Artificial Intelligence
Prof. Mausam
Department of Computer Science and Engineering
Indian Institute of Science Education and Research-Delhi

Lecture-67
Decision Theory:
Non Deterministic Uncertainty
PART-2

So the first thing you need to do is list your possible actions because that is your space, that is the space in which you are playing, that is the space in which you are making a choice. If it is not there, it should not even be part of your thought process ideally.

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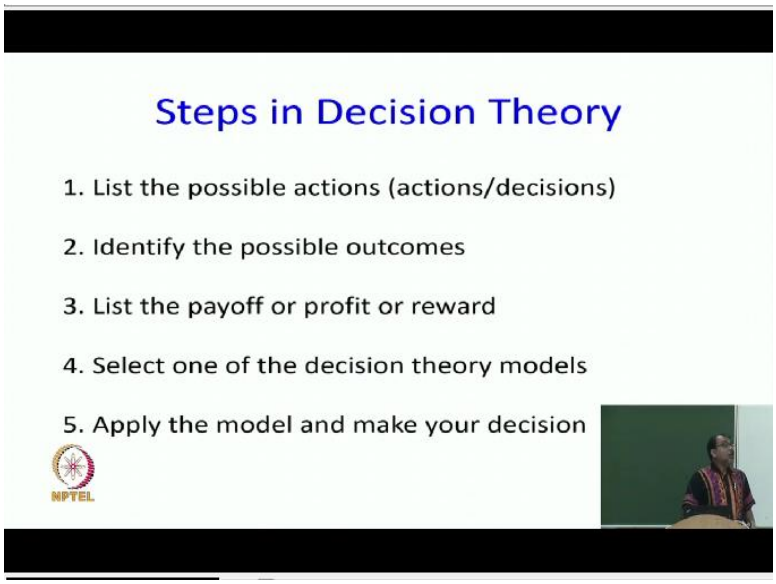

Of course, now with each action, you identify the possible outcomes and list the payoff profit or reward, same thing we can call it in different ways, payoff or profit reward cost, cost is negative of reward, you can list that out. And then based on the knowledge that you have select one of the decision theory models and apply the model and arrive at the decision. This is how at least you will do it in the machine. And in order to develop this, we will do this running example.

Now I want to point out that this running example is I do not know by whom. But I found these set of slides online. And I looked at these slides, and they were beautiful. And this was by

somebody from NPS. So I took those slides and I kept finding the person who has written the original slides, but I could never find it. So all the credit of this class goes to this person.

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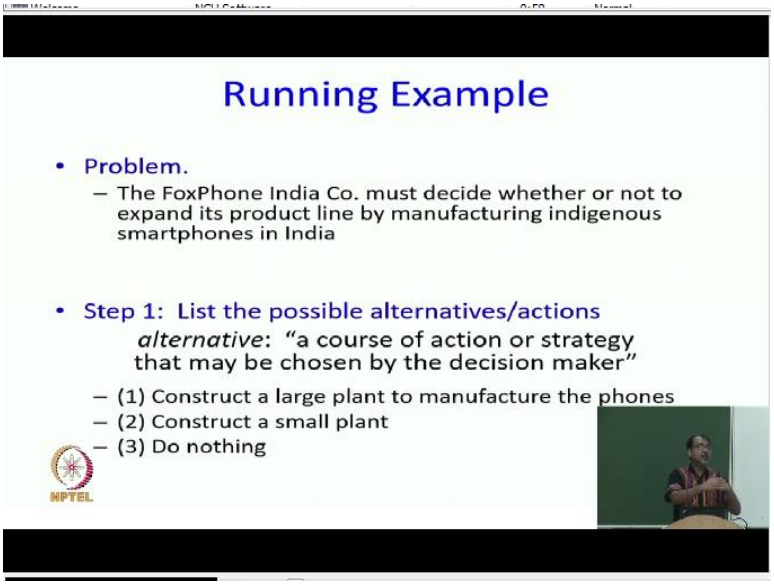

Now it is a beautiful example. And of course, I have adapted it for this class. So, this is the problem that we will be studying the FoxPhone India company, it is a new it is a company which manufactures phones must decide whether or not to expand its product line, maybe it only makes these feature phones by manufacturing indigenous smartphones in India. So you know, you have this company.

And you want to decide should I start manufacturing smartphones, okay. And let us say you can take technology from somewhere and your job is whether to manufacture or not to manufacture. And there are 3 alternatives that you can have, alternative is a course of action or a strategy. That may be chosen by the decision maker, it is in decision makers control. Let us see you have 3 options. Build a large manufacturing plant to build lots of these smartphones.

Build a small manufacturing plant and do nothing. Just chill. Now we have listed all the possible actions, now let us just list all the possible outcomes and to list outcomes we define what is called a state of nature.

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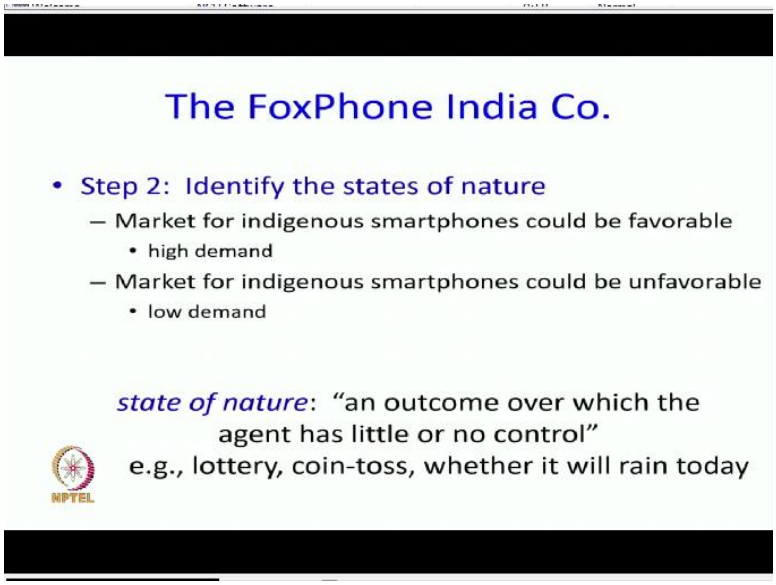

A state of nature is something that is not in your control. It is in nature's control. And sometimes you may not even know exactly what is going to happen like coin toss, unless you are do not need you do not know what is the nature going to give you heads or tails or and so on and so forth. So in this case, there are 2 states of nature market for indigenous smartphones could be favorable.

Lots of people suddenly say Oh, Indian smartphones let us buy them or the market could be unfavorable, they say that I already have my Redmi or my Samsung, and I do not want to buy the cheaper smartphone, right and you do not know exactly whether the market is high or market is low demand at this point.

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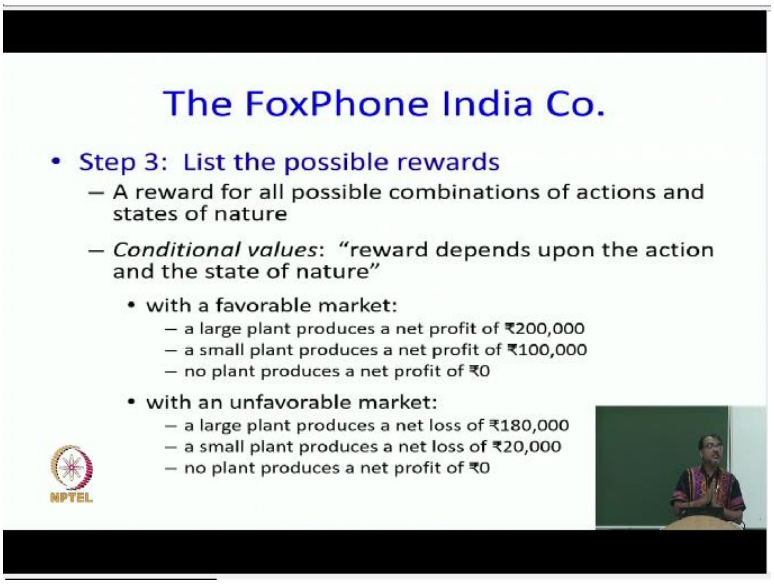

Now the next step is that you list all the possible rewards, right. So reward for all possible combinations of actions and states of nature, right, so these are conditional values, the rewards depend on what you did, and what the state of nature was. So for example, in this case, let us say if it is a favorable market, you make a net profit of you know 200,000 for the large plant, 100,000 for a small plant, and of course, if you do nothing, you get nothing.

On the other hand, if you put a big large plant and the market is unfavorable, you make a net loss of 80000 for the large plant, or 20,000 for a small plant. And of course, if you make nothing thing there is no loss.

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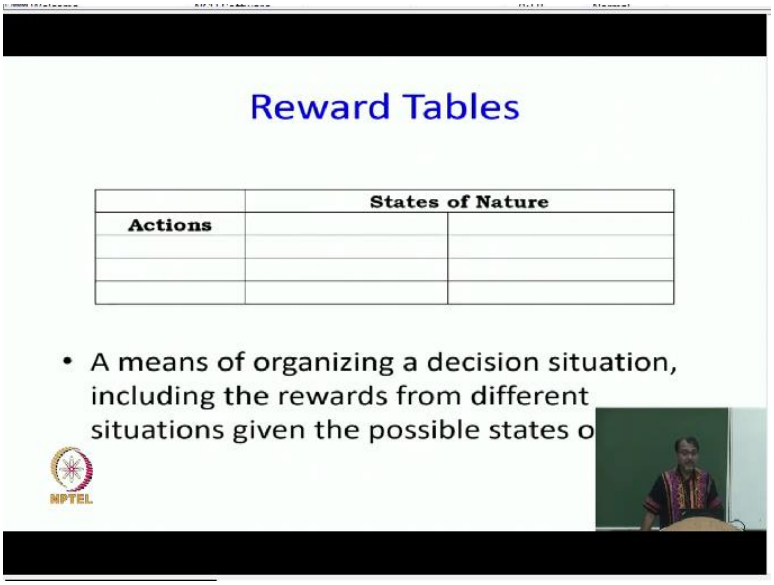

So we can write it down in reward table. It is a way of organizing this decision situation, right. And so let us say on the various rows or various actions, and various columns or the states of nature.

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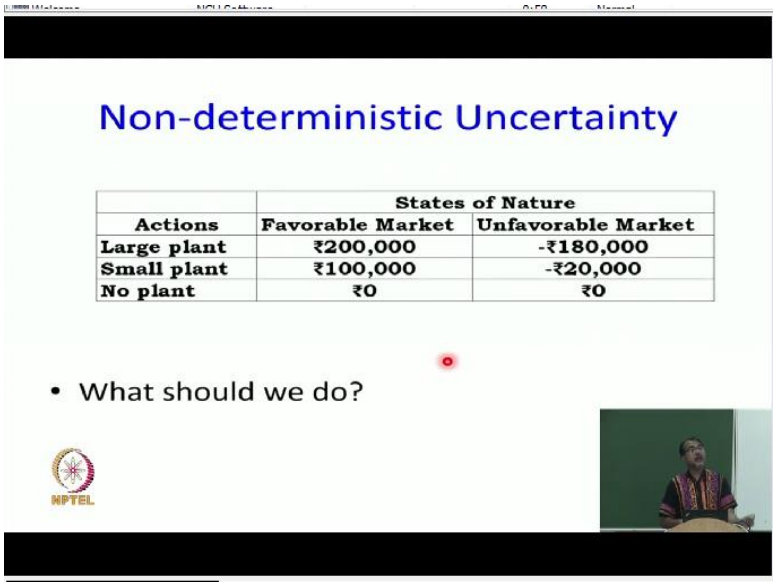

So let us say this is how we have summarized all this information, that your actions could be large plant, small plant, no plant, a market can be favorable or unfavorable. And these are the reward values in each case.

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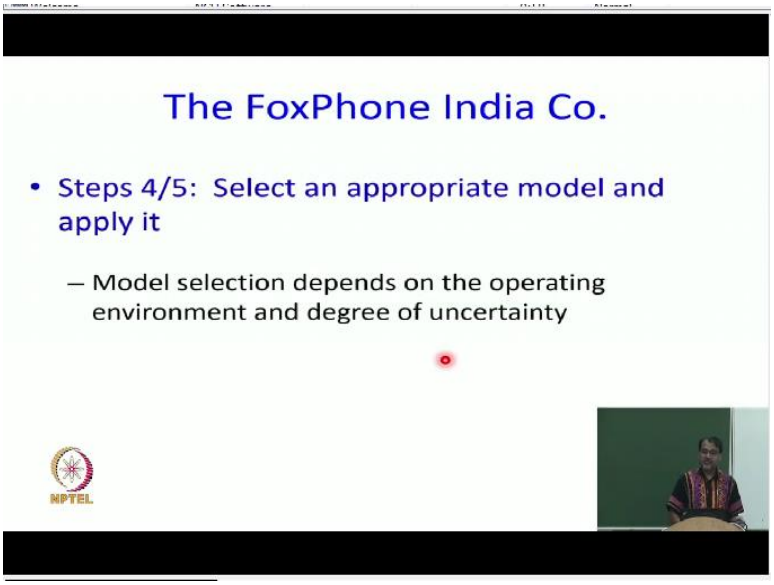

And now your goal is to select an appropriate model and apply it. And your model selection depends on the operating environment and some degree of uncertainty.

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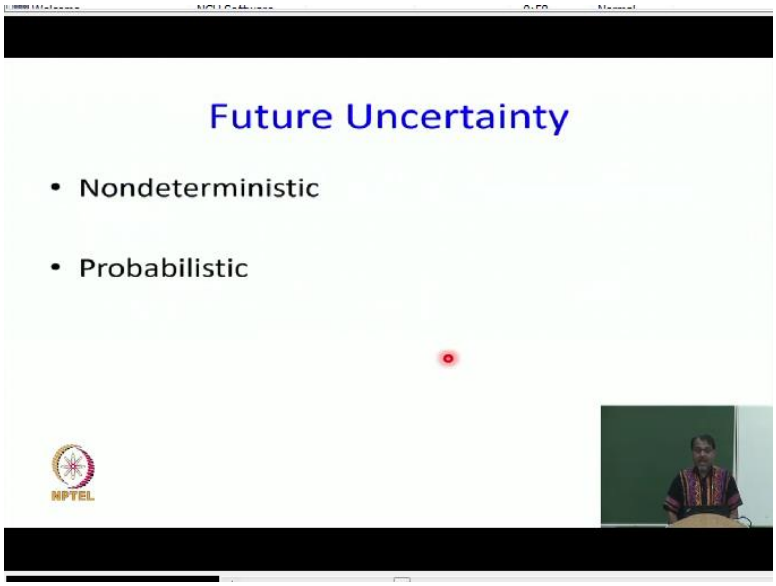

And your uncertainty can be of 2 types. And of course, the more interesting one is probabilistic, but let us spend a few minutes talking about another kind of uncertainty called the non deterministic uncertainty, where I do not know whether the market is favorable or unfavorable. And I do not give you any more information than that. I just do not tell you anything more than this. So, this is the table that you know and nothing else.

And so the question that I would want to ask you before we move forward, in today's class, that I will ask you a very simple question. So please you know everybody should participate. What would you do if I gave you this set of numbers, and I did not tell you anything. What would you do, how many of you will make a large plant, one person only one person will make a large plant. How many of you will make a small plant, many of you will make a small plant alright.

And how many of you will make no plant whatsoever, these people are people would like IIT Delhi you cannot plan mega, they are just happily chilling right, happy to you know, enjoy the sun and lay down on the bed. Why would you do nothing just out of curiosity. It has no loss. Very good, so he says what is your name Harkidhan, Harkidhan says, look, making no plant has no loss right. You cannot lose if you do not play. Why would you make a small plant, anybody yes, expected loss. What is your name, Prathis. Prathis right, Prathis says expected loss of a

small plant, now important question. Can we compute the expected loss for computing the expectation what do we need to have, do we have probabilities.

Wow, why would you make a small plant yes. What is your name is Hash okay, so Hash says the loss is not too much, but the profit is commensurately, disproportionately high. So that is a better balance for me and we will try to see if we can operationalize that kind of an intuition in terms of a model. But now why would anybody make a large plan just out of curiosity yes okay, okay. So so, that is a convoluted argument specific to smartphones and so on and so forth.

Let us abstract everything out. And let me just give you this table right. We are working on AI and not just exactly on smartphones and management here, right. So if I only give you this table, why would anybody make a large plant, the answer is simpler. What kind of a person would be okay, like this is a good way to ask this question. What kind of person would make the large plant, optimistic person, the person the green shirt what did you say.

Who has large amount of funds, but look, I am not even giving you the where who has large amount of funds, That is a good point okay. What is your name, man says somebody who was potentially when you take it to spare. That is another way of thinking about this. And we will talk about this in the next class towards the end. But many of you understand the intuition that an optimistic person will probably make a large plant.

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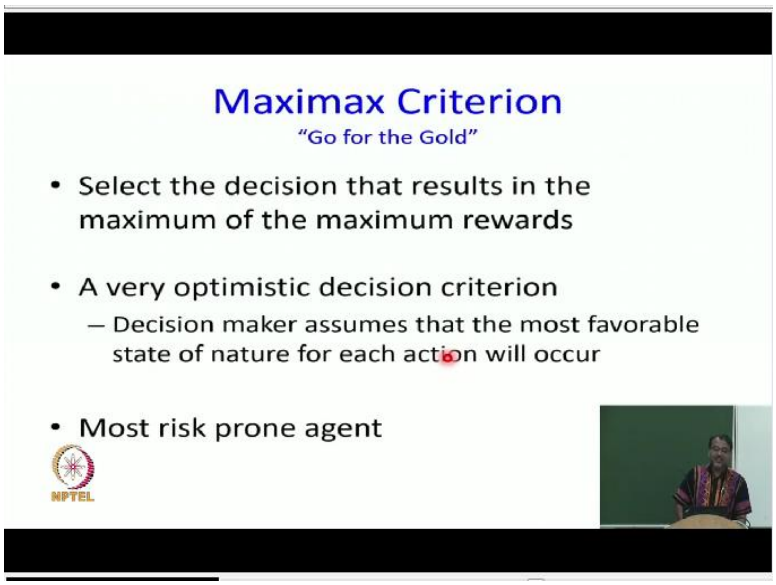

So one criterion for making decisions is go for the goal. Do max, Cmax, let us understand what maxi max means. Look, we do not know any probability distribution. I have not told you anything more. So you can only you do not know whether the market is going to be favorable or unfavorable . So you say, well, market is going to be the best. People who get married are like that okay, that is not true.

But if you marry for the fifth time, then you are definitely like that. The last 4 months we are not good. Next fifth one is going to be the best okay, sorry, serial entrepreneurs who fail and restart the company I thought we like that. That is something you will appreciate more. So, people who say that look, I think the world is beautiful. I think the world will just play their cards exactly to make sure I get the maximum reward.

So let me choose the most favorable option for each action and then figure out which is the best action. This is the agent which is most risk clone, this agent embraces risk, it says that I am willing to jump down the cliff in the hope that in a suddenly a parachute will open up for me and I will be saved, not saying this is taking it too far but that is sort of the optimism that this person will have.

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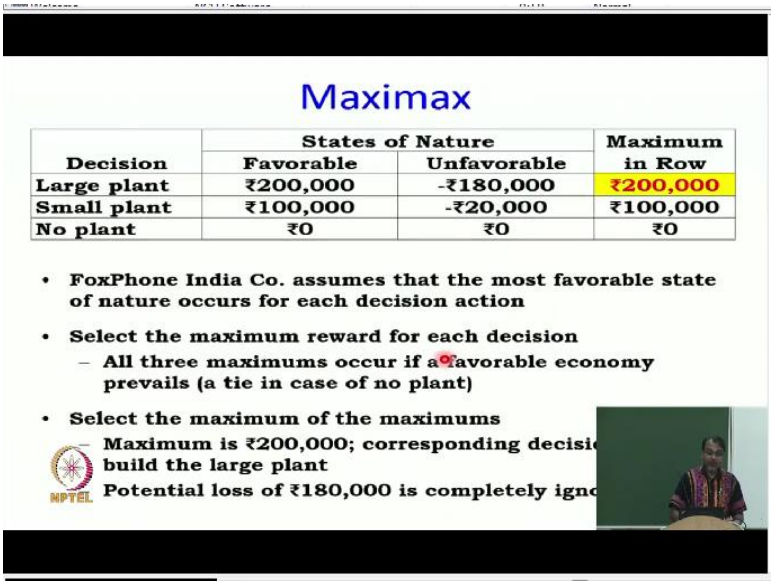

So what would it do. It would say for large plant what is the best I can get. What is the max of that row. Well the max happens when market is favorable okay, I will make 200K, for a small plant where does the max happen, the max happens into the market is favorable okay I get 100K. For no plant where does the max happen actually does not matter however the market is I will get 0.

Now I know that in the best case for large plant I will get 200, for small plant I will get 100 and for no plant I will get 0 which one should I take, max of these maxes, that is why it is the maxi max criteria. And therefore this person will make a large. Everybody with me okay, simple stuff right. Now like we have the maxi max for the optimistic guy or girl. What do we have for the pessimistic person, minimin, minimax, maximin come on think and say why would it be mini min.

It is going to say the world is going to be worse and I am also dumb. I will choose the worst outcome of those. That is now how the world why would you do it or she. So, it is going to feel that the world is going to be terrible for me. My luck is bad. Whatever I will do, the world will get together and defeat me. So let us pick the option where I get the least defeated, but least defeated in this case would be max right.

So this is the most risk averse agent. It is as if it says I just do not want to take risks. I do not want to live in a home which has you know, a roof because the roof may fall down tomorrow. I just do not want to work on the road because a random car may come and kill me. Can I am taking this too far, but you see what is. So maxi min criterion says for each row, find the minimum, because that is the worst that can happen to me right.

So large plant the worst happens when it is unfavorable - 180K, small plant - 20. No plant 0 and which is the best thing to do of these 0, because that is the best I can do right. It is in my control. So in my control, I will take the best action, but afterwards, the world will be terrible alright. Of course, nobody is completely optimistic and completely pessimistic right. So we want to come up with some criteria which are in the middle, right, which is not completely on the 2 extremes.

Can you come up with some such criteria or criterion profit to loss ratio. That has nothing to do with this. The issue is that we do not know what is going to happen as the market is going to be favorable or unfavorable. So, we can have like at which probability of luck like what. Yeah, so, you can sort of say that, look, I have many criteria. I do not know what is the probability distributions are in absence of any knowledge of whatsoever the best I can do is consider them equally probable.

So, I take all possibilities if you have 5 states of nature I take all 5 states of nature and say each of them are equally likely.

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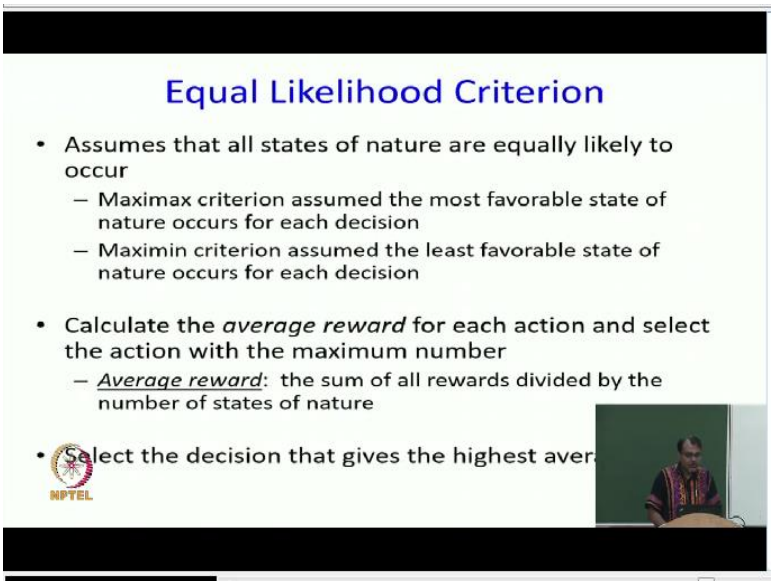

And this is called the equally likelihood criterion. In this case you compute the average reward that you will get for each action right. And in this average reward.

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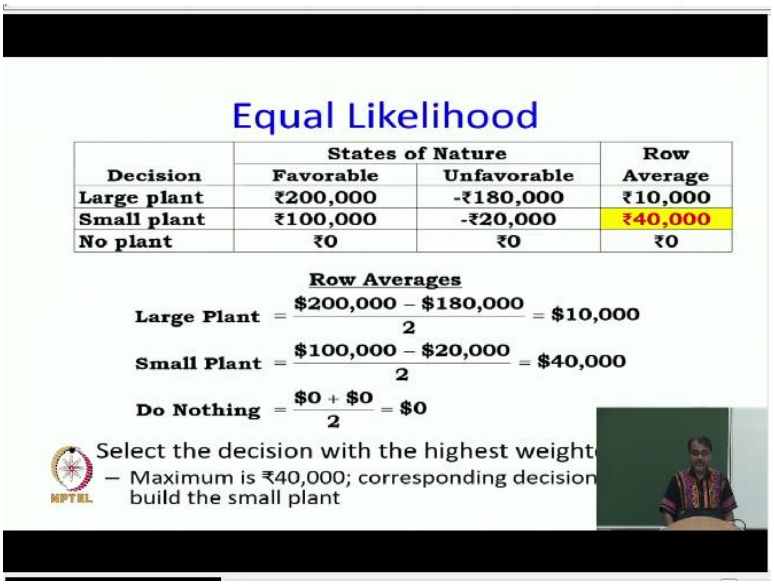

For example, if you do large plant you will get 200 - 180 by 2 because there are only 2 possibilities, small plant 100 - 200 by 2 and 0 plant 0. And you say, okay, in this case, I am going to do a small plant because that has the best average. And so, intuitively, what you are saying is that you know, lots of things can happen, I know nothing, so I will consider them equally right.

Another way of thinking about is that there is an optimistic person which always thinks of the best, there is a pessimistic person which always thinks of the worst, I am a appointed optimist. And this is what is called the criterion of realism.

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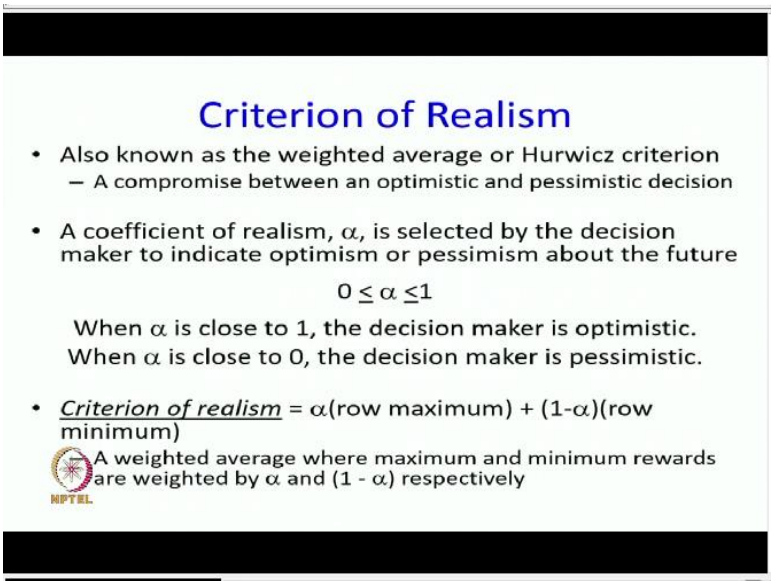

I am a realistic person, but a realistic person, which is sort of towards 80% optimist, right. So then I will have an alpha, which is the coefficient of realism. And it is sort of my balance between optimism and pessimism. And what I am going to do is that for this criterion, I will take alpha times the row maximum. So, I will say that without with alpha probability, the best thing is going to happen to me.

And with 1 - alpha the worst thing is going to happen to me and then I will compute the weighted average like this.

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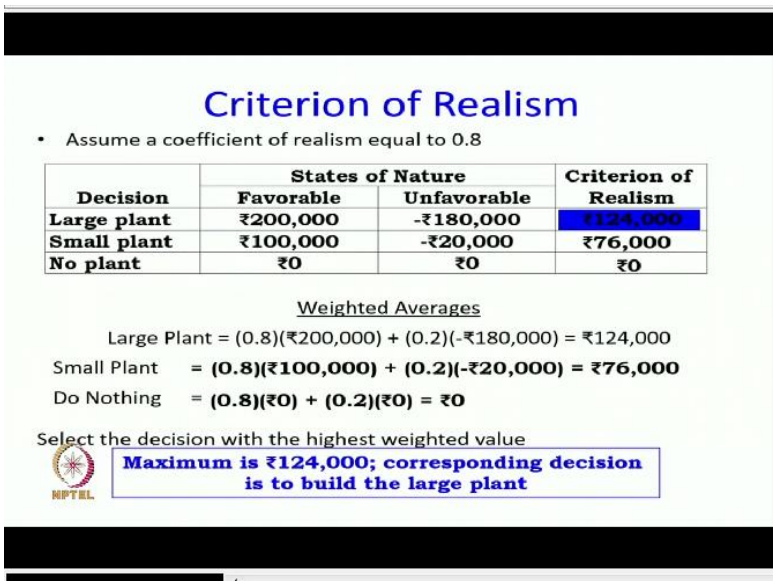

So, in this case, the large plant would be 0.8 times 200 - 0.2 times 180 gives me 124. Small plant will be pointed times 100 - 0.2 times 20 that will be 76 and no plant would be 0. And in such a case where my coefficient of realism is 0.8, I will pick the large plant. Yes question. Then if there are 5 possibilities, then if they are 5 possibilities then second, third, fourth will be ignored in this particular for every row.

For every row, the best in the worst will be pinned, and the other 3 will be ignored. Again, because I am not telling you any probability distribution, you have to come up with some mechanisms to just make a decision and I will tell you the last one, and then we will talk about the more interesting case, which is the uncertainty case. So the last one is minimax regret.

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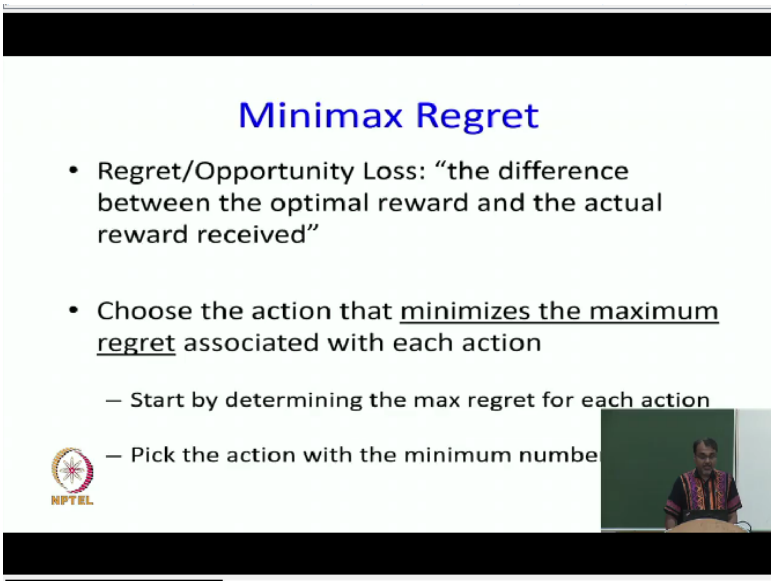

So what is the regret. Regret means I took a decision and then I got to know that oh, I could have made so much. And so I feel bad. Oh, I made the worst decision, right. So this is how humans are. They take a decision later life will give them something else and then they look at the history and they are unhappy, oh, why did I do that. So the difference between the optimal that I would have bought after knowing the information minus the actual that I got, that is the regret.

And I want to minimize the maximum regret. So I want to say that I will pick an action, which minimizes how much I am going to feel bad in the worst case okay.

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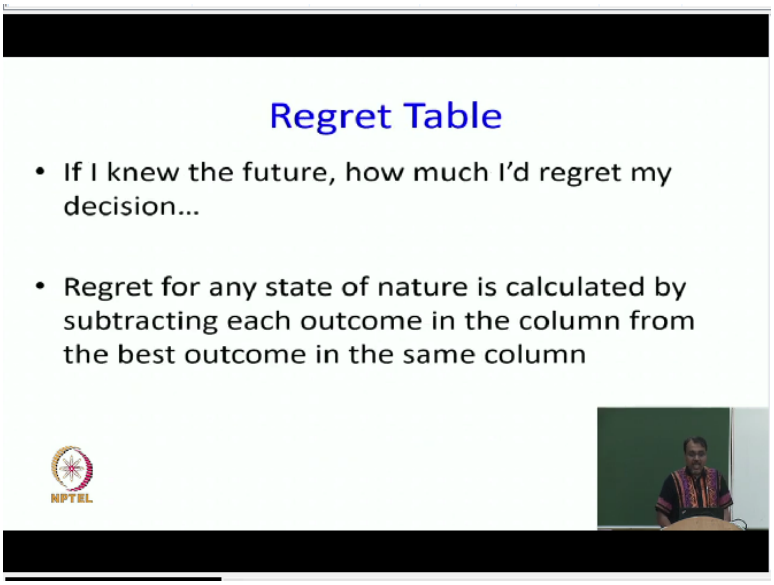

And so what I will do is I will create a regret table. So what is the regret table. If I knew the future how much would I regret my decision. So, regret for any state of nature is calculated by subtracting each outcome in the column from the best outcome in the same column. So let us look at this example.

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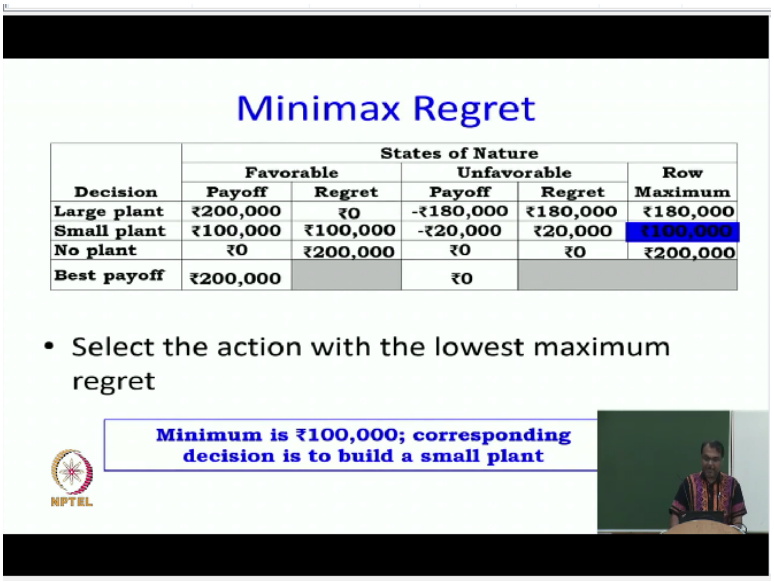

And that will make it clear. Suppose the market turned out to be favorable. Suppose the market you do not know this right now, but suppose the market turned out to be favorable, what is the best you can get 200, suppose the market turned out to be unfavorable what is the best you can get 0. So now in both of these situations, that is what is your best the optimal after you know the information, but of course you do not know the information now.

Suppose you decide to make the large plant and the market turned out to be favorable would you regret anything, your regret would be 0. Why because you could have gotten 200, you got 200 there is no regret. On the other hand, if you made the small plant, how much would you regret, you would make 100K because you get 100 and you say oh, I could have got 200 if I did the right thing. So you will regret for the 100K that you lost.

And if you make no plant, how much would you regret 200. So this is your regret table for the situation when the market turns out to be favorable. Similarly, you can do it for unfavorable, if

you make large plant how much would you regret 180, you are gonna get regret 180K, you are gonna lose 180, you are gonna feel bad about the 180 that you lose.

Similarly they get 200 for the small, 20 for the small and 0 for no plant, now what is the maximum regret you can have if you decide to make the large plant 180 in the worst case you can feel bad about 180K that you have lost. For small plant in the worst case you may feel bad about the 100K that you have lost and for no plant you may feel bad about the 200K that you have lost. And you pick the decision that minimizes this maximum regret.

You pick small plants, you say I will pick small plant because depending upon how life turns out, I will feel the least bad in the worst case okay.

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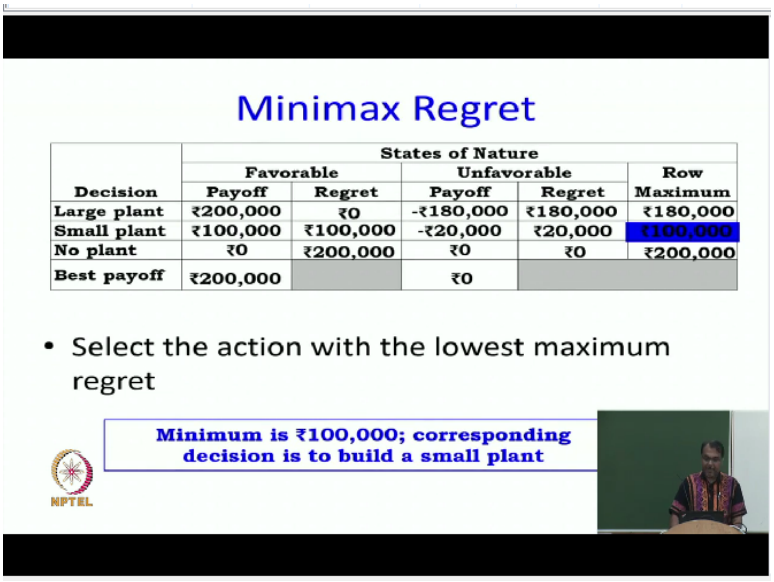

So this is the summary of our results. If we are extremely optimist, we take the maximax criterion, we decided to build a large plant, if we are highly pessimist and we say that we do not want to lose anything in the world ever, we decide to do nothing. If we are in the middle where we say everything is equally likely, we know nothing. That is called the equally likelihood criterion. There, we build a small plant.

If we are realistic, but pointed optimists, then we build a large plant. If we decide the minimax regret criterion, then we build a small plant, now it is hard to decide, which is the right

mechanism that you would choose to make a decision, but at least it gives you a framework in which to make decisions when your uncertainty is non deterministic.

But of course, the uncertainty is never non deterministic. There is always some probability distribution associated with it. Yes right, so Rabav asked a very good question, what if I do not have one reward, I have many different objective functions that are all playing with each other. And we have discussed this in the last class very briefly, there are 3 ways to do this. See, **it is** it is called the multi objective problem or multi objective optimization problem.

In a multi objective optimization problem, you take 3 different approaches. approach number 1, you create a single objective, which is some combination of these multi objective. So, for example, a linear combination, you say, okay, the cost that I lose is 0.8 + you know, 1.2 times something else + - 0.5 times the amount of health that I would lose by building a small plant or large plant whatever it is right.

So you can create one single objective function, and then you can try to optimize that right and we will call that single objective function. So, that now becomes a single objective part, right and we often have to deal with it right. We cannot say that this flight to Goa is 100,000 or 10,000 rupees okay 100000 is bad, but 10,000 rupees looks expensive, but it said 10 o'clock, whereas at 6 o'clock flight is 6000 rupees.

Now, what is the incremental, rewarded I get by getting some sleep and not taking the 6 o'clock flight. Now we have to trade them off and figure out what is the right thing for me and we cannot usually write it down as a single objective function. So, therefore, in the real world, when people have to deal with this they do what is called utility elicitation or preference elicitation. So, they will give you one case that you have 2 flights 6 o'clock, 10 o'clock, this is 6000.

This is 10,000 which one will you take, then they will give you another scenario, one flight 8000, 6 o'clock, one flight 10,000 10 o'clock which one will you take. Then they will change the times, they will give you lots of these scenarios, and you will decide which one you want to take. And

therefore, they will then learn a single objective function that sort of matches your value function okay. The other alternative is to say that my maximum cost is 7000.

Now, let me optimize the amount of sleep or my minimum sleep is 5 hours. Now let me optimize the price. So this is called constrained optimization, where I put constraints on many objective functions, and then I want to optimize the other objective function. First of all, these make the problem harder. And secondly, where do you find these constant pressures or it is not very clear. Last but not the least you say, okay, I cannot choose between sleep and cost.

So I will output several flights where no one is dominated by the other. It is a Pareto optimal set right and then I will let the human figure it out. So that is what the AI system is not making any commitment on which how to build a objective functions together. So these are the 3 ways. Typically, you know, we this is a full different branch of study within AI, so we are not gonna talk too much about.