Design Verification and Test of Digital VLSI Designs

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Module - 4
Temporal Logic
Lecture – 2
Introduction and Basic Operators

So, in last class we have been introduced a logic call temporal logic, why who which we can capture or we can express the timing behavior. Now, we are going to see in details how, what is required in temporal logic and how we can express the properties in temporal logic. So, basically this has specification like which we are going to specify the properties about system. Because in model seeking have said that we have been three components; one is your model of the system, second one is your specification, you have to provide the specification of the system, and third is a verification method logic. So, basically we are going to look in the second component today specification, how to provides specification and for that we are going to use temporal logic.

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So, already I have mention that in temporal logic basically we are going to capture the timing behavior of the system. Secondly, if you look into the notion of timing, we have being two type of notion; one is your branching time and second one is linear time. Again, I have mention about that we can capture timing in two different way; one is your discrete and second one is your continuous. Again, I mention that how to specify that behavior, again we can look into two different ways; one is your qualitative and second one is your quantitative. In case of qualitative, we just specify that infuse something is going to happen or not or in quantitative we can specify that is after some with of times something is going to happen in the system. So, in this lecture basically we have going to talk about qualitative issues of temporal logic.

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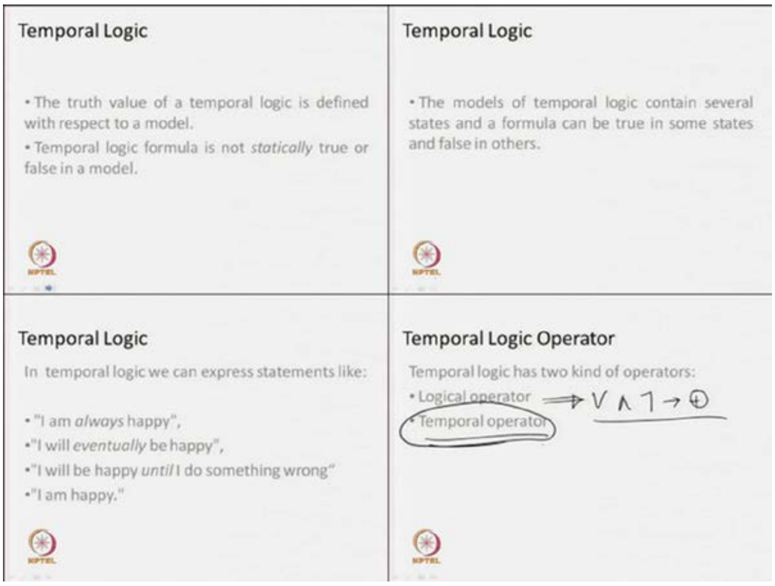

So, we have already introduced about the logics, we have talked about for proportional logic operatic at logic or highered logic and now we are coming to the temporal logic. So, when you are going to look in the two fellows of a temporal logic formula; basically the two fellows of temporal logic formula is define over a model or all are it is with respect to model. We should have a model and it is, in this particular model we are going to defined the meaning of those particular temporal logic formulas.

So, temporal logic formulas is not statically true or false in a model. In some of the state it may be true or in some of the state it may be false; so it is not statically true or statically false like the Adel logic call proportion logic and predicate logic. So, it is a system dynamic behaviors; in some state it will be true or some state it will be false. So, for that what requires in earlier model, the models of the temporal logic contain several state and a formula can be true in some states and false in others.

Now, what we can represent in temporal logics? That we can expect the statement like. I am always happy. So, it says that it says about the state of my mind; I am happy, I am always happy. On the other hand it says that, I will be eventually happy. That means, if sometimes I will be happy or it says that I will be happy until I do something wrong. So, I will remain happy until I do something wrong. So, these are the always things that we can express. Now, we will see this is simple example I am giving but we will see what we can do?

Now, temporal logics have been two kinds of operators; one is your logical operator and second one is your temporal operator. Already, we have talk about the logical operator, where we are having that disjunction, conjunction, negation or say implication or xor; like that all those logical operator that we have that in other logics can be used here in temporal logic also. Along with that we are having another kind of operator which is known as temporal operator.

So, this is the basic things that we are having; now we are going to see what are the temporal operators that we have in temporal logics. So, we know the meaning of other operators, logic operators; the meaning is similar to the similar with respect to the Adel logic. Like you have here proportional logic or in predicate logics; so we will see what are the temporal operators that we have temporal logic.

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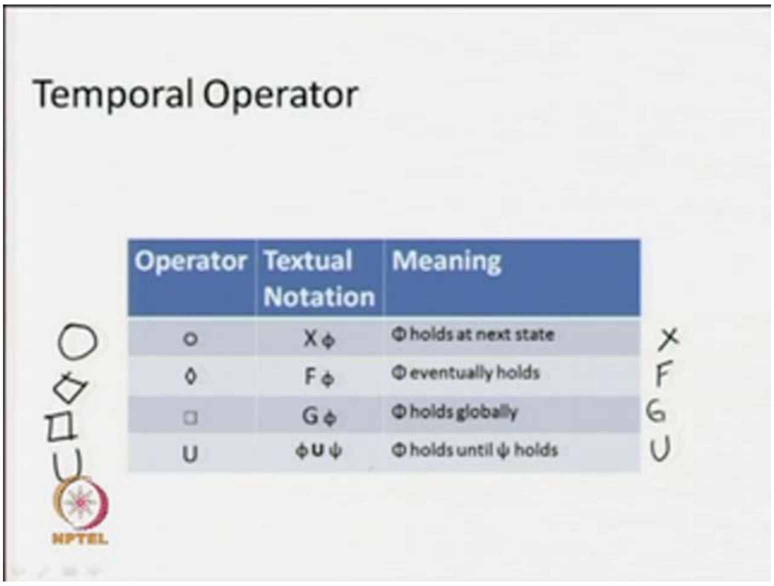

So, basically here we are going to have four different type of operators. We say these are temporal operators and these are basically related to or going to talk about the timing behavior of the system. So, one is your next operators, so this next operators is basically represented by Circle. Second one is your eventually the operator, it is represented by diamond. Third one is globally always which is represented by box and another operator we are having that until which is represented by U. In this case u next state future n your global; these are your preliminary operator, it is going to takes one particular formula.

But until is a binary operators it takes two formulas, two formulas and it says the talk about the meaning with respect to this formula. So, in your classical notice on use this particular symbol circle, diamond, box and until. But in textual notation this next operators is represented by X, eventually the operator operators is represented by F, globally or always operator is represented by B and this until operators is represented by U. So, in case of x phi if you write that means in next state phi holds. If I say that f phi it says a the formula phi holds eventually; that means in future in some future state is particular formula phi will holds. So, G phi which is globally phi which has a psi is always true in the system; that means in all state phi is true.

And, phi until psi; so it is a binary operator we having two two a formulas, phi and psi. It says that phi remains true until psi holds; that means phi remains true until psi holds. In that particular we say that phi until psi holds in a particular states. So, basically we are going to look for this particular four temporal operators and we will seen now, how we are going to represent them and how we are going to defined the meaning of this operators.

Now, already I have mentioned that temporal formulas or meaning of temporal formulas in fact that over a model which is infinite sequence of states. So, in a system we are having a number of states and the basically we are having a infinite sequence of state, if you talk about reactive system. So, we are going to define the meaning of those particular temporal formula over the states of a model. So, how formally what we can define? We can say that given a model M; so we are going to talk one model M and a temporal formula phi.

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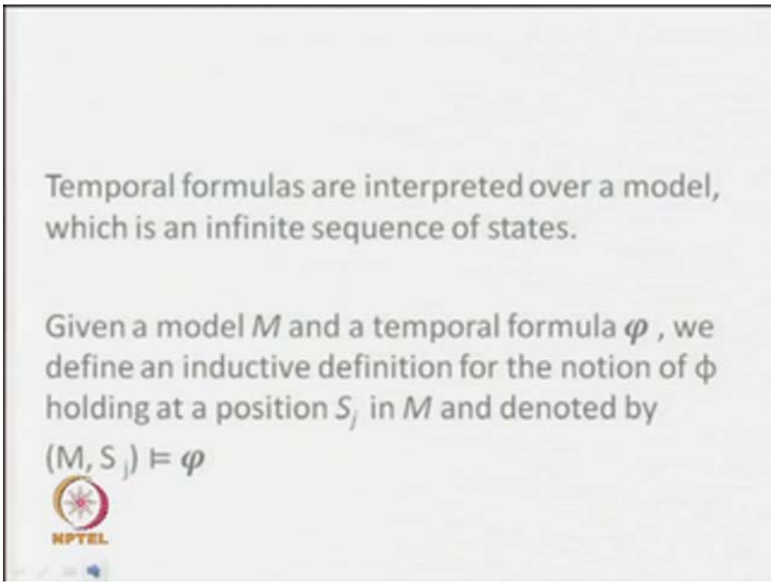

So, we are going to talk one temporal formula phi; so we will take this two component, one model M and one temporal formula phi. We define an inductive definition of the notion of phi holding at a position S j. So, in the system we are going to look position S j in M and it is donated by M S j models phi. It says that in a model M in a state S j, the formula phi holds and we say that M S j models phi. So, this is the notion state of two fellows of a temporal formula is basically defined in a model and in a states; we are going to say it holds or not. But there are having some other notion also in the lecture I am going to elaborate those things.

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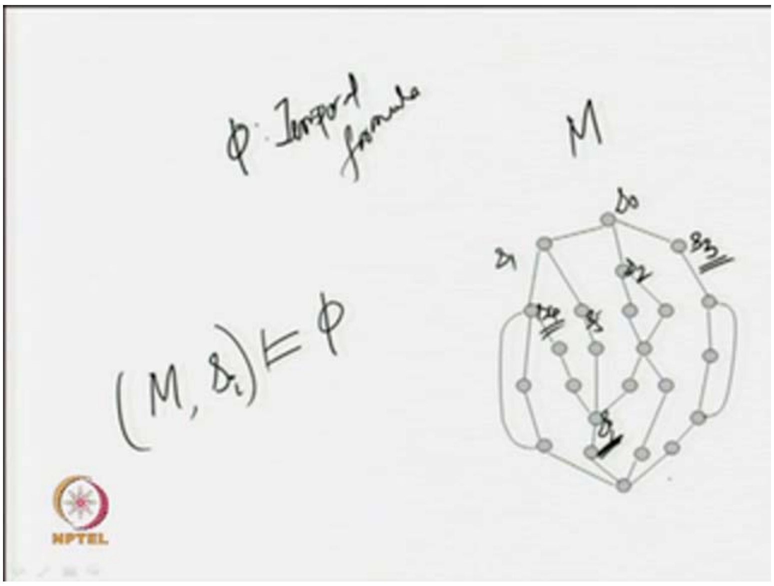

Now, in this particular case we saw said that this is a state conjunction model; so we can have defines state call S 0, S 1, S 2, S 3, S 4, S 5 like that we can name the state of those particular model and if you consider about some particular states say S i. So, this is the model M and we are basically concern about this particular state S i. So, we say that this model M in this particular state i, it models the temporal formula phi; if phi is a temporal formula. So, we are having a temporal formula and we want to check the fellows of this particular temporal formulas. So, we have to check with respect to model M and we are going to define in a particular state S i. So, in this particular case we say that M S I models phi; it means, in state S I model phi holds. This is basically internal meaning of that fellows about temporal formula.

So, in this particular case I am saying that phi holds in this particular sense. But it does not mean that phi holds in the entire system, it may be falls in this particular S 3 or it may be falls in S 4. So, that is why I already mentioned that fellows of a temporal formula, basically the punch over the state in this particular model. In some state it will be true or in some state it will be false.

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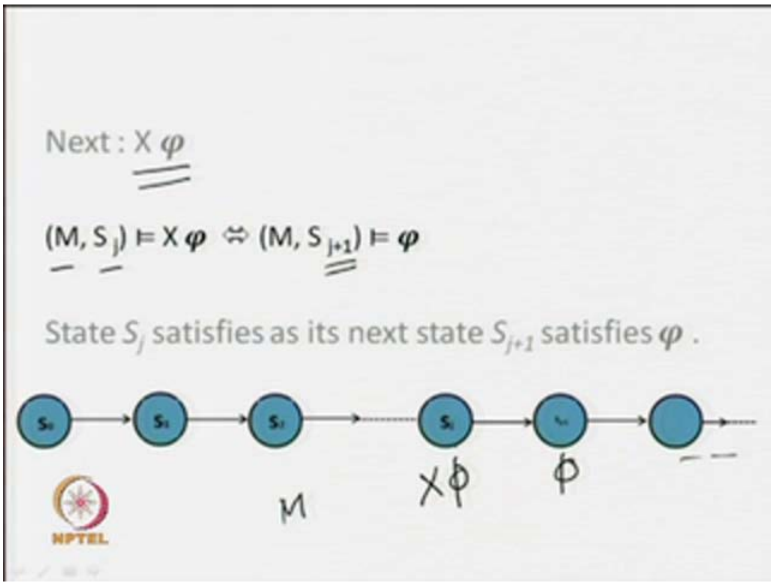

Now, we are going to see the operator by operator; how, what is the meaning of these things. So, first we are going to look in to the next operator. So, basically we have already mentioned that in textual notation next will be represented by X. So, this is the operator next phi; that means the temporal formula X phi it says that next phi. So, what is the internal meaning or how we are going to define the meaning of this particular formula? So, in a model M in state as a it models x phi, provided in a very next state S j plus 1 phi holds. So, if in a particular state phi holds, then what will happen? In a previous step we are going to say that next phi holds.

So, basically if I am going to look in to this particular model, it is going from as state S 0, S 1, S 2 like that S j, S j plus 1 and orders; so what will happen? If a level this particular S j plus 1 phi; it means that phi holds in this particular state S j plus 1 of this in particular model M. So, by meaning of this things now we are going to say that where this particular formula next phi holds; then will find that we will find that the state S j models in x phi. Because when we come to this particular state S j during the transition time, then we will find that from S j wherever I am going in the next state phi holds; so we say that x phi holds in this particular state S j. So, this is the meaning of these particular next operators. So, x phi holds or x phi through in a particular state provided next state phi holds. So, this is the meaning of your x phi and you are going to define the meaning with in a model.

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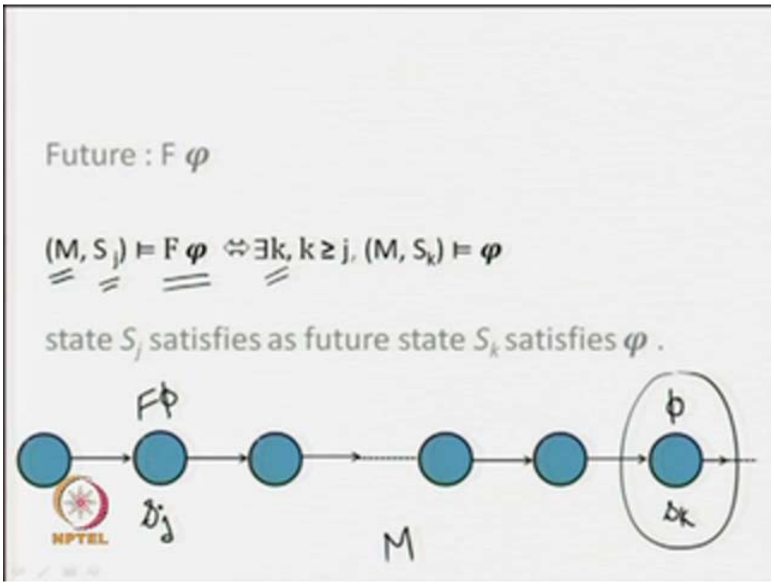

Now, next operator we are going to look for eventuality or future f phi. Now, how you are going to define; now you see the meaning of these things; we can say that in a model M in state S j F phi holds. So, this is a temporal formula, F phi; so (( )) phi holds. Basically, it says that we are going to get some state K in the model; where K is greater than j, if you are going to look in particular state j. And, it says that model M in state K that phi holds. Now, in this particular model say this is my model; I am going to look for one particular state say S j, say this is a state we are looking in to it. In this slide we are going to get some states say S k and say S k is level to this particular point; that means, it says that in S k, phi holds.

So, when we are going to say that F phi holds in a particular state here phi is also another temporal formula. So, we are using a temporal operator phi, we are going to say in which state F phi holds. So, if in future we are going to get a state where phi holds, then we can say that in that particular state F phi holds; so we can mark this particular state with F phi. So, what it says? If I mean state S j, in this transition line I am going to get some state in piece of where F phi holds. So, we can say that F phi holds in that particular S j. So, that is why I am saying that exist some k, where k is greater than j in my timeline; where M S k models wise. So, if we are going to get such type of sub state in my model, then we can say in my model in the state or in the state I am looking in to the F phi holds in the particular state. That means, in future we are going to get a state where F phi holds; so F phi holds in S j.

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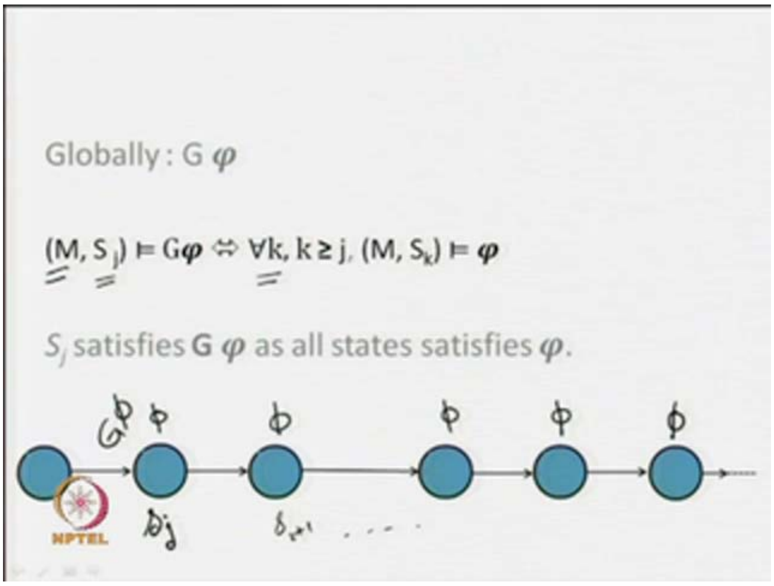

Now, next operator is your globally, G phi; so whether globally phi holds or not? So, again a similar way we are going to define a model M and we are going to look a particular state S j and we say that M S j model G phi provided we are going to get some

K for all K; K is greater than equal to j, M S K phi. That means, all states where all state S k, where k is greater than j; if it (( )) then we say that G phi, S j model G phi globally phi holds. So, basically if I look in to this particular state S j, now we are having this particular state as your S j plus 1 like that we are having.

So, if all the states if we have model (( )) says that k greater than equal to j; so phi must be true in this particular state also. Now, we are having infinite sequence, so basically we have to look for all the states that whatever coming in future. So, it is basically reason about an infinite system but will come down in a state that everything can be represented by final state model. So, our number of state will be final but they will be repetitively measures; so we are going to get infinite sequence.

So, in that particular case form a particular state S j if all are the state in future that phi holds, then we can say that g phi holds in these particular states. So, basically now I can level this particulars state with G phi. So, we will say that G phi holds in S j because in all the state can that can be reachable for that particular S j that phi holds; so this is the meaning of globally operators. So, globally phi it says, globally phi holds in a particular state if phi holds all the state in this particular system. So, this is the meaning of G phi. And, next operator is your until.

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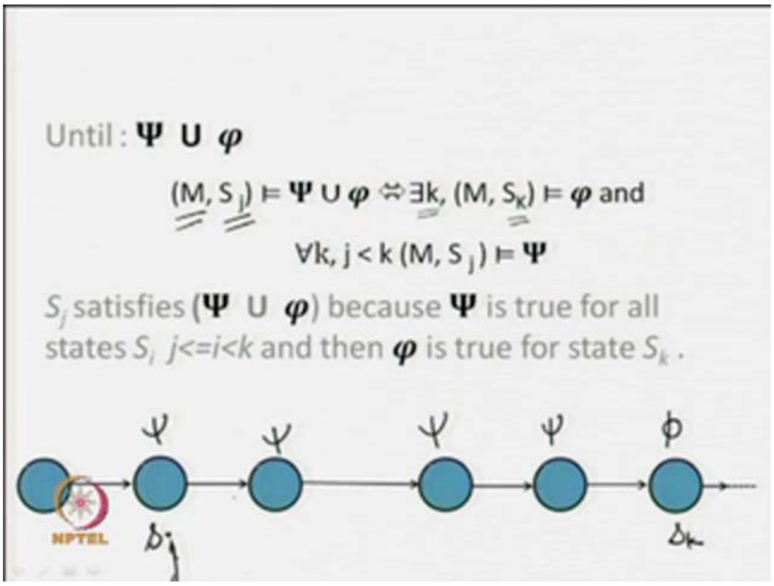

We said that already I have mentioned that until is a binary operator having two operant; one is your psi and second one is phi. So, it is psi until phi. Now, what we are going to say that again we are going to take a model M, we are going to take one particular state S j, we are going to see that whether instate S j of model M psi until phi holds or not? It will be holds if we are going to get some k, they are exists some k we are going to get a state k; S k. So, in the model M set S k phi holds over there and for all the other k, where this thing j is less then k; that M S I model psi. So, in this particular case we are going to say that, that particular state holds psi until phi. So, basically we can say that I am looking into particular states S k, if models this particular phi; so in this particular case state phi holds in your state S k.

Now, we are looking some other state say S j. Now, we have to see whether S j holds psi until phi holds in S j or not. It says that in future we are going to get some S k where phi holds and in all the state in between them, where coming from this particular S j to that particular S k, psi must be true. So, there must be level by particular psi. So, in that particular case we say that S j models psi until phi; that means psi remains true until phi becomes true. So, in S k phi is true and all the state that is going from all the states that we are having from S j to S k, for psi must be true. So, in that particular case we state that S j models psi until phi. So, this is internal meaning of until operator.

So, in this thing model I am going to mark this particular state S j by the formula psi until phi provided, if satisfied fulfill this particular behavior that psi remains true until phi becomes true. So, this is are state meaning of your until operator and we are defining the meaning of this until operator in models. So, basically now we have seen this particular four operator; one is your next stat operator, second one is your eventually operator, third one is your globally operator and fourth one is your until operator. So, these are the four temporal operator that we have. And, in classical notation we just represent X by your next state by your circle, than diamond is for eventuality, box is for globally and U is for until.

So, if you look into saying that if we are in a particular state whether something is going to happen in future, whether in next state or in all the state in the future. So, basically this particular four operator that I have introduced basically forms the future logical; it is a future temporal logic. We are going to reason about the future behavior of the system. I am in a particular state say the system is coming in to your particular state.

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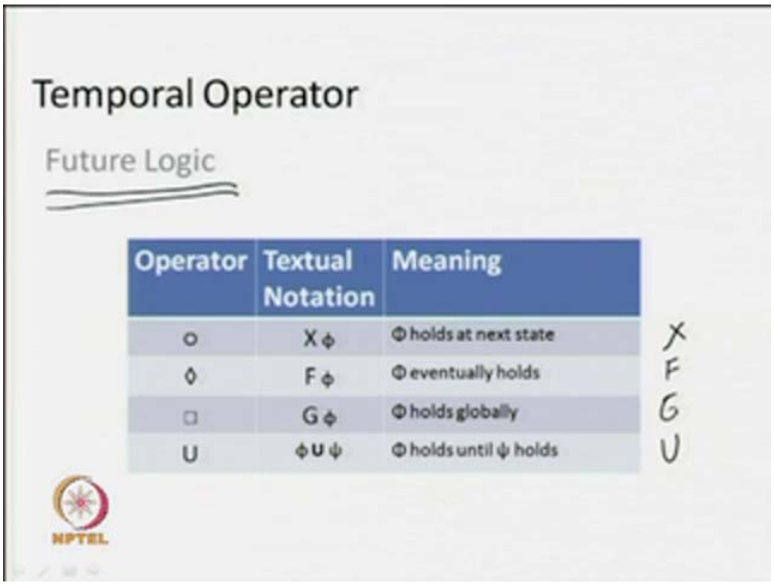

Now, from that particular state, how system is going to behave in future? So, if we are going to reason about this particular behavior, then we are going to look for this particular future temporal logic; so this is future temporal logic. Since, we are taking about future temporal logic; whether we are having something called past temporal logic? Yes, indeed we have. So, we will see that what are the notions of the past temporal logic?

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So, in case of your past temporal logic like your future temporal logic, here are also we are going to define four temporal operators which correspond to those particular four future temporal logic. So, first one is your previous; we are going to talk about the previous state. And, this is notationally we are going to represent it by say; for future, we are having this particular circle.

So, in case of previous that circle in between we are going to use the symbol tilde there. It says that this the previous operator that means past temporal logic, we are going to the look for a previous states. So, eventually in past, so basically in future we are going to have this particular diamond. So, in the inside diamond we are going to write this particular tiled and basically it is going to talk about this or it is going to represents this eventuality in past; eventually in past operator.

So, similarly, we are having a globally past or globally in past; so if can it is the same symbol we are going to use. Then box is use for your globally operator. So, box intact inside this box, we are going to put this tiled inside the then we are going to say this is the globally in past operator. And, back to it is again corresponds to that until operator; so back to is represented by this particular symbol. So, now we can reason about the past behavior of the system also. So, in that particular case what happens? You have to look for the past temporal logic.

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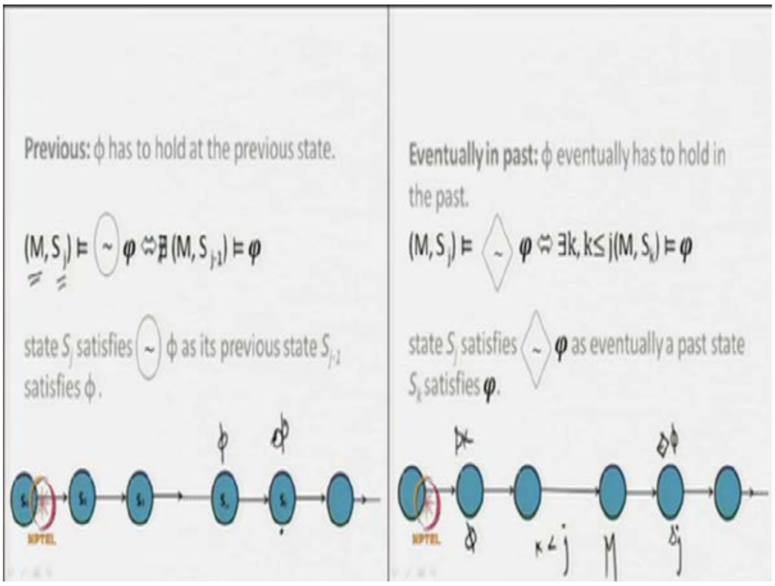

So, again like your future temporal logic, you are going to have this particular proprietor, previous, eventually in past, globally in past and back to. Now, usually say that like future temporal logic, now we are going to look for the meaning of your past temporal logic.

So, that previous operator, we are going to look for a particular model M and you are going to look for state S j. Now, when we can say that state in S j of model M with the formula previous phi holds? That we are going to get a state, this is a wrong symbol it is not reverse. If M S j minus models phi, then we can say that previous is true in that particular state S j. So, one we are going to say that in this particular case whether we are going to look for a particular state M S j plus 1; say this is your phi holds over here, then you can say that in this particular state S j your previous phi holds.

So, this is the way that we are going to say that. In if previous state say that phi holds and you can say that in that particular state your previous phi holds state. So, this is the meaning of a previous state. So, if we are in a particular state, whether if in a previous state something happen; then we can say that previous phi holds in that particular state.

Now, so second one is your eventually in past. So, this is similar to your future. So, in case of future, we are saying that if you are in a particular state, in future something going to happen or not. In case of as eventually past from a particular state, we are going to say that whether something happen in the past or not. So, basically if I say I look into the particular model M and say this is the state S j; now whether this particular state S j model some eventually in past phi holds or not. Then in this time line we are going to look for some state and say that these particular states say phi holds, in this particular state say S k and where k is your less than your j. Then we will say that eventually in past holds in this particular state S j. Now, we can say that this particular state models with eventuality phi.

So, just see that meaning is corresponds to future logic. So, if you are in a particular state you are going to see in future something holds or not. But in case of past, you are going to state that something happen in the past or not. So, this is the way that we are going to of the meaning. So, we are going to define the meaning of those particular past temporal logic operator with respect to the model also.

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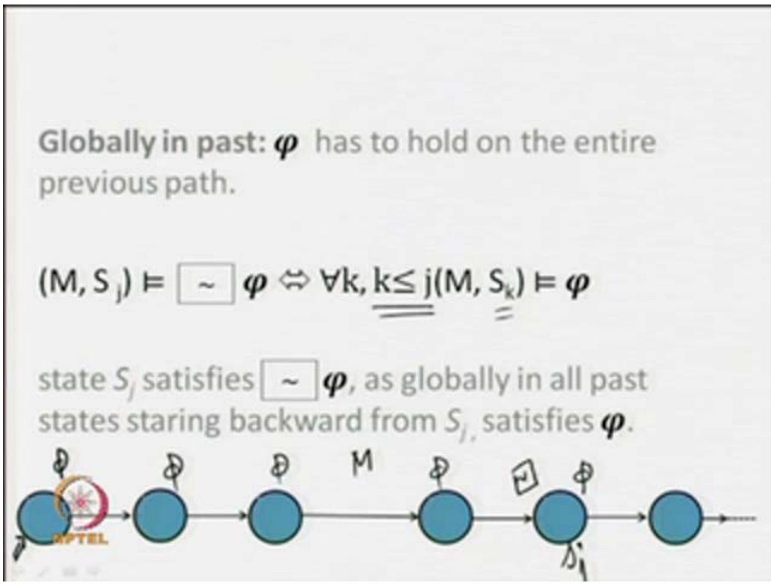

Similarly, next will be our globally in past that means whether something happens globally among. So, meaning is something similar to all globally operator in future but here we are going to talk about similar way. Say this is my model M. So, you are going to look for particular state S j; whether in this particular state S j it is a model globally holds or not. So, in that particular case, in past we are going to have something but again restricted to some other initial state will go off to that particular initial state on; because if you talk about as like in future, we are having infinite sequence of state. So, that means it may happens infinitely in your past also but always we can say that we are going to design about from a particular state; that is the starting of my model, starting of my system, so we will go up to that particular points.

So, we are going look for all k such that k is less than j and say all those particular state S k this phi holds. That means, if I say that from S k all those particular state phi holds just say that I am starting from this particular state. Say that this is the, I am going to look for this particular behavior only then in all those particular state from S j if phi holds. Then we can say that globally in past phi holds; so we can say that we can mark this particular state globally in past operator phi. So, this is the meaning of past operator. So, we are going for a particular state and from that particular state we will see all the state in the past. And, in if that particular formula holds in all past that as than we can say that globally in past phi holds in that particular state.

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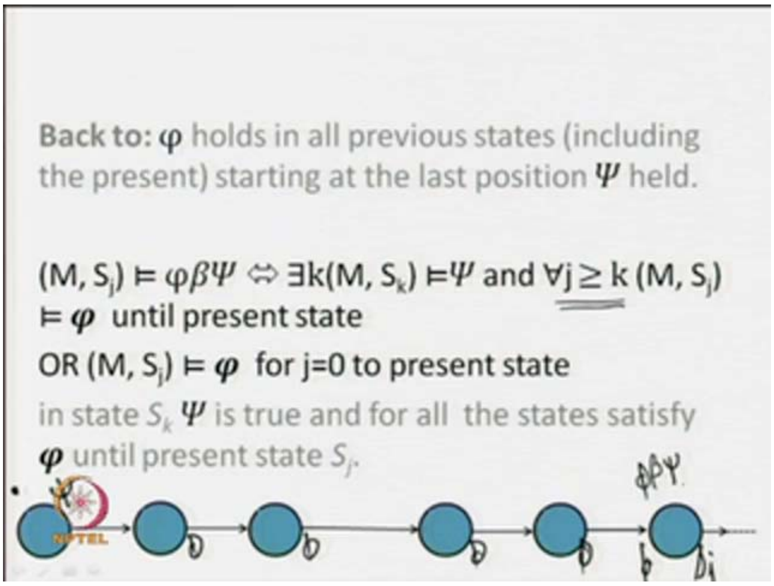

Similarly, the last operator in this particular series is your back to phi; which says that back to phi which say that back to phi will say that it holds in a but say that phi holds in previous state; including presence, starting at a last position, phi holds. That means, again since in past we may have in infinite state but will restrict to some initial state; say they say that we are going to look from this particular state. And, we have say going look for particular state S j. Now, we are going to say that whether in this particular S j back to phi holds or not?

So, similarly, say in future something holds or say until operator phi until psi. So, here phi back to psi; that means we are going to get some state this particular model in past that psi holds and the all states between S j and psi that phi holds. So, if you are going to get this particular model; say in some state psi holds in the past. And, the from the particular state to the state that of my consign S j in all the state in between this two state, if phi holds including S j. Because you are saying that j is greater than k, then what we can say that; in the state S j this particular back to holds. That means, phi back to psi holds in this particular state; so we can mark this particular state with phi back to psi.

So, we just said that, now we are trying to capture behavior of timing system. So, when we are in particular state, we are coming to particular configuration, from that either we can reason about the past behavior of the system or we can reason about the future behavior of the system. In case of your past behavior of system, we are going to use the past temporal logic.

So, we are having this particular four past temporal logic operator previous, eventually in past, globally in past and back to. If you are going to listen about future behavior, then we are going to look for future temporal operator which is your next state; eventually, globally and until operator. So, these are the four operator; basic operator we are going to discus about it. And, we have seen how we are going to define the meaning of those particular four operator and we have seen that define the meaning of those temporal operator. We are going to define with respect to model; so this model will come from our system.

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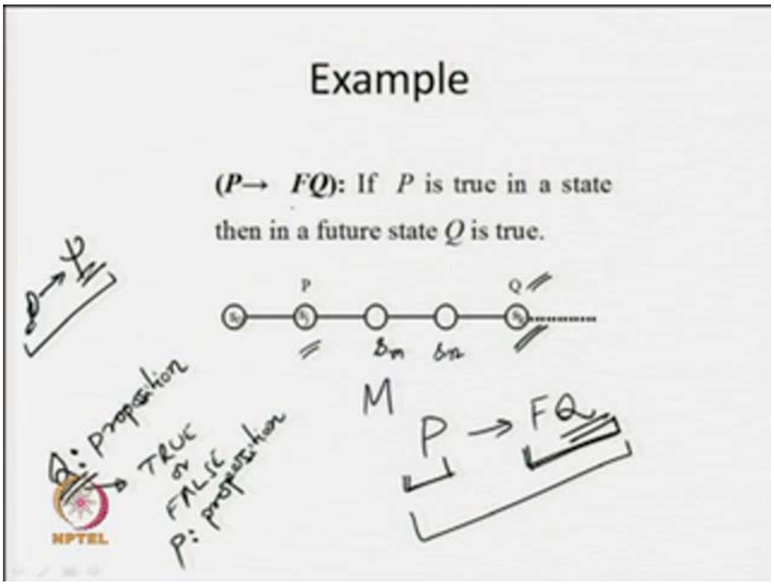

Now, just look for some example in this particular scenario. Now, we are saying that P implies F Q. So, this is the formula, say now I am giving a formulas; so here P implies F Q. Now, what does it mean? So, here Q is a proposition, basically we have to see that Q is proposition; that means it is the atomic proposition that truth values of your Q may be either true or false. So, all atomic proposition will be treated as a temporal formula. Because we can see what are the temporal formula? So, all atomic proposition will be treated as a temporal formula. So, with temporal operator we can again construct a temporal operator.

So, here in this particular case I can say that Q is a temporal formula. Now, F Q that means say that in future Q holds or not. In this particular case, I will say that this F Q is also a temporal operator. Similarly, P again it is an atomic proposition. So, that means here I am going to walk with two atomic proposition; one is Q and second one is p. since, P is an atomic proposition it can take two values; either true or false. So, atomic proposition will be treated as another temporal formula. So, this is another temporal formula.

So, in that particular case say P implies F Q; that means P implies F Q. This is the application operator that we have in your logic and we said that this can be used in a temporal logic also. So, one temporal operator implies another temporal formula; so these whole things will be treated as a temporal formula. That means, it may be something like that phi implies psi. So, here phi is temporal formula, psi is a temporal formula; so phi implies psi is a temporal formula. So, P implies F Q will be the temporal formula; where F is the temporal operators futures. Now, we are going to say that P implies F Q; if P is true in a state, then in future that Q is true. So, this is the meaning; now, how to going to get the meaning of this particular formula? Always you are going to define it with respect to in your model. Now, consider this particular model M; so we are getting a state S k where Q is true. So, this particular state S k; k is true. And, another state we are having say S j where P is true.

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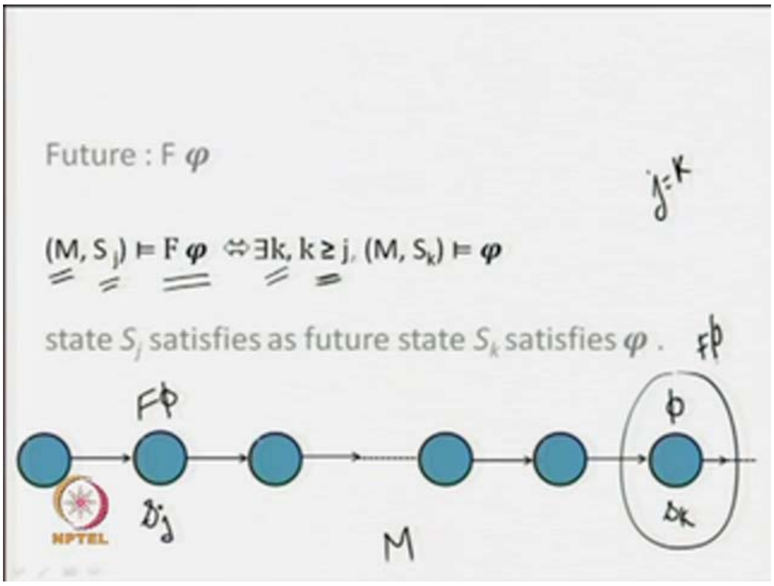

Now, in this particular model say we are going to have this particular state; S 0, S j, S k and two some model, I can say that some S M and S n. Now, this formula P implies F Q, where it is true. So, if you look into it, again I will come into particular notion if you look into the future behavior it says that if you look into this particular definition again, future uses that where I am having this particular things.

So, here you just see that if S state, S j; I am going to say that in state S j F phi holds, we are saying that here exists some K where K greater than equal to j and M S k models phi. If they see this particular symbol here greater than equal to, that means it may happen that; what will happen in case of M? j equal to K. Because that means if psi holds in a particular state itself, according the definition what I can say that F phi holds in this particular state also, because this k equal to j. So, we will come to the particular point that on again. This notion is having some significance, so that means again we say that by looking into the definition saying that if phi holds in that particular state then

So, now come back to this example. So, since as per this particular definition if phi holds in this particular state, then we can say that F phi holds in this particular state. Now, when I come to this particular state n, then from here in a future state phi holds. So, I can say that F phi sorry F Q, here I am using this particular symbol Q; F Q holds in this particular state also. When I come to this particular state S m, from here also I am going to get state in future where Q holds. So, I can say that F Q holds in this particular state also. When I come this particular state S j just say that from S j we are going to get in future when Q holds; so I can say that F Q holds here also.

When I come back to S 0, we will find that from S 0 in the time line I can go down and in future I am going to get state S k where Q holds. So, similarly, I can say that F Q holds in this state also. Now, we know the meaning of these things know that logical operator say say P implies Q. So, if this is the (( )) of operation that means if P then Q; if P is true than Q must be true. But if P is false, we are silence; so if P is false then what where about the fellow Q? We are going to say that P implies Q is true over here.

So, here if we look into this particular model, so here F Q holds; so P is not true over here but if P is false and Q is true then also we are getting to say that P implies Q is true. So, basically we can say that in S k your P implies F Q is true. So, in the similar reasoning I can say that in S n, S M and S 0 that P implies F K is true. Now, what about

S j? P implies F Q; again it will be true in this particular state, because now P is true. If P is true, then Q is true; then we are going to say that P implies Q is true in that particular state.

So, basically in this particular state S j also P implies F Q is true. That means, in this particular model we have seen that in all the states P implies F Q is true. So, basically if state defines then we have to look for each and every state. And, when you are going to talk about a particular formula, say like that P implies F Q; now we have to look for truth values of its components. Now, here we are having components say one temporal formula Q, we are forming another formula F Q, then one temporal formula P. That means, we should know the truth values of this particular temporal formula in each and every state, than only we can talk about truth values of this particular whole temporal formula. So, this is the way we are going to look into it.

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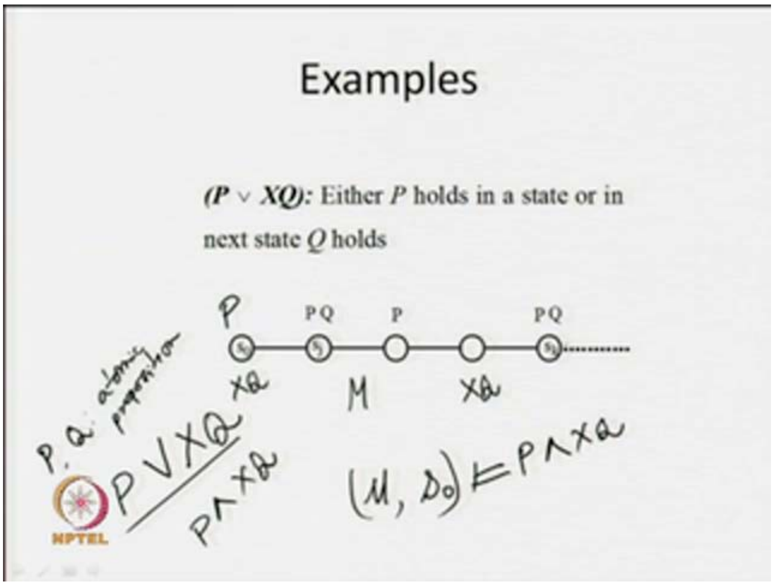

Now, another example you see that it is saying that P or X Q. Now, in this particular case now again similarly, we are having P and Q as our atomic proposition. So, like that what we can say now? P or X Q. So, P is a atomic proposition; so P is a temporal formula, Q is a atomic proposition. So, Q also temporal formula, X is a temporal operator. So, next state X is a temporal formula. So, these three temporal formulas connected by this all operators. So, this is also temporal formula. Now, we are going to look for the truth values of this particular temporal formula. Now, in this particular case either P is true or

X Q is true. Now, in this model if I am having these things; so the similar I can say that P is true over here and Q is true over here. So, in this particular state, for I can say that X Q is true over here. What about this particular state? Because X Q is not true, because Q is not true in the next state.

So, X Q is not true; so here Q is true. So, in this particular state I can say that X Q is also true over here. Now, since this is your P or X Q, now we will see that all the state either P is true or X Q is true. So, here I am going to say that P or X Q is true in all the state. Now, if I simply sense this operator form P and X Q, then what will happen? We are not having any state were both are true. So, in this particular state I say that this P and X Q is false in all the state. Now, I slightly modify the, these things leveling of this particular state the mentioned in all the state. I say that in S 0 also P holds.

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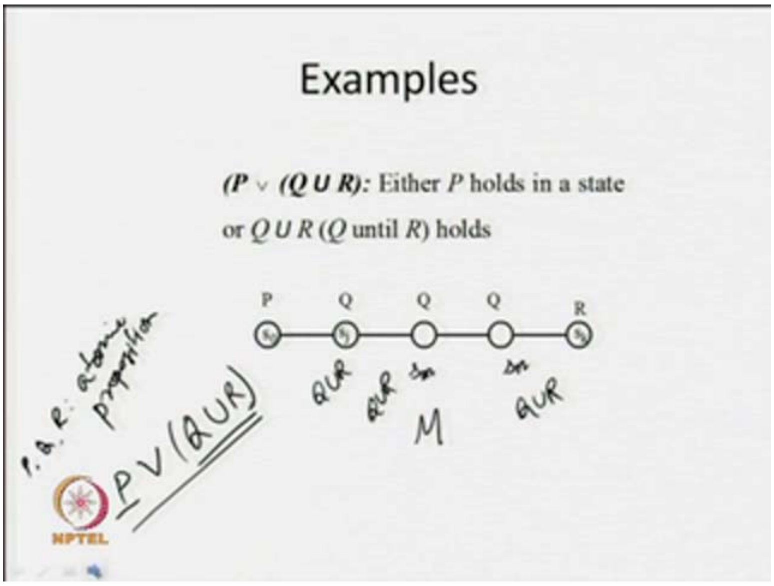

Now, in this particular case you say that in S 0, we will find that both P is true and X Q is also true. That means, we can say that this is my model M. So, in this model M, in state S 0 that P and X Q holds. So, we have say that in this example, what we are getting? We are getting only one state S 0, where P and X Q is true; but in all other state P and X Q is false. But if we talk about P and X Q, we will find that in all the state P or X Q is true. So, this is the way now we have to prove for the truth value of each and every temporal formula and each and every state. And, then only you can talk about the truth values of particular of a temporal formula in a state; so it is state dependent. In some state it will be true; for some state it will be false.

Now, come to this particular state; again we are having now we are having P until P or Q until R; that means, we are having P Q R S as your atomic proposition. We are starting with P atomic proposition and we are saying that P or Q until R. So, Q and R are temporal formula; so Q until R is a temporal formula, P is a temporal formula. So, P or Q until R is a temporal formula. So, now what we are saying that either P holds in a state or Q until R holds. Now, if you look into particular model M, then we will find that in state R, S k thus R is true and when I come back to this particular state say I will just level it as say S m and S n. So, in this particular S n, we will find that Q until R is true. When you come to that S m, you will find that Q until R is true. If you come back to this particular state S j, you will find that Q until R is true. When you come to S 0, you will not find that Q until R is to over because Q is not true when this particular point.

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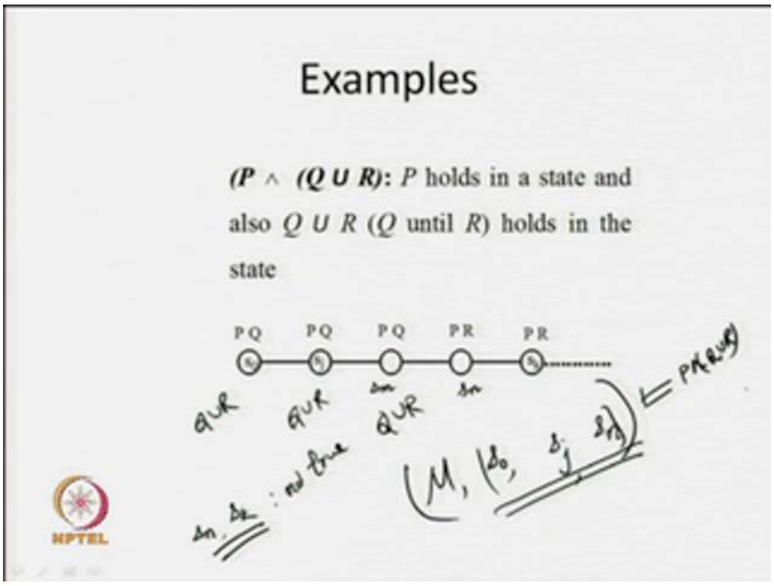

Now, when you are a going to look for P or Q until R, you will find that in this three state S j, S m and S n; Q until R is true. So, P or Q until R will be true in this particular three state when you come to this particular S 0, since P is true we will find that P or Q until R is true. When you come to this particular S k, then we will find that neither P is true nor Q until R is true over here. So, will not we will find that P or Q until R is false in this particulars state S k. So, this is the way that we are going to look for the truth values.

So, we are just looking into some examples to get the meaning of those particular temporal operator.

So, now we think we can look into it say when I can talk about P and Q until R. So, this is similar in earlier case we are having P or Q until R; now we are talking about P on Q until R, so P and Q until R. So, similarly, we can inspect this particular thing, Q until R. So, here we are going to find out now say that because I am going to look into it. If we look into that P and R is true over here; so what is the status of Q until R? Q until R is false over here. I am going to talk about Q until R; so if you look here that Q until R is false over here. If we look into this particular state, Q is true until R is true. So, we can say that Q until R is true over here. When I come to this particular point, again you will find that P is true, Q is true until R is true; so I will say that Q until R is true over here. When you come back to this particular state, again you will find that P is true Q is true until R is true; so I will say that Q until R is true over here. When you come back to this particular state, again we will find that P is true, Q is true, Q is true until R is true; so Q until R is true over here also.

So, again as such I am going to give that S m and S n to this particular two state. Now, we have to select P and Q until R. So, in that particular case both P and Q until R must be true in a state. So, here we are going to get the in S 0 both are true, in S j both are true. When we are coming to this particular case in your S n, both P and Q until R is true; so you will find that in these three states P and Q until R is true. When we are coming to state S n and your S k, you will find that, that P is true in both the cases but Q until R is not true in these two particular states. So, you will say that S n and S k; so this formula is not true basically it is false.

But in these three states, I can say that it models this particular formula P and Q until R. So, in the model M, if I am going to take this particular state of state; we will find that in this three state P and Q until R is true. Now, this is another formula that I am talking. So, we are going to have this circle and inside that I am having that tilde; that means this is the previous operator. So, previous operator means this is an example of past temporal logic. So, in that particular case we say that P and previous Q. So, we have to see whether P is true and the previous state Q is true.

So, if we look into these particulars this is simple model that we are having. So, we say that here if I am in this particular states S k, we will find that previous state Q is true; so I can say that previous Q is in this particular state. When I am in this particular state say again I am marking it as S m and S n. So, when I am in your S n we will find that in previous state you Q is true; so I will say that previous Q is true in this particular state.

Similarly, when I come down to this particular state in S m again you find that previous Q is true. Similarly, in S n I will find previous Q is true. So, we are talking about P and previous Q is true. So, we are going to get that. This is the starting and I am saying that in past we cannot go beyond certain points; so we will say that we are having a some start state and we are going to starting from S 0. So, here the previous Q is not true because we are saying that we know the behavior from this particular point. We do not know the behaviour of the system prior to this particular state; so here we cannot talk about that whether the previous Q is true or not. So, here I am not going to say P and previous Q is true but in this particular fourth state S j, S m, S n and your S k; that P and previous Q is true. So, this is the way we are going to truth values of about temporal formula.

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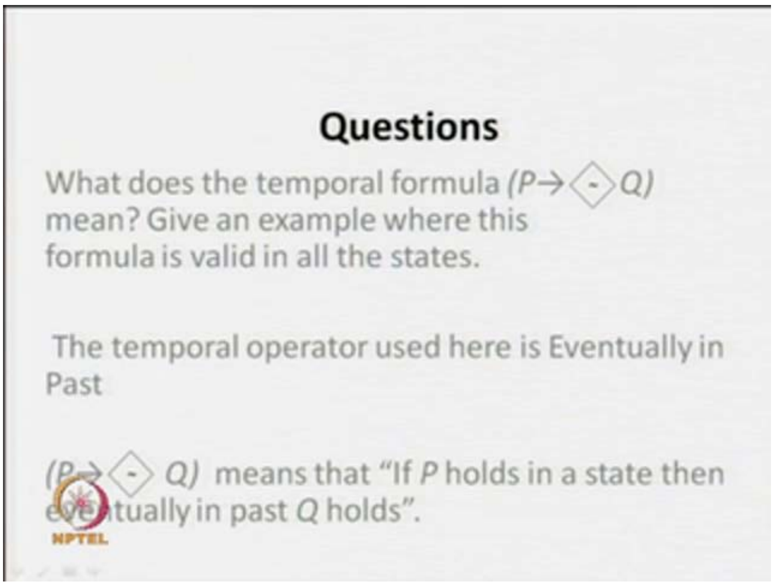

Now, we I will give you some questions; let us see further what we are going to do. What does the temporal formula P implies the tilde inside box that Q mean? Give an example where this formula is valid all the states. Now, say I am here; in this question what I am giving? I am giving a temporal formula P and I am getting a symbol; this is basically diamond and inside of that diamond having a tilde then and Q. So, what this particular symbol means? I think you can very well identify this symbol means basically eventually in past. So, diamond is future; if I write tilde in diamond, then we say this is eventually in past. So, we are going to say that going to look for this particular thing eventually in past P holds or not. If P implies eventually Q; so which has a what does this temporal formula P implies eventually in past Q means. So, that means we are going to say that if P is true whether eventually in future past Q was true or not.

So, give an example of where this formula is spell it in all the states. Now, if we talk about such type of question that means give an example that means we have come up with the model. And, we have to see that in this model we have to look for a truth values of this particular formula in this model and we have to say whether this formula is true or not. Now, we have to come up with an example where this particular formula is true in all the states. So, now this is basically what it says that P implies tilde inside diamond Q means if P holds in a state, then eventually in past Q holds. So, if Q holds in a state then eventually in past Q holds.

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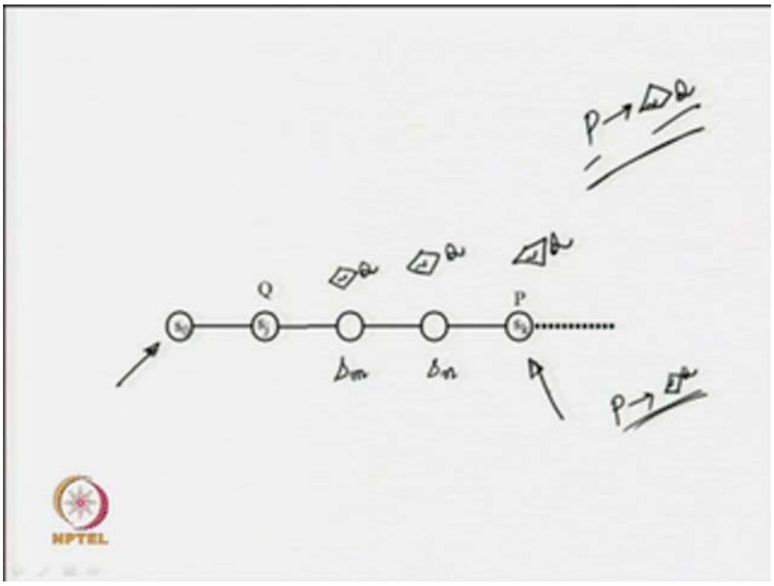

So, it is a very simple model that I have come up with this things. So, we are talking as state S k and beyond that in things we are going to have to say just thing that as k is the current states; I am going to look into it. That 0 is the starting point of my system and thus as I (( )) I am going to give the name of this two things S n and S m. And, what was my formula? It says that P implies eventually in past Q means. So, basically we are going to look for formula P implies eventually in past Q holds. Now, have to look for this particular formula, first say since Q is a atomic propositions; so eventually in past is also temporal operator. P is a atomic proposition; so it is a temporal formula, so even we are looking for temporal formula, so eventually in past Q holds.

So, if you look into S k, if you go in the past direction then we find that we are getting state S j, where Q holds. So, I can say that eventually in past Q hold in these particular states. So, similarly, in S m and S n also, eventually in past Q holds. Now, in S j I cannot say this thing because we do not know, because Q was not to in S 0 and again S 0 we cannot rejoin about this thing; because we has saying that this is the starting point. Beyond that what was happen we did not know about these things. So, this is the formula we say; now P implies eventually in past Q holds. So, if P holds in a state and then eventually in past in Q must holds; so if you look into the S k this formula is holds over here. So, this is the state.

So, similarly, but if P is false we do not have any meaning over there. So, we not require to look for the truth values because if false implies because you not an implication if P is false their formularized true in this particular state. So, in your S j, S m and S n this formula also in all those particular state P implies eventually in first Q holds.

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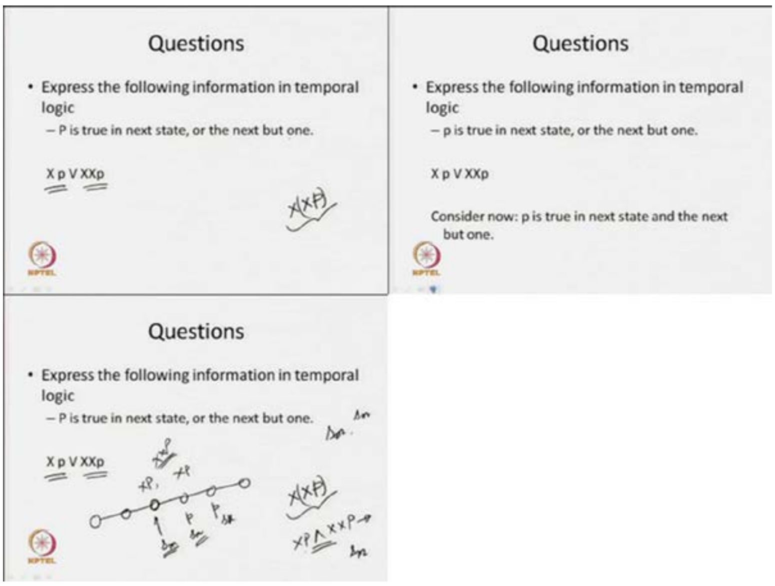

Now, look for another (( )) I am saying that express the following information in the temporal logic. So, basically we are going to design about that we are having some behavior. Now, whether you can express these things in temporal logic or not, we are saying that express the following information in temporal logic; please clue in next state or the next but one. Here, you just think that P j atomic proposition. So, we are talking about one atomic proposition P or since atomic proposition or temporal formula so it can be replaced by any temporal formula, phi also. So, that means, P is true in next state or in next but one. So, now what it can be expressed in temporal logic? Let see; we can say write the formula for this particular statement, X p we are going to say that in next state P holds and X X p.

So, you see that next to next state P hold. So, what I say that? P is a atomic proposition; so it is a temporal formula. X p, next state P holds; so we say that this is also another temporal formula. (( )) with an I am writing particular next operator X (X P). So, again so this thing is also temporal formula. I say that P is true in next state or the next but one; so we say that X X P is basically next but one and X P is your next state. So, now in this particular case, now what will happen? Now, we are going to look into it. Now, we can say hot is a truth values of this things; then again we have to look for one models, so if I am going to have some model like that.

So, this is the model say in this particular state say P holds; I mean P holds. Then what will happen? If you come to this particular state, we will find that X P holds over here, in this state also X P holds over here. If I come to this say I can say that this is your S m, S n and say S k. If I come back to S n, then what will happen? We will say that next state P holds, a next state and next but next state also P holds; so here X X X P also holds in this particular state. So, I may be in all these combination; so we can say that X P or X X P hold in this particular state. Similarly, since I am having this X P or X X P; so in S n also this particular formula also, what happens? I can say that in S m and S n this particular formula X P or X X P holds.

So, if I replace this formula why say X P and X X P that means in next state P holds or an next but next state P hold. Now, since I am having this particular (( )), then we will find the this particular formula holds only in the state S m; it is not true in S n, because in S n X P holds but X P X P not holds. So, that is why I am getting only S m, where this X P and X X P holds.

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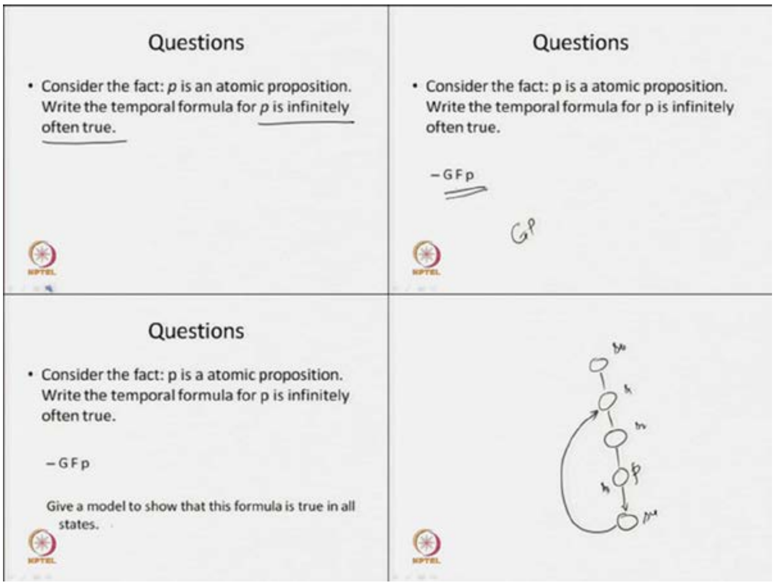

Now, another question you just see that you can think about it. P is an atomic proposition; now I am saying that write the temporal formula for P is infinitely often true. I am talking about P is infinitely often true, that what does it mean? So, it is infinitely often it is coming; that means it is true and after sometimes again it will be true. So, in temporal logic you can write this behavior in this particular concept, globally F p. So, infinitely open; that means globally it should come. What globally it should come? In case of P holds; so we are saying that P is infinitely often true. It is not like that, if G P; if I say that globally P is true, it is different and infinitely often it is true. That means, it is always not true, but infinitely often it is true. It is coming true after some instance of them; so that is why we are saying that globally F P holds. So, it is infinitely often it is true.

Now, can you give a model for this particular (( )) where it is true in all the state; because globally infinitely often it is a true. So, it is we can construct it because if we know the behaviors. I am going to just give an example to say I am having a state machine something like that and say that in this particular states P is true. Now, say that these are the states S 0, S 1, S 2, S 3, S 4; say I am coming after very small models. So, globally infinitely often that means P must to infinitely (( )); what happens? I can make a state like that; now this things is this interest S 1, S 2, S 3, S 4 is repeating.

So, in this particular case you say that whatever you are in fuser that we are going to get a state where P is true. That means P is not globally true but infinitely often it is true. So, that is why we are saying that globally F P is true; when we say a P is infinitely often it is true. So, this is now in this class what we have seen? We have seen what are the temporal operators that we are having. We are having two type of operator logic; one is your past temporal logic and another one is your future temporal logic. In both the cases we have seen the co operators; the basic operators. So, one is your next, eventually, globally and until and we have seen how to define the meaning of those particular temporal operator. We are going to define the meaning with respect to your model.

So, we have seen what is the meaning of those particular operators, and give a some clear idea about those particular temporal operator. I have given some example and along with that and we have discuss some question also. So, with this today I am going to wind up. So, in next class we are going to look for a particular temporal logic. Now, we are having with this four particular temporal logic operator; we are having several kind of temporal logics. So, we are going to look for a particular class and we will discuss about that particular class, and (( )) we are going to use that particular class.