Artificial Intelligence
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Lecture-68
Decision Theory:
Probabilistic Uncertainty and Value of Perfect Information
PART-3

Okay, so I want to quickly talk about probabilistic uncertainty and then we will continue this in the next class.

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So probabilistic uncertainty is when I not only know what my future states of action, but I also know some probabilities associated with okay. So now we can come back to thesis point. If I give you a probability with each state of nature, what criterion can you use expected reward. So now you can take the expectation and create an expect reward or so you will have to maximize the expected reward or you can minimize the expected regret.

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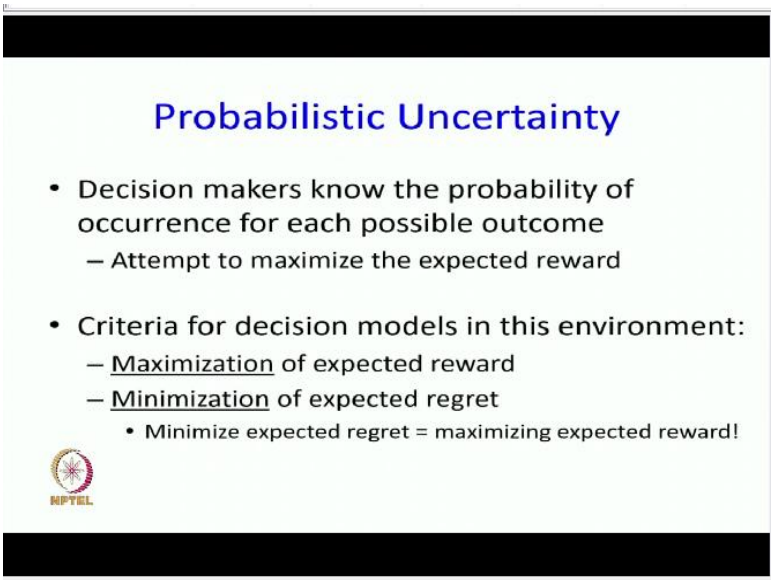

It will minimize the expected regret. The good news is that for this setting, one step decision making problems. Minimizing expected regret is same as maximizing expected reward. And you can check that for this particular probability.

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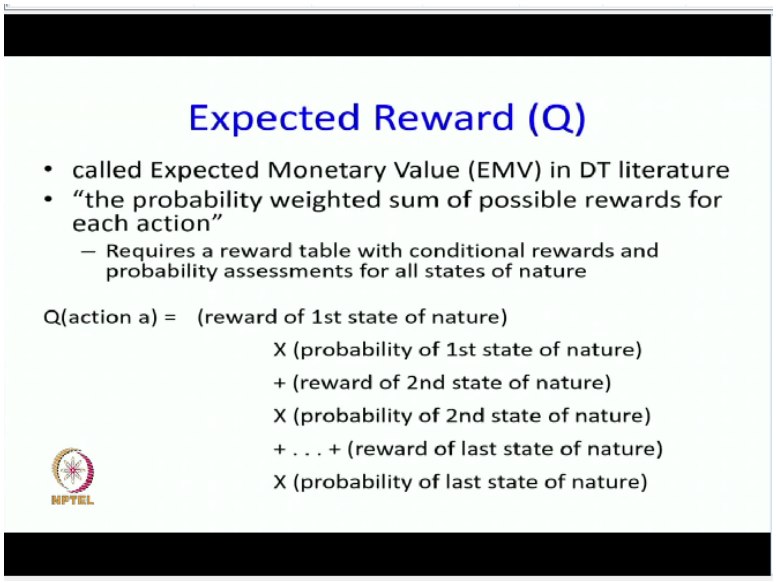

I am going to associate expected reward of an action with the letter Q okay. So Q of an action will be reward of first outcome, times probability of first outcome, plus reward of second outcome, times probability of second outcome and so on, so forth.

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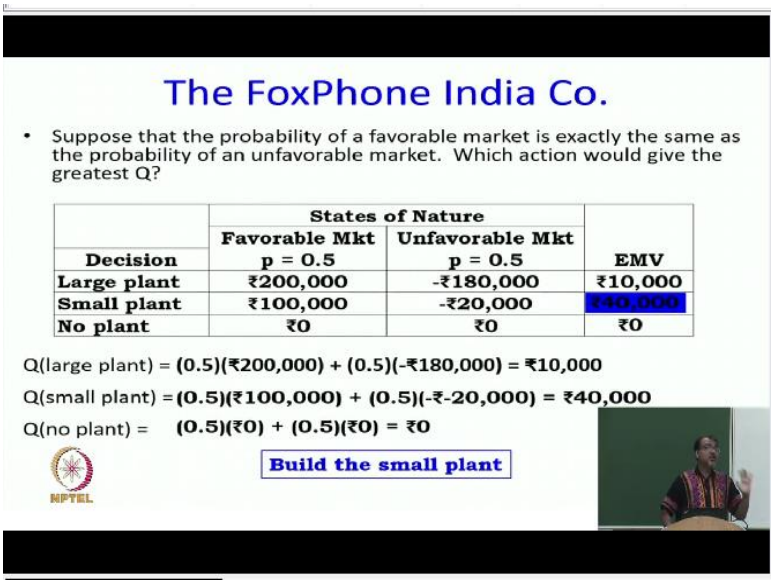

And so let us say a probability I did not know much, so I only said that favorable and unfavorable markets happened that probability 0.5 each. Then for large plant it will be 10000 expected value. Some people call it expected value. Some people call it expected reward. Some people call an expected monetary value okay, same thing in our case, for small plant it would be 40000 and for no plant it will be 0.

And then I will pick the maximum on the expected reward. And that will be 40000 which is small amount. Now everybody with me, this is easy stuff, right. But I want to spend even one more minute on this and make sure that you also look at this visual way of thinking about this.

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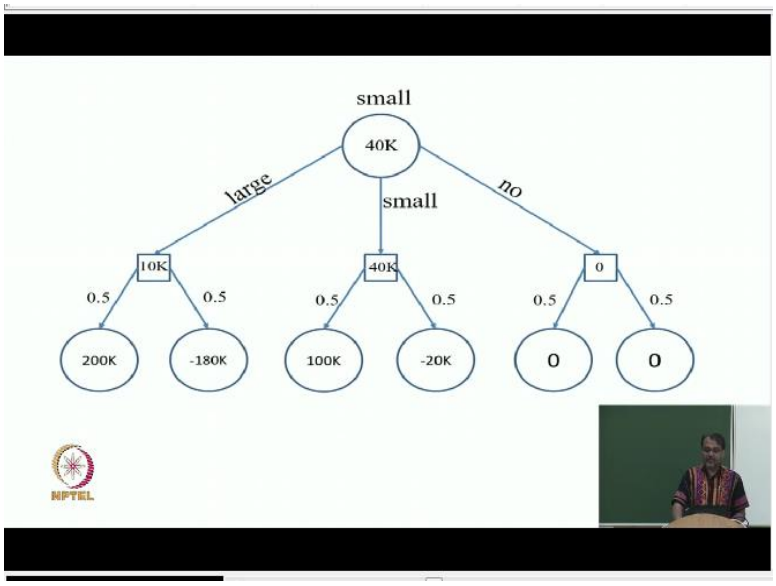

Because when problems become harder, this visual way will be your guide. And I am saying this with all my experience. This is you at the top, you in a circle, because you have to choose an action when you have to choose an action it will be a circle. You are in control. And what are the actions, large plant, small plant, and no plant and that they will be arrows. But once you make an action, multiple things can happen which are not in your control.

And that would be a square. And in this case 2 things can happen favorable or unfavorable. And you also know the probabilities, they happen with probability 0.5. So you make this graph and then you say, okay, when large plant happens in favourable market happens, I make 200, when large plant happens and unfavorable market happens, I make - 180K and so on so forth. I list the rewards and then I back them up.

You can think of them as expectimax, if you remember expectimax. And so I will first compute the expected value. And that will go in the circular node or the square node, it will first go in the square node. I will back it up okay. So, for the first square node, it will be 10K, for the second square note 40K and 0. And then in this circular node I am going to take the max and that will be 40K. And my answer would be build a small.

Now this seems obvious to you, but you shall see how this would become more and more interesting as problems become harder and harder. So now I am going to leave you with a question today, which I will answer in the next class. And that question is as follows.

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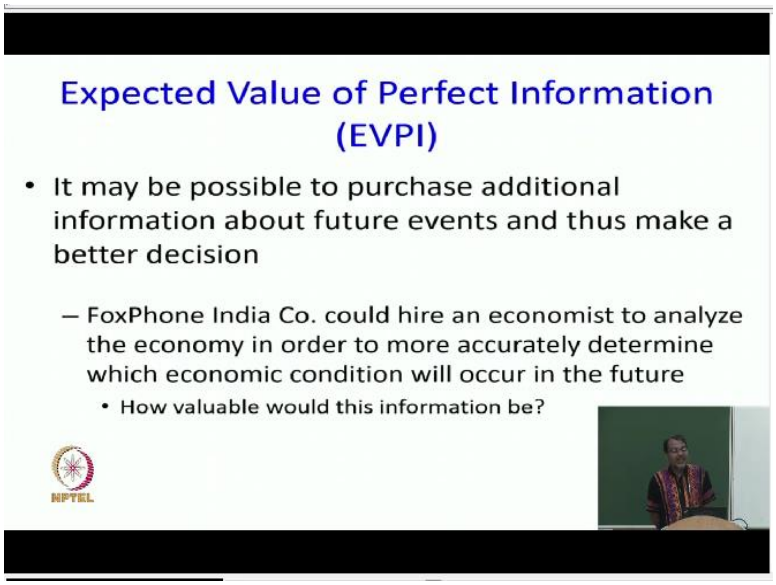

Let us suppose. That somebody comes to FoxPhone company, let us say this is an economist who do market service, who does market service. And this economist says that I will analyze the economy for you, I will go into the villages, figured out how many people can pay yours, cheapest smart phones. I will go into the urban area. They will figure out how many people might be okay with cheapest smart phones and so on and so forth.

And I will do this survey for you and I will tell you whether the market is going to be favorable or unfavorable. You said, wow, that is great. But the question you have to answer is how much should you be willing to pay this economist for this information. So I want to make sure everybody understands the question okay. The question is FoxPhone company could hire an economist to analyze the economy in order to more accurately determine which economic condition will occur in the future.

Whether it will be a favorable market or it will be an unfavorable market. And for now, let us say that this economist tells me the truth. When the economist says it is favorable, then it is definitely favorable. When the economist says it is unfavorable, then it is definitely unfavorable okay. So the economist gives me perfect information. And the question I am asking is, what is the expected value of perfect information.

In other words in this setting how much, what is the maximum you should be willing to pay the economist. Of course an economist comes to you and a market researcher comes to you or an MBA comes to you and says you do not decide on the large, small, plant or no plant right away. Give me a little bit of time. I will go do the survey. I will go into the villages, I will go into the urban centres, and I will come back and tell you what is the market for indigenous smart phone's.

Is it favorable or is it unfavorable. And the question then I have to answer. So I will say, okay how much would you charge for such an information. And the economist will say okay, I want to charge you C rupees. The question I have is, should I hire this economist or not right. How much is the maximum C I should be willing to pay for this information and this is what is called the value of information, often denoted as VOI.

Some people also call it value of perfected information VPI. If what the outcome is going to be perfect, so if the economist says it is favorable, I am going to trust the economist so much that yes, I will believe the market is favorable and the market is favorable and vice verse then they will call it value of perfect information. But of course in the general case it might just change my probability distribution right.

And we have to deal with it somehow. So the question you have to answer is how valuable is this information in many terms. And how do we even go about. Analyzing this, a few people have thought about it I am very happy about the person at the far and yes, what is his name. Shivam yes, right. So the first thing that he says which is right. Is that let us suppose we do not have this additional information. Then this is our tree.

We do not know whether the market is going to be favorable or not and we will be making our decision based on this tree. And therefore, in expectation, we stand to gain 40K, good. This is what we would have if we did not have this additional information. So now the question is, if we had this information, how would you think about somebody else. There was another hand from that. Yes, what is your name. Yes, yes.

So if we have this information, we have to compute the result with this additional information. And it is easiest to think about it pictorially and I will reemphasize the importance of this pictorial graph, because this graph really, really helps you streamline your competitions. If you make this graph is a good chance, you are not going to make a mistake. This graph has to be made chronologically**.**

So what is the first thing I would do. I would say hello economist, go and do the survey. What did the second test that is going to happen. The economist does their survey and comes back with an answer. And then what do I do. Based on their answer I decided my best action of plant, no plant or small plant or whatever. This is the chronology. This is important. So my first step is to ask.

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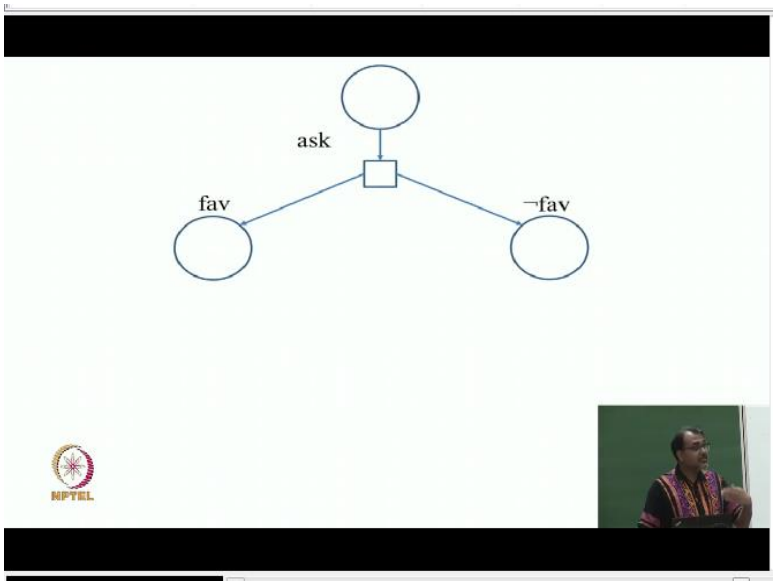

It is to say hello, go do the survey. Come back with the answer. Now what answers can he come back with favorable or not favorable, are they in my control. They are not in my control, I mean nobody else control the economist goes do, they search and then comes up with the answer. But A priory I do not know they will come back with and I have to deal with both of them, because it is not in my gut. So it is an OR node or AND node. It is a circular node or a square node, it is a square node.

Also called an AND node, and these kinds of the graphs are called AND OR graphs, typical graphs are OR graphs, these kind of graphs are called AND OR graphs okay. So therefore, you can see that there is a square node and I have 2. Now why are the other 2 nodes circular nodes. Because based on their outcome, I am going to make a choice. I am going to make a decision. It is now in my control what I do based on their visit okay, so this is how I am building this graph.

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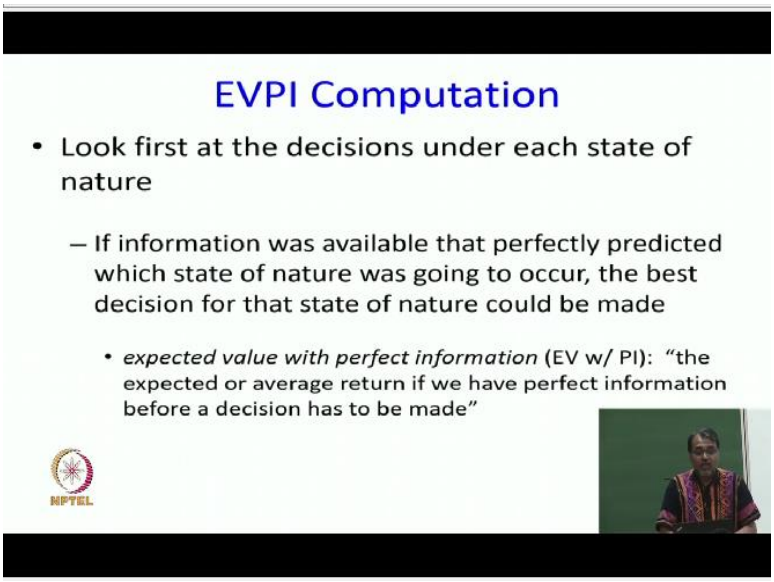

So the first thing I am going to do is I am learning to compute what is called expected value with perfect information. With perfect information means that if I knew what is the perfect information, what am I going to do. But I do not know what is the perfect information going to be, so I will have to take expectation over the various information I will get right. So it is the expected or average return if we have perfect information before a decision is made.

So intuitively, if I tell you that the market is favorable what are you going to do, large plant. If I tell you the market is unfavorable, what are you going to do, no plant. Now notice that this decision is going to happen after I get this information. This is where we have rolls flip earlier. I was making this decision and I did not know the outcome of favorable or unfavorable. Now I know the perfect information. It is either favorable or not favourable and I am going to then make a decision afterwards right.

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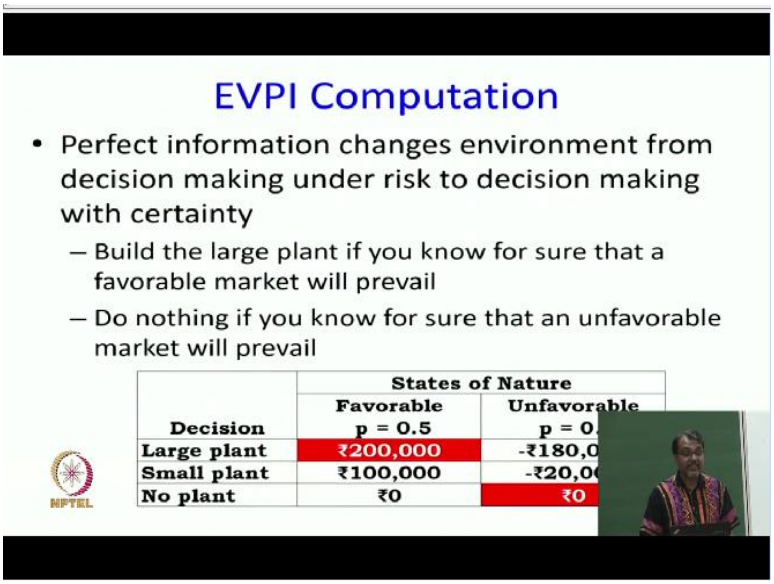

So therefore, if the market is favourable I stand to gain 200K. If the market is unfavourable, I stand to gain 0.

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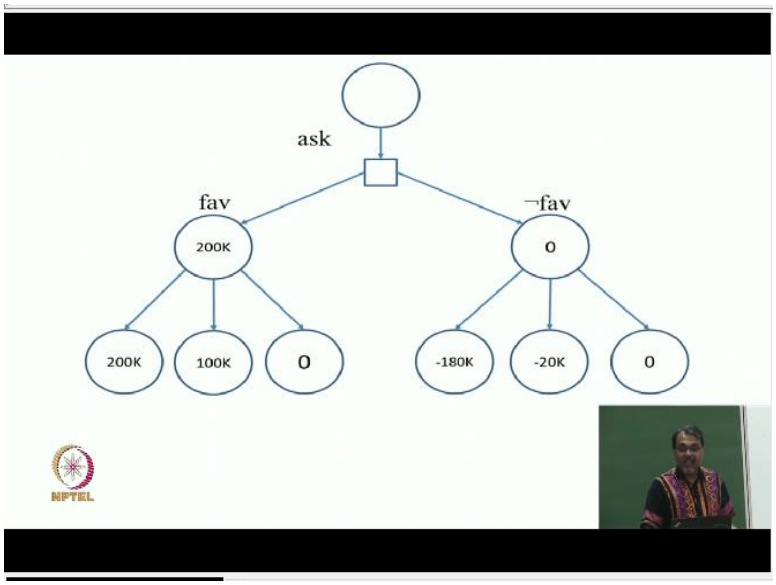

And pictorially, what is going to happen is if the economist tells me market is favorable, I will still have to make a decision between these 3. And if the economist says market is unfavourable, I will still have to make a decision between these 3. But now I can take a different decision because these are 2 different nodes. One is under the condition that economist said favorable, one is under the condition economists say not favorable.

And so the value of the favorable edge, I mean, the favorable node is 200K the value of the not favorable node is 0. However, I am making all this competition today. I have not even asked the economist yet. Today I do not know what is the economist going to say, is the economist going to say favorable or not going to say not favorable. So I cannot make a decision based on 200 or 0. I have to take an expectation with whatever probability distribution I have at my disposal right now.

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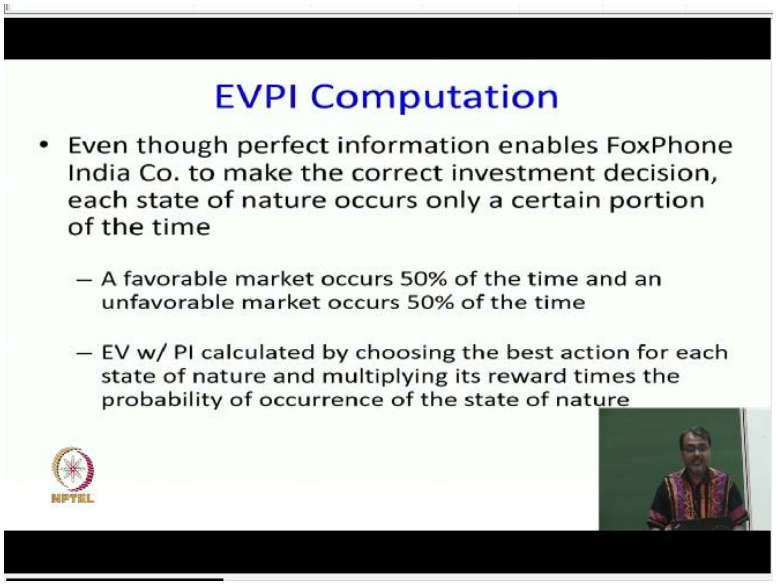

And the probability distribution is a favourable market might be return 50% of the time, and unfavorable market might be return 50% of the time. So this is value with perfect information when information is favorable to 200, value with perfect information, then the information is not favorable 0. But expected value with perfect information happens when this information is not available to us.

And that would be calculated by choosing the best action for each outcome and multiplying its reward times the probability of occurrence, which would therefore be 100K. And if you look at it in the graph, basically, we are seeing is that favorable could happen with probably 0.5. Unfavourable could happen with probably 0.5. And therefore my expected value with perfect information and expectation always happens at the square node right in this case it would be 100 so far, so good.

But my question still has not been answered. My question was, how much should I be willing to pay this economist for this information. Somebody was in spoke. And now should become more or less clear to, let us say, what is the max I should be willing to pay the economist. Somebody was in spoken. Yes. Yes. This person orange shirt. What your name, Path yes, 60000 and why 60000. The best we can get is 100000 with the economist in the loop.

And if you do not have this information, we get 40K, that we computed last in the last version. So therefore, we will be silly to give more than 60K to the economists for this information, because that means that we will end up losing more money by taking that action.

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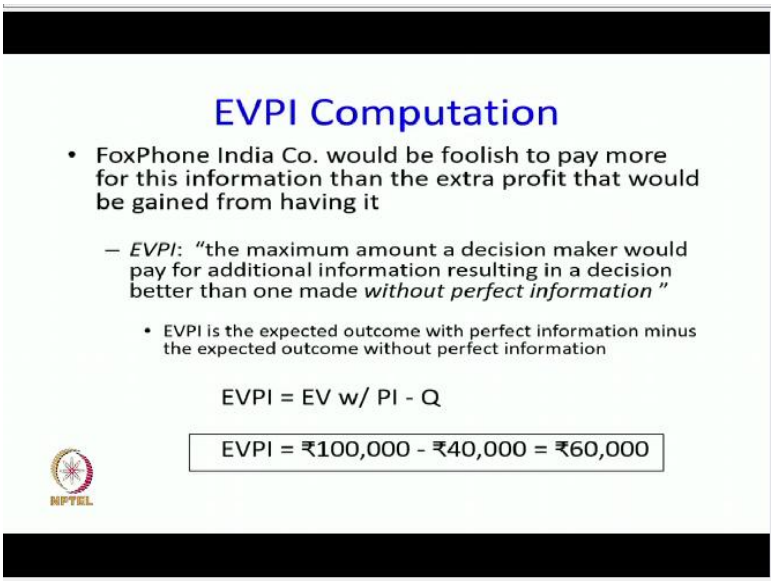

So this is exactly right. Expected value of perfect information is expected value with perfect information minus the expected value without information and this equation sort of stays forever.

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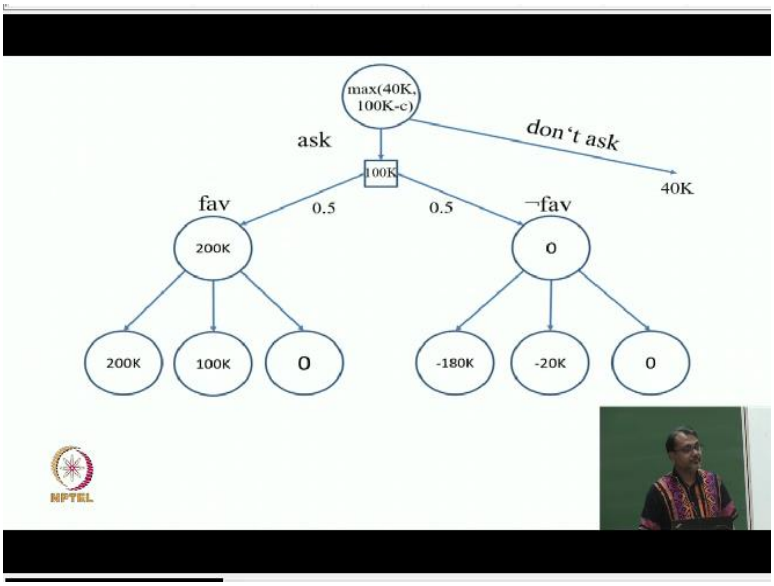

And in terms of the graph, what is happening is that. I can also not ask the economist that is my other possible edge. And I know that if that happens, I will make expected value of 40K and let us say economist charges C. So I had my OR node I have to do max of 2 numbers. One is 100K - C, where C is what economist charges and the other is 40K. And of course, the left edge is picked if 100K - C is greater than equal to 40K. And that means C equal to 60K okay.

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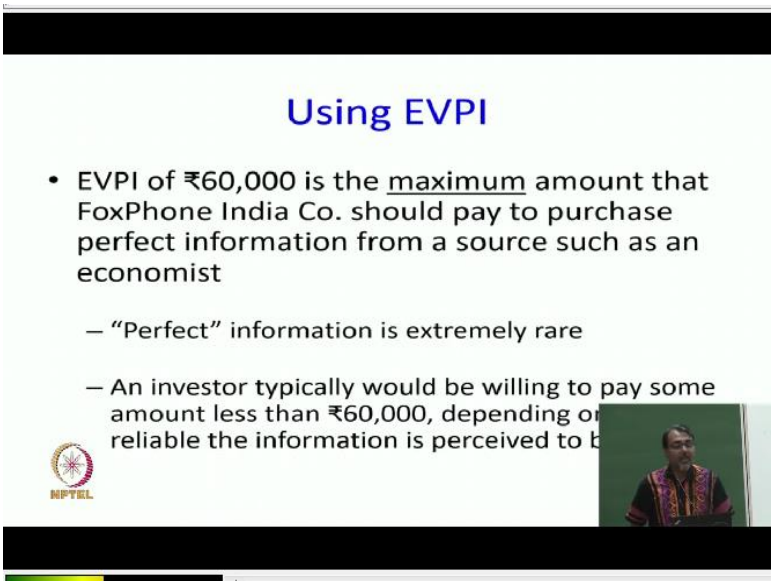

So expected value of perfect information is 60000 in this case, it is the maximum amount that FoxPhone India company should pay to purchase perfect information from a source such as economist. But of course in practice you may not really pay 60000 because perfect information is rare, you do not know exactly what is going to happen. In fact, after they come up with the

information, we may build up plant and they may, you know, suddenly Jio will combine in the middle. And if that happens, then the whole probability distribution changes right.

I have a company which was doing really, really well. It is a start up company which was doing amazingly well before you Jio came into the market. And then suddenly they have become sidelined because whatever things they were planning to do and they were proposing to do Jio does it for free more or less. If you on Airtel today, at least I do not know if you guys have been using mobile phones for the last 4 or 5 years or you are too young that you only started using the when Jio came into the market.

But there was a time when mobile phones I mean, the plans are expensive. I have been on Airtel for a very long time. And then suddenly Jio came into the market and in the next few months, I suddenly find Airtel prices to get slashed. This is what happened when somebody is giving lot of subsidy, everybody else has to somehow flawed and they cut the profits, but made sure that the customers do not go out.

And other companies started merging Idea and Vodafone and BSNL and all there are many, many such companies. They were more now they are being merging, I do not remember who are all merged in. So to summarize, many other things can happen, and so you in practice will not give the absolute maximum amount. But this calculation allows you to figure out what is the absolute maximum that you should be paying the economist.