Artificial Intelligence
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Lecture-69
Decision Theory:
Expected Utility vs Expected Value
PART-4

Okay, now before we move on to the next set of lectures, there is a small fun stuff that I would like to do with you guys, just for fun, I would like everybody to participate. Now we have been assuming that expected value is the right numbers, but is it**.**

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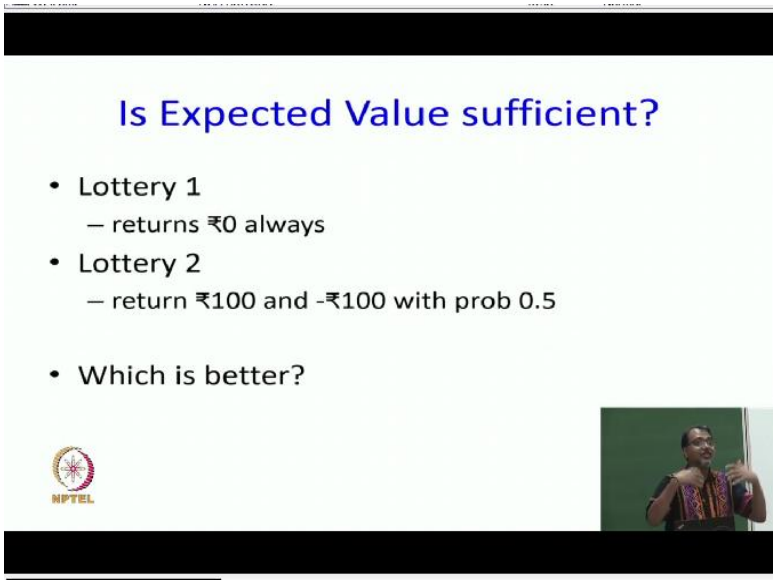

So let us start playing this. I do not want to be the professor who encourages the students to play lottery games, but if you were too good you is the question. So there is a lottery 1 which gives you 0 rupees every time. There is a lottery 2 which tosses a fair coin and says heads, I give you 100 rupees, tails you give me 100 rupees okay, everybody has to raise 1 hand. How many of you will play lottery 1, 1 2 5 6 7.

How many of you will play lottery 2, the whole class okay, what is the expected value of lottery 1, what is the expected value of lottery 2, in this world of our class, which one looks better

lottery 1 or lottery 2, lottery 2. How can that happen, what is going on. Now let me push this a little bit.

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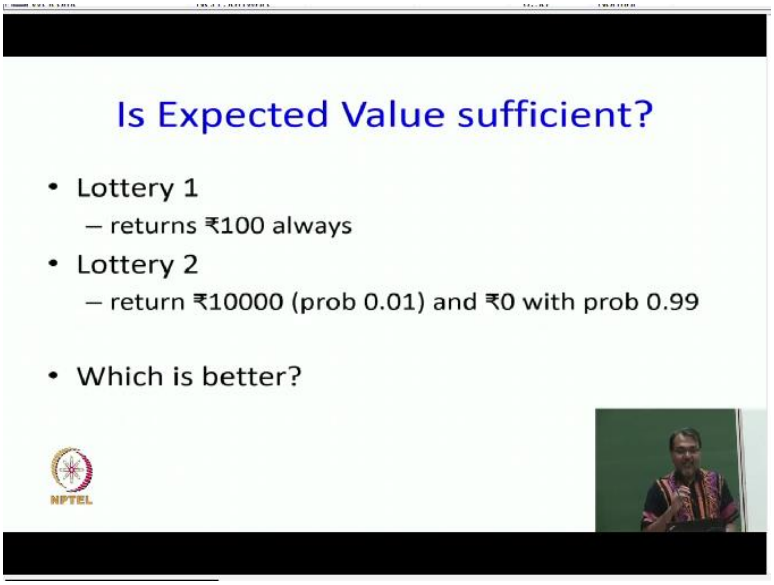

Lottery 1 always gives you 100 rupees you can only play it once. It is important to say you can only play it once right. You know why yes my people, lottery 1 always gives you 100 rupees, lottery 2 gives you 10,000 rupees with probability 0.01 and 0 with probability 0.99, expected value of both the case is 100. How many of you are going to play lottery 1, wow, more than half the class.

How many of you gonna play lottery 2, some numbers right 30 of you, 20 of you okay, now the worlds have shifted. It is not like more determinism is better. See what is going on. In the first case, lottery 1 was deterministic second case, right and also in the second case lottery 1 was deterministic. In the first case we chose the random one in the second case we chose the deterministic one, okay.

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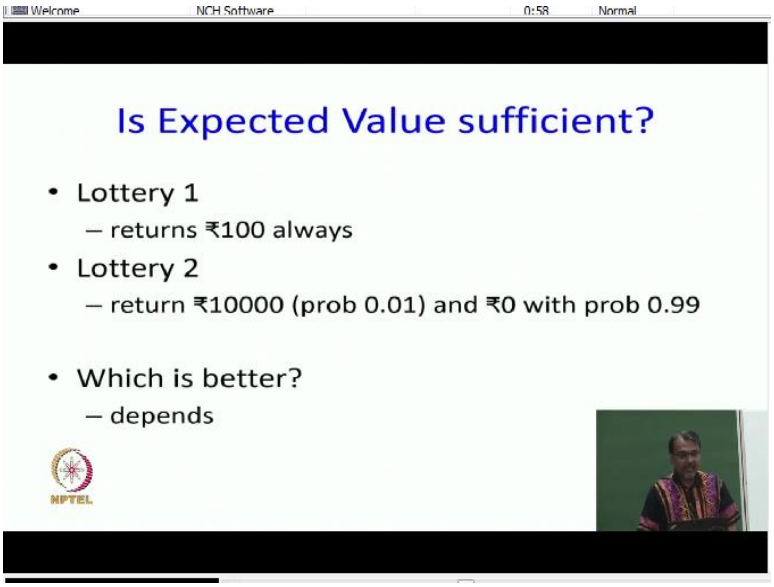

Now, as you can see, which is better, there is more depends in the answer right, this flip has been shifting. Now, I will push this further, I will say lottery 1 you will get 3000 rupees and lottery 2 you will get 4000 rupees with probability 0.75, but I will take 500 rupees with probability 0.25. Now, who is going to play lottery 2, 4 people, you are going to play lottery 1, everybody, right So completely shifted.

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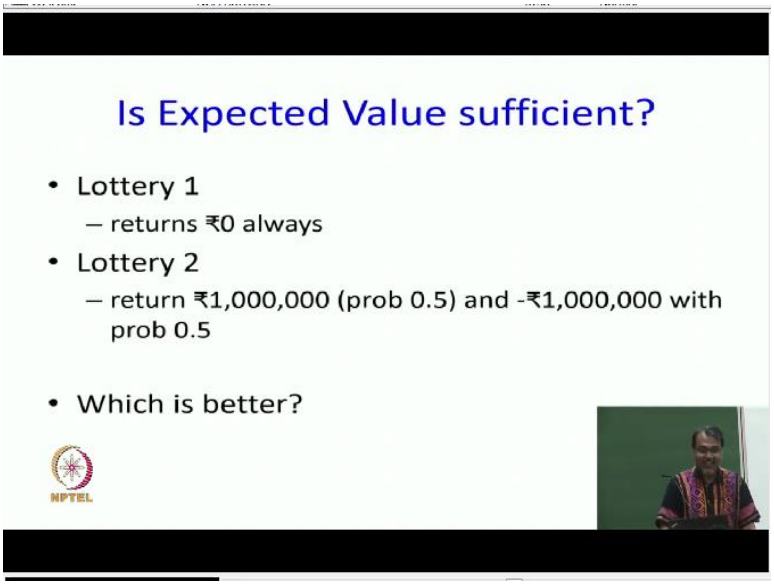

Now, just for the fun of it. Let us say lottery 1 gives you 0, and lottery 2 gives you a million rupees with probability 0.5 and a million rupees takes away a million rupees with probability 0.5. Who is going to play lottery 2 only the French guys. Because they are doing conversion, right

Oh, it is only few you those who cares right. You know I used to do this in dollars when I was teaching at NCITEL.

And I was teaching a professional master's class. This is a class where you know, people are working in Microsoft and this that and the other. And again, I asked this question who is going to win a million dollars or lose a million dollars there was 1 person who was determined to raise their hand. It is like losing a million dollars. Who cares I would possibly gain a million dollars that will increase my right.

What is going on okay, let us this is a good time to figure out what is going on. Yes, so who has a theory, what is going on. First of all, have I proven that expected value is not the right thing yes right. Even though we will work with expected value for the rest of this class, at the back of your mind, you should always remember it is not about expected value in the real world okay. So what theory could you give yes, Divya I am sure you have not spoken in a while.

It is about the degree of risk you are willing to take. And the risk that Bill Gates takes is gonna be at a different level then the risks that I will take and hopefully is going to be at a different level that the risk you take with your money, not your parents money, and then it will switch possible right. And how do we operationalize risk there okay. And there are many different ways to operationalize risk.

One way to operationalize risk is to say oh, I will just take a decision based on the minimum, because minimum is the risk, in some case a minimum is - 1 million, in one case the minimum is - 500. In one case the minimum is 0, but that will not give you the a satisfactory solution. If you absolutely had to choose a minimum, you will never want to, you know, climb mount Everest.

If you always had to choose the minimum you will not live in your house because the roof will fall down, you will not drive on the road because somebody will kill you. So, we do not make decisions based on the minimum. You could say it is about variance. You could and that there is a legitimate way of thinking about it, those problems become much harder to solve, but yes, it is possible.

What is something simpler we could do, you have a hint. So, how many of you have taken an economics course. So people have taken an economics course should have some idea at least if you have done some micro economics, you should have some idea of where am I getting to. There is a very famous law in economics, called the law of diminishing returns, that is relevant here.

So a law of diminishing returns, says, you know, if you have taken if you have eaten 2 rotis in the third roti is less important to you, then if you had not eaten any roti in the first roti, right. So, is being compared. And same for money, I guess. But if you had to think about modelling that particular law in our world, what are we saying , if you are following law of diminishing returns, then are you a risk averse agent or the risk prone agent okay, that was too fast.

So we will get to the answer to that. So basically what happens in economics and often we can use this if you are using a model for decision making over humans is that we do not work with rewards, we do not work with money, we work with the utility over rewards.

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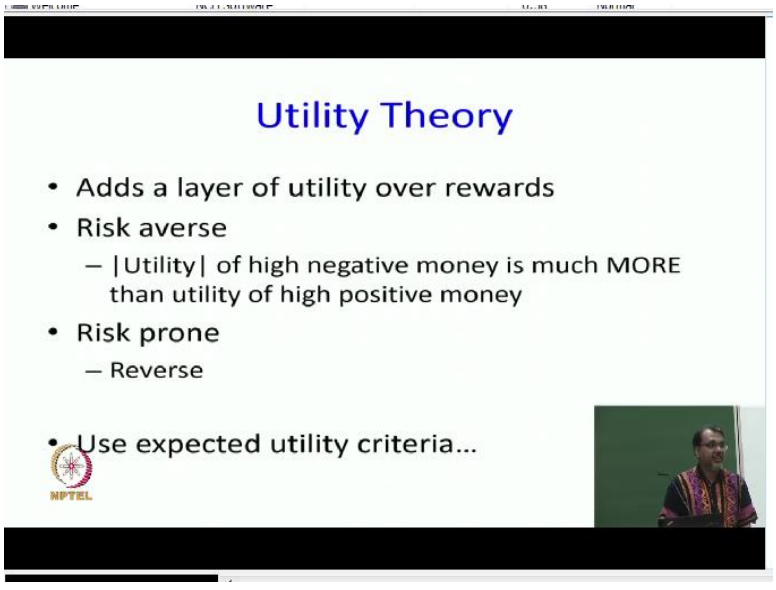

So, we define a secondary function called the utility function, which is a function that takes in the money value and gives in some sort of calibrated normalized value called the utility and if you are risk averse agent if I take 100 rupees from you, versus if I give you 100 rupees, you will

feel more pain in taking away 100 rupees then the joy you will feel by getting 100 rupees. Whereas, if you are risk prone agent.

Then I can take 100 rupees from you and you will not feel as much pain as the joy you will feel when you get 100 rupees. And if you are the risk neutral agent then it will be exactly equal. And by enlarge our Indian society is a risk averse society. This is because most of us come from a middle class mindset and our parents have ingrained to us that how important is earning each you know, paisa and how careful you have to be with your spending decisions and so on and so forth.

And therefore, the utility of negative money, the magnitude of it is much higher than the utility of same amount of positivity right. However, different societies look at money very differently and you will find that different societies have more risk taking abilities sometimes than us or maybe worse also. So, therefore, instead of doing expected value, people use expected utility as the criteria. So, utility is sort of expected to normalize all this and then you can take an expectation on top.

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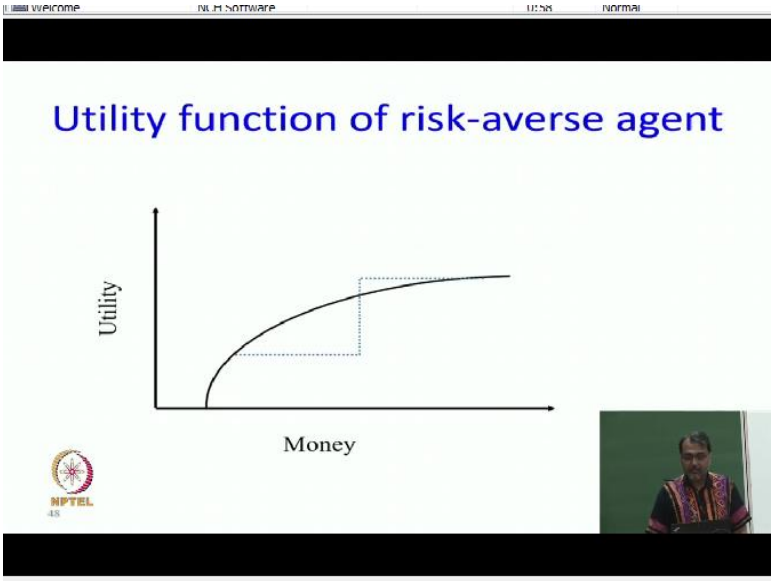

So, this would be the utility function of risk averse agent, which basically says is as money goes negative, the utility becomes very highly negative. It is sort of going down very fast right Negative utilities going down very fast. But if you keep giving me more and more money

beyond the point, yeah who cares if I made my first million, that is amazing. If I made my second million, that is very good too.

If I am making my third million good. That would be a utility function for the risk averse agent. And these dashed lines sort of show you that if the x axis is same, then early on, the utility is much higher later the utility continues to diminish. So this is also following the law of diminishing returns.

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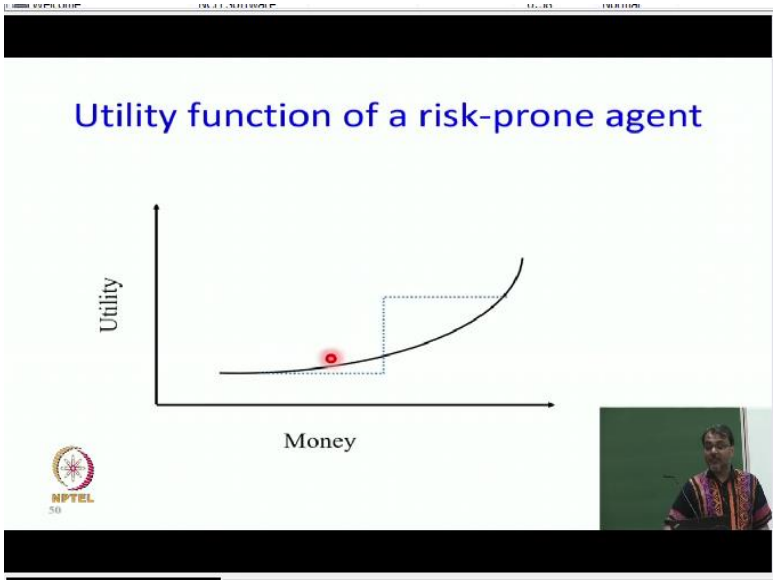

On the other hand, the utility function for the risk prone agent will look something like this. It will say that the 10th million is much more important than the first million, right and there may be some situations where people who show risky behaviour will probably be following this kind of utility function.

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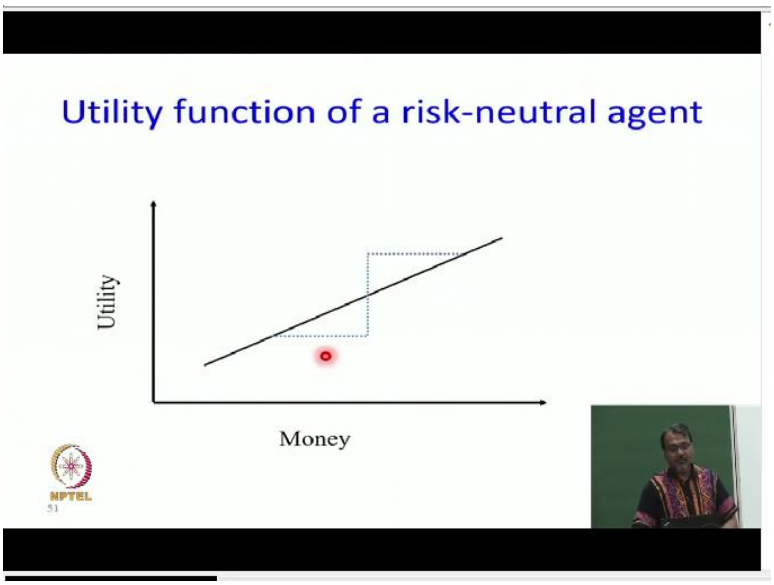

And of course, if you are neither risk averse nor strong you will be risk neutral and then expected value will be same as expected utility. So, for the rest of the course, we will still work with expected value. But at the back of your mind you have to realize that you may not use pure rewards. You may use another utility function on top of those rewards if you are working in a space where agents can be risk averse or risk prone, but as long as there is neutral, this is good enough. Any questions from this alright.