Artificial Intelligence
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Lecture-57
Bayesian Networks: Reducing 3-SAT to Bayes Net- Part - 5

So, ahm I want to do one last thing which I am very fond of. And this is to show you that Bayes Net encompasses satisfiability 3-SAT. It is pretty obvious if you actually think about it you can come up with this production but in the spirit of making it quick, I show you the result.

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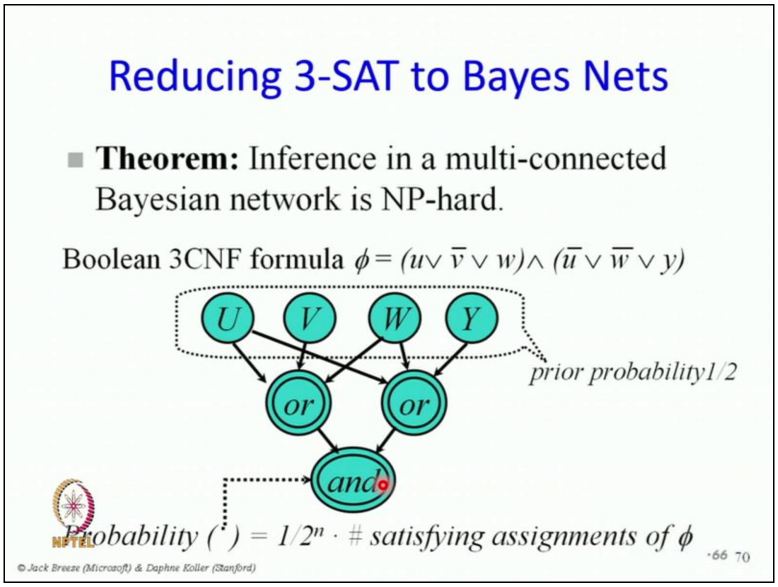

Let us say if I give you a CNF formula, U, V or not W and not U or not W or Y. Let us say I want to create a base net which solves the Sat top. What is the SAT problem to figure out if the formula is satisfiable or not? Can we create a Base net and do inference of a base net so that the answer to the base net solution gives me the answer to the satisfiability problem. So, how do you do it? You take each, logic and make a random variable out of it.

So, U V W and Y, they all random variables and you give them a prior probability of 0.5 to be true and prior probability of 0.52 to be false. OK Then, you take each clause and create an or node here. So one or node be U, or V or W, one or node would be not U, not W or Y. This also needs a conditional probability table, for your conditional probability will be zero, if U is false, V is true and W is false and one otherwise.

It is basically saying you are representing the all function in the conditional probability table. And then you have an and function here, which basically says that this is an and function, if all of them are one I am one if any of them zero then I am zero. So, now what have you done? We have been given a conditional probability table for each node in my base net and I have also given the probabilities for each of the initial random variables.

Now, I do Bayesian network inference. What is the inference I do. I say what is the probability of and? Nothing about what does and give you tell me. When would probability of and be 0 if the formulas are satisfied? This there is no way to make probability of and node 1. If probability of and is 1 what does the tell me? All configurations make and true, that is not going to happen. But in in in usually it may be a number between 0 and 1.

So what does the number represent and this is very interesting because all the original variables and dependent will have all possible configuration captured in this. And if there is only one solution to the whole problem probability of the and will come out to be one by two power n. Exactly two solutions probability and come out to be 2 hour to the end and 64. So what is very interesting is not only is base net is telling you that formula that is satisfiable not it can do something bigger. It can count the number of solutions of the SAT formula.

If you can do Bayesian network inference, not only can we do that we can also figure out how many solutions at particular SAT formula has.

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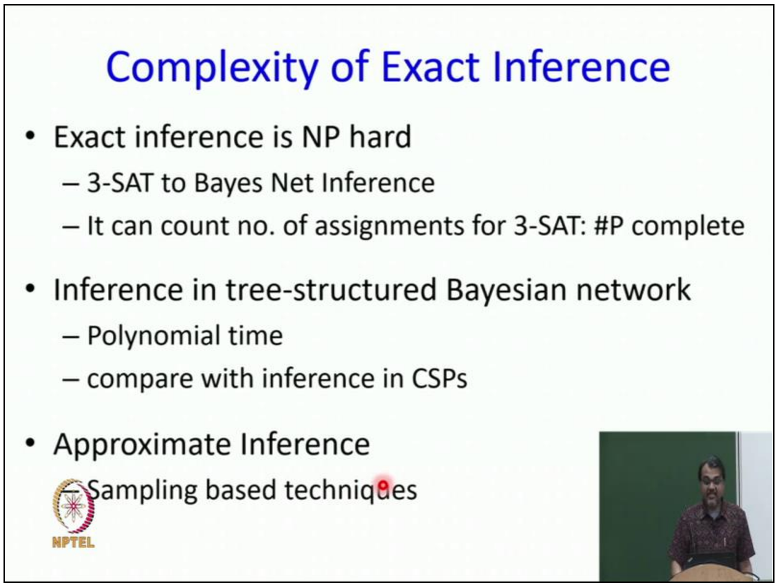

And that particular problem is by the way called model counting. So SAT is NP complete problem. But if you are looking at the model counting problem that is what is called the sharp NP complete. NP means counting the number of solutions of the NP complete problem. So Base net unless all the hierarchy collapses, Itna more advanced complexity class then NP complete problems then NP is sharp pic. Base Net has always has more information than that of sampling.

So, that is the first thing. So, exact inference has to be NP hard to get away from exponential. Second we like in CSP is if it was a free, we could do it in polynomial time there is something called a poly tree structure where if it is a poly tree structured Bayesian network, then you can do inference polynomial. And like you have cycle cut sets in CSP, you have probabilistic that cut sets in CSP.

You have tricks of looking at the structure and doing things with that we have equivalent tricks here. We have variable elimination here. We have bucket elimination there. You can do your full in a deep analysis of comparing CSP and Base net and you will find the so many similarities that it's not even to find it is there. They are just two sides of the same coin though they are in different complexities.

The next thing to do after today's lecture, next lecture is it will talk about is approximate inference algorithm, which is better than its source of constant time, because if you spend more time, you will get a better approximation. Ok. So, that is what you will do in the next class. It uses the general principle of sampling and we can stop here. Thank you.