Design Verification and Test of Digital VLSI Designs
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Module - 4
Temporal Logic
Lecture - 4
Syntax and Semantics of CTL - Continued

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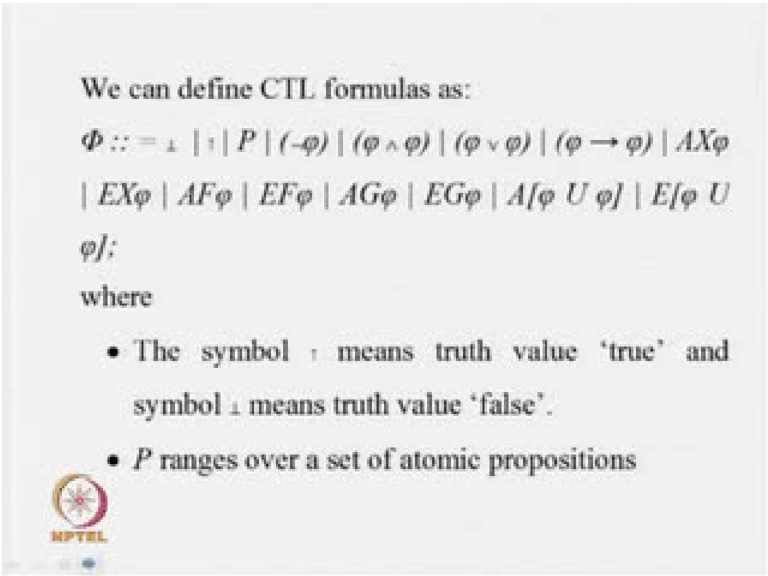

So in last class we have introduced a temporal logic call CTL, Computational Tree Logic. So what happens? We have seen, what is syntax of CTL? And we have discussed how to define the semantics of those particular CTL. So what happens? In syntax we have seen that, if P is a atomic proposition or set of atomic proposition; then all atomic proposition will be treated as your CTL formulas, along with that truth values true and false. And we have already seen that, if we are having your logical connectives like your an odd implication with all those connectives; we can again from CTL formula provide that, phi is the CTL formula.

On the other hand, we are having 4 temporal operators; basically we have discussed about 4 temporal operators; next state, future state, globally and until. So with these particular 4 temporal operators, we can from CTL formula in conjunction with your path quantifier A and E. So we are going to get these particular eight different forms of your

CTL formula; A X phi, E X phi, A F phi, E F phi, A G phi, E G phi, A phi until psi and E phi until psi.

Now, what will happen in last class? We have seen, how to define the semantics of those particular your logical connectives. Today we are going to discuss about the semantics of those particular temporal operators, along with your path quantifier A and E; and already I have mentioned that in CTL, we are going to deal with state formula that means, temporal operator preceded by your path quantifier then it becomes a state formula. We are going to define the truth values of temporal formulas, with respect to state of a model.

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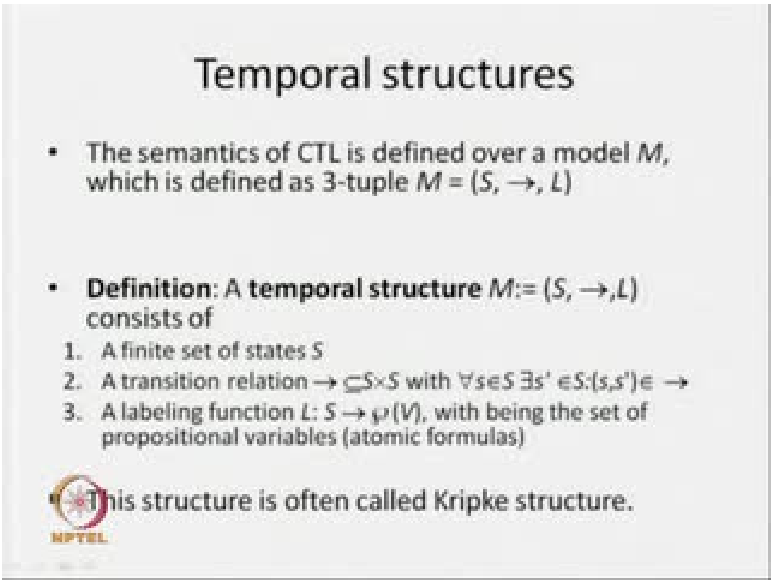

Now, how we are going to define your semantics; already I have mentioned that, to define a semantics we need a model; we call it CTL model or temporal structure or Kripke structure because, it is having three components; set of states, the transition relation and a labeling function. Already I have mentioned, what is labeling function? It is basically related to your atomic proposition, which are true in that particular state and we are going to label those particular state with the help of those particular atomic propositions.

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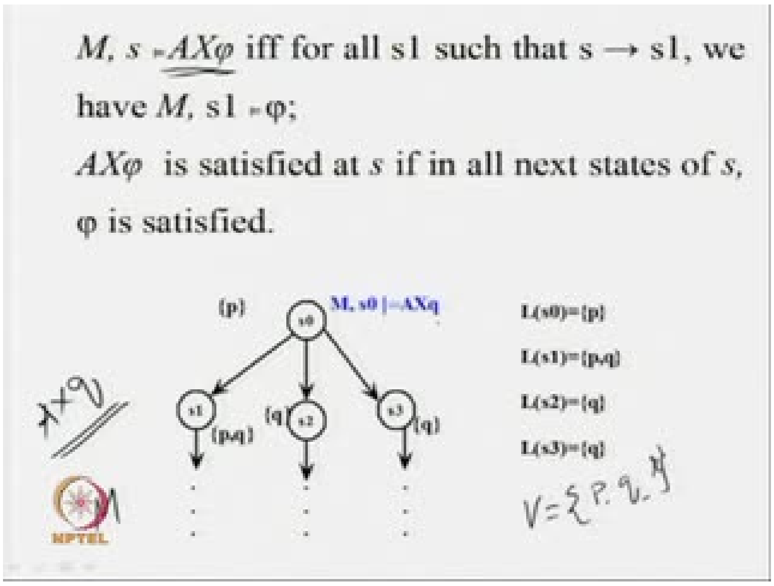

Now we are going to see how we are going to define the semantics of our temporal operator. Now first we are going to consider A X phi; this is your in all path in next state phi holds and we are going to define it in a state a, in a model M state s. We are going to said at M s models A X phi, if for all s 1 such that we are having a transition from s 1 to s 1, we have M s 1 models phi.

So you just see that, we have going consider one particular state that s. We are going to look for all next state s 1 and such that, this one in s 1 phi holds. So in next state operator, what will happen? We are going to see in next state, whether something holds or not? If we told, then we are saying that A X phi holds in this state. So A X phi is satisfied at s, if all next state of s phi is satisfied.

So just look into this particular model said, we are going to look into this particular model and these are the labeling function say L s, L s 0 is p that means, we are having a set of atomic proposition V, where I am having p q r; these are the three atomic proposition. So we are having that labeling function; it state that in s 0 p is true, in s 1 p and q is true, in s 2 q is true and s 3 q is true.

Now, we are going to look for formula A X q. So what does it means, in all paths in next state q holds or not? So we are going to look into this particular state s 0 will see; come to your s 1, this is a next state s q is true at the particular state because, it is labeled with q; if you come to s 2, again we will find that q is also true over here; similarly in s 3, q is also true. So we can say that in s 0, A X q is true. So you say that, m s 0 models A X q. So this is the way that, we are going to define the semantics of our formula A X phi. So the semantics of meaning is define with respect your model.

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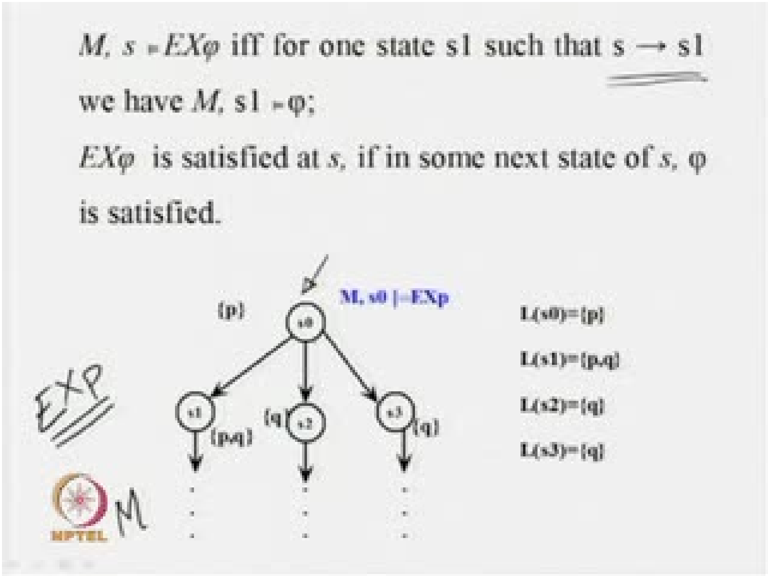

Now the next operator is your E X phi, that means they are exist a path in next state phi holds. So it is similar to your previous one, but here we are going to concern about any path not all paths. So if for some state s 1 such that, there is a transition from s to s 1. When we have M s 1 models phi that means, we are going to get at least one transition s to s 1; that means s 1 is a next state of s and if you find that in s 1 phi holds then we said that, M s models E X phi. So that is why you say that, E X phi is satisfied at s; if in some next state of s phi is satisfied.

Similarly, we are going to look into this particular model M. We are having this M labeling function l s 0, l s 1, l s 2 and l s 3 and atomic propositional p q only two atomic propositions, we are having p q. Here we will say that from s 0, we are having a transition to s 1; where key holds again s 2 again s 2 on all the process q holds, what here p holds in your s 1; so we are going to look for the formula E X p, whether there exist a path in next state p holds.

If I looking into your transition from s 0 we will see that, there is a state called s 1; where this particular formula p holds. So we said that in your state, this particular s 0, E X p holds. So we write it that in model m state as s 0, E X p holds; that means E X p is true in this particular state s 0. So this is about the behavior of our next state so, we are having two operator in temporal logic, one is your A X phi and second one is your E X phi.

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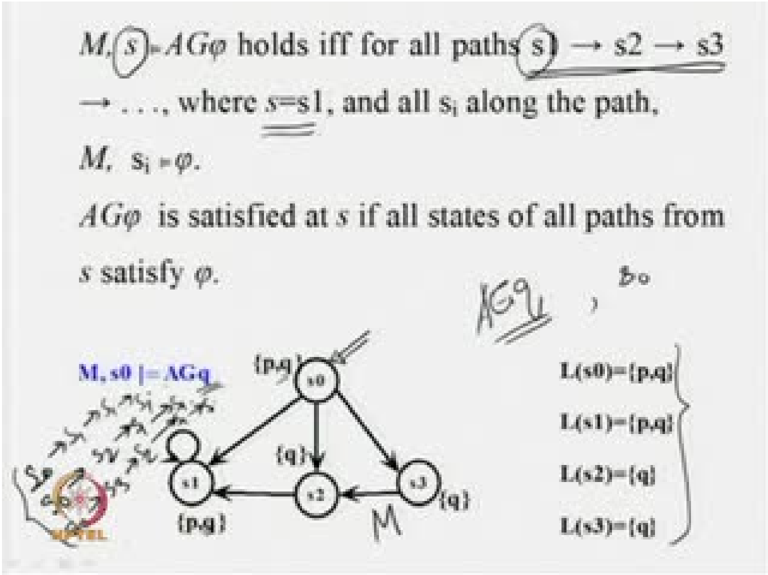

Now, next formula that we are going to look into our logic is your A G phi; G is a global operator and A G phi it says that, in all path globally phi holds or not. So you said that if it is true in a particular state, we say that in model M state s A G phi holds. So when we said that A G phi holds, if for all paths that we are going to consider all paths like that, we are having a transition from s 0, s 1 to s 2, s 2 to s 3 and like that, we are having an sequence of states.

Well we are going to say that s equal to s 1 that means, we are concern about this particular state s and we said that in all paths, that starting state s 0 is equal to your s and all s i along the path. And now we are going to consider all s i, all other states along the path such that M in this model M s i models phi that means, phi holds in this state s i and what is this s i? s i is the all state along the path; that means what it says that, A G phi will be true in a state provide that in all states along all the paths that phi holds. So you just say that, A G phi is stratified at s if all states of all paths from s satisfy phi.

Now again come to this particular model M; just say that this is our model M. We are having four state s 0, s 1, s 2 and s 3 along with this labeling function; where we are having that atomic proposition p and q. Now we are concern about these particular formula A G q and we are going to look into state s 0. Now that means, when we concentrate these particular state s 0, we will see the paths say it is having three different transitions so, we are going to get three different paths starting from this s 0; one will go something like that from s 0 to s 1, then s 1, s 1 like that.

Second one will go from s 0 to s 2; then from s 2 it will go to s 1, then it will remain in s 1 like that and another path we are having it will go from s 0, it will go to your s 3 from s 3 it will go to s 2 and from s 2, it will go to s 1 and like that it will remain in s 1. Now these are the three possible executions says that, we are going to get in this model. Now we are going to look for all such type of path s 1, s 2, s 3 like that; where s equal to s 1, now my s is your s 0.

Now from here, we are going to see all the possible state and will find that all those state s 0 and s 1 that q is true, it is labeled with q; when you come by this particular path s 0, s 2, s 1, s 1, s 1 it is having a self loops so, it will go to from s 1 to s 1. In all the states we are going to get that q is true over here and in third path, again we are going to s 0, s 3, s 2, s 1, s 1, s 1 like that and in all the states we are going to get that q is true; that means, all possible state in all paths will see that or we have seen that, phi is true or q is true in this example. So you can say that, A G q is true in this particular state s 0. So you said that in model M in state s 0, A G q holds. So this is the way, we are going to look for the meaning of your A G phi.

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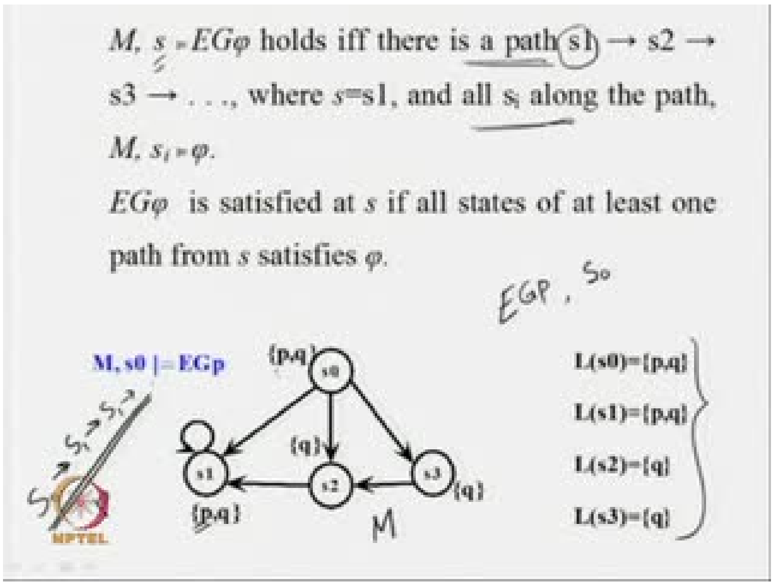

So, next formula that we are going to get is your E G phi. So it is again the global operator, but we are having the path quantifier as your E; that means, we are concern about any path, we are not going to look for all the paths. So that is why you say that, in model M in state s it models E G phi; that means you said that, E G phi holds if there is a path. Now we are going to look for any path such that, it is going from state s 1 to s 2, s 2 to s 3 like that.

Well, s equal to s 1 that means, a starting state of the path is model state; that we are looking for and all s i along the path that phi is true. So M s i models phi, where s i is your all states along the path. So that means, if we are going to look get a path, where in all states phi is hold; then we said that, E G phi holds in the states starting state of this particular path, that means s. So you said that, E G phi satisfied at s; if all states of at least one path from s satisfies phi. Now again just look into this particular model M; we are having this particular labeling function l s 0, l s 1, l s 2 and l s 3 is p q, p q, q and q.

Now we are going to look for this particular E G phi, E G p s state s 0; it means there exist a path globally p holds. Now again like previous case, we are going to get this three different path, s 0 to s 1, s 1 to s 1 and like that; again s 0 to s 2, s 2 to s 1 and it will remain in s 1; again, s 0 to s 3, s 3 to s 2 and s 2 to s 1 and in remains in while. Now we are going to look for E G p. Now, when we start from s 0, we are getting path to your s 1 in s 1, you have seen in that p is true that means, when we are going from s 0; it is your p is true then, we are coming to s 1 again p is true; then it will remain in s 1 like that. All the path if you see, here in this execution states from s 0 to s 1, s 1 to s 1, s 1 to s 1 that p is true.

But, if you look into this second path s 0 to s 2 to s 1; we will find it state s 2, where q is not true. So that means, in this particular path globally p is not true. Similarly, if we go for the path s 0, s 3, s 2, s 1 then, in state s 3 and s 2 that p is not true; that means, in this true path p is not true globally, but at least you have got one path s 0 to s 1 from s 1 to s 1, where in all the state p is true; that means, we can say that in this particular state s 0, that formula E G p is true. So we said that in model M state s 0, E G p holds. So that is why you say that, M s 0 models E G p; so we are getting at least one path where p is true globally and that is why you said that E G p is true in state s 0. So this is basically globally operator that means, in all state it must be true.

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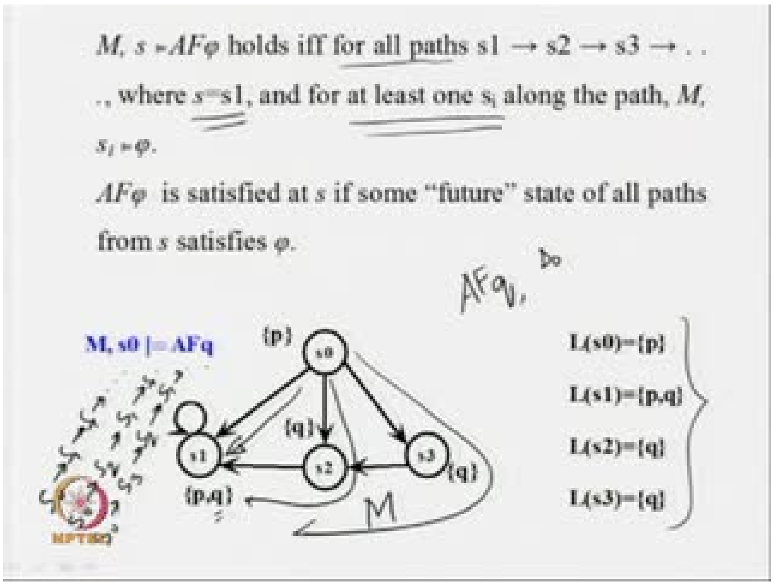

Now that next temporal operator is your F, future; already you have seen that, in some future state are eventually something must hold. So we say that you in model M s, it models F phi or F phi holds in that particular state s provide that for all paths. Now since we are going to have this particular path quantifier s; so we are going to look for all such type of path going from s 1 to s 2, s 2 to s 3 like that, where s equal to s 1; that means the starting state of all those path is the state of our concern, that we are looking for this particular state s and for at least one s i along the path. Now you have seen that, at least one s i along the path M s i models phi.

So you just see that, we are going to look for all paths in all paths you have saying that, eventually phi must true. That means we are going to look for some state whether phi holds over here or not? In that case, if this is going to happen in all paths; then we are going to say that, A F phi holds in this particular state s. Now again look for this particular model M, we are having four state s 0, s 1, s 2 and s 3 and this is the labeling function of this particular four state s 0, s 1, s 2 and s 3.

Now, we are concern about this particular formula A F q in state s 0. Now again we are going to get the path s 0 to s 1, s 1 it will remain in s 1 because, we are having a self flow; similarly, we are going to have s 0 to s 2 and we are having a transition from s 2 to s 1, then s 1 like this. Again in another path from s 0, this is going from s 0 to s 3 from s

3; we are having a transition to s 2 from s 2, s 1 like that. So these are the three possible paths.

Now we are going to look for E A F q that means, whether we are going to get a future state where q hold; obviously, you see that when I go by this particular path, we are getting a state s 1 where q is true. When I will go by this particular path, again will find that in s 2, q is true; again if I follow this particular path, we will find that again q is true in a state; that means, in all three paths in some future state we are going to that q s true. So you just say that, here it happens to be in the very next state, but it may happen that in next state it is not true; but in some future state it is true. Then we are going to say that, in all paths in future q holds so, we say that M s 0 models A F q. So we are going to look for this particular formula q, which is true in some future state along all the paths.

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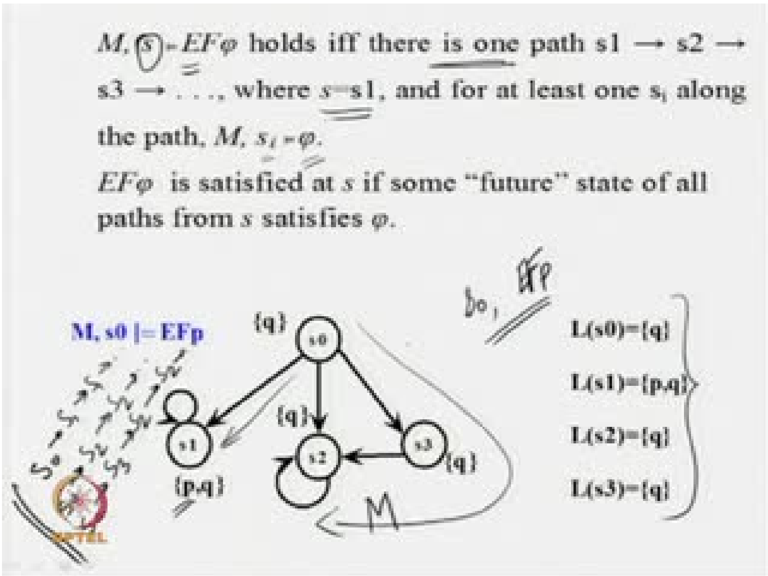

Now next formula is again with this future operator, but with this path quantifier E that means, there exist a path in future phi holds. So again it is similar to this thing we are going to concern about any one path; we are not going to look for all the path, but if it is true it any one path, then we are going to say that E F phi is true.

So again we will say that, we are going to look any one path going from s 1 to s 2, s 2 to s 3; where s equal to s 1 that means, starting state of the path is equal or is the path state of our concerned s and for at least 1 s i along that path and s i model phi that means, phi holds in some state s i along that particular path. In that particular case, we are going to say that E F phi holds in that particular state.

Again look for this particular model M and my state of concern is your s 0 and the formula is your E F p; there exist a path in future p holds and this is the labeling function that we have q, p q in your state s 1 in s 2 q is true and s 3 q is true. Now you just see that, we again we are having those particular path s 0 to s 1 to s 1 like that; we are having an influence sequence, second path is from s 0 to s 2 from there is transition from s 2 like that I have straightly sense the model.

Now, again third path we are going to get s 0 to s 3 from s 3; we are having a transition to s 2, then we are having the transition from s 2 to s 2 like that. These are the three possible paths, that we are going to get from this particular state s 0. Now if you look these two paths, say from s 0 to s 2 and it will remain in s 2 so, we are going to get the sequence s 0 s 2 from s 2 to s 2 like that; in none of the future state p is true that means, in this particular path we are not going to say that, p will be true in future.

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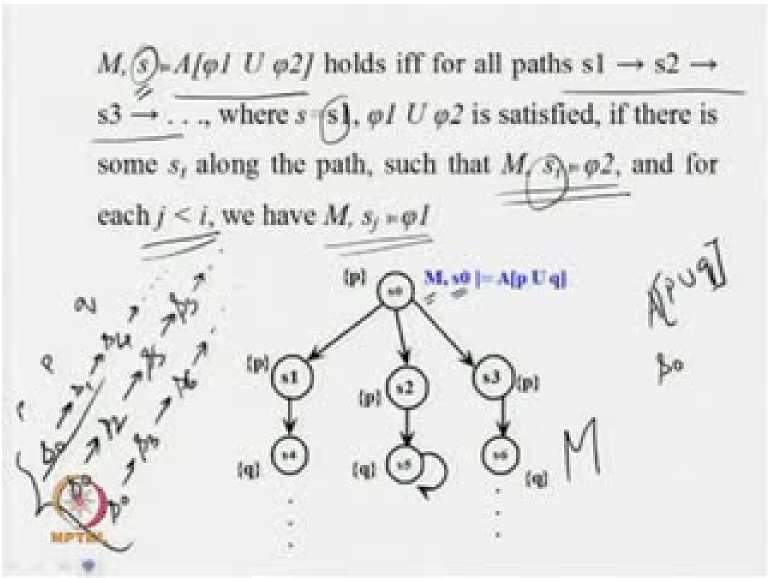

Similarly, the path s 0, s 3, s 2 if we consider this particular path again we will see that, we are not going to get any future state where p is true, but if you follow this particular path s 0 to s 1 to s 1 like that, then that s 1 is marked with your p that means p is over here; that means, we are going to get at least one future state where p is true. So due to this particular path we say that, in s 0, E F p is true. So this is the way that we are going to define the truth values of E F p.

Now, the next operator is your until because, already I have mentioned that, the three operator that we have next state future and global, these are unary operator it works with one particular formula, but until is a binary operator. So it works with two temporal formula so, we are going to say that, phi 1 until phi 2 and this until operator must be preceded by path quantifier, either A or E in case of your CTL.

So that is why, we are going to look into this particular formula A phi 1 until phi 2; that means in all path phi 1 remains true until phi 2 becomes true. So we said that, these formula holds in a state s of model M provide that, for all such type of paths that we are going to look having a transitional from s 1 to s 2, s 2 to s 3; where this particular starting state of the path s 1 is equal to the state, that you are looking for and we said that phi 1 until phi 2 is satisfied, if there is some s i along the path.

Now we are going to look for those particular paths such that, M of s i models phi 2; that means we are going to get some state s i, where phi 2 is true and for each j is less than i. So we are going to get one particular state s i, where phi 2 is true. Now look for all other state s i where, s j where j is less than i; we must have an s j models phi 1 that means, phi 1 must be true all those particular state s j.

Now just consider this particular model M over here and we are looking for this particular formula A P until q in state s 0. Now in this particular case you just see that, if I am having the just now execution state something like that, from s 0 it is coming to s 1 from s 1 we are coming to s 4. So we are having an execution state s 0 to s 1, s 1 to s 4 like that, it is an infinite path. Second one, we are looking for this particular path s 0 is having a transition to s 2; s 3 is having a transition to s 5, in s 5 we are having a self group so, we are having a transition to s 5 like that. Again we are having another path from s 0, this is your s 0 is going to your s 3 and from s 3 We are having a transition to s 6 and it is having a path form s 6.

Now if you look into this thing we are concern about A P until q; that means, we have to look for all these three paths and we are going to look for one future state where q is true. Now in this particular first path, we are going to say that q is true in your s 4 and in that particular case, if q is true in some state then we have to look for all preceding state of those particular state in that particular path and we have see, we have to see whether p holds over there or not? So will find that, p is true in s 1 and p is true in your s 0.

That means in this particular path p until q is true, with the similar reasoning you can look for the other two paths also s 0 to s 2, s 2 to s 5; we will find that, p remains true until q becomes true in this particular path and similarly, in third path also will find that p remains true until q becomes true. So we have seen in all these three path p until q is true that means, we can say that, in is that these three paths are coming out from these particular state s 0. So we can say that, in state s 0 of model M A p until q is true; this is the way that, we are going to look for the true values of the formula A phi 1 until phi 2 or A p until q.

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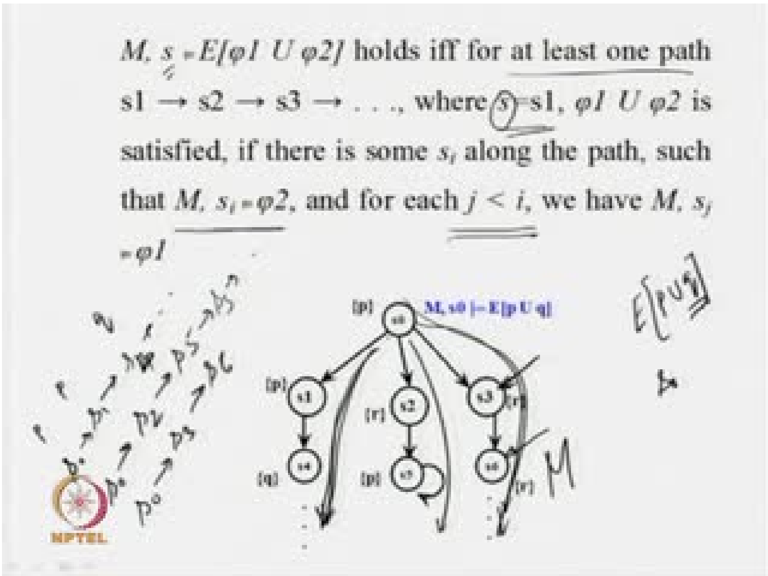

So similarly, now we are going to look for the last formula, where it is with until operator at the path quantifier E; that means we are concern about any path there exist a path so, that phi 1 until phi 2 holds from this particular in a particular state s. Again in this particular case what will happen? We are going to look for at least one path such that, it is going from s 1 to s 2 from s 2 to s 3. Where s equal to s 1 that means, the starting state of this particular path is our concern state s such that, phi 1 until phi 2 is satisfied in this particular group; What does it means? That means, we are going to get a state s i, where phi 2 is true and all the preceding state in the path from this s i phi 1 must be true; that means, for each j is less than i M s j model phi 1.

So, this is similar to your phi 1 until phi 2, but here we are concern about one particular path, any particular path. That is why we said that, there exists a path with E quantifier. So again just look for this particular model M. We are having this particular label, this is with respect to your labeling function. So we are going to look for the formula E p until q in state s 0. Again similar to our previous case, we are going to get three path s 0 to s 1 to your s 2 then, s 0 to sorry this is your s 4, then s 2, then it is going to s 5 and it remain in s 5 then, third path is your s 0 to s 3 then from s 3 to s 6.

Now, we are going to look for p until q so, if you look into this particular path at least since it is a infinite sequence, but we are not getting any states where q is true; similarly, if you look for this particular path, this is again going from s 5 to s 5. So there is no possibility that, we are going to get any state where q is true. So here we may get somewhere q is true, but in this particular state previous state, that p is not true at least here p is not true.

So if you look into this particular path will find that we are getting s 4, where q is true and it is having two particular state s 0 and s 1 so, in this two preceding state p is true. So that means we are getting this particular path, where p until p is true, but in other two path p until q is not true. Here it is obvious that p until q is not true because, we are not getting any state where q will be true, but since in this particular path we are showing an infinite sequence that means, in far behind we may get a state where q is true, but at least we will find this particular state where p is not true; secondly s 6 p is not true that means, p will not remain to until q becomes true.

So it is also avoided that in this particular path, we do not it is not going to satisfy E p until q. So the in this particular case at least we have got one path so, from that we can come to that in this particular state s 0, p E p until q is true; so this is the way that we are going to define the meaning of our CTL formula.

Now till now what we have discussed? We have seen one particular temporal by logic, which is known your CTL computational tree logic. Generally is the truth values of temporal logic is defined over, if model which is having transition and states, but in case of CTL basically we have already said that, this particular model can be unfold to a tree and the semantics is basically define on a tree. So we are saying that, this is your computational tree logic.

And we have seen that four temporal operator, that we have along with two path quantifier A and E; we are going to get a defined possible temporal operator in case of CTL and all temporal operator must be preceded by a path quantifier and due to that reasons the formulas of CTL becomes our state formula. So it defined the truth values of a formula with respect to a state and this is due to the presence of this particular path quantifier.

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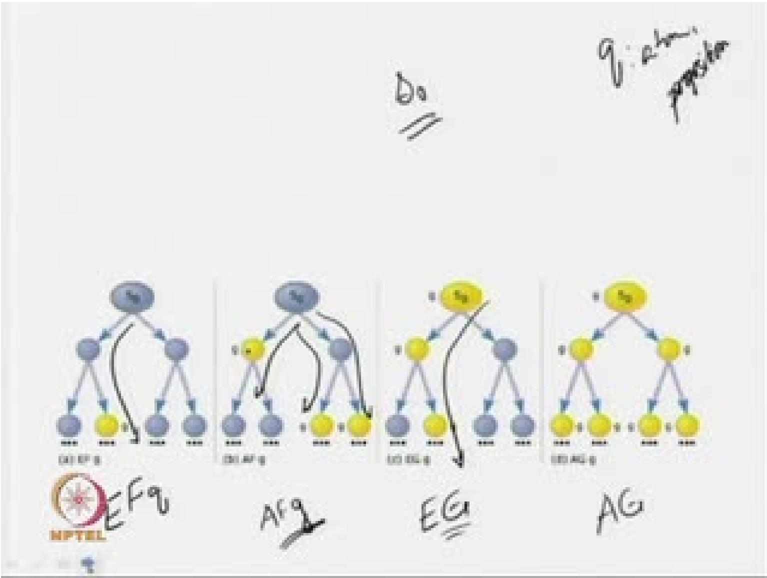

So now just see that, what will happen? Just I am showing here one diagram that we are looking for some formulas showing fix to really say we are concern about this particular state s 0; we are having that atomic proposition q, q is an atomic proposition. Now in the first diagram tree, we have look for there exist a path in future q holds; wherever q is true just we have colored it with yellow so, will find at least one path where in future q is true. In second one I have said that, A F g sorry this is true G. So in this particular case we will find that whether in all path in future, we use to unknown. So we are looking for this particular path, it is starting from at least we are getting one state where it is true and other two path we are having this way. So in future we are getting this particular q is true so, we can say that A g, A F g is true; that is your there exist a path whether globally something whole or not.

So this is the path where everything is marked with yellow that means, this property is true in all this particular state of this particular path. So we can say that E G is true and forth one is your A G all paths globally that means, this particular formula is true all the states you just see that; if it is marked like that, then you can simply said that A G is 2 over here because, in all path globally something is true that particular G is true so, that is why it is marked like that. So by looking into these particular tree structures very well you can see whether particular formula is true in a particular state or not here. We are going to look for this particular state s 0 starting state of this three eventually.

So like that we can see whether particular formula is tree, true in a particular model or not. Now after defining the syntax and semantics of CTL formulas, now will see some example to make it more clear; before proceeding pardon because, you will be knowing this.

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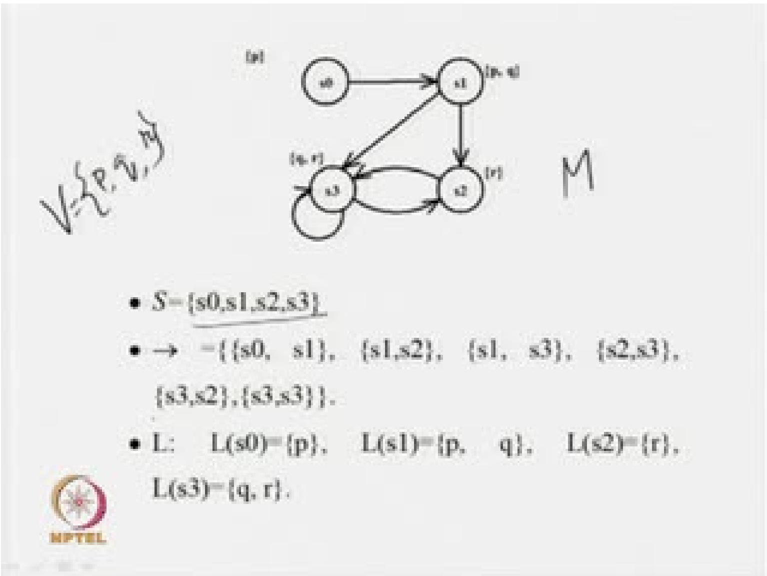

Now just see that, consider this particular model M; it is a model what is model is having three components basically first one is set of states, second one is the transition relation and third one is your labeling function. So in this particular model L, say s is a set of state so, we are having four particular state s 0, s 1, s 2 and s 3 and this transition relation how many transition relation we have? 1, 2, 3, 4, 5, 6 total six transition relation we are having.

So first one is having a transition from s 0 to s 1, then s 1 to s 2, s 1 to s 3 then, s 2 to s 3 from s 3 to s 2 and we are having a self group s 3 to s 3 and if you see that, we having one more condition; where it says that, the transition relation must be complete. What does it means? It says that every state must have a successor state. If you look into this particular model, you will find that all those particular four states are having a successor state; because, from all the state we are having at least one outgoing as s.

And this is the labeling function, now what will happen in this model; we are having the set of atomic proposition p as your p q and r. These are the state of atomic, these are the atomic proposition that we have. Now the labeling function says that L of s 0 p, what does it means? That means, the atomic proposition p is true in s 0 and indirectly it means that, the atomic proposition q and r are falls in this particular state s 0.

Similarly in state s 1, the labeling function says that p and q that means the atomic proposition p and q is true in your s 1. Labeling function s 2 says that r that means, it says that the r is true in this particular state s 2 and labeling function of s 3 says q and r; that means the atomic proposition q and r is true is s 3, but atomic proposition p is false over here. If it is false, we are not directly marking it, but indirectly it says that q is false. Now we are concern about this particular model and these are the three components, now will see some examples over here with respect to this model.

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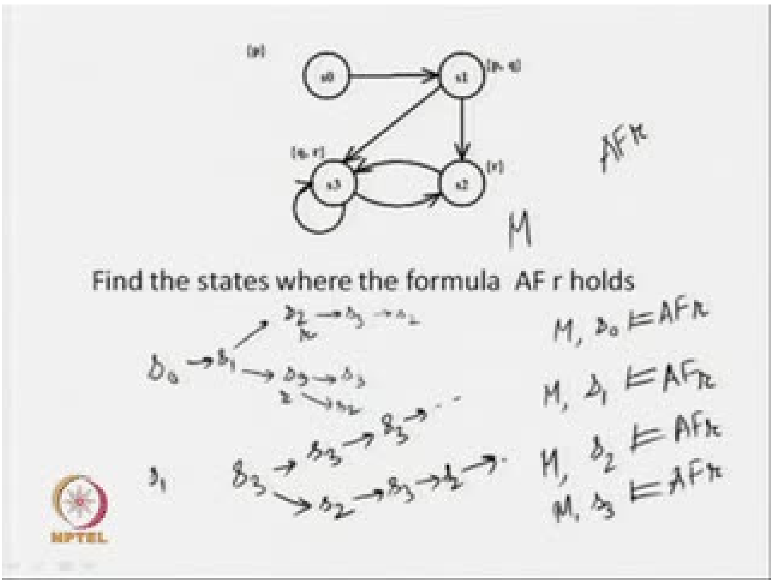

Now the first question I say that find the states, where the formula A F r holds; that means, in all path in future r holds. Now we have to see it is a state formula, now we are going to look for the state s 0 and they said that, this is my model M. So in s 0 if I go for s 0, it is having this particular path only one path is eliminating and we are getting that s

1 going from s 0 to s 1 then from s 1;. I am having two possibilities, either I am going to s 2 and s 3. Now if, these are the and again from s 2, it is going to s 3 and from s 3 again it is coming back to s 2 like that and from s 3 it remain in your s 3 or it may have another 1 also, it may go to s 2 like that, but now you just see that in all path in future whether r holds or not. So if you see look for all those particular paths will find that in one future state this s 2 r holds and we are going to get one future state r holds; that means you can say that in this model M s 0 that in all path M future r holds.

Now similarly, you look for the state s 1; if you look into this particular state s 1, then will find that it is having two paths and in future path in future state r is true. So in s 3, r is true and s 2 also r is true so, in the entire possible path in future, we are going to state where r is true.

So we can say that in this model M s 1, A F r is true. Similarly, when I come to your s 2, then will find that form s 2 it is going to s 3, where r is true coming back to s 2 and again going to s 3 that means, all path we are going to get state in future r is true. So again we can say that, in your M s 2 it models A F r. Now when you come back to this particular state s 3 so, basically what will happen? From s 3 we are having a transition to s 3, s 3 like that or we can have that from s 3; you are going to s 2 coming back to s 3 again going to s 2 like that. So we will say that, all those particular examples in future state r is true.

So will say that in model M is s 2, it also models A F r in all path in future r is true. Now individually you have gone to each and every state and we have found a these formula A F r is true in all those particular four states. So this is a way that we have to look into it because, now CTL formula the state formula, that true values of CTL formula is define with respect to a state only.

Now this is a same model I am having over here and now we are going to see the formula find the states, where the formula A G A F r is true or not? What does it means, in all path in future r holds and it says that, in all path globally whether these particular formula holds or not? So now what we can say that, r is an atomic proposition so, will say that r is a CTL formula so, we know the truth values of r in each and every state; this is with respect to your labeling function, then F is a temporal operator so, F for each says that in future r holds or not and A says that, whether in all path in future r holds. So this is A F r is also CTL formula; we can say that, this is your phi is CTL formula.

So if phi is a CTL formula, then will find that A G phi is also CTL formula. So that means, A G A F r is a CTL formula. Now we have to say, whether this CTL formula is true in all states or not, whether it is true in some states or all state in this particular case or not. Now basically you just say that, I am coming to that state s 0; now will see that from s 0, whether it holds in all possible states in all possible direction or not. So just let us go back to my previous example; in previous example what we have seen that, this A F r is true in all those particular four states, we have seen over here. Now what does it means? Since A F r is true in all state that means I can label those particular states with this particular formula A F r.

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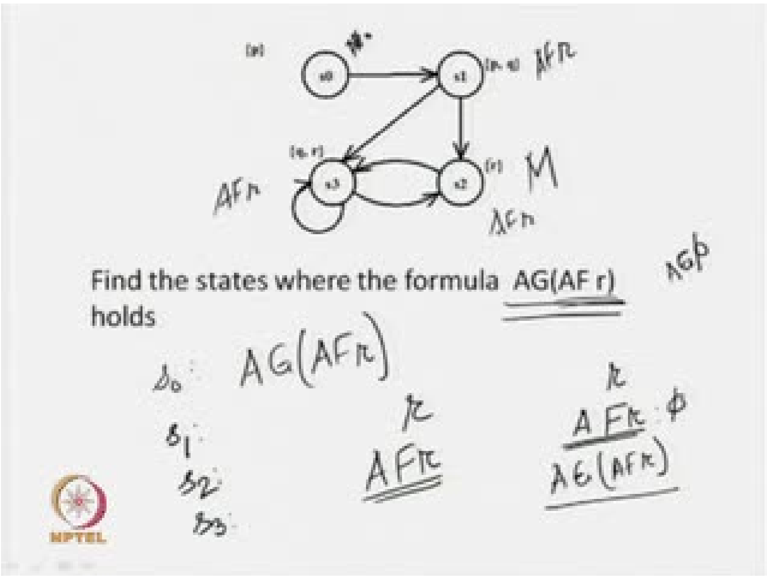

So now you can say that in state s 0, A F r is true in state s 1; A F r is true in state s 2; A F r is true in state s 3; A F r is true. Now, it says that in all path globally A F r is true; now from s 0, if you go in all possible direction will find that A F r is true. So we can say that, in s 0 in all path globally A F r is true. So with similar logic, what we can say that? When you start from s 1, then we look for all possible path, all possible combination and wherever I will go, I will find that A F r is true everywhere. So we can say that, A G A F r is true in s 1 also, which similar logic we can say that, it is true in s 2 as well as it is true in state s 3 also, that means in all those particular course that of this particular model M A G A F r is true.

So when we are going to look for the truth values of a particular CTL formula. First of all we have to know the truth values of is sub formula. So basically here, if I consider this particular formula A G A F r it is having two sub formulas, one is r; we must know the truth values of r in this all states. Then another sub formula we are having A F r before going to A G A F r, we must know the truth values of A F r in each and every state; then only we can look for the truth values of the given formula A G A F r. So after analyzing this particular model, we have found at A G A F r is true in all those particular four state.

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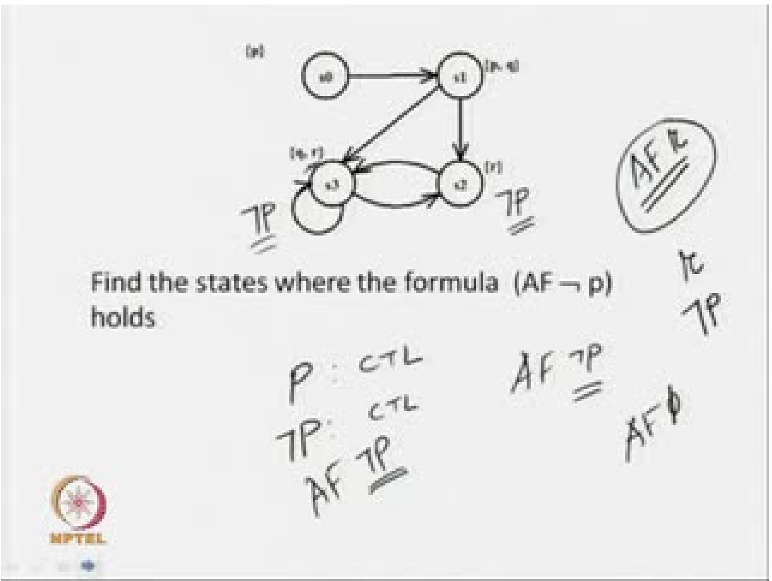

Now this is simple one that I am again saying now, we are going to look for A F knot of p. So this is similar to our A F r only; just we are saying that, it is knot of p. Now how I am going to look into with say, p is a atomic proposition so, p is a CTL formula because, its atomic proposition is a CTL formula. Now if phi is a CTL formula, then knot of phi will also be a CTL formula. So I can say that, knot of p is also a CTL formula.

Now if we are having any phi as a CTL formula, then will say that A F phi is also a CTL formula; so we will get that A F knot of p is also a CTL formula. Now what about this particular knot of p? You are going get the truth values of knot of p from our labeling function only; if it is not label with a particular atomic proposition will say that, these particular atomic proposition means falls in that particular state.

Now you just see that in your s 0 p is true so, p remains 2 over here; in s 1 p and q is true, that means p is true over here. In s 2 it is labeled with r so, what does it mean? It says that, q and r is not true over here. So we can say that, knot of p is true over here similarly, in s 3 q and r is true; it is labeled with q and r so, it is not labeled with q. So we are going to say that, knot of p is true over here.

Now you just see that, already I have analyze this particular formula in all path in future r is true or not. When we have analyzed this particular formula, we have found in all those particular state A F r is true because in future, we are going to get one state, when r is true. So this is the same behavior we are going to get, say in s 2 r is true similarly, knot of p is also truth value; in s 3 r is true, here also knot of p is true. So the behavior of this model is same with respect your the atomic proposition r and knot of p, since A F r is true in all though a four states of this particular model. Similarly, we can compute that, that knot of A F knot of p is also true it all those particular four states of this particular model. So this is the way that we have to see, whether some CTL formula in true in some models or not.

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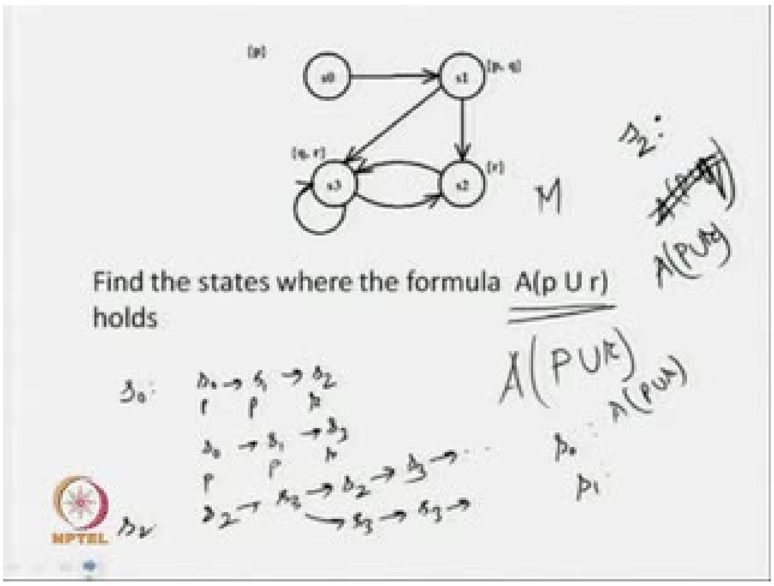

Now just look for this particular model, again that this is the model M and we are consign about this particular formula A p until r, p remains true until r becomes true. So again we have to see all the states so, from s 0 what will happen? It will go to s 1 say in s 0 p is true, s 0 p is true then s 1 p is true, then we are going to get s 2, where r is true; this is one execution path. Similarly, another path we are going to get s 0, s 1 and s 3. So again in s 0 p is true, s 1 p is true and s 3 r is true. So we are getting some state in future, where r is true and in all the preceding state of this particular path all those particular path p is true. So we can say that in s 0, that A p until r is true.

Similarly, if I look into s 1, it is simpler than the previous one because, from s 1 we are getting two state s 2 and s 3, where your r is true and the preceding state in s 1 p is true; so you can say that in s 1 also, p until r is true. Now we look into state s 2, so it is going from in s 2, if you say we are having a path from s 2 to s 3, s 3 to s 2 and s 3 to s 2 to s 3 like that, we are having this particular path along with that when I am coming to s 3; again it with remain in your s 3, s 3 like that. So these are the possible execution states.

Now when we go for your s 3, then from s 2 I am getting these are the execution trace from s 3 we are getting these are the execution trace because, one is sub set of other word; we can say that, this is the sub sequence of this particular path from s 2. Now what will be the truth values of your p A, p until q? If you look sorry we are concern about in all path p until r, this is the formula.

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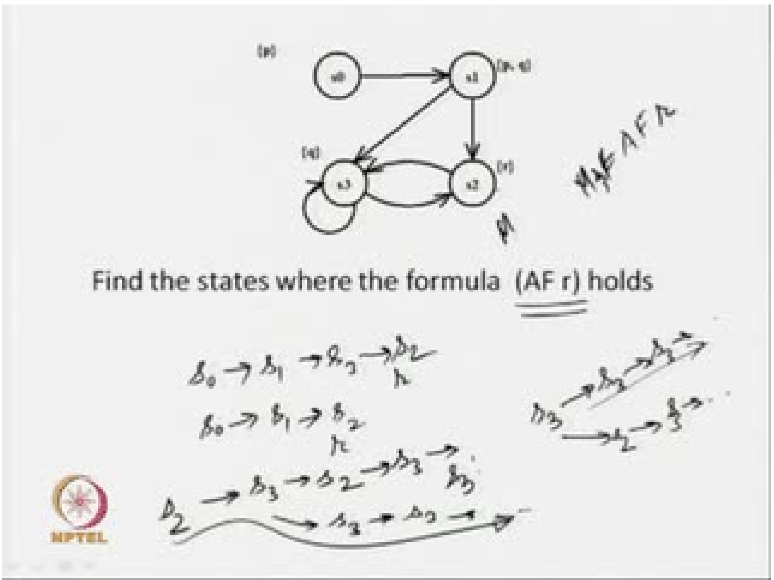

Now what are the truth values of this particular formula in state s 2 and s 3? If you look into it, you simply say that p is none where it is true in all this particular path. Now, how or what will be the truth values of your A p until r in state s 2 and s 3? I will explain it, after explain some more things to you; will see what will be truth values of A p until r in state s 2 and s 3? So I am not saying anything whether it is true or false at that particular movement; I will come back to it.

Similarly the next formula again you see that, I am talking about in all path in future r holds or not. Now here, I have slightly sensed a labeling function in s 3; I am having only q so, we have talking about A F r. Now again similarly if you say that, this is the same model slight difference. Now if I come to this particular point A F r, then you will find that from s 0, we are going for all execution path and we will find that in some future state; we are going to get r is true because, this is a part where r is true, this is another path where r is true. So for basically s 0, s 1, s 3, s 2; so r is true over here or s 0, s 1 then s 2; so r is true over here. So we are getting two possible paths and in future state r is 1.

Similarly from s 1 also, it is sub path of this particular path coming out from s 0 so, again we are having that it is true over here. Now what will happen in this particular case, if I am going to look for s 3. So we are going to get a path from s 3, it will remain in s 3 like that or maybe it will go from s 3 to s 2, s 2 to s 3 like that. So if you go this particular path will find that in future r is not true; so that means, this particular formula A F r is not true it in model M, in state s 3; it does not model A F r because, we are getting one path where r in not true.

Now, when we start from this particular state s 2, then what will happen from s 2? We are getting going to s 3, then we are going to s 2, again s 3, like that or maybe it is having that it is from s 2, we have going to s 3, then it will remain in your s 3. So here you just see that, in this particular path somewhere again we are coming back to s 2 and r is true over here, but if you go this particular path s 2 is not true. So in this particular case what happens? I am not going to whether we can conclude that, whether this A F r is true in s 2 are not? Because, we are getting this particular path where in future, we are not going to get any future state where r is true, but I am not going to say whether, A F r is true at that particular state s 2 or not.

I will come to you like that previous example I have say that, I am not going to say anything about the truth values of that particular formula in that s 2. Similarly, for A F r, I am not going to talk anything about now truth values of this particular formula in s 2, but I will come back I am see what will happen now?

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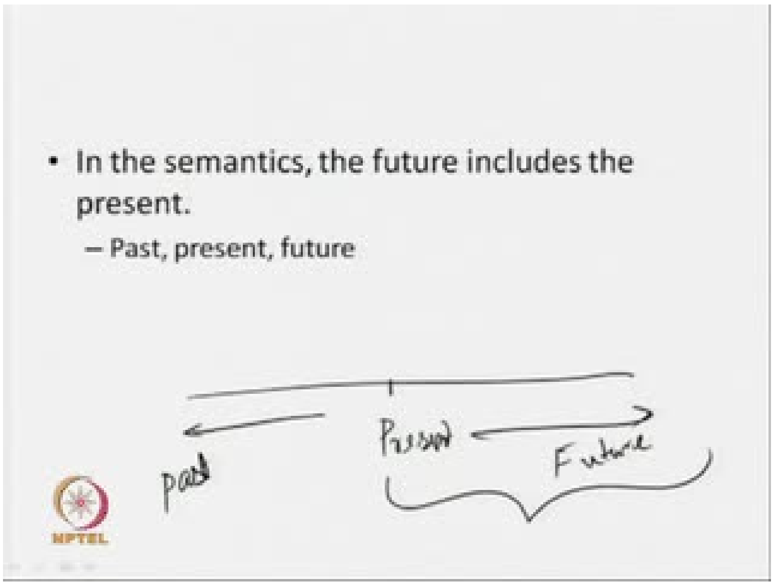

Now in this particular case, now what will happen? Now we are going to talk about the timing behavior of our system. Now if you look into the time line, basically we are having three particular tenses past, present and future. So if we are in a present state, then we can region about what is, what happen in the earlier state of past or we can try to design our that, what is going to happen in future? But, now I am in the present state. Now what will happen? Now we are having three these things past, present and future, but the way we are defining the semantics of CTL; it says that, the future includes the present that means, if you look into this particular timeline; if this is my your present state, these are my past and from present I am going for future; then what will happen? The way that we have the semantic of CTL, it says that the future includes the present; that means the present states also included in the future behavior. This is because of defining the syntax, that we have defined the syntax. So that means, we are having only two nouns; one is past behavior and one is future behavior when future behavior, the present is included.

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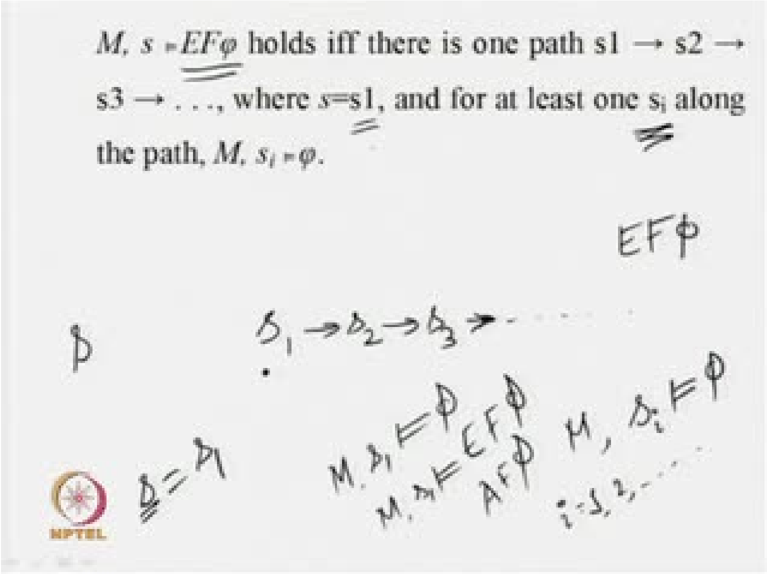

Now see why it happens? One simple example I am going to say that, we are going to look into that E F q. How he have defined this particular E F q or E F psi. So we are going to look further transition s 1 to s 2 to s 3 like that, we are going to look for one transition path and we are looking for one particular state s and what we say that s equal to s 1 that means, our state of concern is s and we are saying that, this is s equal to s 1; that means, starting state of these particular path and for at least one s i. Now we are going to look for one s i, where M s i models your phi.

Now we are going to look for one s i, but in this s i we are not excluding this particular one. So I made where is from now 1, 2, 3 like that, all possible values conveyed because, we are not excluding this particular 1 over here. So what will happen? That we are getting M s 1, if it models phi then what will happen? As for this particular definition, M s also models this particular E F phi. So according to this particular definition, M s 1 also models E F phi.

So see this is due to the definition of the semantics. So if a particular formula holds in a state will say that, E F phi holds in that particular states also. Similarly, A F phi will also holds in that particular state if phi holds in that particular state because, this is the way we have define the semantics because, we have not excluded this particular s 1 from this particular any assign. So this is the way that we have defined so, that is why that present state also includes in the future behavior.

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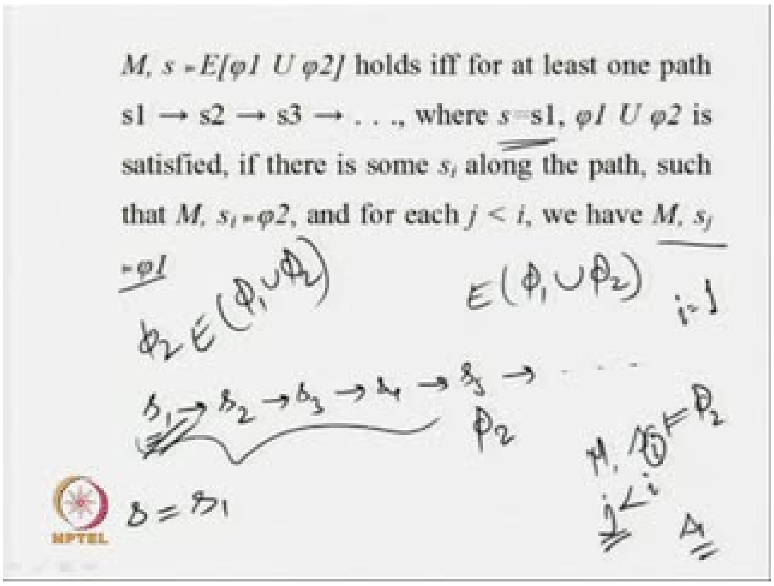

Now see the another one that E phi 1 until phi 2. Again what happens? We said that, we are going to take any execution trace s 1, s 2, s 3, s 4 like that s 5. Now what we are saying that? We are concern about a particular state s and we are saying that, s is equal to s 1; the starting state of this particular path. We are saying that s 1 and what we are saying that Phi E, phi 1 until phi 2 will hold in this particular state s provide that, we are going to get some s i which models your phi 2. Just say that, here in this particular phi 2 models and all j less than i, we should have M s j models phi 1. So phi 1 must be true in all those particular states.

Now what is this particular j? You just say that, j is all preceding states of this particular part. We are not excluding s 1 from here, we are not saying that all j i apart from s 1 is true. So in this particular case what will happen? It is not over here, but we are going to talk about this particular i, when I talking about i; we are not excluding one from this particular i also. So i maybe 1 also so, i may be one also. So in this particular case what will happen in s 1? E phi to holds at that particular state; then what will happen? We will find that, as according to this particular definition of particular semantics E phi 1 until phi 2 holds in this particular state s 1 also.

So see that, that means if phi 2 holds in a particular state will say that, E phi 1 until phi 2 holds over here. Similarly, A phi 1 until phi 2 holds in that particular state. So this is because of the definition of this semantics. So the semantics that we have define over here, says that the present state includes in the future behavior also, but if we want to exclude the present behavior from the future, then accordingly we have to define the semantics. Now you just take it as a task or as a homo and you just try to define the semantics of this CTL formula, where the future behavior is going to exclude the present behavior.

Just see it is simple the way I have explain over here because, here i equal to 1; we have not to exclude it. So what will happen? If i equal to 1, then it says that in s 1; if phi 2 holds then will get that E phi 1 until phi 2 holds in s 1. Now i have given your task what you have to do, try to define the semantics of CTL formula in such a way that, the future behavior excludes the present scenario. So how to define the future semantics for that CTL formula, it is simple task you can do it.

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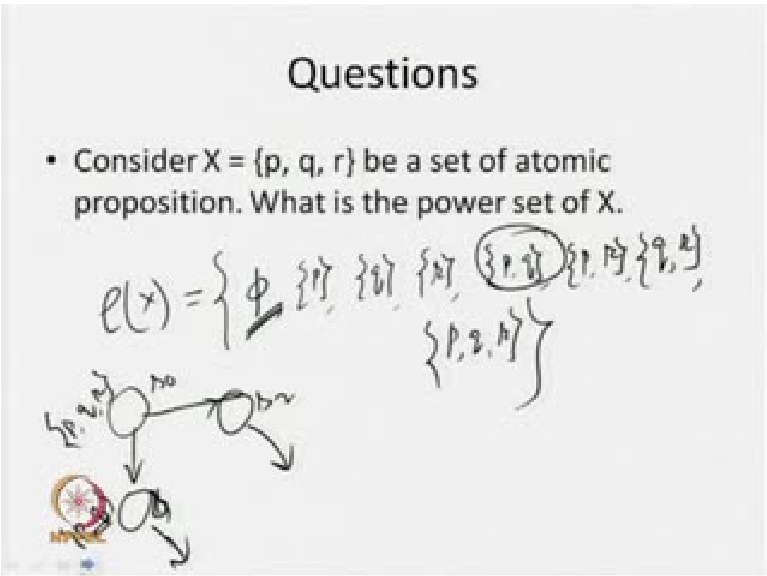

Now in this particular lecture, will going to see some questions, simple question I am going to consider. The first question I am saying that, consider a set of your atomic proposition x, where we having three atomic proposition p q r so, this is the set of atomic proposition. Now we are saying that, what is the power set of x? Why you need it because, the labeling function we have seen that, the states will be label by the member of the power set of our atomic proposition. So it is just very simple, now if x is your p q r, then power set of x will remain; how many elements will have? I think you know it is very well that we are having eight elements. So one is yours the null set, then only p, only q, only r, maybe p q, p r or q r and the complete set p q r. So this is the power set of x; 1, 2, 3, 4, 5, 6, 7, 8. These are the all possible combination and we say that, this is the power set of x.

Now, when we look for any state say, if you are having state transition say s 0 to s 1, s 1 to s 2 like that; then if you say that, this particular s 1 is labeled with say p and q. So what does it means, that atomic proposition p and q is true in this particular state s 1, but r is not true. So what is the p and q? It is a member of these particular powers; that means the states will be labeled by the member of power set of our task given atomic set. So x is set of atomic proposition p q r so, these are the component of this power set and the labeling function is going to pick up any member of this particular power set because, it says that these are the atomic proposition true in this particular model.

If s 2, if say that it is not marked with any atomic proposition that means, what says that we are going to take this particular member of phi; it says that it is null set, none of the atomic proposition is true in this particular set s 2 and if says that p q r is marked over here in as way it says that, all these three atomic proposition is true over here.

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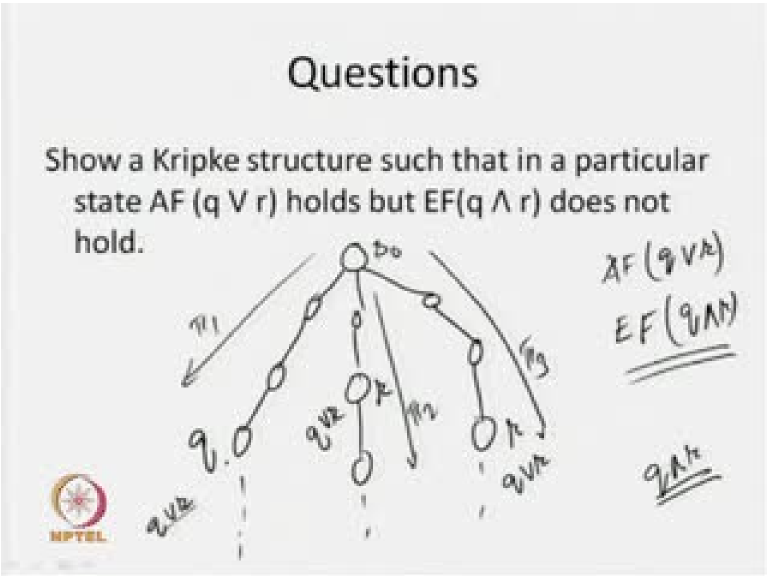

Next question; what I am saying that, show a Kripke structure such that in a particular state E X p or r holds, but E X p q and r does not holds. That means, what we are looking for say there exist a path in next state either q or r holds or I am saying that, but there exist a path in next state q and r is not hold. So this is very simple say I can say that, this is the state s 0 since it is your next state, we have to look for one particular next state only.

If I say that it is labeled with q and it is labeled with r, then what will happen? There exist a path in next state q or r holds. So if you say that since q is true over here, what we can say that q or r is also true over here. So we can say that at least we are getting one state so s 0, there exist a path in next state q or r is true, but if you look into this particular next state, it is q is true but r is not true; in this particular state r is true, but q is not true. So q and r is not true in any one of this particular state. So that means in this particular model in s 0, there exist a path in next state q and r is not true.

So we are saying that, show a Kripke structure such that in a particular state; in all part in future q or r is true, but there exist a path in future q and r is not hold. So what we are saying that, in all path increase q or r is true, but there exist a path in future q and r is not true. You just look for this particular models say s 0, say that we are having three possible path.

Now here if you see these particular things, then what will happen? We are having three execution traces from s 0, this is the first path say phi 1, this is the second path phi 2 and says this is the third path phi 3. Now if you follow this particular path we are getting one state in future, where q is true. So I can say that q or r is true in this particular state. Similarly in phi 2 also we are getting one state where r is true so, in this particular state I can say that q or r is true over here. Similarly in the third path phi 3 we are getting one state where r is true, I can say that q and r is also true over here. So in all these three perform s 0, I am having three possible paths; so in the three paths we have seen that, we are getting one future state where q and r is true. So we can say that in s 0, A F q or r is true.

Now look for a second formula, that there exist a path in future q and r is true. If I am having this particular labeling, then you will find that none of this particular path in none of the state's q and r is true. So that means, we are not getting any path where in future q and r will be true. So that is why you say that, this particular formula E F there exist a path in future q and r is holds in this particular state s 0.

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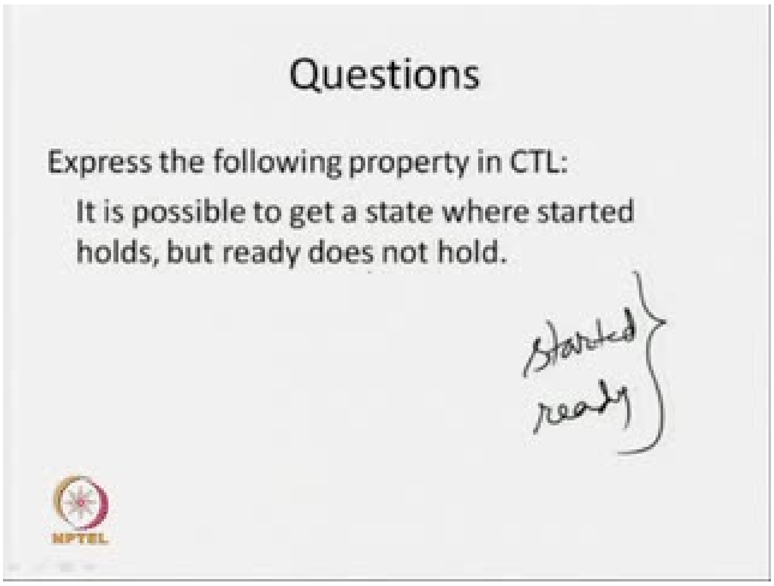

Now just look for another way of looking into the CTL formula; what I am saying that, express the following property in CTL. Is it possible to get a state where started holds, but ready does not holds. So if you look into this particular statement, what will happen? From here we can capture two key words, one is your started and another one is ready; that means, what we can say that? These two are my atomic proposition. So we can haves some signals in my systems, where the value of this signals maybe either 0 or 1; if it is 0 you can say that it is true; if it is one you will say that it is true; if it is 0 will say that it is false.

So these are the two signals we can see in my system; that means, we can map these two as my atomic proposition of my system. Now we have to say that it is possible to get a state where started holds, but ready does not holds; that means, we can think something like that we have design a system and after that it is going for operation. Now we have started machines, but it is not going to that ready state.

So it is you have started it, but it is not going to the ready state; so whether we have to design about such type of property. Now whether these property can be express in your CTL or not. So is it possible to get a state where started holds, but ready does not hold.

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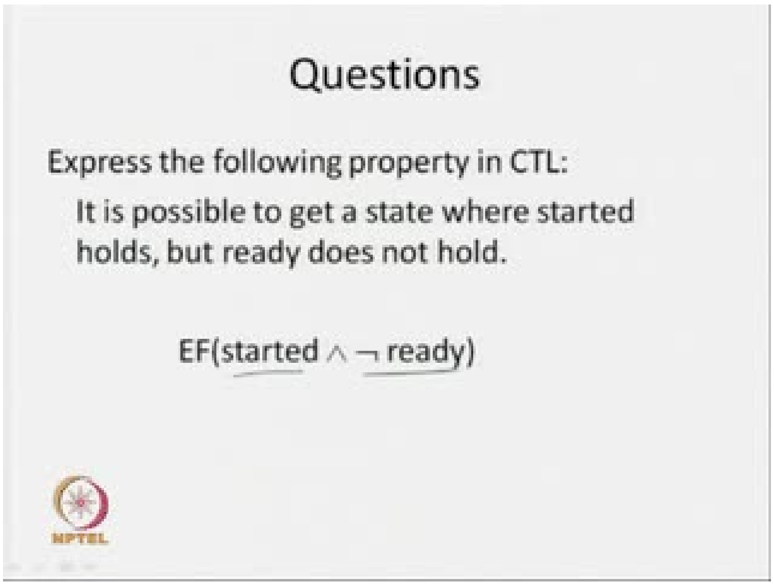

So in find that we can express this property in CTL something like that; there exist a path in future whether started is true, but ready is not true. So this property we are going to captured in CTL like that. There exist a path in future or started is true and ready is not true; so this is the way we can capture. Now if we are having the model of the system, now we are going to look for the truth values of these particular formulas and will see, what are the states that formula is true? And what are the states this formula is false?

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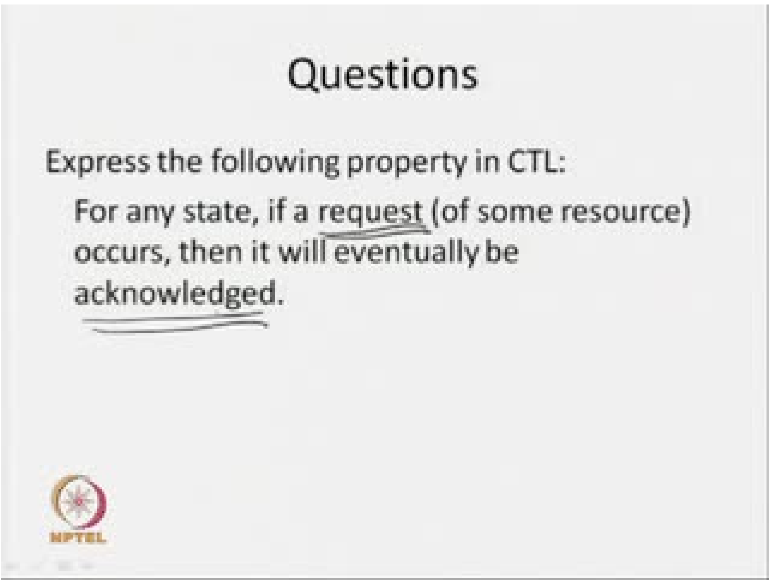

So another property just you see that, express the following property in CTL; the property says like that. For any state if you request of some resource occurs, then it will eventually be acknowledged. Now you see that, if you are going to look for your distributor system or maybe concern system, what will happen? Same resources will be shared by defined processes. So what we are saying that, if we are requesting for a particular resources. It maybe said memory or it may be something else also, it maybe some other devices also; if we are requesting from some resources, then it will eventually be acknowledged because, if say if I am going to excess said memory, eventually I should get it; that only I can progress. If I cannot get it then what will happen? I will go into the third reason.

Now when we design a system, that system must satisfy such type of property. Now this is we know the meaning of this particular property; now when we are going to formally verify somehow, we have to capture it formally. Now we are going to see whether this property can be captured in CTL or not. Now again you just see that, here we are having two key word, one is your request and second one is your acknowledge; that means, we can say that some process or some process might have placed are request. We say that, this is signal if it is based are request then, it will go to active why and if it is not requesting it, then it is active loss.

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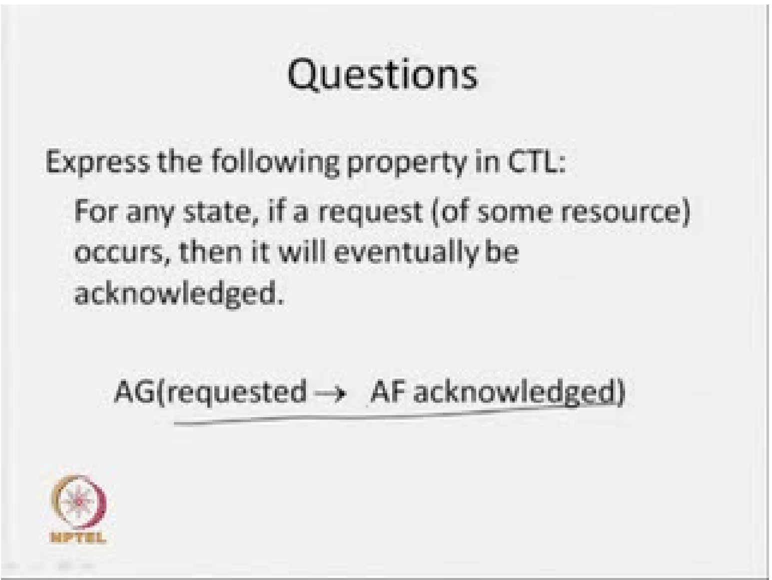

So request can treated as your atomic proposition; it's values are true and false. When it is that resources granted to the process, then what will happen? You can say that it has been acknowledged, then again you can say that acknowledged is a signals; it value is either 0 or 1. So we are we can say that, this is also the atomic proposition in my system or of my model. So we are getting two atomic proposition request and acknowledged. Now with the help of these two signals, we will define this particular property in CTL.

Now what we can say that, if somebody has requested form some resources; it should implies that in all path in future, it must get the acknowledgment and these properties should whole globally in the system. So that is why I am saying that in all path globally, if it is requested then it should say that, it should get an acknowledgement where about the system would go. So that is why I am saying that, in all path in future it should get the acknowledged. So in all path globally it must be true.

So if in a particular state it requested happen, then form that particular state wherever you go in all paths in future, you should get that acknowledgement. If there is some paths, where we are not getting the acknowledgement, then what will happen? If I follow that particular path, that system is going to get starvation because, it is not going to get the resources. Now this is the way that we have to look for the properties about system and you have to express in your CTL and we are going to use look for the truth values of those particular CTL formula in our system, say whether it is true or not.

So this is about our one particular class of logic, temporal logic calls CTL; Computational Tree Logic. We have seen how to define the what is syntax of your CTL? How to define it and what is the meanings. So we have seen the semantics of those particular CTL formulas and secondly we have seen, how to define the semantics? It is with respect to a model and second we have seen that the CTL formulas are basically state formulas, if the truth value defines over states of a model.

So with this I am stop here today; in next class we are going to see some of the notable equivalence of our CTL formula, which will help us for verifying our system.

Thank you.