Design Verifications and Test of Digital VLSI Designs
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Module - 4
Temporal Logic
Lecture - 5
Equivalence between CTL Formulas

Ok, till now we have seen about temporal logic and we have particularly discussed on kind of temporal logic, which is called CTL; Computational Tree Logic. Till now we have seen, what is the syntax of CTL and how to define the semantics? So, while define a semantics, we need a model and a meaning of those CTL's formulas define over a model; which is basically known as your Kripke structure.

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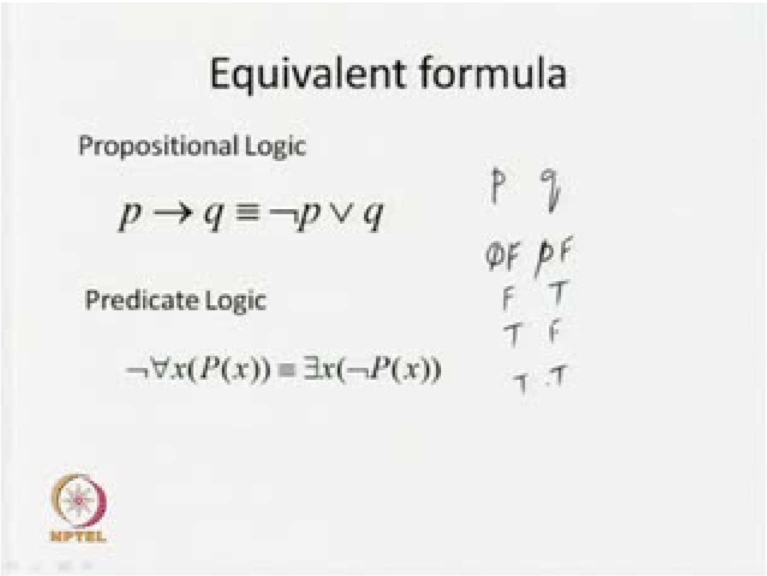

Now today, we are going to see some equivalence between CTL formulas. Because in logic we have equivalent formulas, in CTL also we are having some "Equivalent formulas". So recap you just see that, in Propositional Logic we are having equivalent formulas and we say that this p impulse q is equivalent knot of p or q. So, what does it means, it says that for same assignment of truth values for p and q both these formulas give me the sample values. If one will give me truth value true, then second one will also give me truth value true; if one will give me truth value false, then second one will also give me truth value false. So it should be true for all possible assignment for p and q.

Now here seems, we are talking about only two variables. That means, we can have that proportional variable p and q they are having four different combination, may be both are yours 0 0, or I can say that true true; or other I should say that instead of 0, I will say that this is false and 0. I will say that this is false, maybe it is false and true, true and false and true and true; these are the four possible combinations we have.

Now if you see these things then, for all possible combination both the formula will keep us thus same truth values. In this case you will say these are the equivalent formula in your propositional logic. Now similar notion of equivalent formula comes in our predicate logic also. In predicate logic we are having this particular quantifiers say for all x and there exist x.

So, one common or basic notion of equivalent formula here we have going to get knot of for all x, P x is equivalent to there exist some x for which knot of P x is true. Ok, So, basically P x is a predicate; it says that for all x P x is true and negation says that we do not get any x for which P x is true. So that means, you may have we should get some x, for which knot of P x is true.

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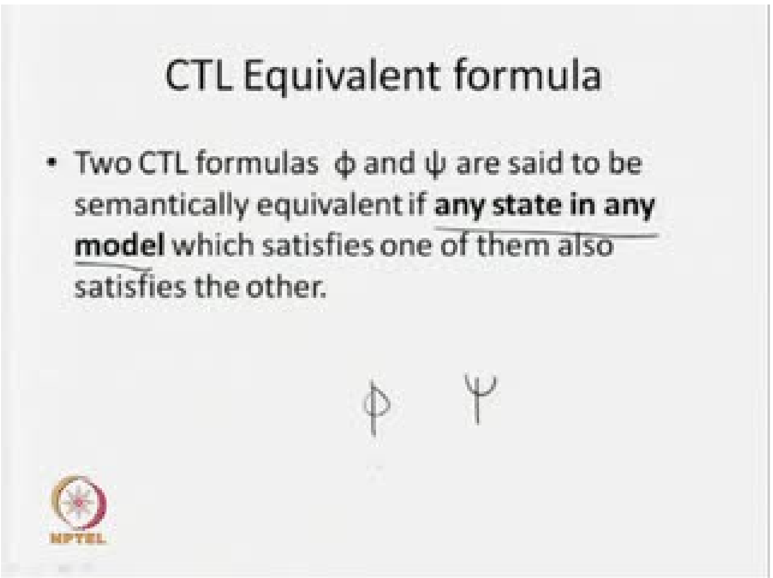

So, this is the notion of your equivalent formulas in our predicate logic. So, like that now we have to see one will say that two formulas in CTLs are equivalence. So, what is the notion of equivalent formulas in CTL? So you can see that two CTL formulas phi and psi, say you are going to consider two formulas phi and psi. So, the truth values of these two formulas phi and psi will be define with respect to models. So, we are going to say that these two CTL formulas phi and psi are said to be semantically equivalent.

If any state in any model, now this is important any state in any model; which satisfies one of them also satisfy the other. So, that means if you are going to say that two CTL formulas are equivalent, there must have the same truth values in all sate of any model. So, if you are going to consider any state, if phi is true then psi must be true in that particular state of any model. So, if you look for any models then both must be true in any state or both must be false any state, then you are going to say that these two formulas are equivalent.

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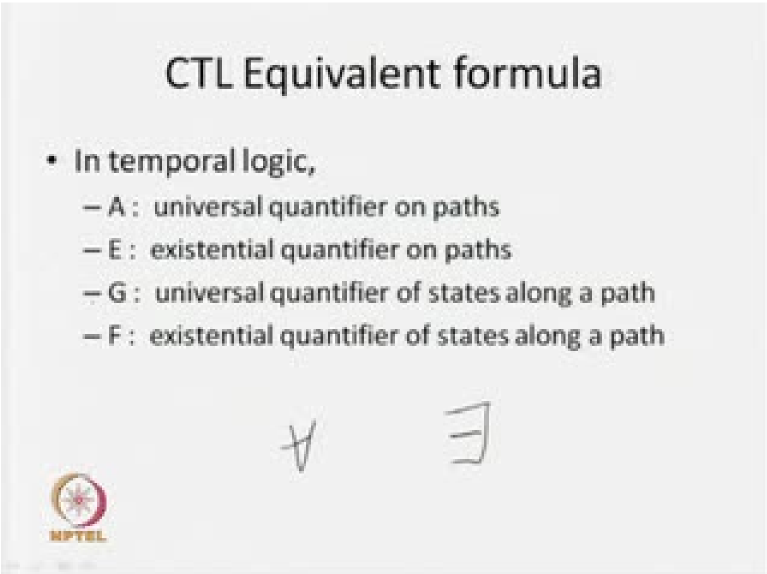

Now, we know the syntax of our CTL formulas, we know the meaning or semantics of CTL formula; now will see what are the equivalent formula that we have in our CTL. So, in temporal logic basically you see that we are having, some temporal operators and some part quantifiers; so it respects to desire your CTL. So, in CTL we are having A; which is called universal quantifier on paths that means, you are going to reason about all paths. E, existential quantifier on paths that means, we are going to look for any paths these are the two path quantifiers. Similarly you are having two temporal operators, one is c and second one is f. What c says that? This is the universal quantifier of states along a path. So, basically you say that G globally it is true in all states in a path and F we say that they are we have going to get this states in future, where some formula or some CTL formulas are true. So, in that case we can say that F can be treated as our existential quantifier of states along a path and G is the universal quantifier of states along a path.

Now, you just see that here we are having two quantifiers A and E. which is basically across path; and F and G, which are basically states along a path. So, these two things can be again you can see, that we are having that for all quantified in predicate logic and there exist quantifier in our predicate logic; and we know that they are having some relationship with negation we are going to get the other. So similar notion will be available with our this temporal path quantifier as well as the state quantifier; A E and f G. So, for that we are going to get some equivalent formulas, so we are going to see those particular equivalent formulas.

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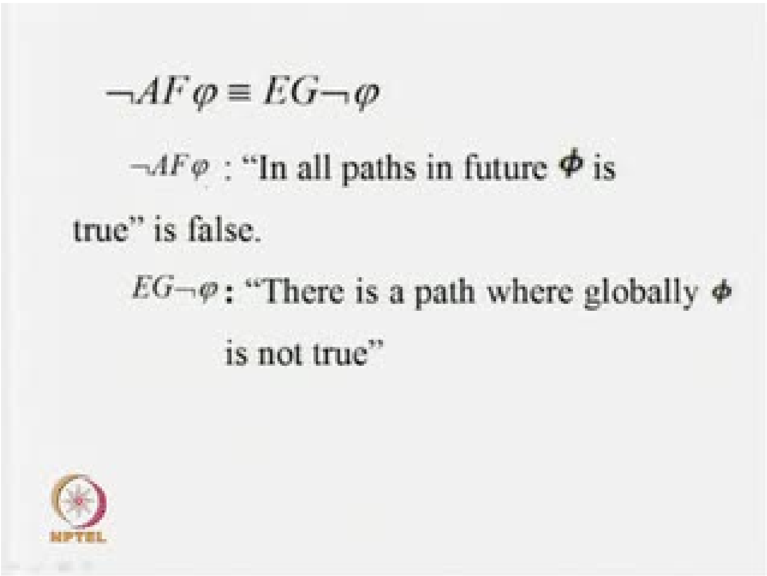

First equivalent formula that is coming over here is like that, knot of A F phi is equivalent to there exist a path globally knot phi. Just see what it means? Knot of A F phi, we know that A F phi; in all path in future phi true. So, if knot of these things means, these particulars step means is false. So, in all path in future phi is true is basically false because we are having this particular negation symbol in point of this particular CTL formula.

Now second path, second formula is says that E G knot of phi. Here is a path where globally phi is not true, so we are going to get one path where globally phi is not true. So now see that, we are saying that these two are equivalent. How we are going to get the equivalent, with the help of small example I am going to say that. It says that in all path in future phi is true is false.

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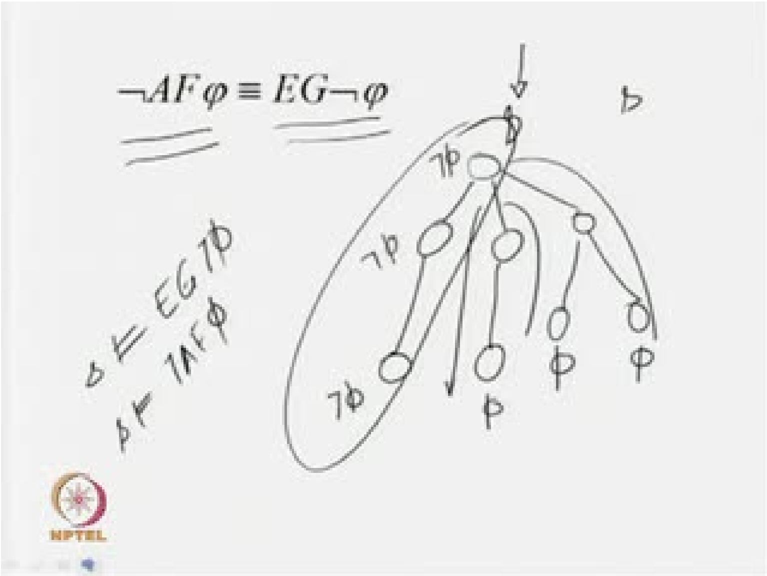

So, basically you just see that we are going to define the semantics over models. So, we think some models something like that. This is simple model, now we have to see that in all paths in future phi wholes. So, that means we should get some state in future where phi is true and we are saying that this is not true. That what means, in all paths we are not going to any future state where phi is true. At least it is not true at least in one of the path. So, basically if you say that if I am writing phi is true over here, phi is true over here, phi is true over here; then in all these three paths, we are going to say that A F phi is true but, in this particular path say we are not going to get any state where phi is true. So, in this case what I am say that A F phi is not true in this particular state says where form we have started.

Now, this is equivalent you are see that, there exist a path when globally knot of phi wholes. So I will say that this is your knot of phi wholes. Basically you see that since not of phi wholes bar may be A F phi may not p true or a F phi may not be true in this particular; but, what will happen in these two order two states. If phi whole over here then F phi wholes in this particular path so, knot of phi must be true over here. So, similarly, in s zero also knot of phi must be true. So, what it says that basically we are going to get one particular path, where globally knot of phi is true. So, since in this particular state diagram in states, I say that these models, there exist a path globally knot of phi wholes. Similarly, you will find that in this particular states knot of A F phi wholes. Basically we are getting one path where in place of phi is not true, so knot of A F phi is true in this particular state. So, like that in this particular state these two formulas are equivalent, both are true.

So, like that we will say, now if you are going to make a any model, if will find that E G knot of phi is true in a particular state. Always you are going to find that knot of A F phi is true; so that is why we are going to say that these two are equivalent. You just see that path quantifier A, which is a universal quantifier it becomes a existential quantifier across path and the existential quantifier among states in a path becomes the universal quantifier along that particular path. So, they are dual of each other. So, in this case I am going to say that these two formulas are equivalent. So, this is the first equivalent formula that we are discussing.

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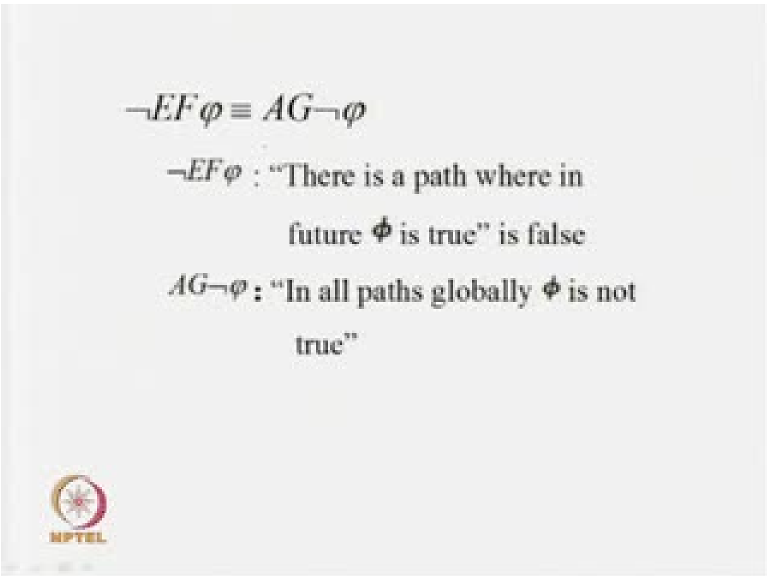

Now see, what are the others equivalent formulas we are having. So, this is another equivalent formula we are having, there exist a path in future phi holes and negation of this one is true and this is equivalent to in all path globally knot of phi wholes. So, what it says that, E F phi; so there is a path where in future phi is true and negation says that this is false basically. And A G knot of phi, it says that in all paths globally phi is not true and we are going to say that these two are equivalent. Of course, in this you can get the internal filling or intuitively you can say that yes; it is true because, in there does not exist any path where phi is true and it is upend up in all path globally knot of phi is must be true.

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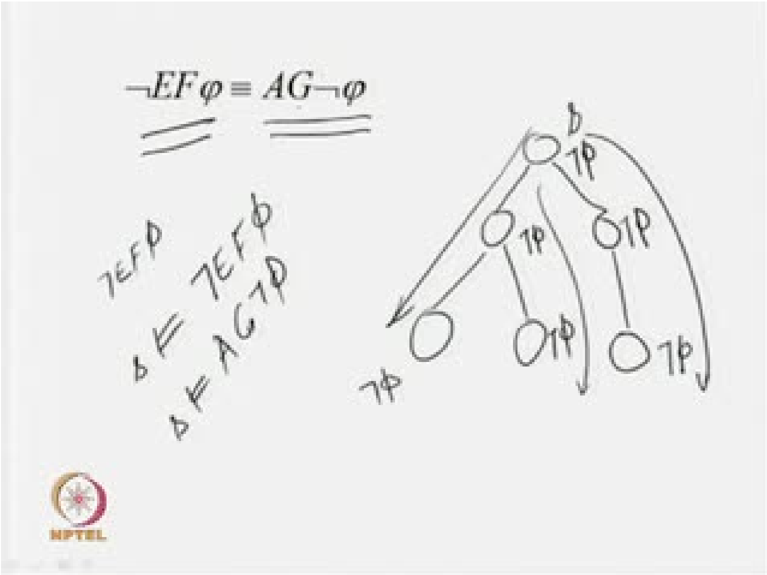

Again just come back to look into some model, because we are going to have define the meaning of this CTL formula of a model. So, you just see that, we are going to say that there exist a path in future phi is true and the negation of this one is true in this particular states. And A G globally knot of phi, so in all path globally knot of phi. Say if I am going to say that knot of phi is true, knot phi is true, knot of phi is true in those particular state. That means in this path knot of phi is true, in this path knot of phi is true; but, if I am going to say that phi is true over here, then what will happen?

We are going to get there exist a path in future phi is true in this particular case, that means knot of there exist a path in future phi is not true along dispersion. To become to have this particular formula through over here, we should have knot of phi over here; then only we say that in this particular path also we are not getting any state where in future of phi is true. So, in this particular states, I can say that knot of E F phi is true. Because they are does not exist any state where in future phi is true.

So, this is while looking in to this particular model, we have found that again in s models your A G knot of phi; because knot of phi is true in all states. So if you consider these two formulas and you take any state of any model; you construct any model, you look for any state. If one is true in a particular state of that model, you will find that the other one is also true in that particular model. So, that is why I will say that these two are equivalent.

So, again you will say that they are having the relationship between the existential quantifier along path and it is reversal quantifiers along paths; and this is your existential quantifier of sates and universal quantifier of sates along a path, so they are having relationship.

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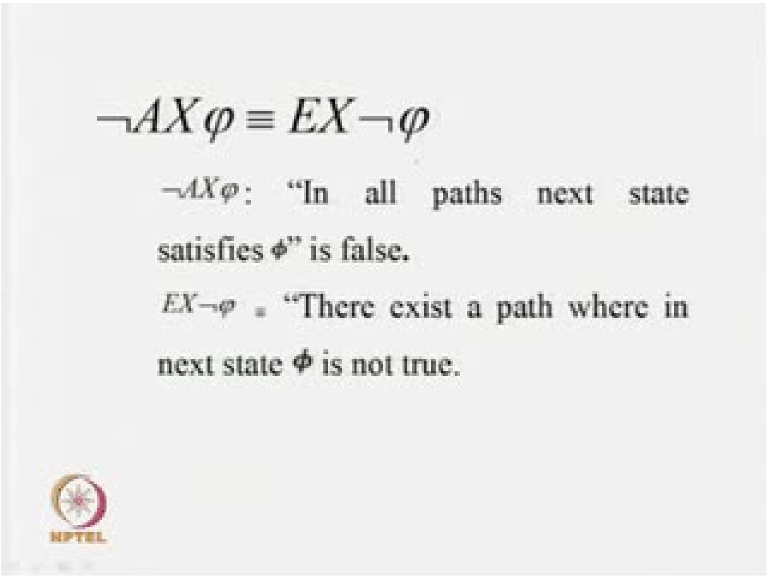

Now another one which is related to your next state operator, so what is the next equivalent formula? We are getting that knot of A X phi is equivalent to E X knot of phi. So, first formula says that in all paths next state satisfies phi is false, that means in all paths that phi is not true in next sate. So, that means second formula say that there exist a path where in next state knot of phi is not true; that means phi is not true. And you will find that these two are equivalent I can just look into a particular model.

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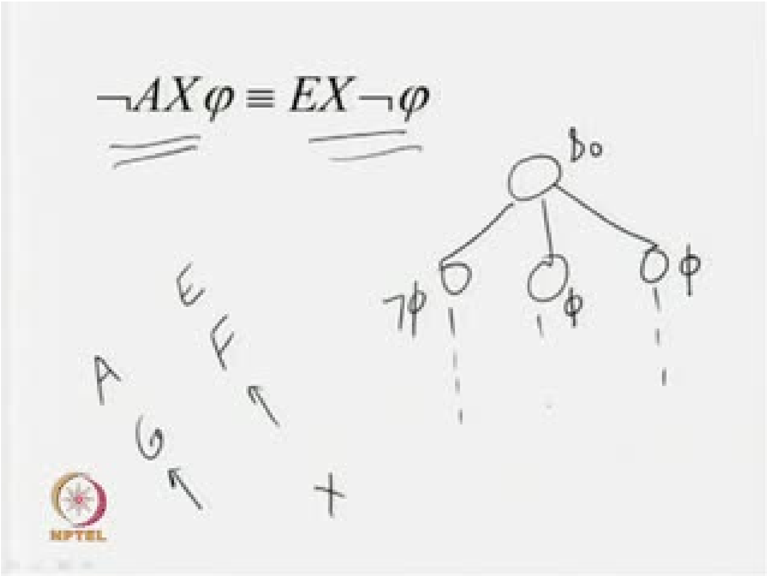

Say if I am having a model something like that. So, in this particular state so in all paths next state phi is true and in this is not true in this particular states zero; then I can say that phi is true in these two states but, phi is not true over here. So, in all paths in next state phi is not true. And this is equivalent you can say that we are going to get there exist a path in next state knot of phi is true. Again now if you are going to look any model, and look for any states of any model you will find that if one is true in a particular state then other will also be true in this particular state. So, that is why we are going to say that these are equivalent formula.

Now, what we have seen now? That we are getting that path quantifier A and E your that state quantifier you can say that state of temporal operator; this is basically globally and you are in future. So, these two can be treated as your universal quantifier and these are basically existential quantifier. And by looking into these formulas you will find that one is a dual operator. So, if A and G are they can be replaceable by E and F but, in next state it is dual of itself because next state is going to replace by next state operator only. But, A becomes E when we are going to take the negation of this particular formula. So, that is these are the equivalent formulas that we are getting in yours CTL formulas.

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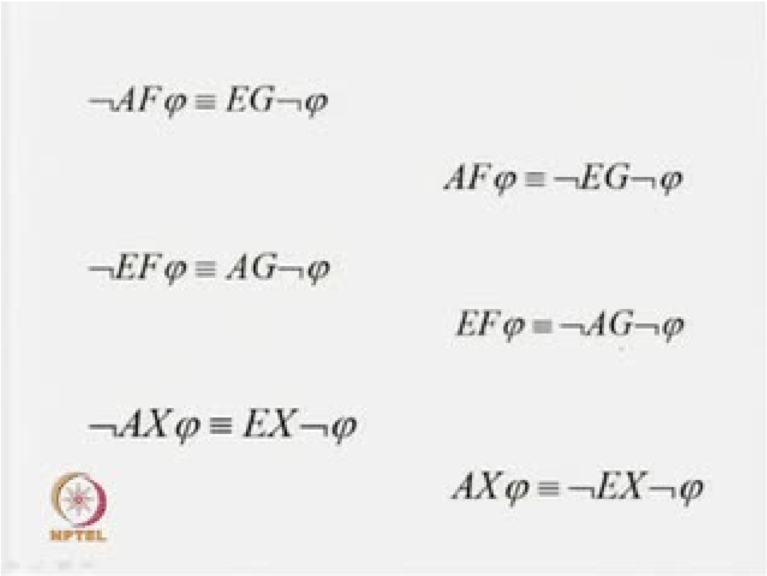

So, in our self we can summarize it like that this is once equivalent that we have got knot of A F phi is equivalent to E G knot of phi. So, if you take the negation of these two formulas, then we will find the A F phi will be equivalent to knot of E G knot of phi; that means the formula A F can be replace or express by E G.

Similarly second one that we have already seen, knot of E F phi will be equivalent to A G knot of phi. So, if you take the negation in both sides, we will say that E F phi, E F pji will be equivalent to knot of A G knot of phi. And third one which is related to your next operator, so knot of A X phi will be equivalent to E X knot of phi. So, in that case if you take the negation in both the sides will say that A X phi will equivalent to knot of E X knot of phi. So, basically you see that A and F can be replaced by E and G or E and F can be replaced by A and G. So, E will be replaced by A and F will be replaced by G. So, this is the things that we have seen till now.

So, these are basically we have going to talk about the operator globally future and next state. We are having another temporal operator which is your until. Now will see whether some of this particular operator can be expressed with the help of until operator or not. Until is your binary operator which involves to CTL formula and others F C and X are unary operator.

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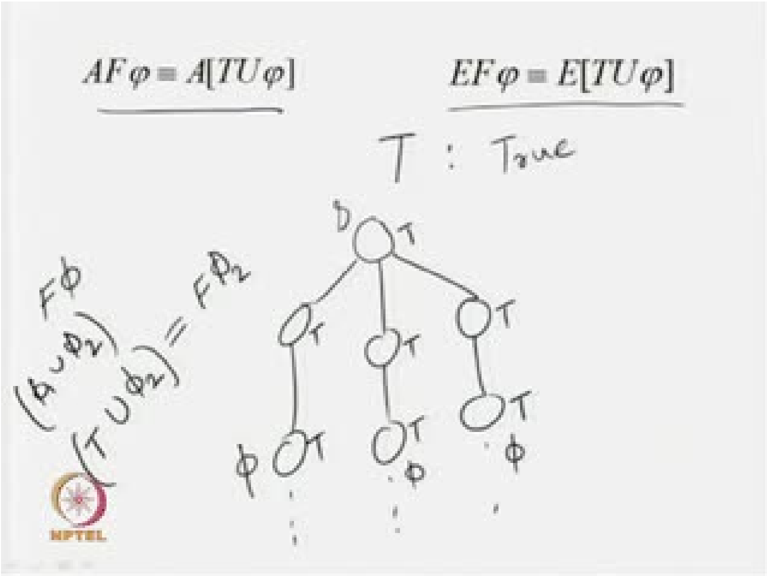

Now you just see that, we are having two equivalent with respect to your until operator; one we have going to say that A F phi is equivalent to A T until phi or E F phi will be equivalent to E T until phi. Here the symbol T, that I have replace it basically it is the top it says that the truth value true.

Now, you say that, when we have define a semantics of your CTL formula; what we have seen that? All state will be merged by this truth symbols true and none of the state will be label by truth symbols false. So, what does it means? That means true is true everywhere this is the notion that we have qualifying our semantics. So, if we take any model say these are the model we having say computational tree we have. Now if you look into this particular model, that means true is true everywhere; that means all the states will be model by this particular truth symbol true. So, that means it will be labeled by these things will say that true is true in all state.

Now, what is the future operator? F phi it says in state in a path, we should get some states in future where phi is true. So, that means if you consider this particular path that phi if phi is true over here that we say that F phi is true in this particular path. So, similarly, if it is true in all those particular path then we will say that F phi is true in this particular states. And what is the until operator? If you say that phi 1 until phi 2, it is says that phi 1 so remains true until phi 2 becomes true. So, in this particular case now I replace this phi 1 by say T top or that truth symbol true and until phi 2. So now, I am going have these things; that means true is true everywhere so, true until phi true. So this is nothing but, we are going to look for some state in future where phi T is true. So, this is basically nothing but, we can say that F phi 2. So, that is why we are saying that, that future operator can be replace with the help of your until operator. So, A F phi will be equivalent to A true until phi and E F phi will be equivalent to E true until phi. So, it is there exist a path in future phi is true, we will say that there exist a path true remains true until phi becomes true. So, we will consider these two as our equivalent formula.

And similarly A F phi will be equivalent to A true until phi. So now you just see that, that temporal operator F can be expressed with the help of temporal operator until. So, these are another two equivalent formulas that we have. So, already we have seen the equivalents formula, which are related to A F E G and A X E X; and now we are getting two equivalent formula which are related to until and future operator.

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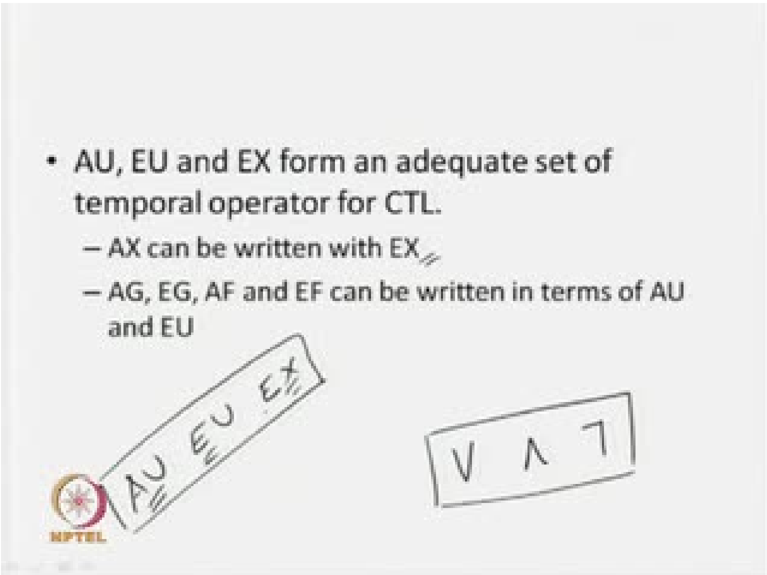

Now, if you look into CTL: Computational Tree Logic, we are having four temporal operators. Just we have discussed four temporal operators and all those temporal operators will be presided by path quantifier and either A or E. So, in this case we will say that we are getting or having all together eight define combination. Now we need method for all those particular eight different combinations, all eight different CTL operators to say that truth values of a formula in a particular state. But, since we have those particular equivalents it may happen there in order to look for procedural for all eight combinations, all eight operators, CTL operators; we can come to your restricted state and after that other operators will be expressed with the help of those particular restricted state.

So, in that particular case what we can say that, A U, E U, E X form an adequate set of temporal operators for CTK. Like your proportional logic or you classical logic, what i will say that? The junction, conjunction and negation forms a adequate set of operators because with the help these three operators you can express any other operator. Like that in your CTL also, now what we are going to say that? A U in all path until operator, E U in all path until operator and E X. These three forms an adequate set of your CTL operators, because other operators can be expressed with the help of these three operators. Like that say A X can be written with E X. So, if I am having E X wherever I can go for A X. Similarly A G, E G, A F, E F can be written in terms of your either A U or E U. So, basically that is why we are saying that these three are sufficient enough two express all the eight combinations that we have discussed till now.

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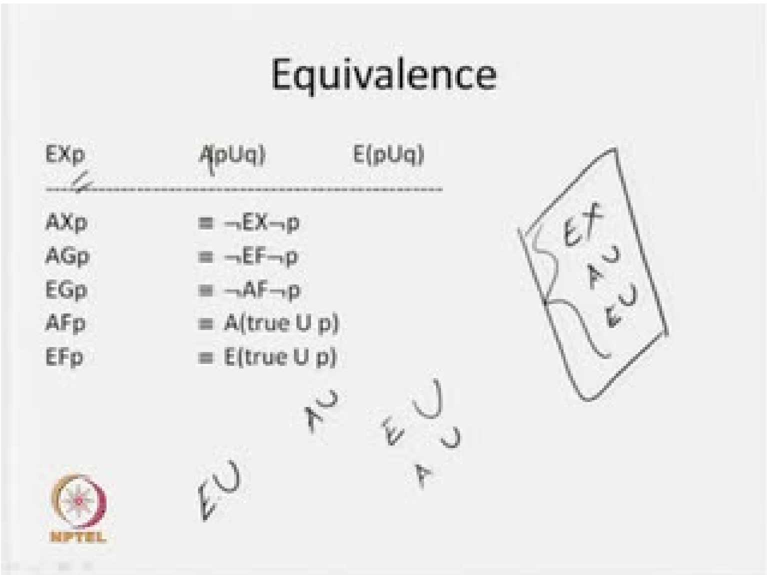

So, how would this is simple example that I am saying that, I am going to take only E X p, A p until q and E p until q. So, this is the adequate set of operator E X, A U and E U. Now since I am having total eight combinations so, what I will have? I am having this, another phi combination over here. So, A X p will be nothing but that we are already seen the equivalence, so I can say that this is knot of E X knot of p, A G p will be equivalent knot of E F knot of p.

So, A X is the replace by E X, E G p is nothing but knot of A F knot of p. So, that G operators is replace by F, again this F will be replace by your until operator say; A F p is your A true until p and E F p is equal to A E true until p. So, eventually all those particular e can be expressed with the help of these three operators.

So, you can say that these are the educate set of operators, that we have and with the help of these three operators we can express other operators also. But, this is not the only adequate set of operators or CTL formula; we have some other adequate set also. But, how we are going to get it? We can you just see that with the help of these things, I need both A U and E U. So, in until operator I need both the combination E and A U.

So, that is why we need these two things along with that we are having this particular either E X or A X we are going to take. But, some of equivalence see that, that E U can be expressed in terms A U or A U can be expressed in terms of E U. Then what will happen? In that case I can just omit one of these two combinations. Then if we can omit one of these two combinations then we are going to get some other adequate set of CTL operator.

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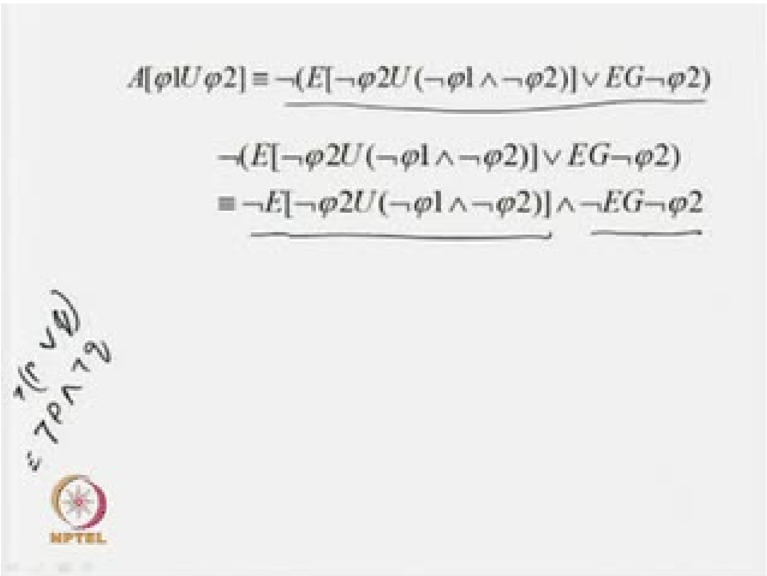

Now whether is it possible or not, let see. So we are having one result. This is says that if this is until operator A until, this A until can be expressed with the help of your E until. This is E until; what it says that? A phi 1 until phi 2, say if you are going to look for this particular formula. So, we are having an equivalent of this particular formula, it says that negation of this whole formula. What is the negation of E? There exist a path knot of phi 2 until this conjunct knot of phi 1 and knot of phi 2 or E is the knot of phi 2.

Till now we have discussed some of the equivalent formulas. And by looking into the construct of the equivalent formula it is upon that s, there indeed equivalent and we can ferial explain it also. But, if you look into this particular equivalent a partly it own clue get it is going to be equivalent, because it is having some complex combination. But, indeed their equivalence, we are not going to have a formal flow to see whether equivalent or not; for that we need some other information also. But, intuitively I am going to establish the result or with example I am going to show that these two are equivalent.

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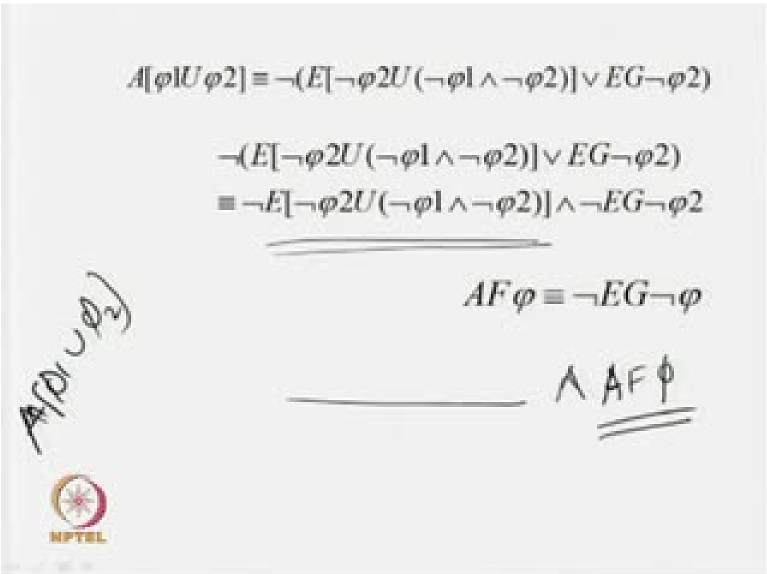

Now, you just now look into the right hand side, that we are having these particular things over here. So this right hand side this expression can be written in some other way. So, what we are doing basically, it is nothing but we have simply using the Demorgan theorem to this particular expression; that expression that I have in our right hand side. We know that Demorgan theorems say negation of p or q is equivalent your knot of p and knot of q. So, this is a Demorgan's rule, so we are using this particular Demorgan rules to this particular expression. So, what we are getting? I am just pushing this particular negation inside. So, I am getting knot of E and this particular portion and this r becomes n negation of this second path.

So, thus using the Demorgans theorem and we are getting conjunct of two particular formulas. Now what is this knot of E G knot of phi 2. So, already we have seen this particular equivalent that A F phi is equivalent to knot of E G phi 2; that means we can say that we can replace this particular second path by A F phi and first path will remains as it is. So, eventfully you are getting this thing.

Now, you just say that A phi 1 until phi 2, if I look into these particular things A phi 1 until phi 2; that means phi 1 should remains true until phi 2 becomes true in all path. On the other what I can say that? In all path in some future state phi should must true, phi 2 must whole and along with that it should satisfy the all other preceding state phi 1 must wholes. So, that is that we must get some future state in all path where phi is true this particular criteria or condition is captured by this particular path; A F phi 2. And along with that now we are saying that in all preceding state phi 1 must be true, so this particular other second condition is captured by this particular path.

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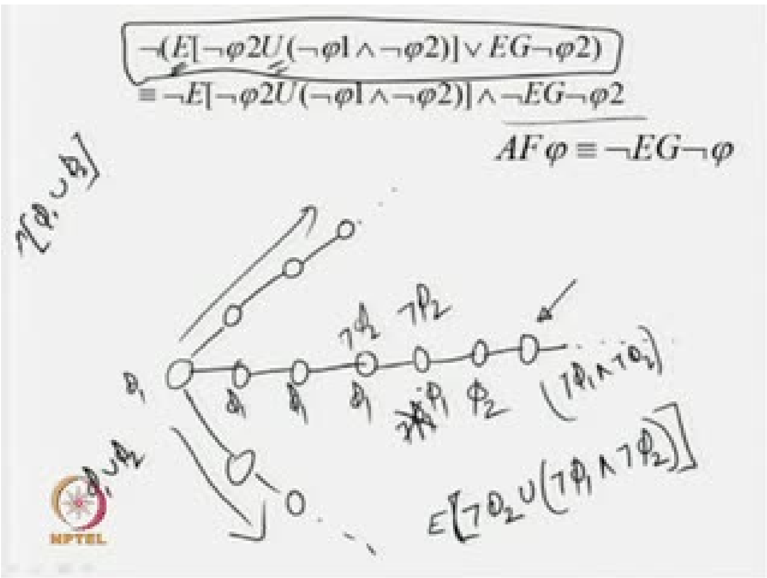

Now will see these things; so basically if we are having this particular expression say this is the first path and this is the second path; so F phi. So, you can look for all possible path whole, you just say that I am having; because this is A until. So, we may have many more existent paths, since A phi 1 until phi 2. So, in all paths this much is true, so will consider one particular path and same thing must be true if all other paths. So, it says that in all paths in future phi must be whole this second path basically. So I will say that in future I am going to have this particular phi 2; that means in this particular path in future I am going to get. And along with that what it must say? That it must true this particular portion knot of E knots of phi 2 until knot of phi 1 and knot phi 2. So, what basically it says that? I am going to get the formula E knot of phi 2 until knot of phi 1 and knot of phi 2.

So, we are having this particular formula and it says that, it must not be true in this particular path; because, I am having this particular indication. So, we are going to see this particular formula, what it says that? Phi 1, knot of phi 1 and knot of phi 2, true at some state and till that 1 we should have knot of phi 2. Now you just see that, I am getting this particular sate say, considering this particular state where knot of phi 1 and knot of phi 2 is true. So, to become this particular formula true then what will happen on all those particular state knot of knot of phi 2 must be true.

And if in all those particular state knot of phi 2 is true, then we will say that this particular path is true. And you should have the negation; so this is this should not be this should not be true over here that means, it should knot it should this particular formula over here. Now, since your knot of phi 2 is true over here so, it is not true at that particular point; so in this particular point knot of phi 2 it will not remain true.

So, negation is true over here, so but, what will happen in this particular state? Say if knot of phi 2 is true over here. So, along with that say if knot of phi 1 is also true, then what will happen? Knot of phi 1 and knot of phi 2 will be true. So, in this particular case what I am going to get, say already we have say that in semantics the fusion notion include this presence also. So since this is both of these are true that means, eventually this formula will be true; but, our objective negation must be true.

Since negation must be true, eventually what will happen? That knot of phi 1 must not be true over here. So, phi 1 must be true at that point because, we need that negation of this particular formula must be true. So, with this same logic if we will find that if knot of phi

2 is true over here, then will find that phi 1 is true over here also because, we need this component must remain true.

So, in this power till this particular phi 2 so in this way find that in all cases that phi 1 will be true. So, that mean we are getting this particular path where phi 1 until phi 2 is true. And with this similar logic we can extent it for all other path and we find a all other path phi 1 will remains true until phi 2. So, what basically we have seen? That A phi 1 until phi 2 can be express with the help of this particular formula. So, which basically involve, E until operator; that means, A until can be expressed with the help of E until or other hand you can say that E until can be expressed with the help of A until also. That means we are having equivalent relation between A until and E until, in all path until operators and there exist a path until operator.

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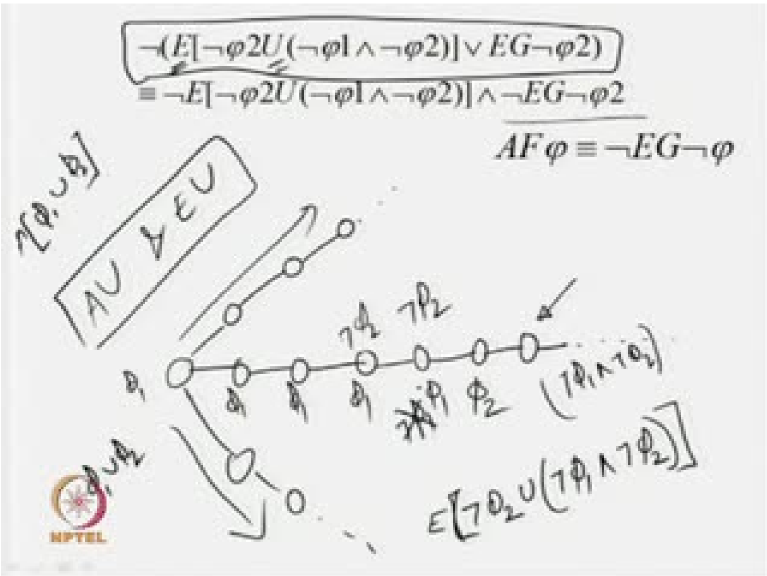

So now with this particular relations that we are having equivalent between A until and E until; so, since we have this particular equivalent between this two operators. Then what will happen? Now we can get some other adequate set of operators.

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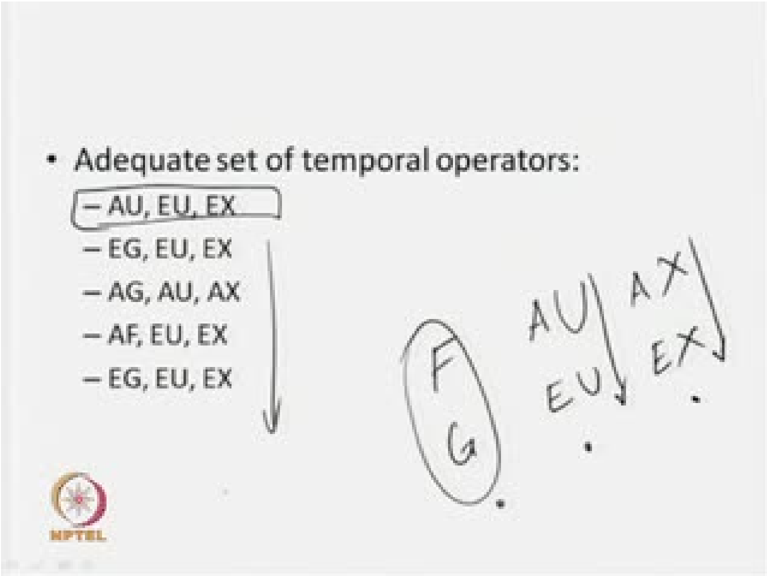

Now what are these things? So, other adequate set of operators temporal operators or in CTL operators you have going to get different sets. So, first one we have already discussed, already we have seen that A U, E U and E X.

Now, what are the other things, you just say that. If I am again until operator, if I know that A U is there, then it can be expressed as E U. Similarly, if I am having that next state operator, say this is your A X and E X can be expressed with the help of A X. So, we need any one form A U or E U and we need any one form A X and E X and other two so now out of that four temporal operators that we have discussed. That until and X already it can be expressed either A or E, now other two operators we are having F and G in future and globally. These are basically existence quantifier of states along a path and universal quantifier of state along a path, so one can be expressed with the other.

So, we can take any one of these two operators, because G can be expressed with F or F can be expressed with G; so will take any one of these. So, within one operator from this state picking up the operator for until and picking up the operator for that F and G, we can now construct or we can get several adequate set of operators. So, that is why we are just listing some of them, so these are treated as a adequate set of operators.

So, taking 1 X E X taking 1 U E U and we have taking just one thing E G; because if I am going to take E G, then it will be expressed with your A F; and one we are having the F, then we can expressed it with the help of your until or these things. So, basically these are the similarly, here also E G then second I am taking A G or I am taking that instead of A, I am taking E U E X and A F or E U E X and E G. Like that we can construct different adequate set of CTL operators. And we can see that we need at least three operators; with the help of three operators, we can express the other phi also. So we are going to get several adequate set of CTL operators.

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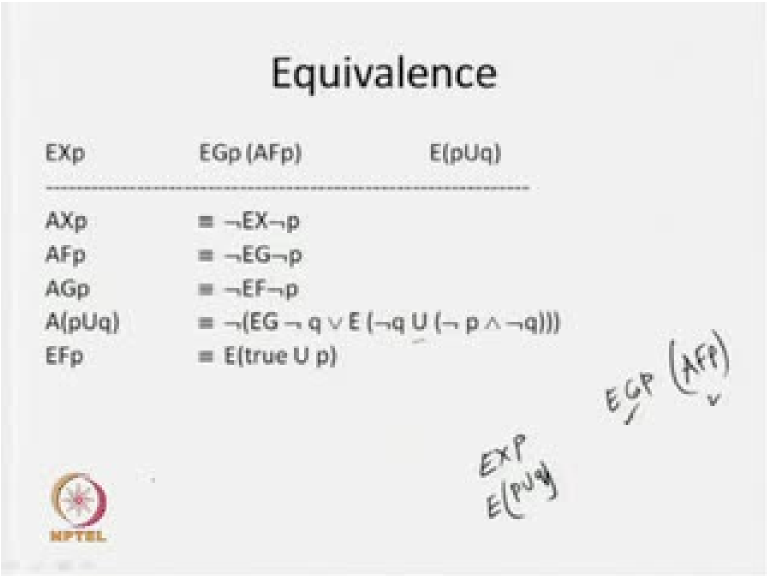

So, out of these particular things I am going say to consider say, something and explain how we are going to see this. So either I am going to take the adequate set of operators say, E X p I am taking that 1 X. I am going to take E U, E p until q and the third operator either I am taking E G p. There exist a path increase globally p wholes or may be A F p; because, I know the E is develop A and G is develop p. Either I can take this one or this one. So, in this particular case you just see that if I know the procedure how evaluate E X p then I can where is a evaluate A X p by looking into this particular equivalent; A X p is equivalent to knot of E X knot of p. We do not have any problem, it is simple.

Now if A F p, then what will happen? You just see that I am having say E G p; then what I can say that, A F p is nothing but, knot of E G knot of p. So, if I know the procedure for E G p, then we can look for your these things A F p or on the other hand if I am having this A F p then I can evaluate E G p. So, that is why I am saying that either of this I can take, so I am having this equivalent A F p is equivalent to knot of E G knot of p.

Now, what will happen in A G p? A G p is nothing but, you can say that knot of E F knot of p; that means, we are having this particular future operator. So, this is the future operator and this future operator can be expressed with the help of A until p, A p until q is expressed with the help of this particular equivalent, that we are discussed negation of this whole formula E G knot of q or E until. And E F p you just say that, E true until p, so we need this particular E F p for E G p; so we are going to say that E true until p. So, if with the help of those particular equivalence, we can say that if I am going to consider only these three operators, then what will happen? Others can be express and similarly if I take E X p E p until q and A F p, then others three can be expressed with the help of these three operators.

So, we must know the procedure to evaluate at least three temporal CTL operators, then other can be expressed with the help of this things. So, this is the power for equivalent formula and we have seen that now we need the restricted state and we are going work with that restricted state. So, this is the notion of equivalent in CTL formula. So, what is the notion? It is slightly define form your proportion calculus and predicate calculus, because here the meaning of CTL formula defines over a model and that is why we will say that two formulas will be equivalent. Provide that in any state of any model if that truth value of one formula is true, then that truth value of other formula must be true. Then only we are going to say that these two formulas will be equivalent.

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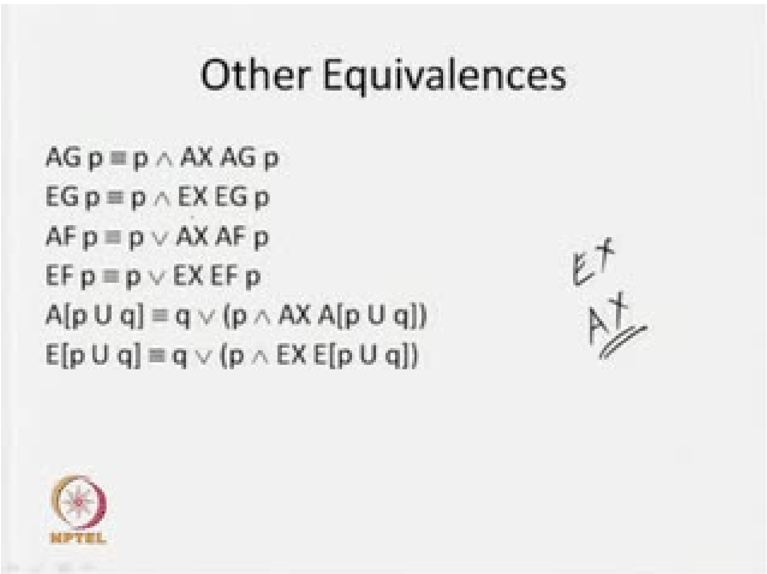

Now this is the notion of equivalence; like that you can find some like written also. But, these are the basic equivalent formula. Now we are having some other equivalence also and we are saying that these are basically some sort of your recursive definition. So, what I am saying, just see those particular equivalence; you are seeing that A G p is equivalent to p and A X A G p. So, A G p is equivalent to p and A X A G p.

Similarly E G p is equivalent to p and E X E G p and A F p is equivalent to p or A X A F p. E P is equivalent to p or E X E F p and like until A p until q is equivalent to q or p and A X A p until q. E p until q is equivalent to q or p and E X E p until q; so see the nature of those particular equivalent. Actually this is some sort of your recursive definition of those particular temporal operators; here we are having six temporal operators. That means, these six temporal operators is expressed by them self along with the temporal operator E X and A X, that next state operator ok.

So, see that we know the meaning of E X and A X, if it is having a all path quantifier; then this A X will come into fixture, if it is the quantifier is your existential quantifier E; then E X will come into fixture. So, say A G p I am having A X E G p, I am having E X. So, all these operators six operators are define by themselves. So, this is some sort of your recursive definition of those particular temporal operators and we are going to have those particular equivalent. Just see those particular meanings how we are going to get it.

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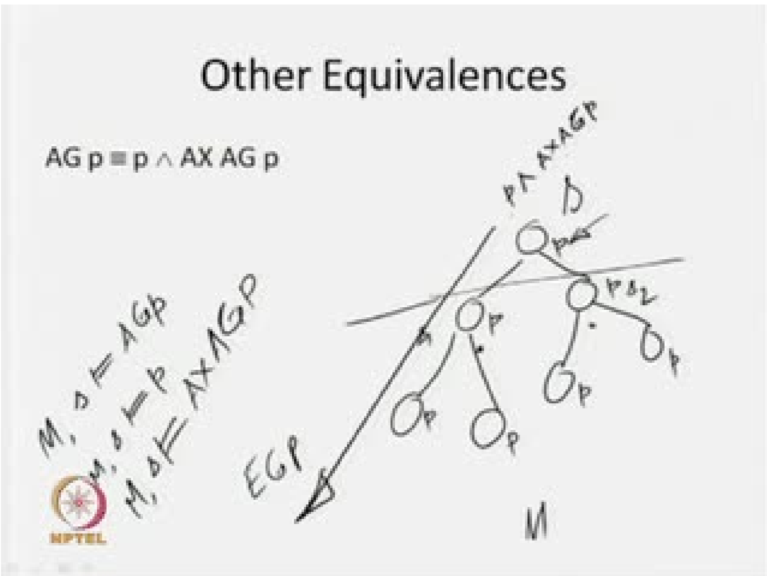

So, first formula we are saying that A G p. Now if I am going to have a model just explaining with the help of model only then it will be clear to you. So, what will happen? Say this is your s. I am going to say that whether in this particular model M in states whether it happens A G p. So, that means in all path globally p must be true; that means you can say that I am going to say that these A G p will true in this particular state if p is true in all those state.

Now, I can break this A G p like that, if I am going to look for this particular state. That means in state M, in states of M it must model p. If p is true in as them, what will happen? We are going to see all the next states over here. What are the two next state of this? This is s 1 and s 2. In s 1 and s 2 now, what it should happen? Again in s 1 and s 2 A G p is must be true. So, that is why we are saying that in all paths in next state A G p must be true. If A G p is true over here, then in both the states that means here I am going to get that in all paths in next state A G p is true. So, that is why we are saying that, if p is true over here then I am going to look for those particular states whether A G p is true or not. Since, I am going to look for all those state that means, in this particular state I am going to say that whether in all path in next state A G p is true. So, that means I am going to say that it will all over here if again states of model F if models A X A G P.

Now you just see that, if both these components are true over here p and A X A G p, then what we can say that? This formula A G p is true at that particular path. So, I think it is clear to you. That we are getting this particular thing if both the components are true, then I can say that A G p is true over here. So, with the similar notion we can go for E G p is equivalent to p and E X E G p. So, this is similar to your dissolve but, instead of looking for all paths it is your E G p. So, you are concern about one particular path, so what you are going to say, p must be true over here and there exist a path in next state A G p must be true. So, E G p is true must be true. So, this is the things that for E G p, p and E X E G p.

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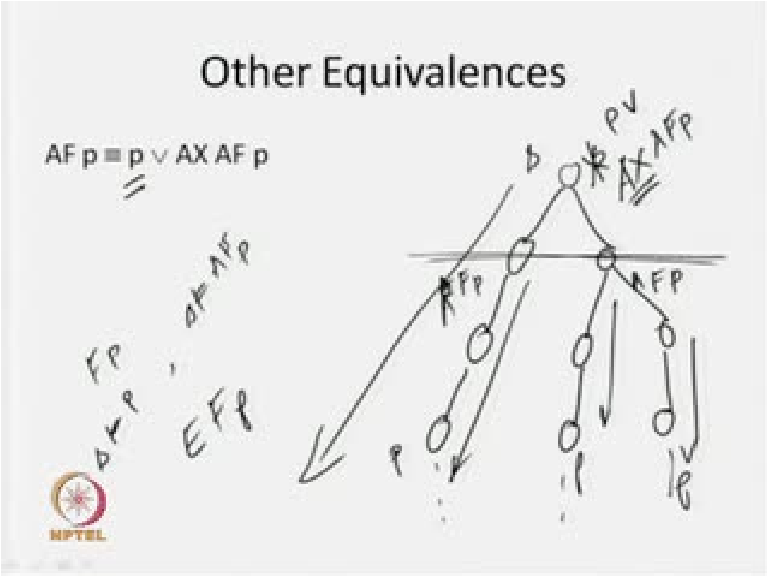

Now similarly, we are having A F p is written as either p or A X A F p. Now again you just see that, similar things that we are having. Now, what will happen in this particular case? I am say that if p is true over here then, I am getting this particular path that A F p is true over here. But, already we have seen the semantics that in this particular notion of time, it says that future includes the present behavior also. That means if p is true here itself, say I am concern about this particular state. If p is true in state that means, if s models p then we can say that s model F p also. If p models here I am going to say that, A F p models. That means these particular decision, it is a decision of two formula; if p is true over here there I am going to say that A F p is true there itself.

But, if say p is not true over here, then what will happen? Then I am going to say that in all path in next state whether A F p wholes or not. So, that is why I am saying that p or these things. It p whole there itself and I will say that A F p is true over here. Say if p is not true then I am going to look for this particular component; in all path in next state whether F p wholes or not. In all path in future if whether in next state A F p wholes or not. When A F p is whole, if I am going to get some future state where p is true.

So, this is the way that we are going to say that, A F p is expressed with the help of A F only; but, along with that particular A X operator. So, similar notion will go for E F p also; in this particular E F p, what will happen? Instead of A we are going to have E. So, instead of looking for all paths we will going to look for one particular path. So, this is the way we are expressing decease, this is another equivalent with respect to your future operator.

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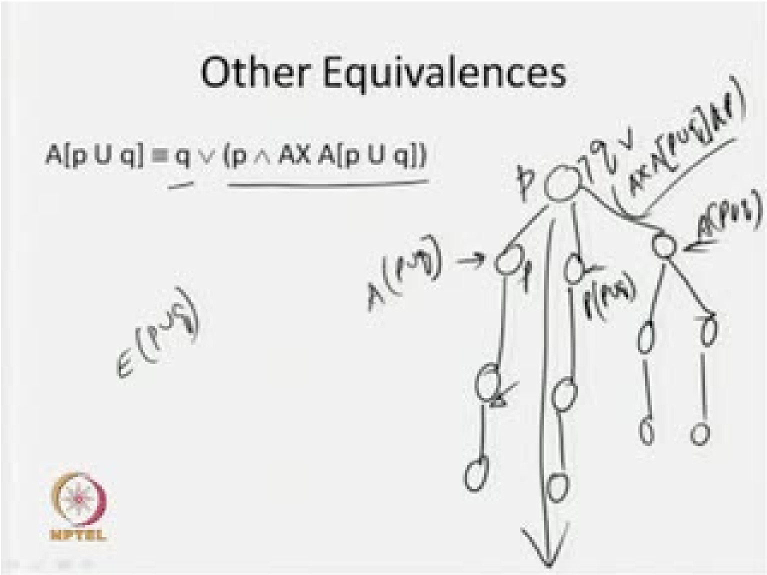

Now, look for the other one, that until operator; A p until q. Again we are saying that A p until q is equivalent to q or p and A X A p until q. You can you just see that particular. So, again what happens? The notion of time if you look into it, we have seen that top. When we are looking into the future behavior, we say that present scenario is also included in future behavior. So, future includes the present. So, that is why we are going to look for A p until q, p remains true until q becomes true. So, this is having again two descents, either first path or second path.

So, if first path is true, that we will say that A p until q is true; that means if q is true here itself, then we will say that A p until q is true. Then we do not look for it. Say if q is not true at that particular point, say if knot of q. Then we are going to look for the second component, A X A p until q and along with that and p. So, what does it means, that means if it is q is true there itself, I will say that A p until q is true. But if it is not true, then we have look for a second path; and what it must do? That p must be true over here, p and this is a p and A X in all next state. What it must whole? It must whole again A p until q, A p until q must be true over here.

So, this is the way that we are going to look into it. Now when I am going to loop for whether A p until q is true in this particular state, again I look for these two conditions;

either first your whether q is not true there itself on not. If q is not true, then I will see whether p is true over here; if p is true, then I am going to see whether in next state A p until q is true here. So, these are another equivalent, that A p until q is expressed with the help of A p until q; but along with your A X operator. So, similarly we are going to get the E p until q, in this particular case instead of looking all possible paths will be concern about any path. If these things happens in one particular operator, we will say that E p until q is true in that particular state.

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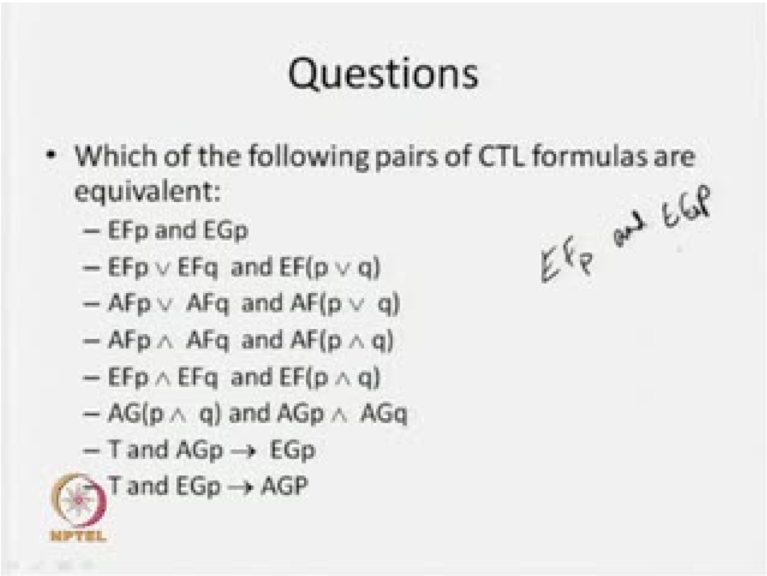

Now, after going through this particular equivalence now just look for some questions. Basically we are going to say, whether some of the some pairs of your CTL formulas are equivalent or not. So, the question I have given over like that, which of the following pairs of CTL formulas are equivalent? Here I am giving some pairs. So, first I am saying that E F p and E G p. Second one I am saying that E F p or E F q and E F and p or q. Third one is your E F p or A F q and A F and p or q. So, like that I am going to check whether those particular pairs are equivalent or not. So, first one you just see that, I am saying that E F p and E G p. So, well I am going to check for equivalent, if that equivalent indeed then what will say that will try to establish this tree logical.

But if they are not equivalent, in that particular case what will happen? We will try to give a contra model. We will give a counter example, where I will say that in one particular state one is true but other one is false and in that particular case we will say that these two are not equivalent. Because I am saying that what is the notion of equivalent? In any state in any model if one is true, then other must be true. Now you see that I am having this particular formula E F p and E G p.

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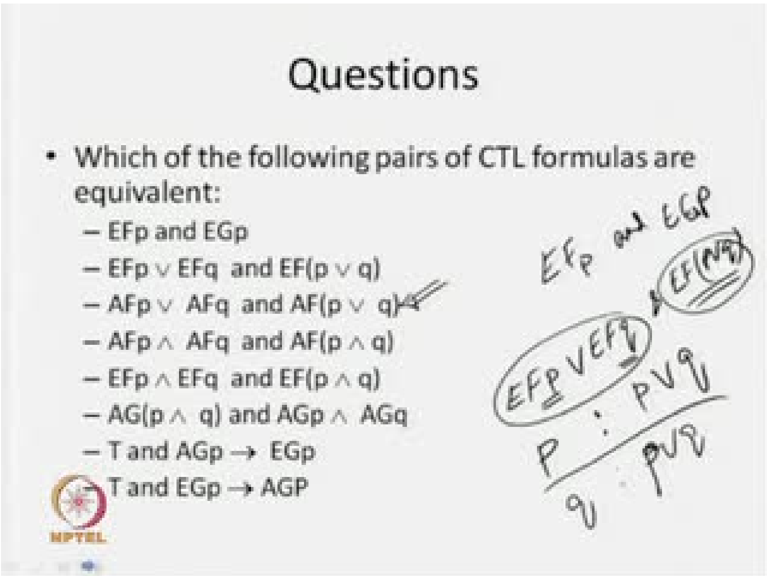

So, E F p and E G p; so in that particular case say I am going to consider one model. I will say that p is true over here. So, in this particular case in this particular states, I will get that s models E F p and say on all other state knot of p is true. So, in this particular model in s, I am getting E F p there exist a path in future p whose. But E G p I am not getting any but, where in all state knot of p will be true. So, at least I am getting coming up with one example, where in this particular state one formula is true but the other one is false. So, in this case I am not going to say that this particular equivalence, So these two formulas are not equivalent.

Now, look for the second formula. We are talking about E F p or E F q and E F p or q. Now in this particular case what I am saying that? There exist a path in future p wholes or there exist a path in future q wholes. And whether this is equivalent to there exist a path in future p or q wholes or not. Way we know that, if p is true then what will happen always p or q is true; because either or it p is true and p or q always true. If p is false then it depends on q. So, if both are false it is going to give me false, if anyone is true then it is going to give me true value. So if p is true, I am going to say that p or q is true; that means if there exist a path in future if p is true, next time I am going to either there exist a path in future p or q is true.

Similarly, if there exist a path in future q is true, than I am going to get a path where p or q is true. Because q is true, p or q is also true; but if they does not exist any path when p is false and q is false then p or q is also false. You just see that, if this particular path is true in any state then this path is also true; where this is false, then this one is also false. So, in that particular state what I can say that, these two formulas are equivalent.

Now, look into third part, it is saying that A F p or A F q and A F p or q. So, this is also look into these things, it is similar to your second formula; but instead of A I am having E. So, this is the same property wholes for this particular all operators. So, these two formulas here also equivalent, you can check it. You just try to check it; I am saying that this third formula is also equivalent. Because, this is the due to the property of this particular or if p is true then p or q is true, if q if q is true then p or q is also true. Now look for this particular fourth operator, fourth equation; A F p and A F q and A F p and q. So, we are going to say that A F p and A F q.

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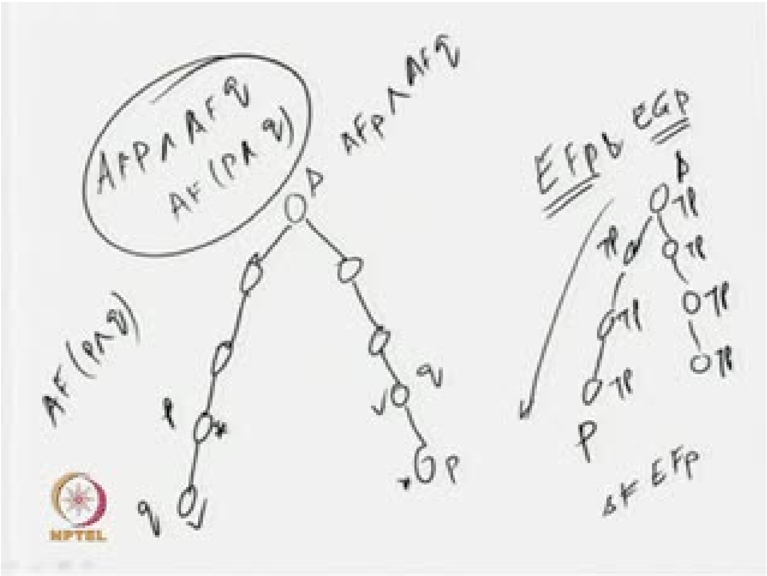

Now, A F p and this is the formula I think only look into it A F p and A F q. A F q and second formula I am thinking that A F p and q. Now you just see that I can look for any model, whether these two things are true or not I am going to check. Just say that p is true over here q is true over here, say q is true over here p is true over here. Then what will happen? In all path if I going to look into this particular things, I am going to say that in all path in future p is true. Because I am if I go along this particular path I am getting this particular state, where p is true if one go along this particular path I am getting this particular state and A F q.

If I go along this particular path this q is true over here, and if 1 go but other path then q is true. So, along all paths A F q is true. But, now if you look into this particular formula, A F p and q, then in along this q path I am not getting any state where p and q is true; because in this particular state either p is true or q is true, so A p and q is not true.

Since, A p and q is not true in any of the future states so, A F p and q is not true; that means these two formulas that I given over here are not equivalent. You understand these things. So, you have to see because the notion of equivalence is in any state in any model if the truth value of one formula is true, then other must be true; this is the notion of equivalent.

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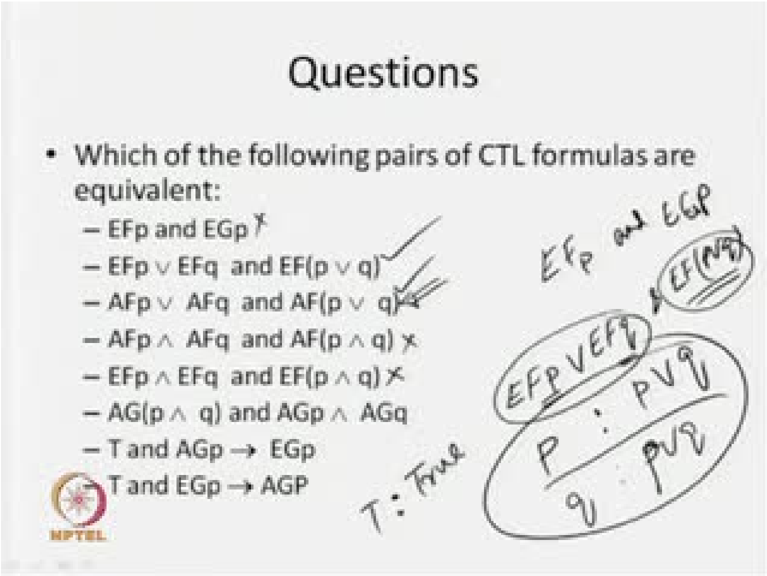

Now, so you are saying that, this is not equivalent, this is not equivalent, first one is equivalent, second and third one is equivalent. Now similarly again E F p and E F q and E F p and q. So, this is similar to the previous formula, instead of your all path, now here I am having there exist a path. Now in the previous example, previous model I have shown two different paths. Now you consider any one of this particular path, you will find that E F p and E F q may be true in a state; but, E F p and q will not be true. So, this formula is also not equivalent.

Now the next formula you see that, A G p and q and A G p and A G q. Now you try it yourself and see whether these two are equivalent or not. This is similar way, if we are going to say that these are equivalent then logically you have to establish it. You can use any properties the way use the property over here, that p is true and p or q must be true, if q is true, then p or q must be true. Like that you look for some of those particular properties and try to establish if they are equivalent.

And if you feel that they are not equivalent, then try to give a contra example. Try to come up with a model, where you will show that come formula is true in that particular state whether other formula is not true. So, if you are going to establish or going to show that these two formulas are not equivalent then you should give me a counter example; or if they are equivalent then, what you have to do? You should logically establish it.

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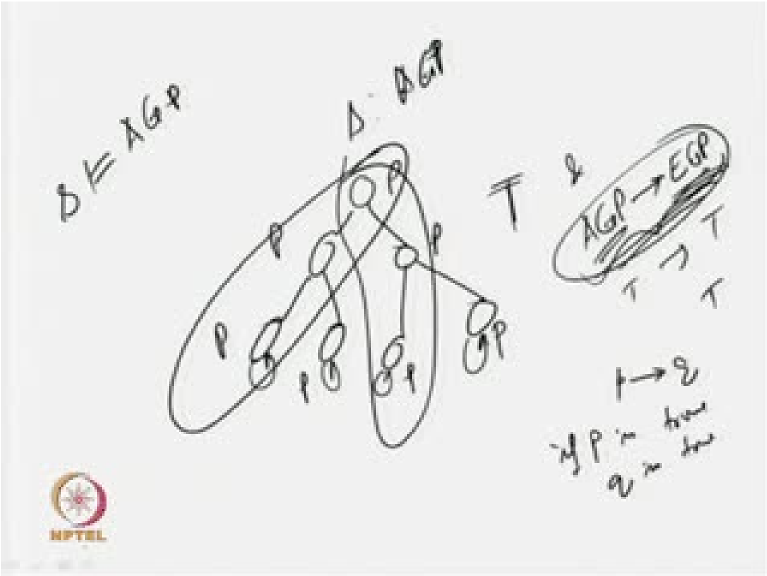

Then next formula I am saying that T and A G p implies E G p and T and E G p implies A F G p. What is this T? This is nothing but their top, I have already say that this is the truth value true. Just I will show one thing, T and second formula you are seeing that A G p implies E G p; that means true is always true. I have to check whether, this is a topology or not. If this is topology, then the truth values will be always true and in that particular case these two are equivalent. Now what will happen? Incredibly I will feel that this is a topology, because what I am saying that? A G p.

So, if I am going to say that that means p is true over here in all state, then I can say that in this particular state A G p is true. Now what is this particular implication, p implies q; if p is true then q is true. That means in all path globally p wholes, that means in states in all path globally p whose. So, in state I can say that it models, all path globally p wholes. Because if I am just going to for a sake of completion, I am going to give a self over here and you see that in all state p is true. So, I can say that e g p is true.

Now, what is a second formula is says that? There exist a path globally p wholes. So, if p true then second path second formula must be true. Now here I am having several paths. now you can such think one particular path; may be this particular path because, you know that if all path globally p is true there exist a path where p is globally true. Because we are going to get any path, where it is show; if this path is true then the second path is also true. So, this is going to give me that truth values true always so, that is why I am going to say that T true and this particular formula; because this is some sort of your topology we are going to get. Because if this is true this will be always true and true and true is going to give me always true. So, that is why I am saying that this formula and this particular truth value true are equivalent.

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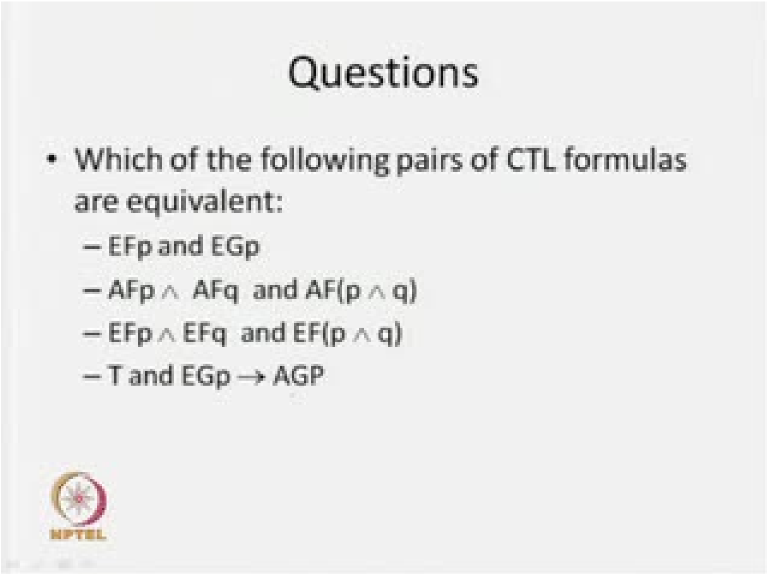

So similarly you can look for the other one also. That I am going to talk about T and E G p implies A G p. You just see whether they are equivalent or not. If they are not equivalent, then you can give a counter example; if they are equivalent then logically try to establish it. With this I will stop here today, again we will be meeting in our next class.

Thank you.