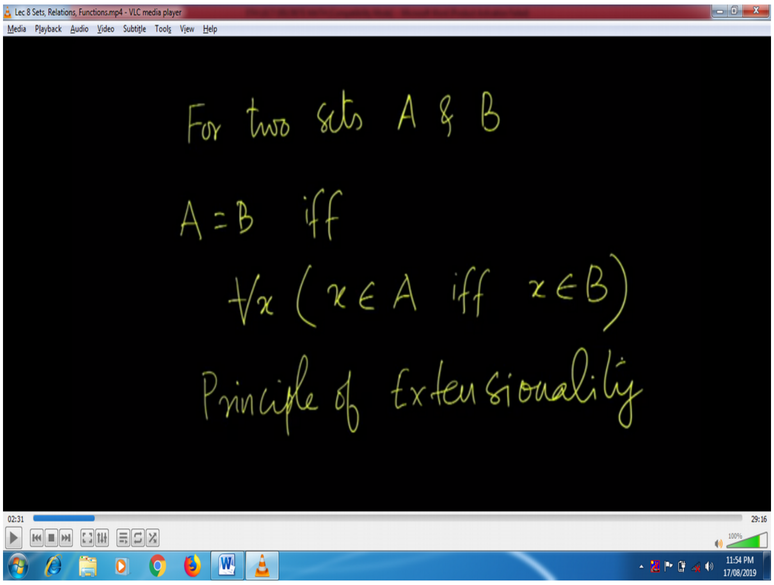

For two sets A and we say that A is equal to B if and only if for every x x belongs to A if and only if x belongs to B In other words two sets are equal precisely when they have exactly the same members. This is called Principle of extensionality. Two sets are equal precisely when they have same extensions.

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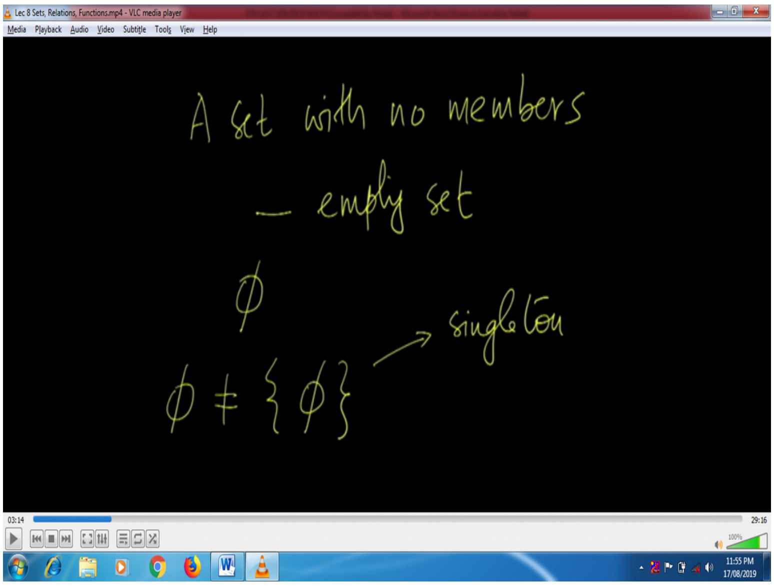

It also states that sets are defined by memberships. There is a set with no members this is the empty set. Empty set is denoted by fi. Note that fi is not the same as the set containing fi A set with exactly one member is called a singleton set. So here fi is not the same as a singleton containing fi because fi has no member it's an empty set. The singleton containing fi has one member namely fi itself. So these two are not the same.

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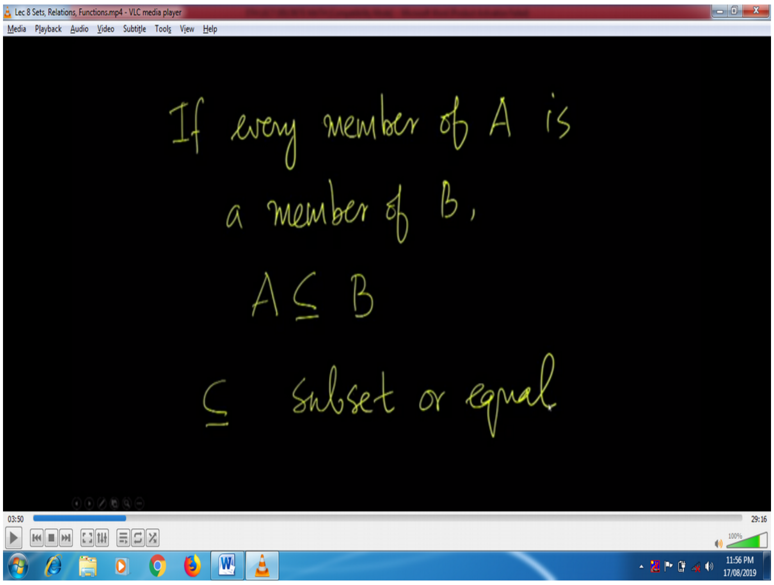

If every member of A is member of B as well for sets A and B A is subset of B Thus symbol says A is a subset or equal to B.

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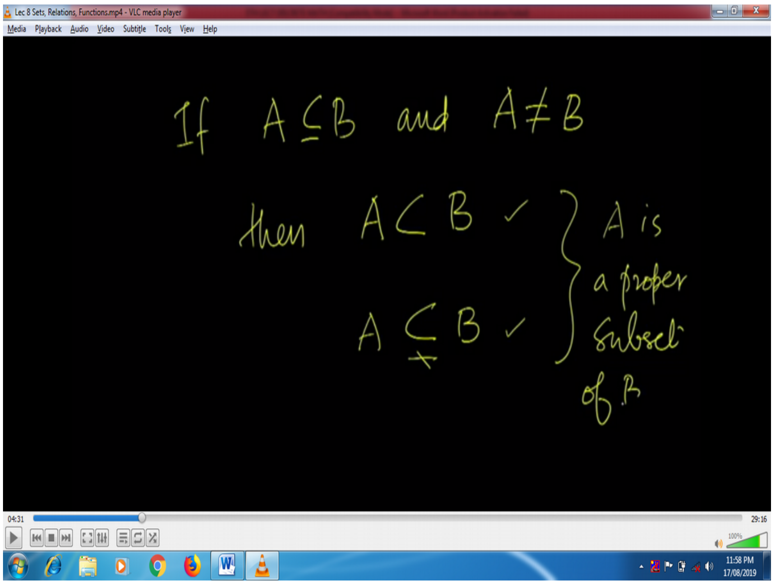

If A is a subset of B and A is not equal to B then we say that A is a proper subset of B and it is denoted like this either like this or sometimes right like this. To indicate A is subset of B but A is not equal to B. So these two notations you will be used you interchangeably. Both mean to say that A is a proper subset of B. Now let us define some operations on sets.

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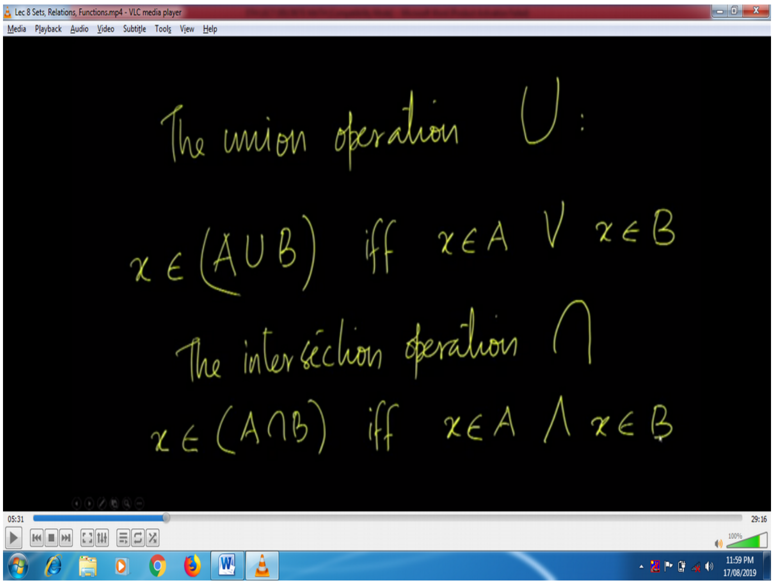

The union operation denoted using this symbol is defined like this. x belongs to A union B If and only if x belongs to A or x belongs to B. Another operation is the intersection operation. This expressed using this symbol we say that x belongs to A intersection B if and only if x belongs to A and x belongs to B.

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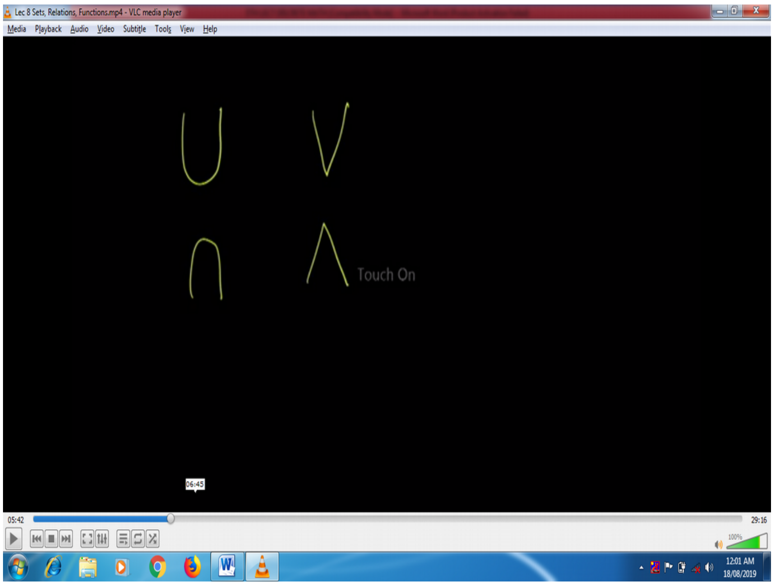

Note it is correlation between the union symbol and the OR symbol the intersection symbol and the AND symbol. x belongs to A union B if and only if x belongs to A OR x belongs to B x belongs to A intersection B if and only if x belongs to A and x belongs to B

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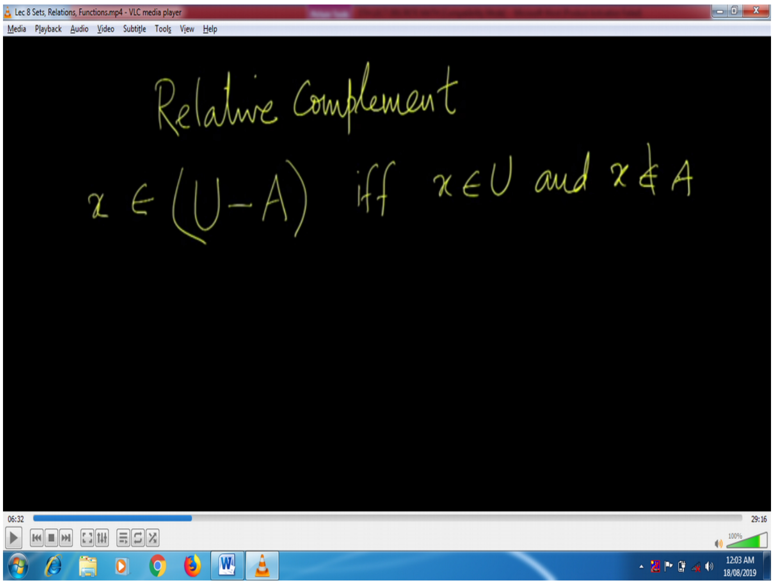

The third operation is relative complement. Complement of A with respect to a universal set A is defined like this x belongs to the complement of A with respect to U if and only if x belongs to U and X does not belong to A.

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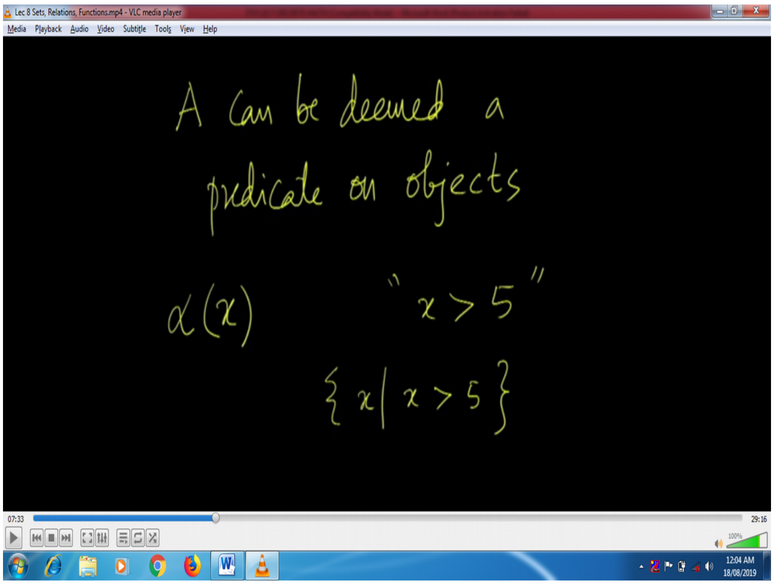

A set A can be deemed predicate on objects. For example consider first order of predicate or any logical predicate with one free variable. So this statement is essentially a statement about one unknown entity x is the unknown entity. For example, it could be a statement of this form x greater than 5 there is one free variable in this. So, you can assume that this is actually a definition of a state it is talking about all individuals that are greater than 5. We write like this. So you saying every one variable predicate we can construct a set. In another words set can be equator to a predicate on objects.

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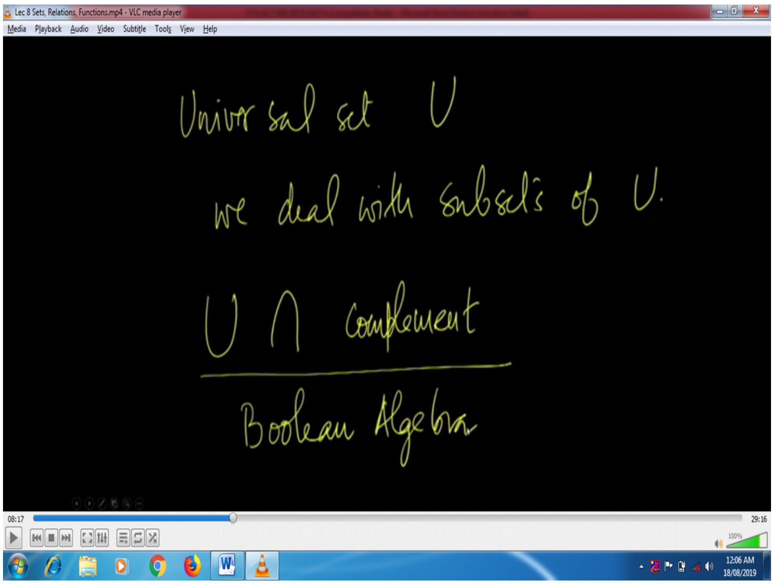

So let us say that we have a universal set U and we deal with subsets of U and we have these three operations Union intersection and the relative complement. The algebra defines by these three operations you can see you in fact Boolean algebra which we have already seen in the module on mathematical logic.

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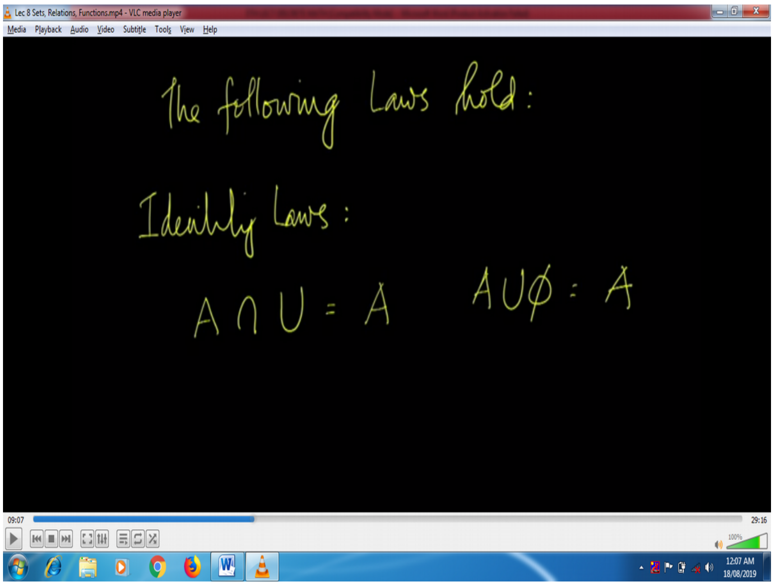

Therefore, the following laws hold by virtue of this algebra being a Boolean algebra. The following laws hold we have seen that analogues of these laws in the context of propositional Calculus the first one is the identity law. Which states that A intersection U the universal set is A and A union fi is A. Which means U is the identity of intersection and the empty set is the identity of union. Then we have domination laws.

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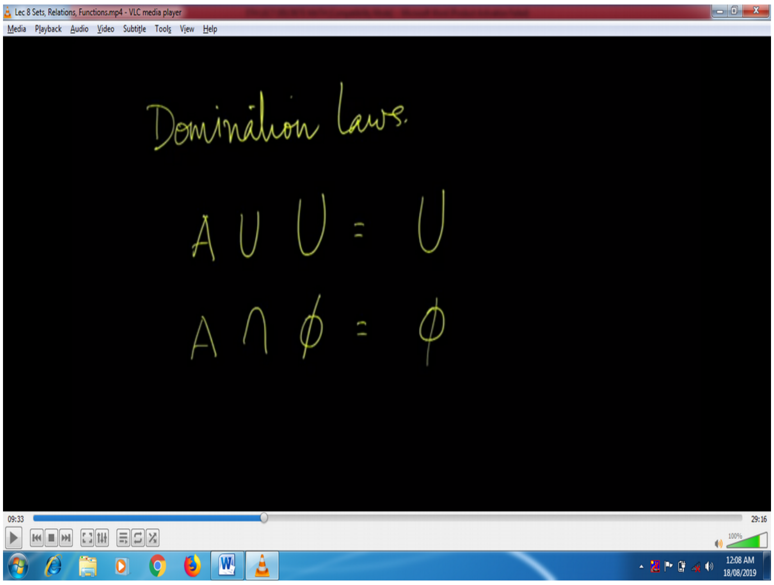

Domination laws says that the universal set U dominates the operation of union that's because A union U is U for any A and the empty set dominates the operation of intersection A intersection fi is fi for any A.

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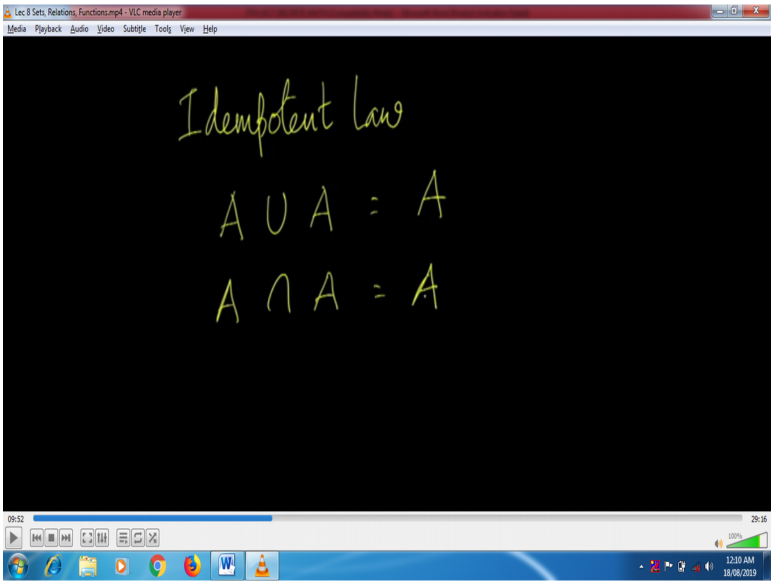

Then we have idempotent law one each for union and intersection A union A is A and A intersection A is A.

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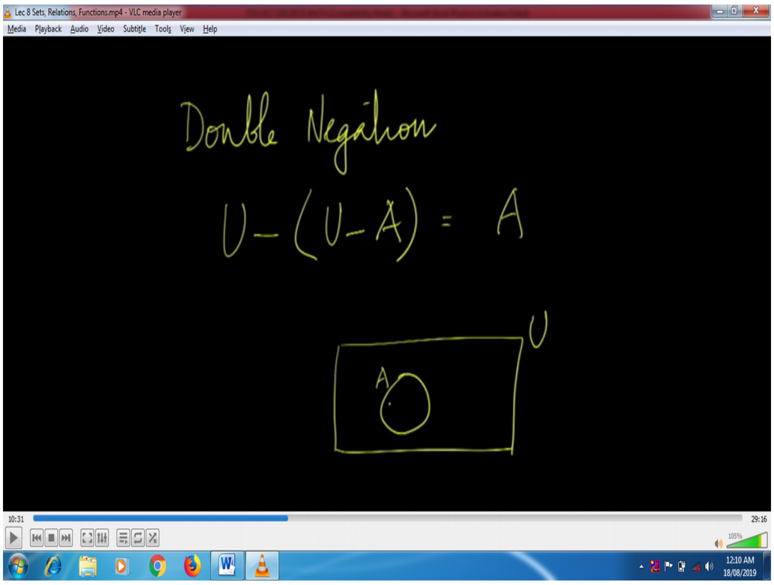

Then we have the double negation law which complement of the complement does the original set U minus U minus A the relative complement of the relative complement of A is A itself .you can draw a Venn diagram to verify this. If the rectangle represents universal set U and the circle represents the set A. U minus A is the elements which are outside of the circle the complement of the elements which are on the outside the set formed by the element which are on the outside of the circle will be the elements which are inside the circle.

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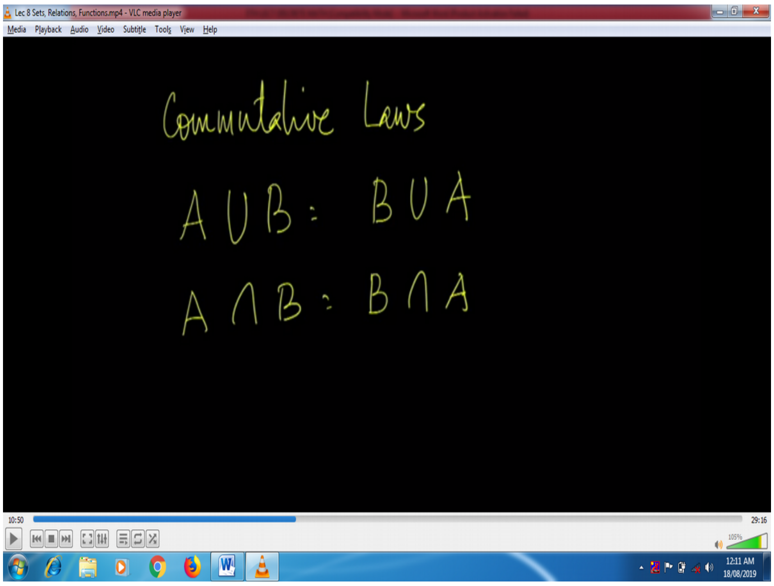

Then commutative laws it says the both the operations union and intersection or commutative A union B is same as the B union A and A intersection B is same as B intersection A.

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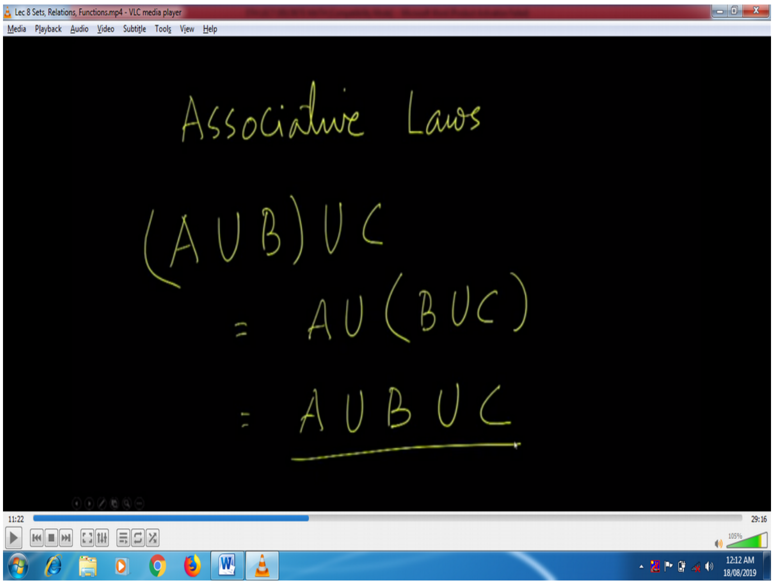

Then we have associative laws. It says both the operations Union and intersection are associated. You can apply them any in any order when you have a sequence of unions A union B union C is the same as A union B union C. Therefore we can avoid the parenthesis and write a sequence of unions in this manner and this can be extended to any number of sets.

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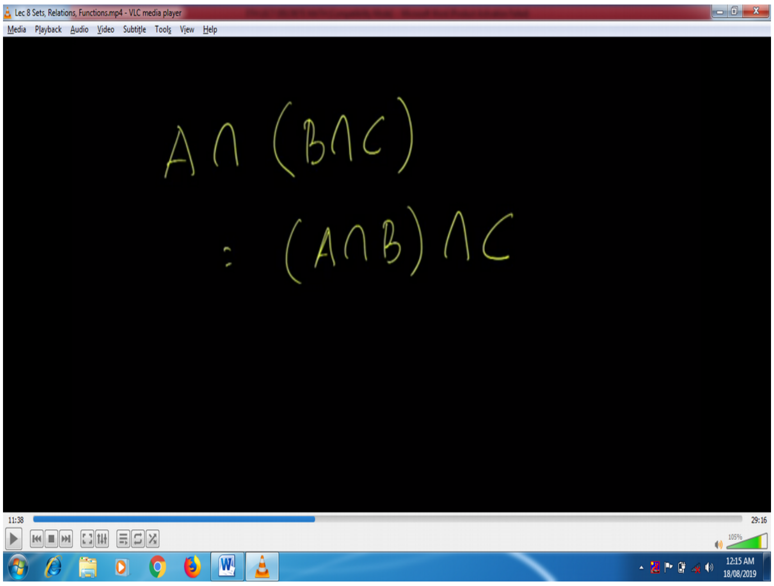

Similarly, for intersection as well A intersection B intersection C is the same as A intersection B intersection C.

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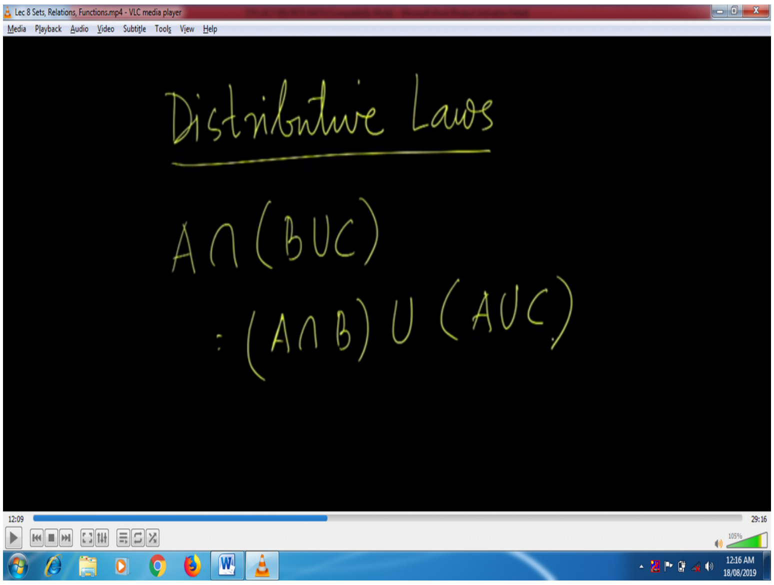

Then we have distributive laws it says that intersection distributes over Union and union distributes over intersection. A intersection B union C is the same as A intersection B union A intersection C.

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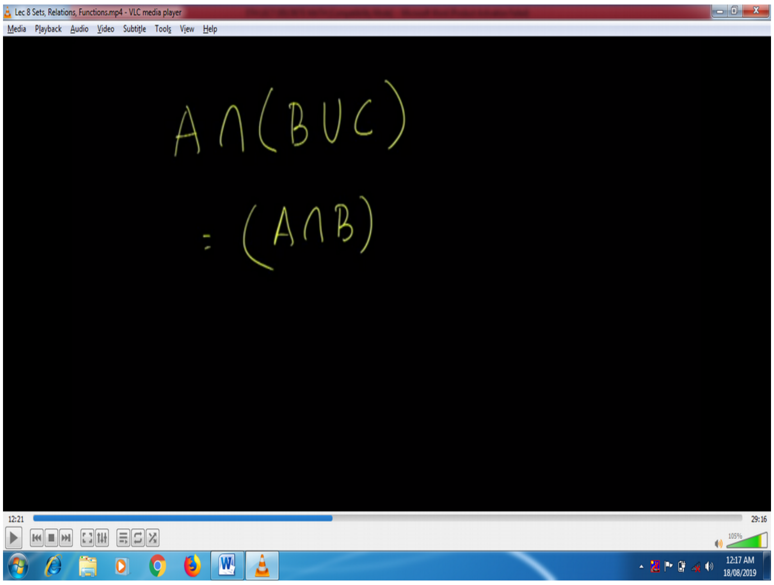

Similarly, intersection distributes over union as well A intersection B union C is A intersection B.

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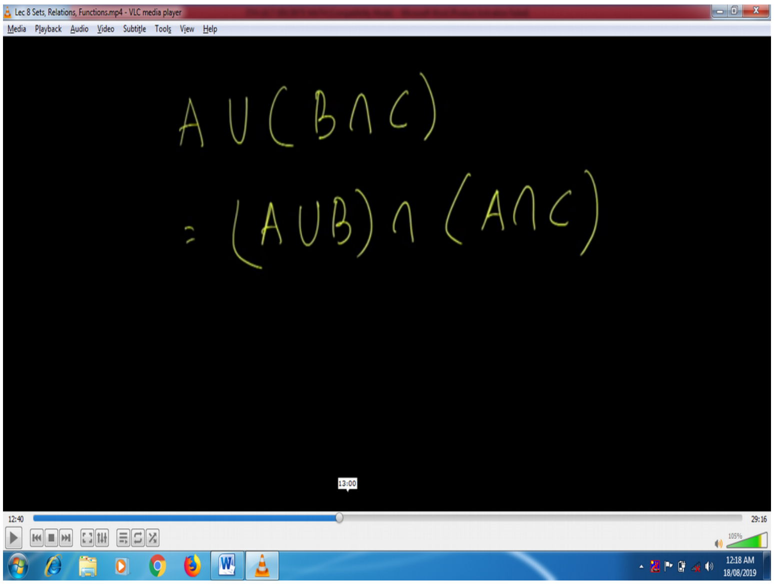

Similarly, union distributes over intersection as well A union B intersection C is A union B intersection A union C.

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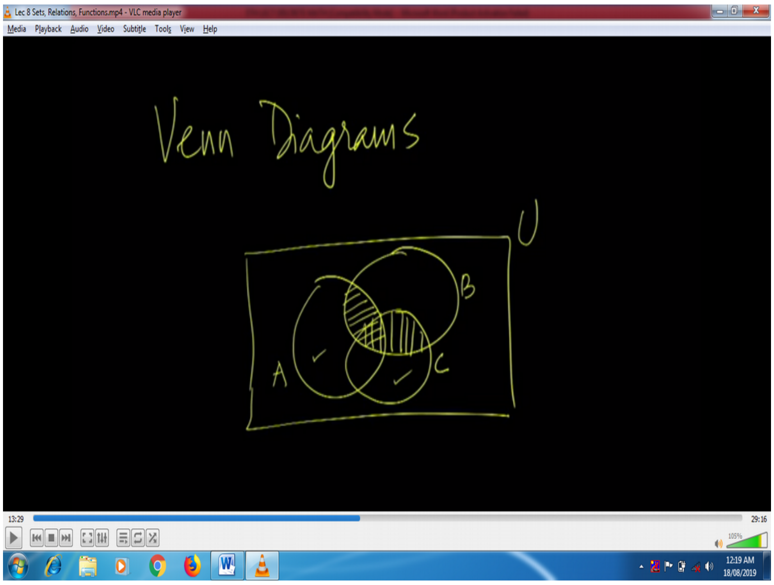

You can verify this using Venn diagrams. You would have studied Venn diagrams in school on. In Venn diagram you prop singly universal set using a rectangle and then sets inside can be represented using convex figures circles typically. So you have sets ABC with possible intersections between them then you can talk about A intersection B B intersection C etc. A intersection B in this case would be this portion and B intersection C would be this part. A intersection C would be the common parts between circles and C.

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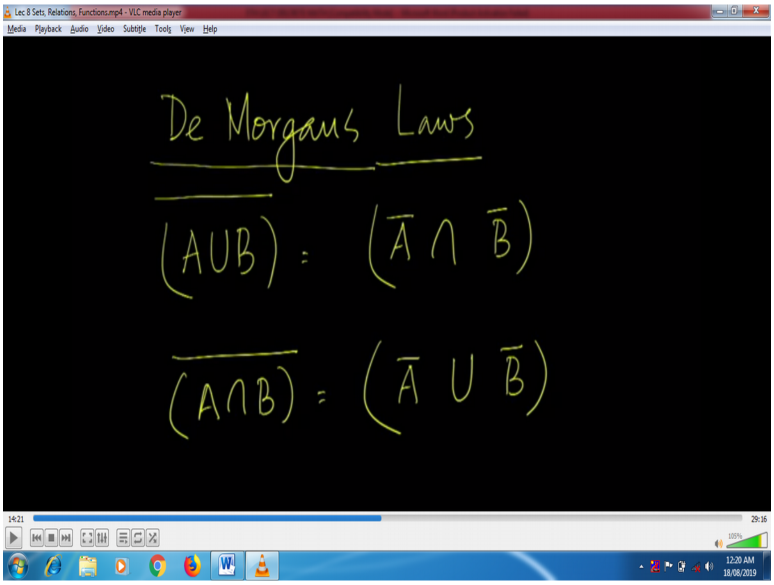

So using Venn diagram you can verify the correctness of all these laws. Then we have De Morgan's laws in this Boolean algebra as well. The corresponding De Morgan's law would be this. The compliment of A union B. Let me denote the relative compliment in this fashion the relative compliment of A intersection complement of B. Similarly, A intersection B compliment would be A intersection union A complement Union B complement. Weather complement is relative complement. So these are the De Morgans laws of set theory.

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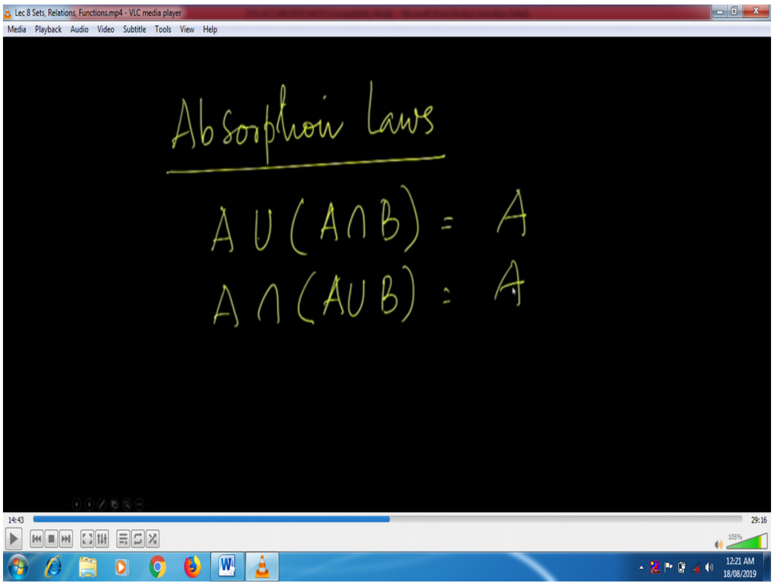

Then we have absorption laws. Absorption laws say that A union A intersection B is A and A intersection A union B is also A.

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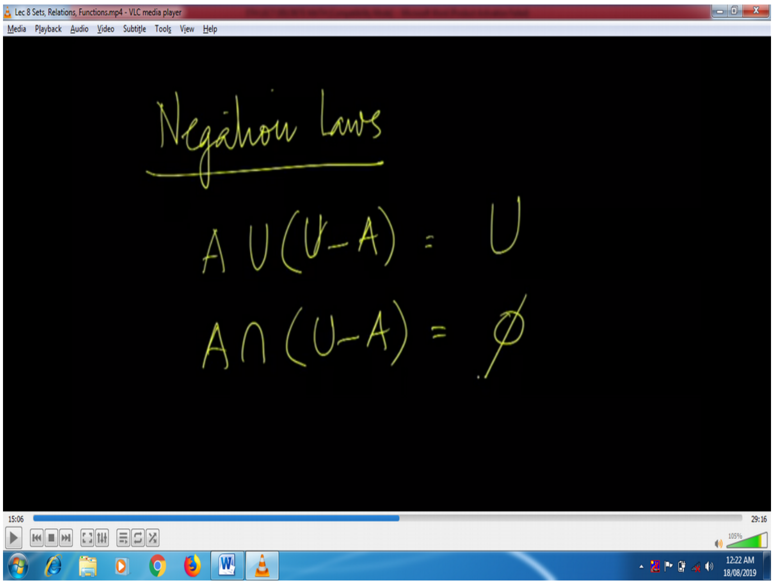

And finally we have negation laws. A union the relative complement of A is the universal set. A intersection the relative complement of A is the empty set. So verify all these using Venn diagrams or logical arguments and living as an exercise to you.

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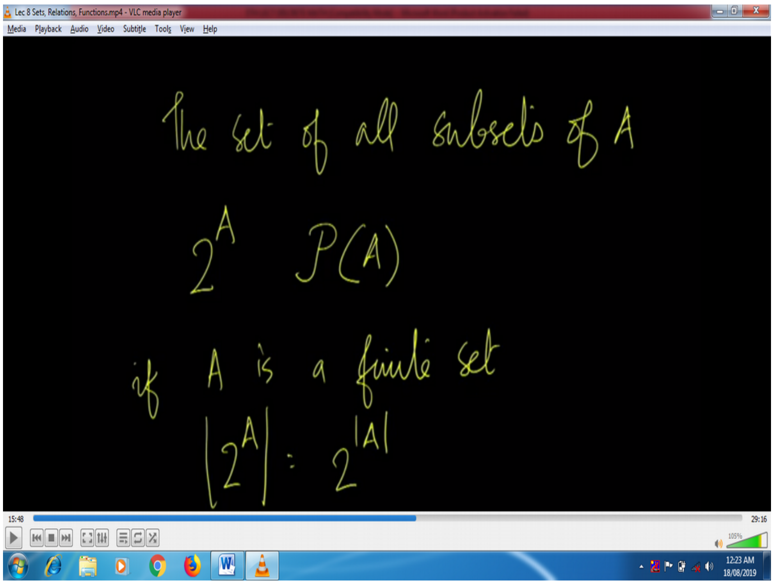

The set of all subsets of A is denoted by two power A or sometimes P of A if A is a finite set the size of two power A two power the size of A. Another word if A has n members then the power set of A has two power n members.

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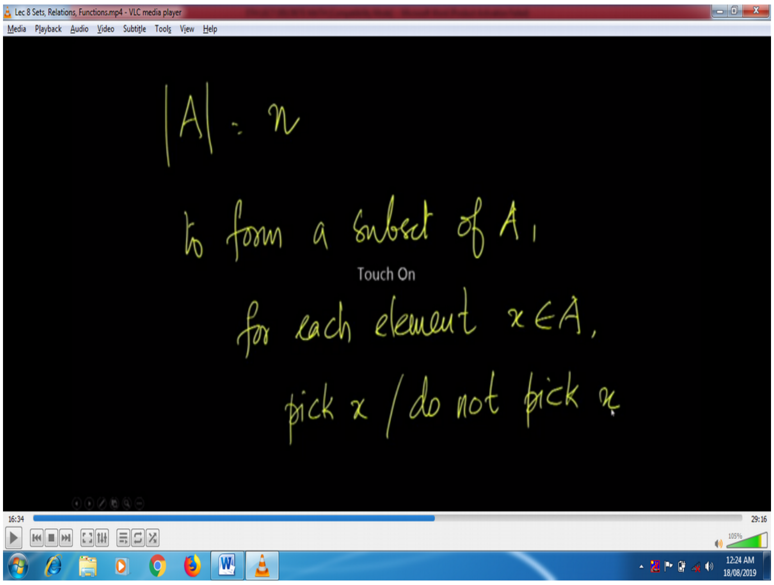

The argument goes like this. If the cardinality of A is n the cardinality set is the number of elements in the set when the set is finite. If the cardinality of A is n then two forms a subset of A for each element of A we have two choices. Pick the element or do not pick the element.

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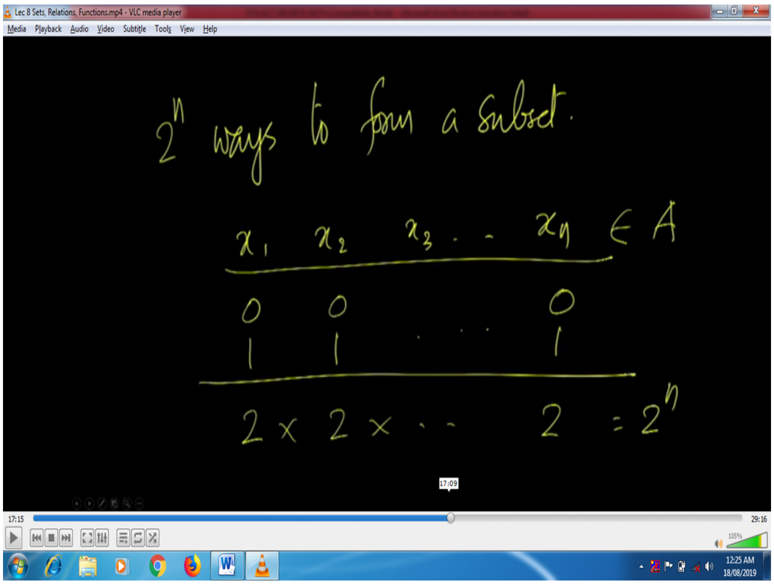

Therefore there are two power n ways to form a subset. That is if x_1 through x_n are the members of A. For each element either I can pick it or leave it selected or leave it for each of the n elements. So there are two choices for each element therefore the total number of ways in which you can form the subset is a multiple of n twos this is two power n. Therefore the number of subsets of A is two power n.

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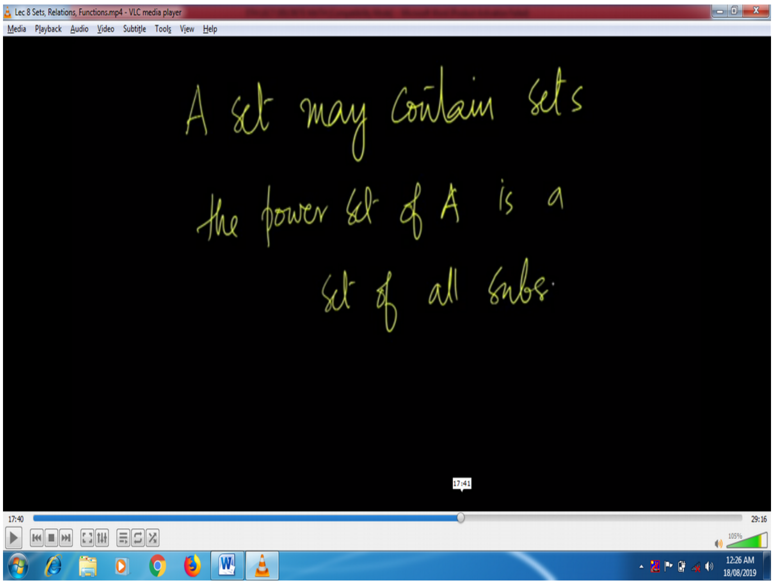

A set may contain sets. We have seen the power set is the set of sets.

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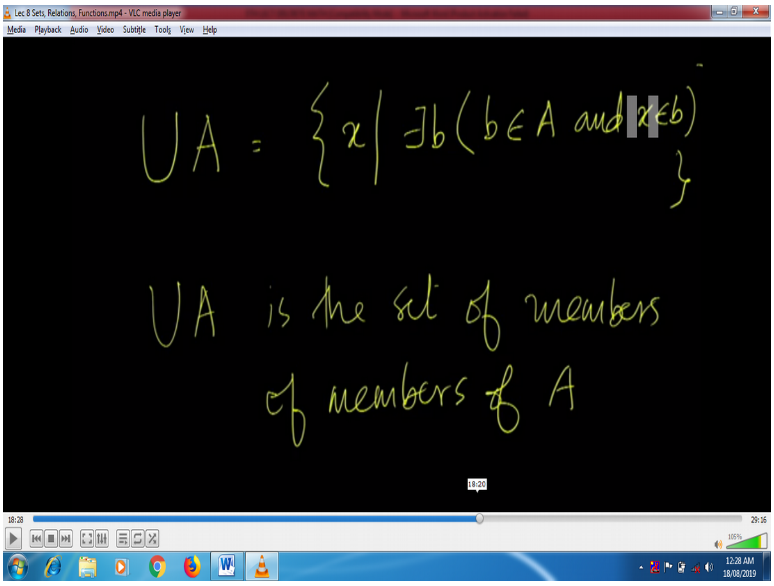

We define the union of A in this manner. It is a set of all x such that access b b belongs to A and x belongs to b. Another word union of A is the set of members of members of A. If B is a member of A and x happens to be member of B then x belongs to Union of A.

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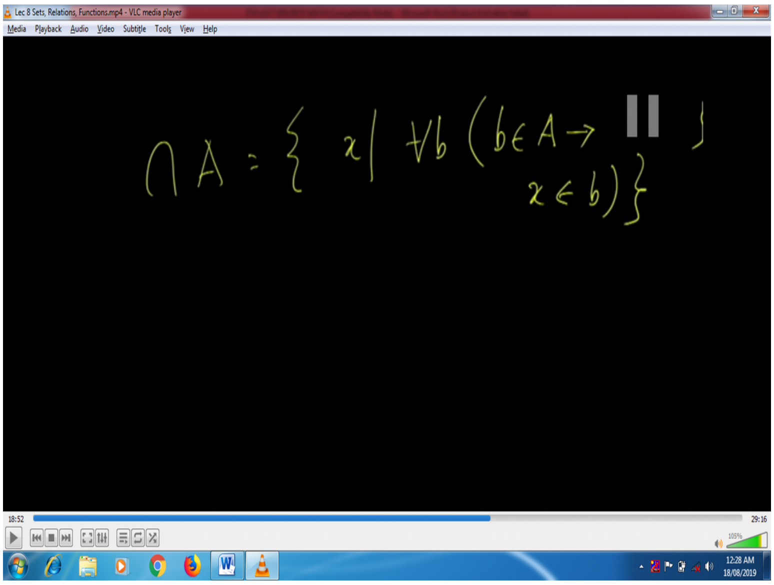

Analogously we can define intersection of A as x such that for every b if b belongs to A then x belongs to b. So intersection of A is the set of those elements which belong to every member of A.

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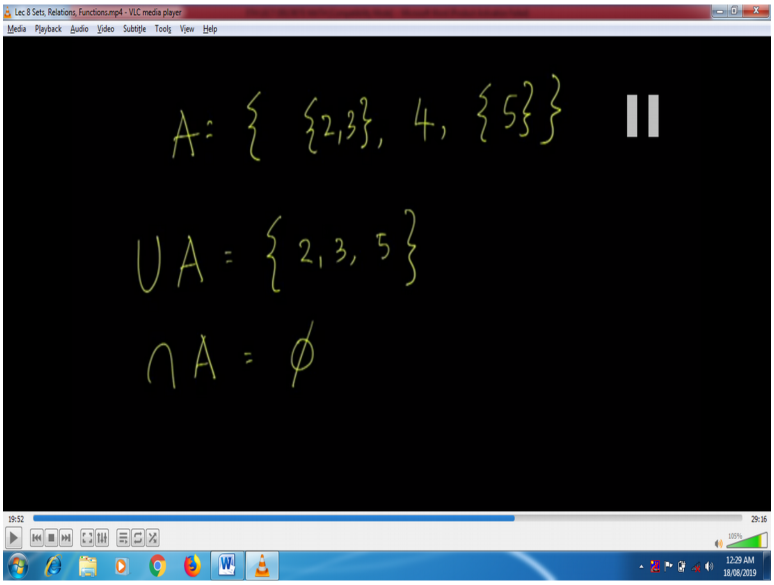

For example A made up of these elements set 2, 3 number 4 and the single term containing just 5. Then union of A would be the set of all members of members of A. Number 4 is not a set. Therefore it doesn't have members but set 2, 3 has two members namely 2 and 3 set 5 has one member namely 5. So union of A is 2, 3, 5. Intersection of A is the set of those elements that belong to every member of A. But since 4 is not a set it does not have any member. Therefore there is no such element there is no element which is the member of every member of A.

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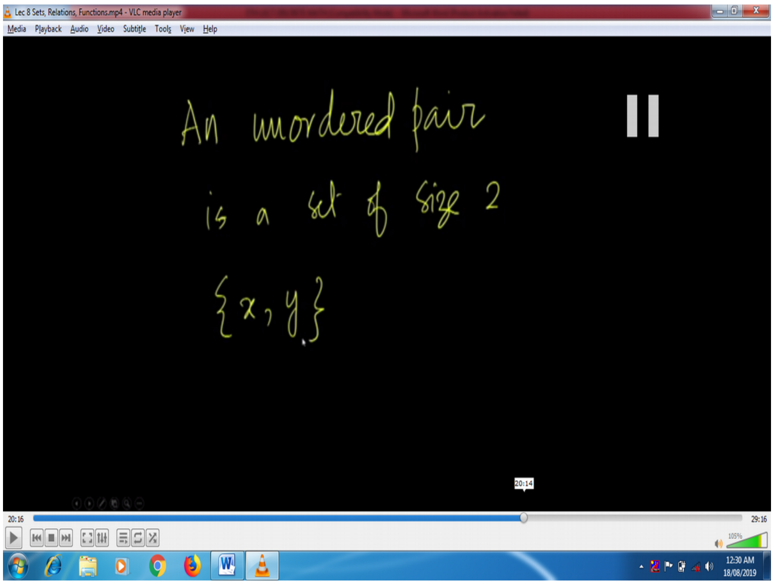

An unordered pair is a set of size 2. A set of size 2 x, y is an unordered pair it has exactly two elements and there is no particular ordering of the elements. That is you cannot say this is the first element and this is the second element these two elements are identical. As per as the membership is concerned. So an unordered set set of cardinality 2.

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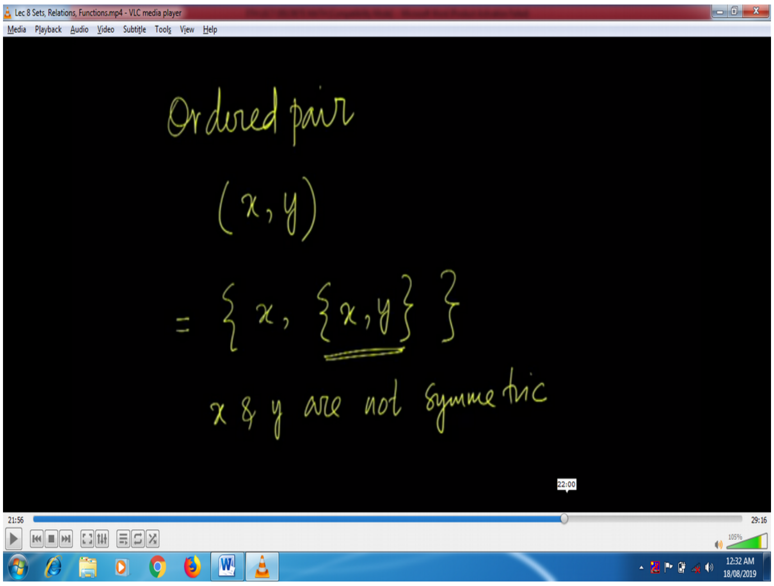

Where is an ordered pair has two elements x and y and these elements are to be ordered you should be able to say that x is the first element here and y is the second element. So x and y are not symmetrically included here but an ordered pair can be defined as a set. So let us define the ordered pair x, y in this fashion. We define the ordered pair x, y as a two element set. In which the first element does x itself where the second element is a set containing x and y the unordered pair.

So the ordered pair define the set consists of two elements the first element is an element from the universe whereas the second element is an unordered pair of the universe and the first element is a member of the second set. So here x and y are not symmetric. In the sense that, x is the first element and it's also a member of the second element. Whereas y is the only member of second element it does not have an independent membership of the circle. Therefore x and y are holding symmetric position within this on ordered pair.

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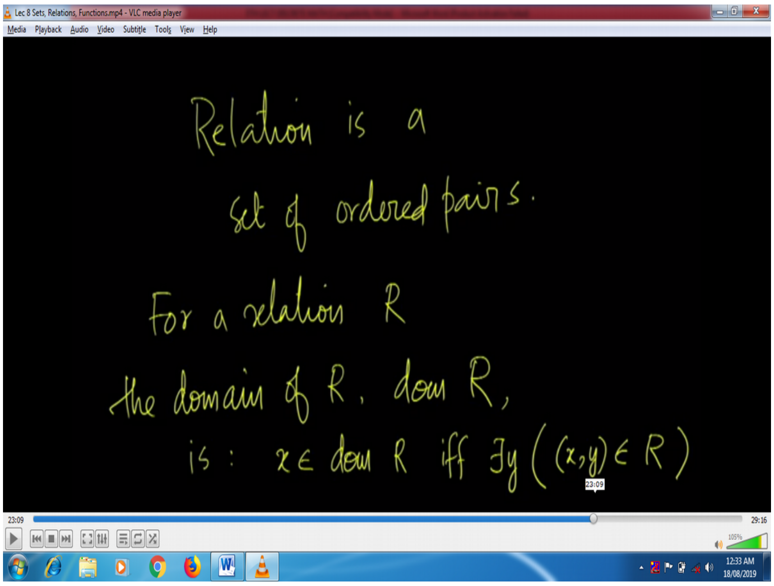

We use ordered pairs to use what are called relations. A relation is a set of ordered pairs. For a relation R the domain of R denoted in this fashion is defined in this fashion x belongs to the domain of R. If and only if there x is y so that x, y belongs to R. There is an ordered pair in R where x is the first component if that is the case we say that x is the domain of R.

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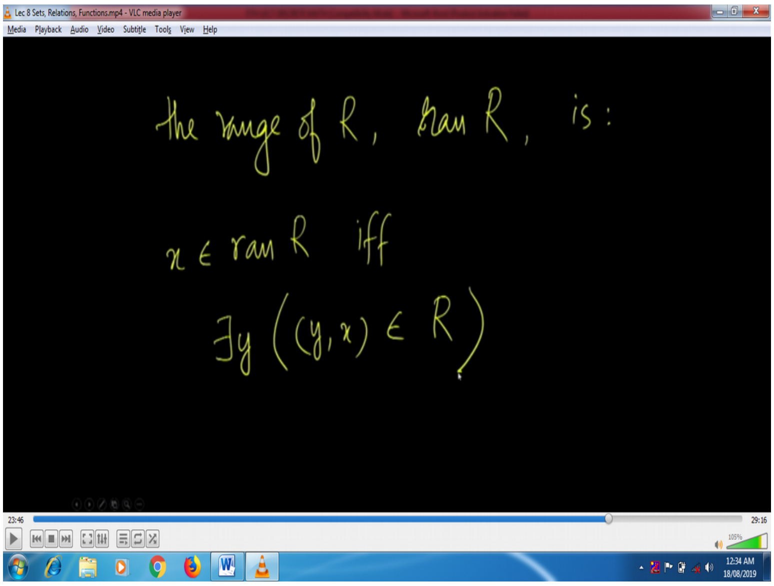

Similarly, the range of R denoted like this. This defined in this fashion. We say that x belongs to the range of R. If and only if x is y so that the ordered pair y, x belongs to R.

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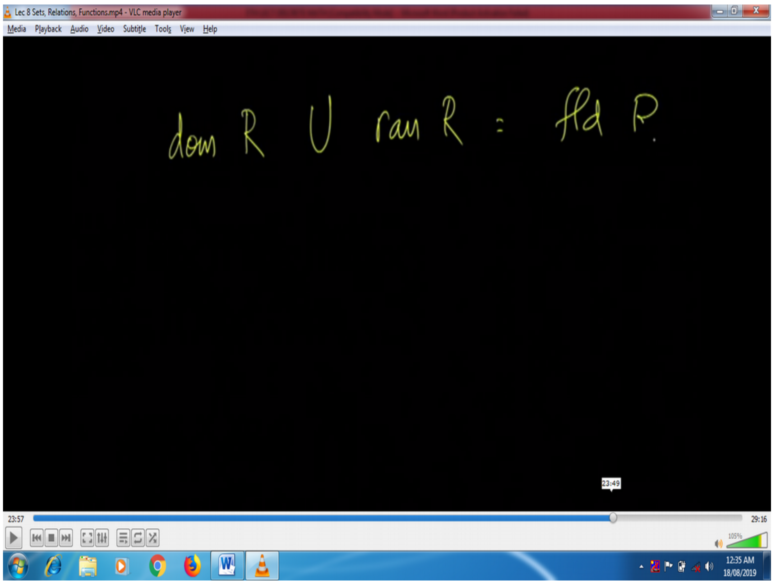

The union of this two the domain of R and the range of R is called the field of R.

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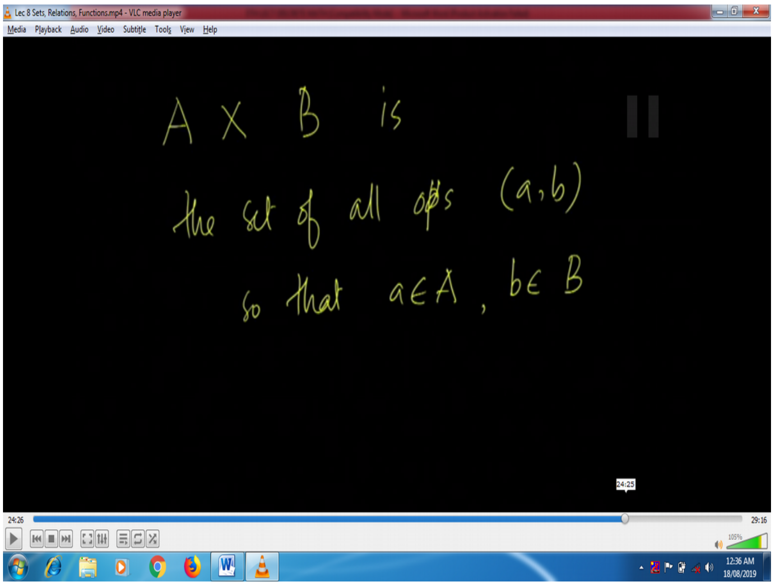

For two sets A and B across B is the set of all ordered pairs a, b. So that a belongs to capital A and b belongs to capital B. That is across b set of all ordered pair a, b so that a come soon the first set A and B come soon second set namely b.

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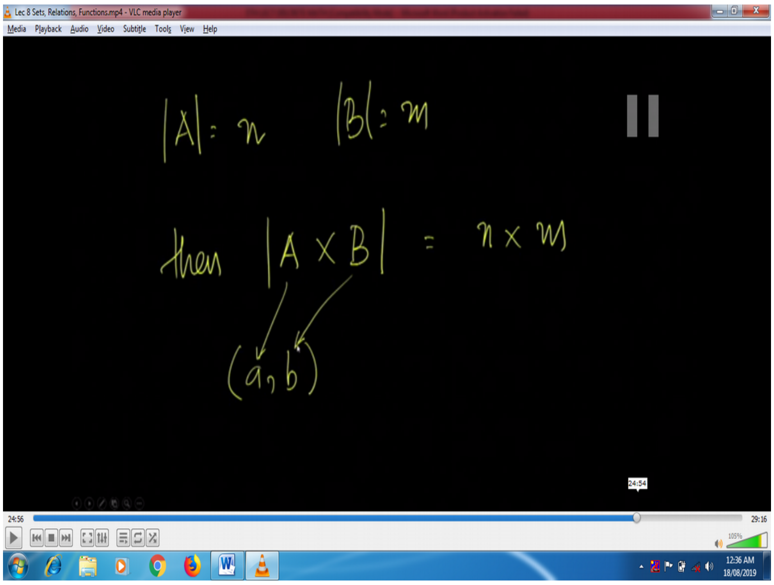

So if A and B are finite and a has a size of n and B has a size of m then cross product of a and b has size of n into m. Where is the number of ways in which you can form ordered pairs with the first component coming from a and the second component coming from b is n into m.

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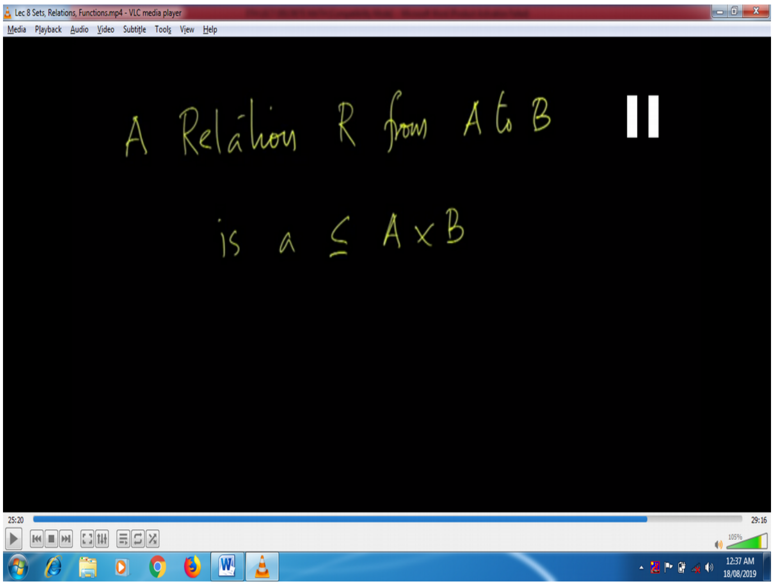

Observe that, relation R from A to B is subset of across A, B. If R is made up of ordered pairs from A and B then R is a subset of across B.

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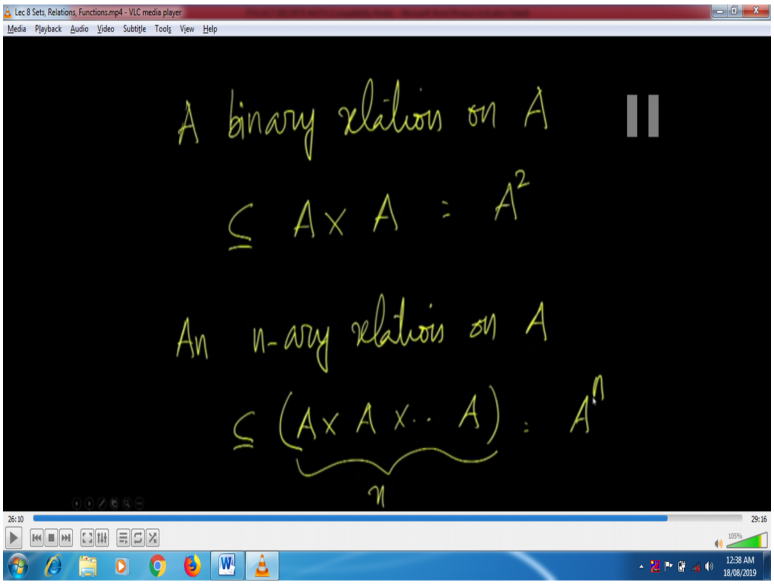

A binary relation on A is a subset of A which is also sometimes written as A square. And n ary relation on A is a subset of cross product of A with itself n times. Which is denoted in this fashion A power n.

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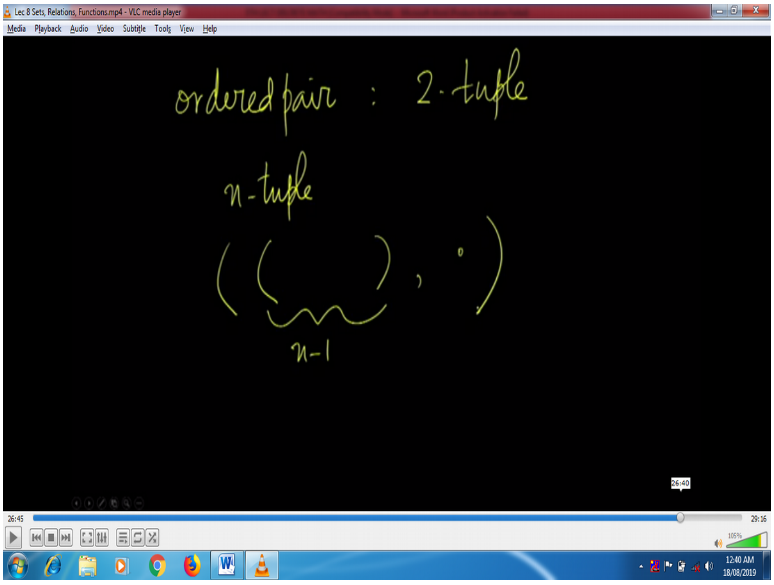

An ordered pair is also called 2 tuple and n tuple. In that sense defined as an ordered pair consisting of and n minus one tuple followed by a single element.

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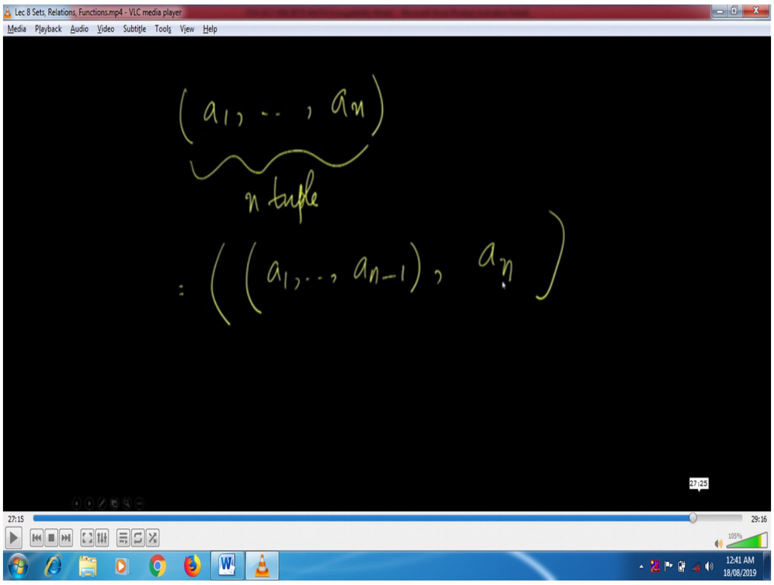

For example, when we have elements a_1 through a_n forming an n tuple. This can be defined as a set in the fashion define an ordered pair first to form and n -1 tuple using the first and n-1 elements and then include a_n in that. So every n tuple can be defined as an ordered pair.

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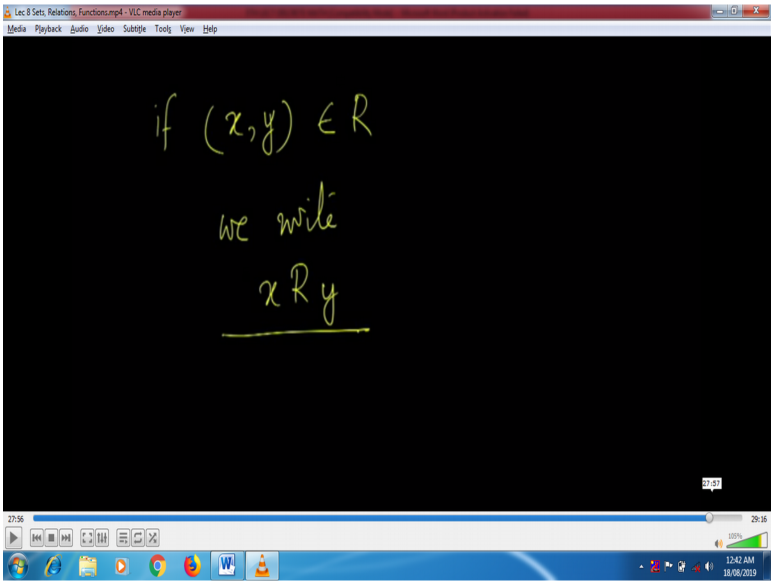

And A power n is the set of all n tuples. If x, y belongs to R we write xRy.

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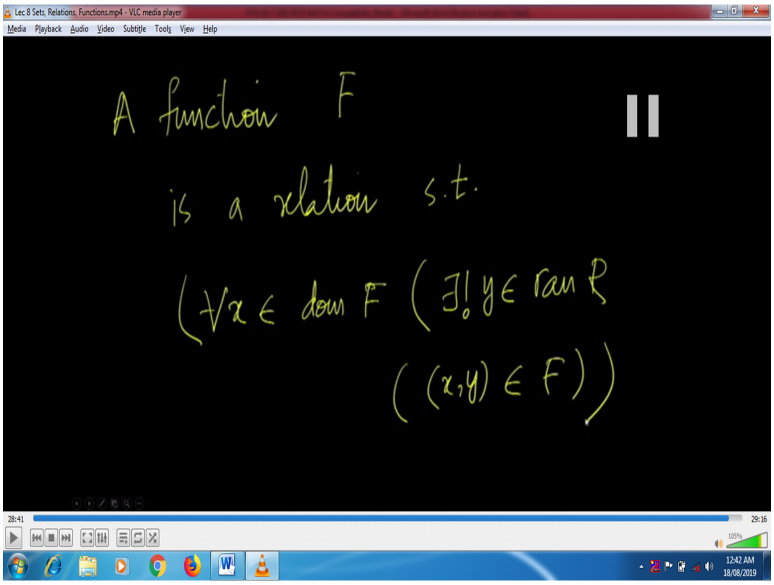

Now, let us define some functions. A function F is a relation such that for every x belonging to the domain there x is a unique y in the range of R. So that x, y belongs to F. We will see more about functions in the next class. That is it from this lecture hope to see you in next thank you.