Introduction to Computer Graphics
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Lecture - 8
3D Viewing

So we have been talking about 3D viewing, the transformations pertaining to 3D viewing. Today we will continue on it.

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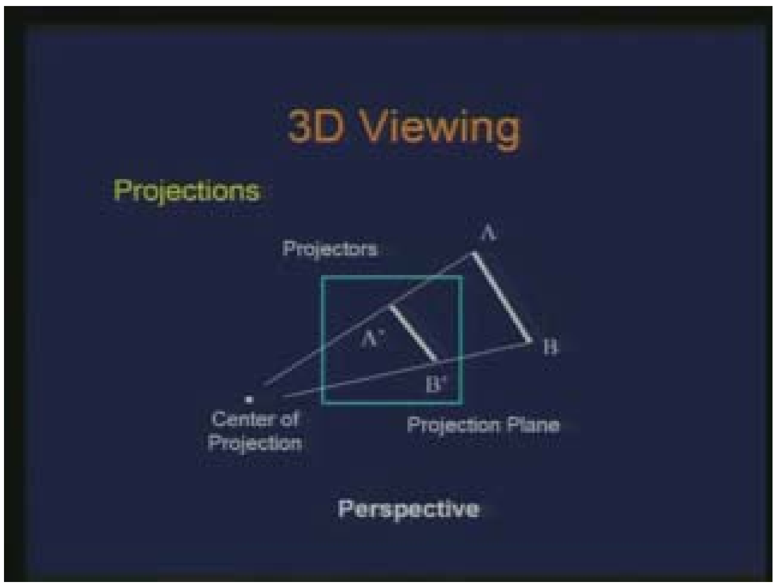

Earlier we basically looked at 3D viewing as a process where there is an object AB here and we have a projection plane and what we get are these projectors emanating from the object on to the projection plane meeting at the center of projection and this is the process of projection. And in this way we get what we call as perspective projection. So A prime B prime is basically the projected image of the object AB. And you notice here that these projectors are not parallel to each other and they are basically meeting at this point which is center of projection.

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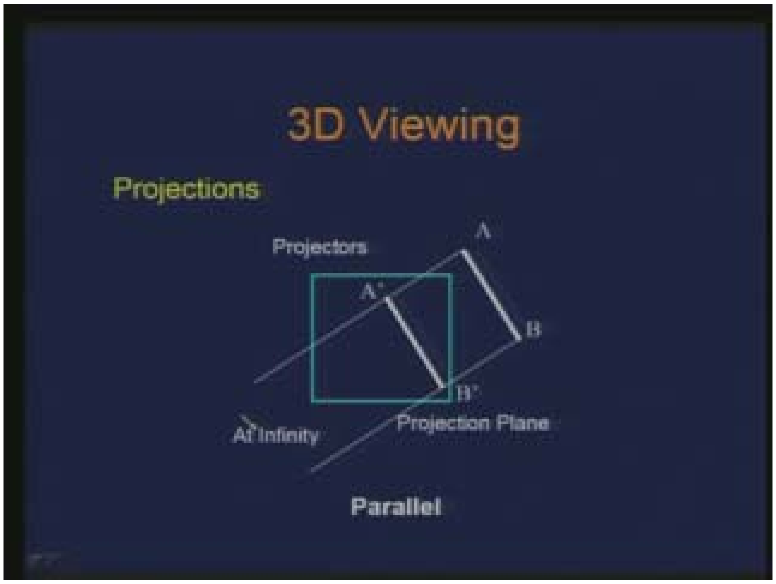

Now the other type of viewing projection is the parallel projection where we have this projectors coming out the object AB parallel to each other and this is again the projection plane. So, in effect what we get is the center of projection now moves to infinity. It does not meet at the point which we will call it as the center of projection. So this is parallel projection and we basically looked at the various types of parallel projections.

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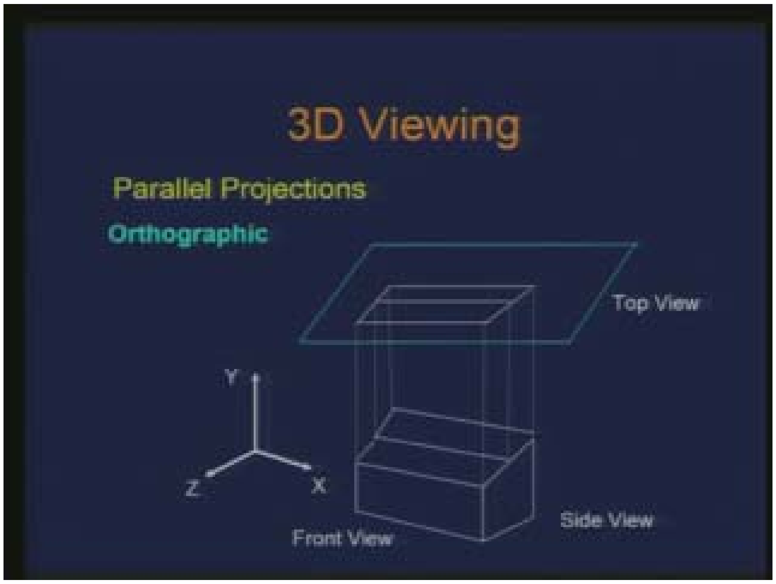

For instance, we looked at the orthographic projection where we basically have an object and the projectors are parallel to each other and they are perpendicular to the principle planes which are actually planes perpendicular to the principle or the coordinate axis. So this is for instance gives us the top view the plane which is perpendicular to the y axis and similarly the front view and the side view we get.

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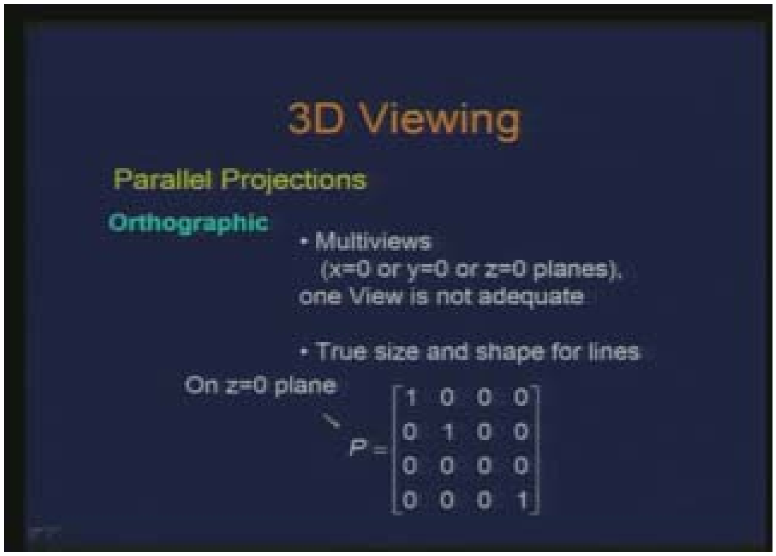

Mathematically we can get this projection again using as a matrix transformation. Therefore this matrix P basically gives as an orthographic projection on Z is equal to 0 plane. What we get basically is this third column and all the values are 0. So we can represent this orthographic projection again as a matrix transformation. So the property here we have basically preserves the true silent shape for the object. So these are the examples.

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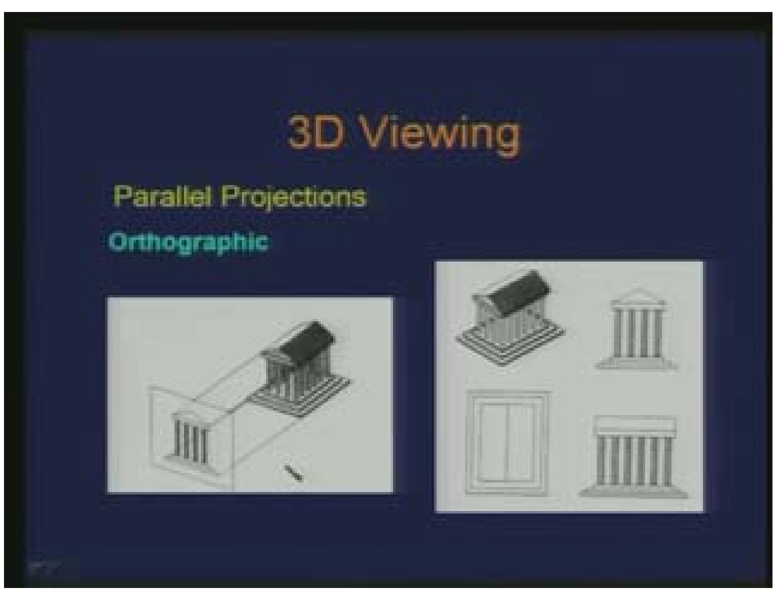

Here we notice that these are the projectors, this is the projection plane and these projectors are parallel to each other and they are also perpendicular to the projection plane and these are the various views we can get of this object such as the front view, the top view and the side view.

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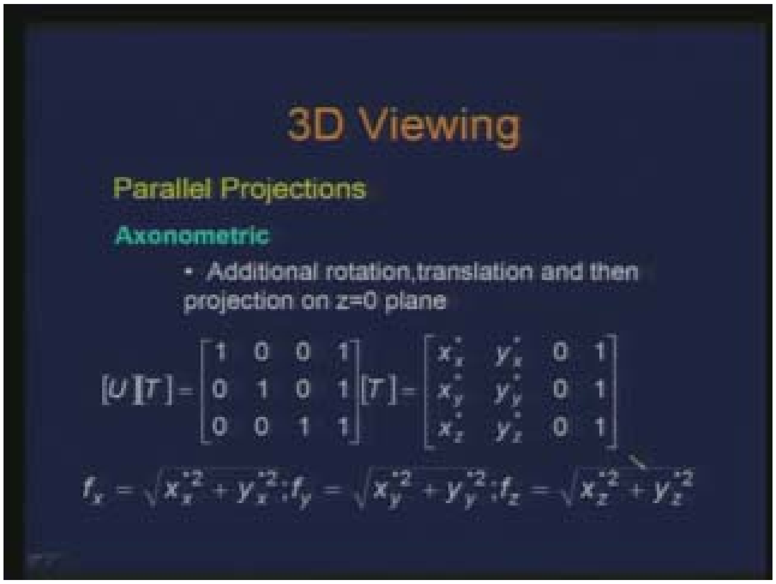

The next category we looked at was the axonometric projections. These are also parallel projection meaning that the projectors are parallel to each other. So the way we can look at these axonometric projections are basically as additional transformation before we apply the conventional orthographic projection. For instance, we can have additional rotations or translation and then we have the projection on Z is equal to 0 plane. Now as a consequence if I consider the unit vectors in the three coordinate axis 1 0 0 1 represented in homogeneous coordinates 0 1 0 as Y axis or unit vector in Y direction and 0 0 1 as unit vector in Z direction and if I apply a transformation T to it what I observe is and then this T is also consisting of the projection of Z is equal to 0 plane which basically means that the third column is going to be all 0.

So what I will get is basically this meaning that this particular unit vector which I considered will get transformed to this X_x Y_x star 0 1 which says that there is some sort of a foreshortening which has happened to the unit vector which I had considered in X direction. And the amount of foreshortening is nothing but this distance in X direction. Basically I just measure these and that gives me f_x which is foreshortening in X. Similarly I get foreshortening in Y and foreshortening in Z. Now depending on the relationship of these foreshortenings I have various kinds of axonometric projections because these foreshortenings could be equal or may not be equal.

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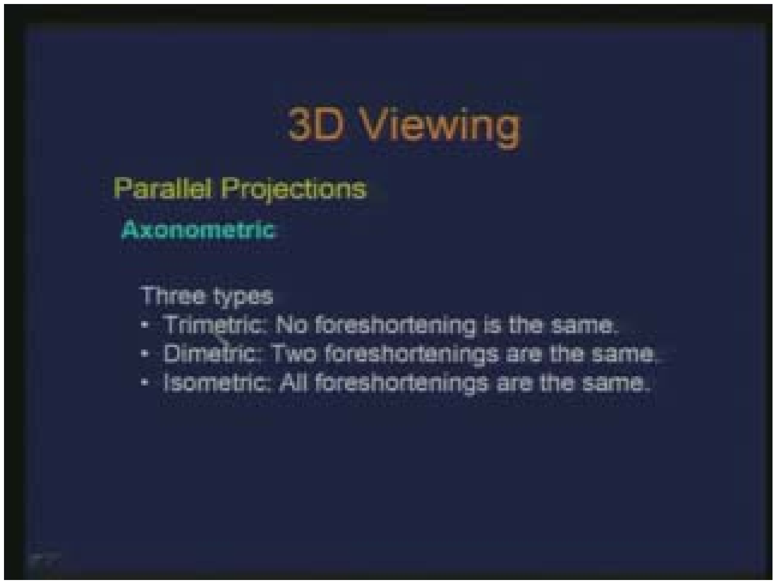

Therefore for instance I can basically define three types of axonometric projection; trimetric where no foreshortening is the same. So f_x is not equal to f_y is not equal to f_z. In diametric two foreshortenings are the same out of these three and in isometric all the foreshortening are the same.

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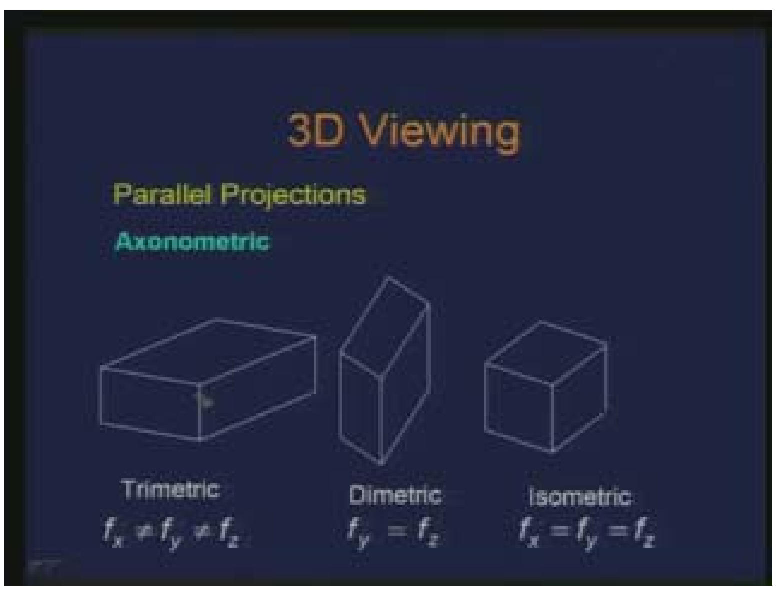

As an example here we have the trimetric projection where f_x is not equal f_y is not equal to f_z. This is diametric so you that this basically equals to this if I had taken the object to start with as a queue. So when we observe the foreshortening these two directions are the same. And in isometric I have all the foreshortening. What basically happens here is that these projectors are parallel, these projectors are also perpendicular to the projection plane. It is just the projection plane which is placed differently as opposed to in the case of orthographic projection where it was kept actually at the principle planes. So this basically adds the visual realism to the 3 dimensional object. When you view the object it looks more like a 3 dimensional object then you would see in orthographic projection. Orthographic projection in isolation for a particular view does not give you any 3 dimensional information whereas these actually help you viewing in 3 dimension. That is the basic advantage we have here.

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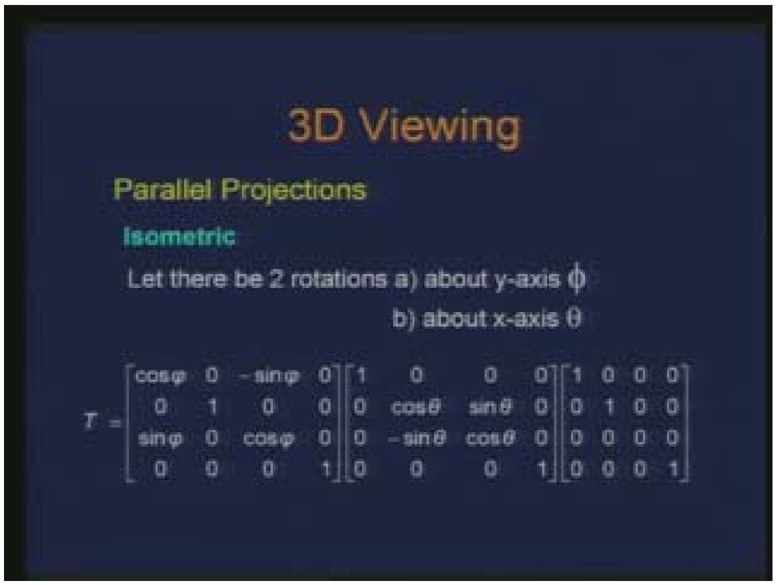

If we are dealing with isometric projection the basic thing is that the projecting plane undergoes some kind of a transformation before we perform the conventional orthographic projection on Z is equal to 0 plane. These transformations are of the kind rotations. So, if I use two rotations the first one is about Y axis by an angle phi and the other one is about X axis by an angle theta. The corresponding transformations I have given, this is the rotation about the Y axis and this is the rotation about X axis and then I apply the projection on Z is equal to 0. So this whole matrix now would give me a transformation which I want to use as an isometric projection.

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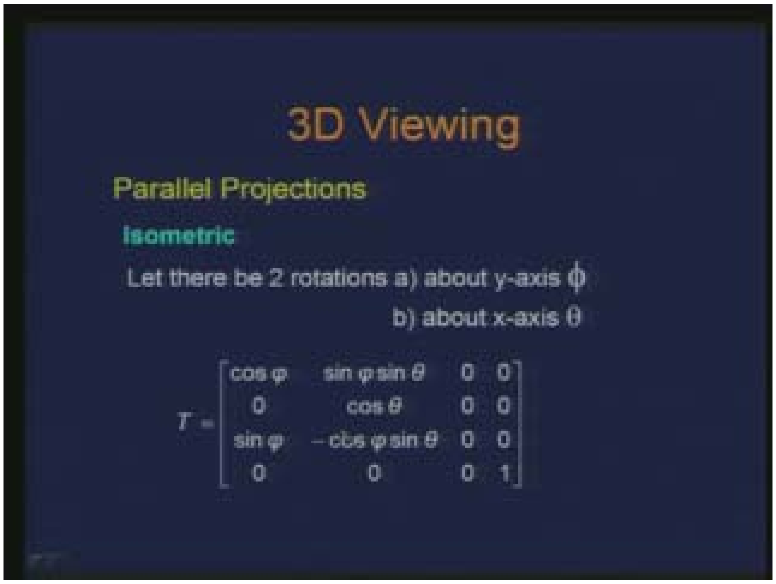

This is the resulting transformation I get after concatenating these individual transformations. It is just a multiplication of those matrices. Now if you go back and see how we obtain those foreshortenings for the unit vectors let us again try to see these foreshortenings with this transformation matrix.

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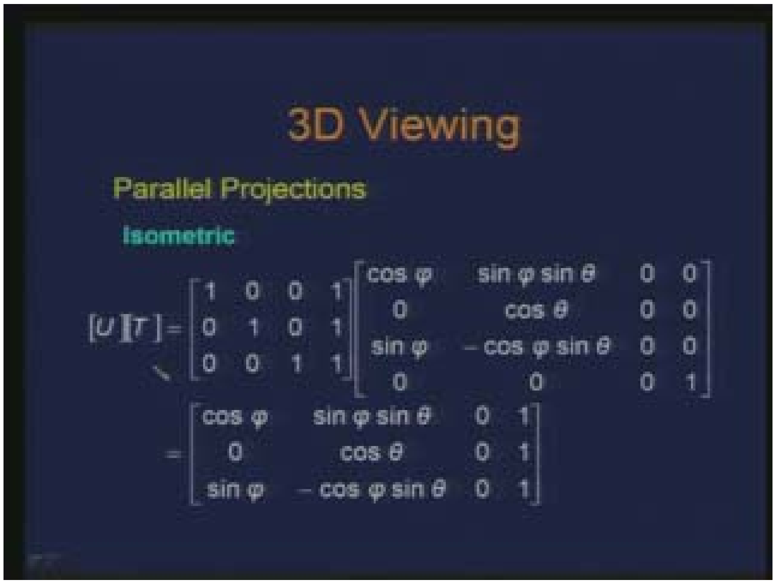

Again I have the three unit vectors and this is the transformation. Once I apply the transformation on to these three unit vectors the modified or the transformed unit vectors are dmos. So the first row gives me the transform unit vector in X, the second row gives me the transform unit vector in Y and the third one gives me the transform unit vector in Z. So these are the three unit vectors transform.

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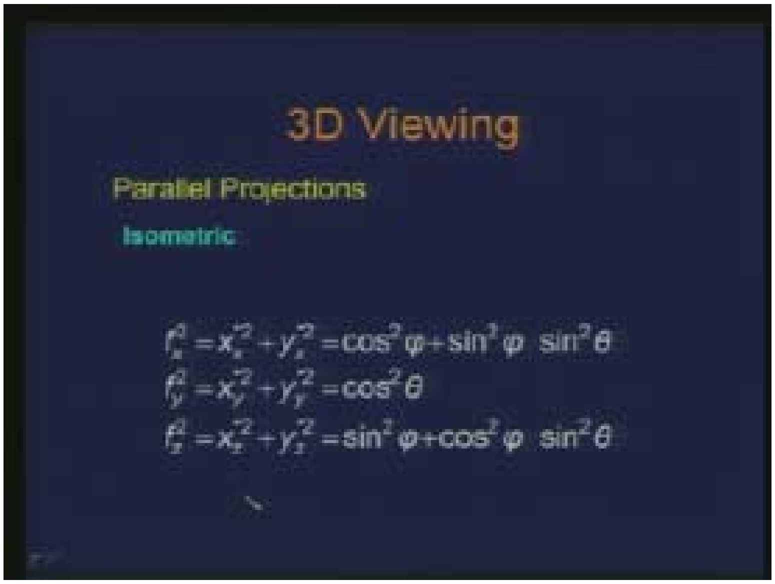

Now, from these I try to compute the foreshortening just in the way I had done earlier. So I get the square of foreshortening in X given by this which are the square of these two entries I have here.

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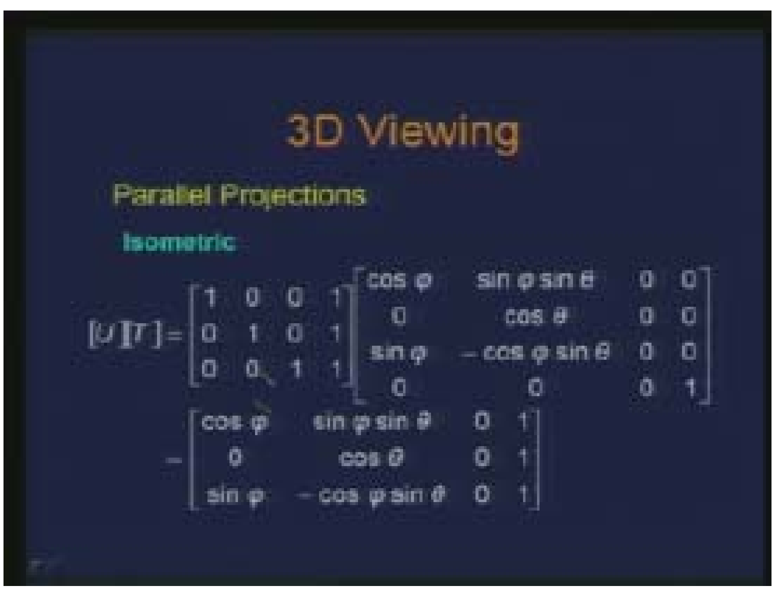

Similarly I have the foreshortening in Y the square of it given here and foreshortening in Z. Now I impose the constraint of the transformation to be isometric which in turn means that these three foreshortenings should be the same which basically means now I have f_x is equal to f_y is equal to f_z is equal to some f which gives me these two equations.

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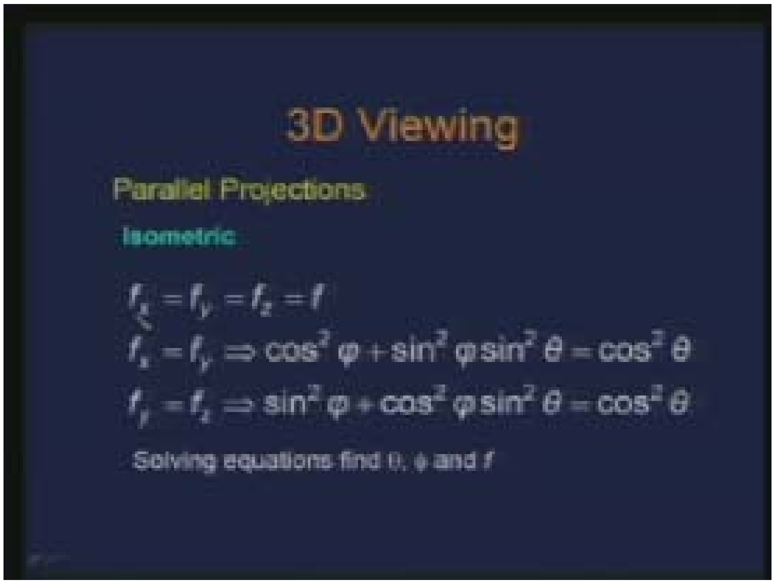

Now I have these three equations where I can solve it for pi, theta and f. So basically it can give me the amount of rotations I should be performing in order to get isometric projection for a given value of foreshortening. Now we move on to the other type of parallel projections. So what were the observations we had in the axonometric projection? It was that the projectors were parallel and they were perpendicular to the projection plane. Now we relax one of the restrictions.

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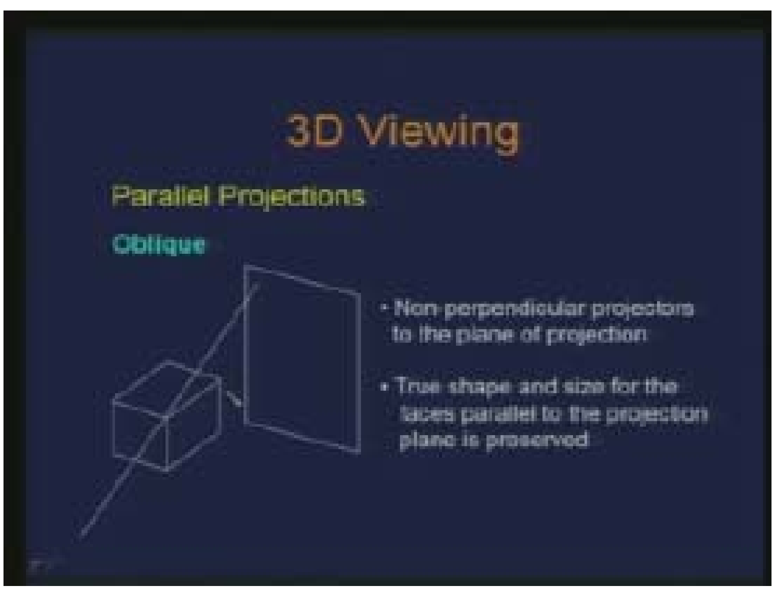

Since these are in parallel projections we will still retain the projectors to be parallel because we want the center of projections to lie at infinity whereas we relax this condition that they need not be perpendicular to the projection. So what this gives us we call it as oblique projection. Here the projectors are not perpendicular to the projection. For instance, if this is the object and this is the projector I have this is not perpendicular to the projection. Therefore now with this property the true shape and size will be preserved only for the faces which are parallel to the projection.

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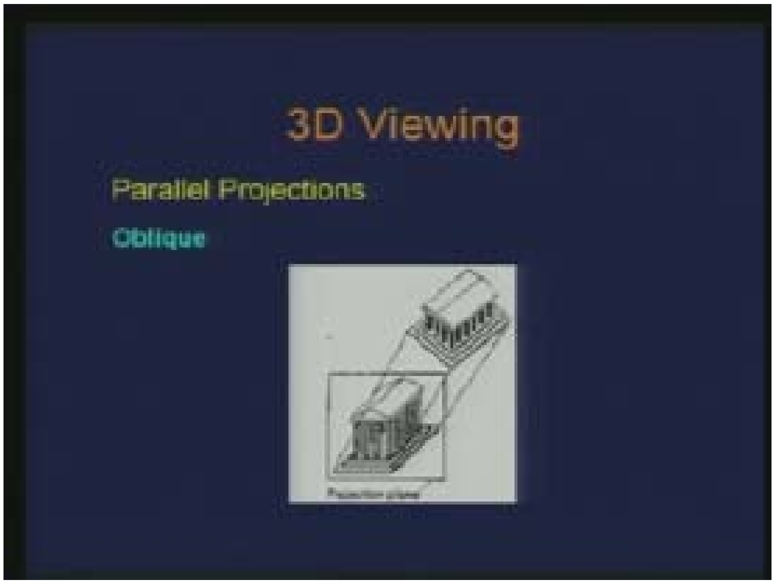

This is again an example here, here these projectors are not perpendicular to the projection plane. And again the motivation of having oblique projection is to add the element of 3D viewing, to make it look like a 3 dimensional object. So this is the example of oblique projection.

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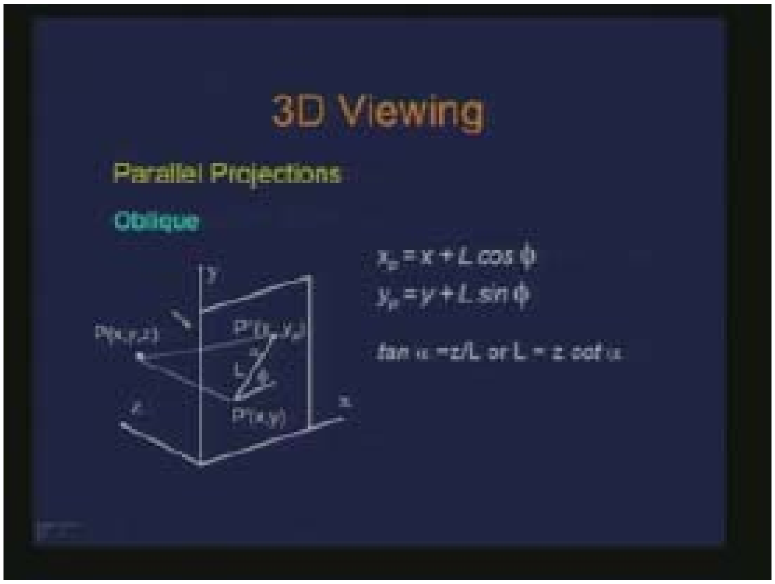

And mathematically what we can observe here is let us say we have a point P(x, y, z) and this is my projection plane the (x, y) plane and this is the projector I have from P to P double prime and now I want to get the value of X_p Y_p which are the coordinates on (x, y) plane of the point P so what I observe is that the projector which is parallel to the Z axis goes to P prime which gives me (x, y) basically so I can represent X_p Y_p in terms of this (x, y). Hence xp is nothing but X plus L cos phi where L is this length from p prime to p double prime and similarly Y_p is y plus L sin pi using this triangle. So phi is basically the angle between P prime and P double prime and X axis on this plane. Now the angle which this projector is making with the projection plane is the alpha. Now for a given alpha we can get many projectors for the same value of alpha so I have to restrict the location of the projector and I am doing that using this phi because it basically gives me a cone. The set of all projectors is basically a cone. Now this alpha is actually related through Z and L so I have this tan alpha given by Z by L or L is equal to Z cot alpha from this triangle. Depending on what alpha we choose we get a particular type of oblique projection for some value of phi by fixing the value of phi.

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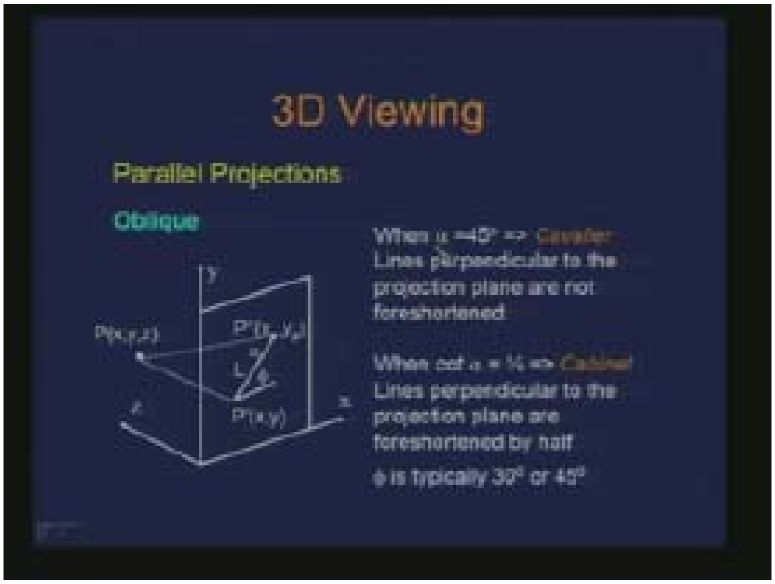

Then we choose alpha to be 45 degree, the type of public projection we get is called a cavalier and what we are observe here is that the lines which are perpendicular to the projection plane are not foreshorten. Remember how L and Z are related. Basically cot alpha is 1. The lines which are perpendicular to the projection plane will basically have the same length whereas if I choose alpha in a manner that cot alpha is half the projection which we obtain we call it as cabinet and here we observe that the lines perpendicular to the projection plane are foreshorten by half for the same reason what we observe in this case because they are related by cot alpha. And the five is actually typically 30 degree or 45 degree. So this is basically the oblique projection. So here we observe that the projectors are at an angle other than 90 degree to the projection.

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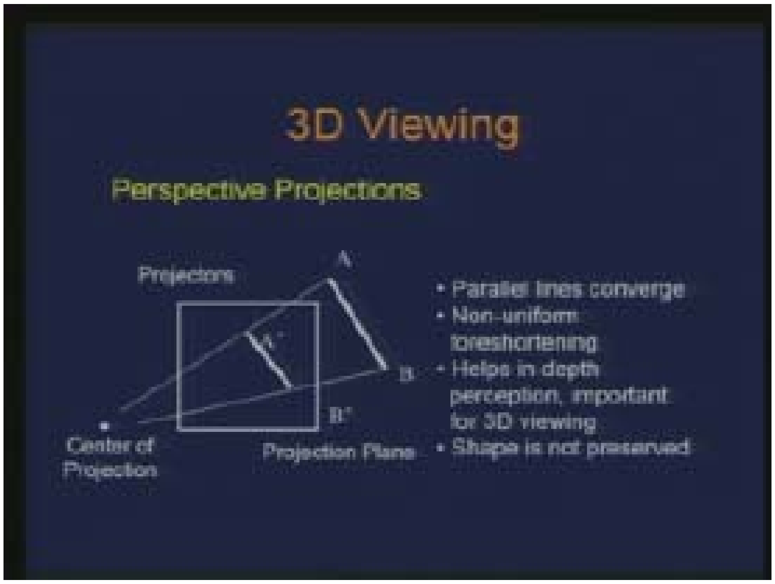

Now we move on to the more general kind of a projection which also matches to the visual system we have which is the perspective projection. And in the beginning we observed that these projectors are not parallel and they actually meet at a point which is the center of projection. So here parallel lines seem to converge and the foreshortening could be non uniform. And in fact this perspective projection helps us in the 3D viewing because it gives you some sort of a depth perception. The front objects seem to look bigger there is a depth queue and the shape is not preserved although the lines map to lines but the parallelism is not preserved and the foreshortening is non uniform therefore the shape will not be different.

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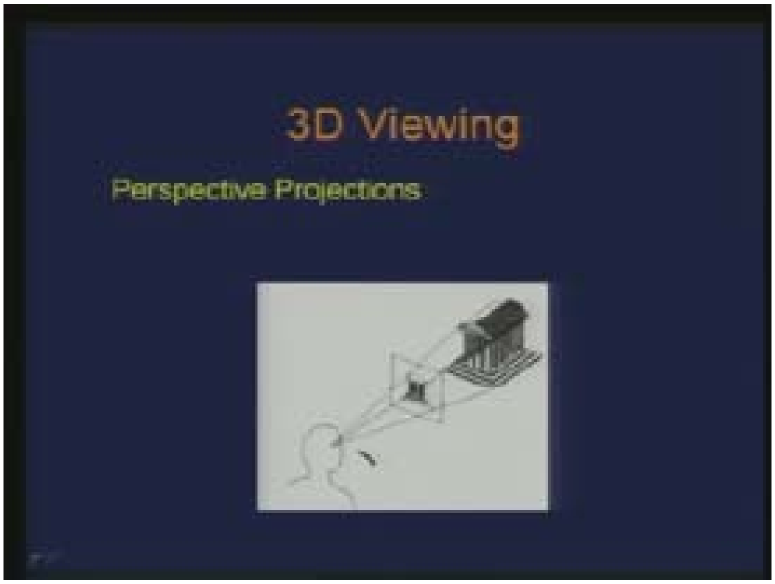

This is the projection plane and this is the object here and the center of projection is sitting here. Now let us try to see it mathematically.

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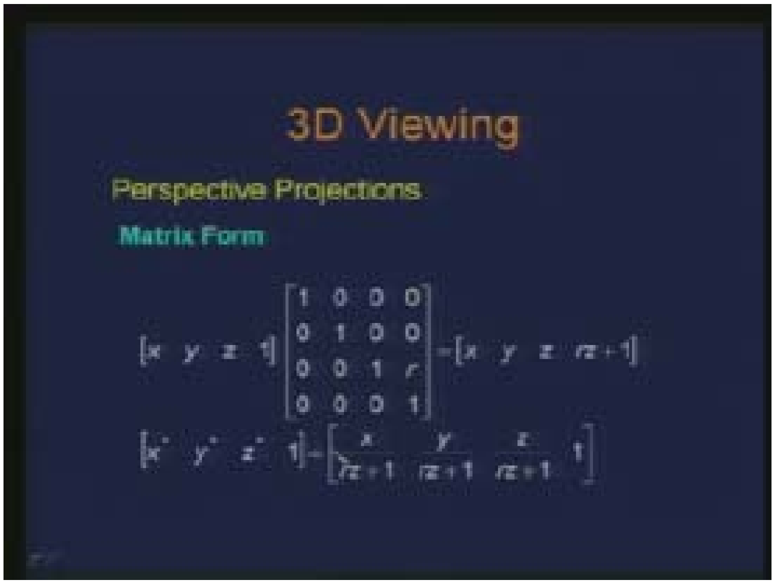

Just as we have seen axonometric and orthographic projections even the oblique projection can be represented in the matrix form because the way you had seen the transformation of obtaining X_p using X_y you can actually write that in the matrix form. So that can also be obtained using a matrix form. So now we try to represent this perspective transformation using a matrix form. Here we have the input given as x y z 1 in homogeneous coordinate system and this is the matrix we have. In 4 into 4 general matrix of transformation we have not been considering the last column.

Now let us consider the last column where we put a non zero value in the third row of this column this r. Now using this transformation the point x y z 1 becomes this in homogeneous coordinates. So, to get the Cartesian coordinates the normal coordinates which we use for our purposes we divide it by the homogeneous coordinates all through and we get this X by rz plus 1Y by rz plus 1, Z by rz plus 1 so this is still 3D. We are basically making a transformation from 3D to 3D.

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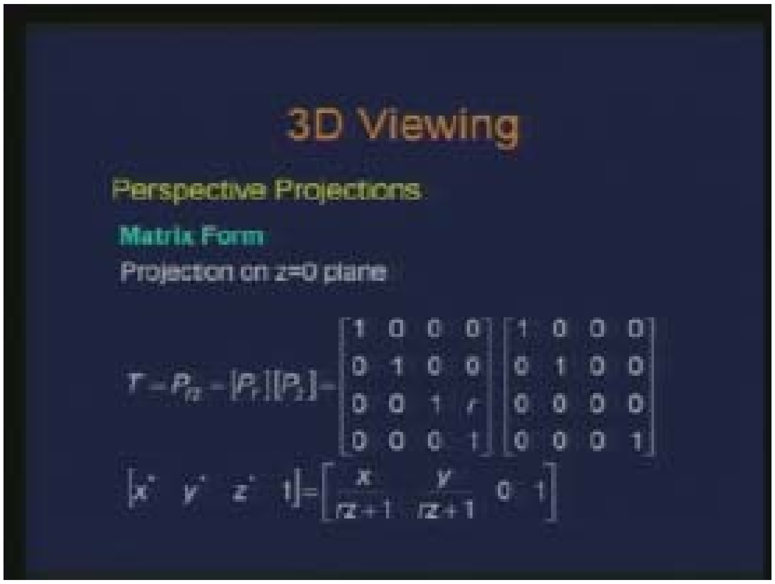

Now if I take a projection on Z is equal to 0 plane which basically means I multiply by this matrix where the third column again is all 0s this would give me this, basically this value becomes 0. Therefore this is the projected point now after having done the perspective transformation.

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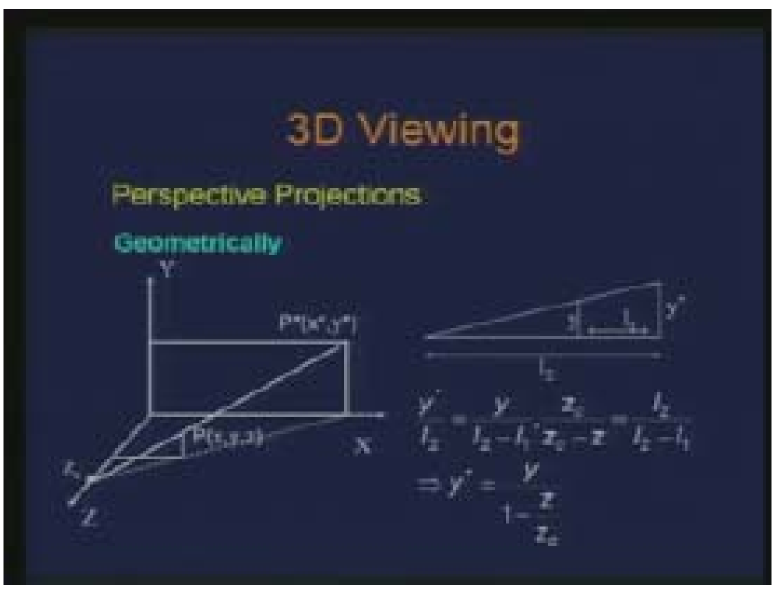

Using a matrix form of transformation we know what we got. Now let us try to see geometrically. So here is your center of projection along the Z direction Z axis located at Z_c and this is your point P(x, y, z) and we are looking at projection on Z is equal to 0 plane meaning (x, y) plane. So I basically project this point through the center of projection on to this plane and I get T star which has the coordinate x star y star and that what is happening as a process when I say that the point p is projected on Z is equal to 0 plane from the center of projection located at ZC along the Z direction.

So now consider the properties from similar triangles and try to obtain the values of x star and y star.

Now I consider the similar triangle first this where this is y star which is already present and this is y which is here. And these lengths are given as l_1 and l_2. So using this triangle I can now get the value of y star. Basically y star by l_2 is equal to y by l_2 minus l_1 and also now using these similar triangles on the X_z plane I have Z_c by Z_c minus Z from this triangle on the base equals to l_2 by l_2 minus l_1. So first I consider this triangle which is there and then I consider these triangles from the base. So this gives me y star is equal to y by 1 minus z by Z_c. Similarly I can obtain x star.

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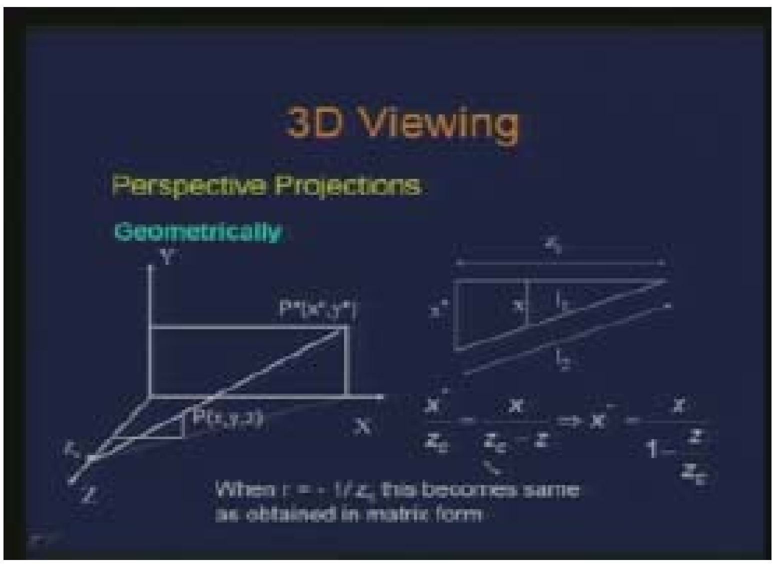

I can obtain X star in a similar way. So X star is given as X by 1 minus Z by Z_c.

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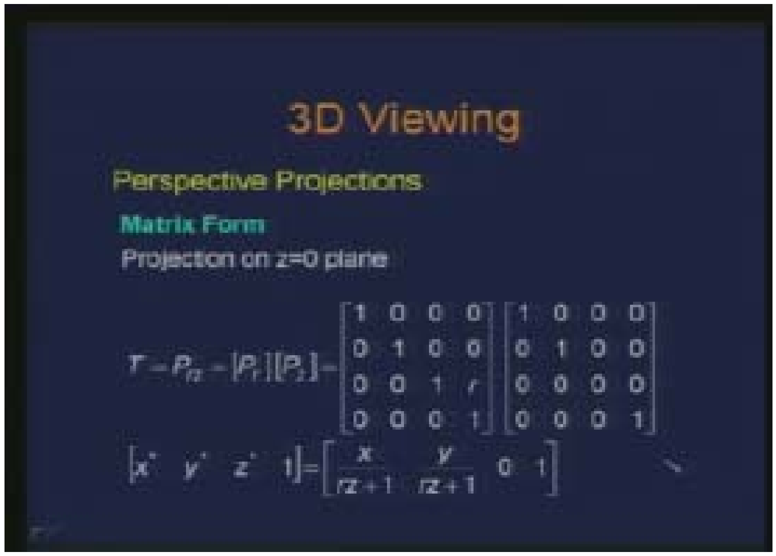

If you go back and see what we have obtained as x star and y star are these values and here what we got is X star as X by 1 minus Z_c and Y star as Y by 1 minus Zc. So, if you substitute r as minus 1 by Z_c they are equivalent. So this establishes the correspondence between the two representations what we do geometrically and what we obtain from the matrix. Therefore if I have r is equal to minus 1 by Z_c I basically get the same transformation which I derive here geometrically. And Z_c is nothing but the location of center of projection along Z axis. Now there is another property which we observe in the perspective transformation.

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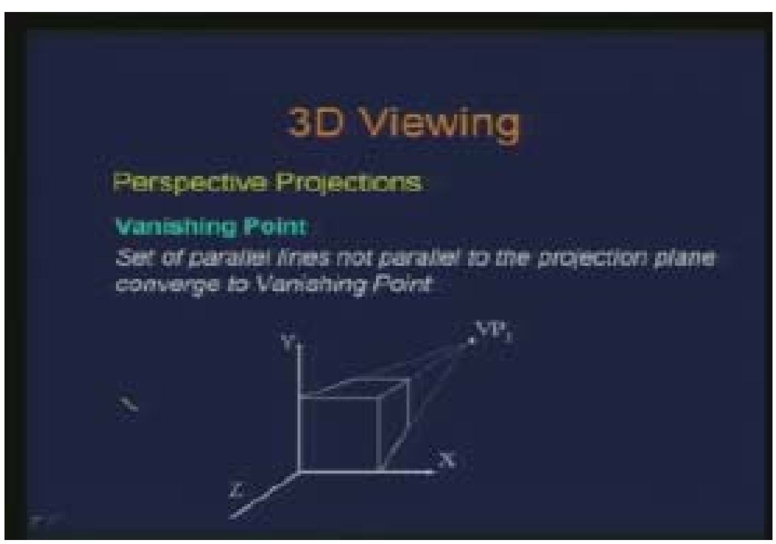

The fact that the parallel lines seem to converge is being observed. Even in the real life scenario we observe that the two parallel lines need to converge, there will be tracks and so on. But in reality they do not converge. So what we get are these vanishing points which are basically the points where the set of parallel lines not parallel to the projection plane converge.

Here is an example where if I look at an object like a cube or parallelepiped I see that the lines which are perpendicular to the projection plane in this case the lines which are parallel to Z axis seem to converge somewhere else. This is what we call as the vanishing point in Z direction. In the world this point is located at infinity. It is just that the transformation which is the perspective transformation making it appear there. So what does it mean? If I want to know this point I can actually take a point at infinity and apply my perspective transformation. That will locate the vanishing point corresponding to the point at infinity. That is what is happening now.

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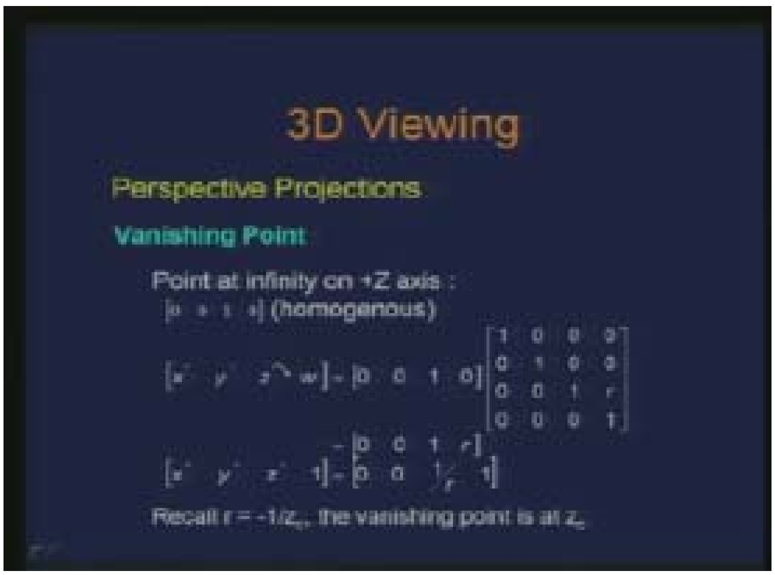

If I consider a point at infinity on positive Z axis, remember the homogenous coordinates enable us to represent the point at infinity just by using the homogenous part of the coordinate as 0. So this is the point at infinity on positive Z axis. Now if I take this point and apply the perspective transformation using this r here in the last column what we obtain is this value 0 0 1 by r_1. This is after homogenizing it after dividing by the last homogenous coordinates I get this. And recall that this r is nothing but minus 1 by Z_c. This is the scenario I am taking where the center of projection is laying along the Z axis at Z_c. So r is equal to minus 1 by Z_c which means that the vanishing point which I am trying to obtain here is at Z_c. So it is an image of center of projection on the other side.

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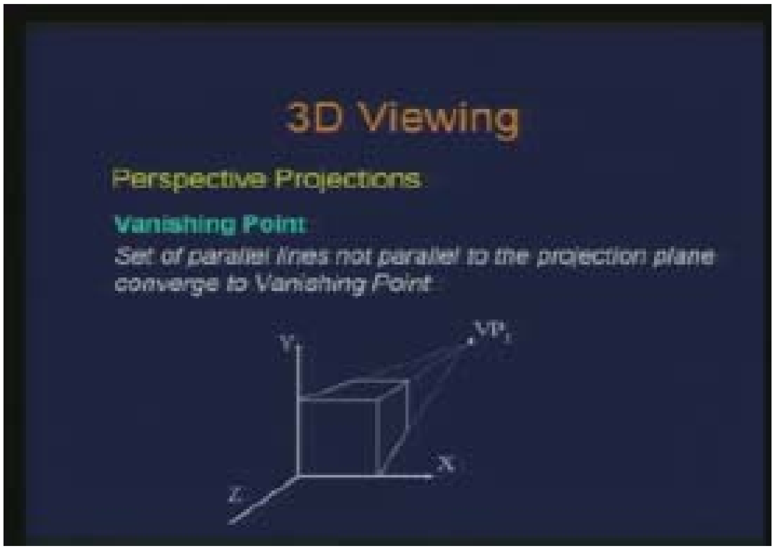

This is after the perspective transformation. But you are trying to locate what image would it have after the projection then it is going to lie on the origin. This is after the perspective transformation I am not projecting it on a 2D plane. The moment I do the projection on 2D plane this point will get there because this is along the Z axis. So this is the before the projection has taken place.

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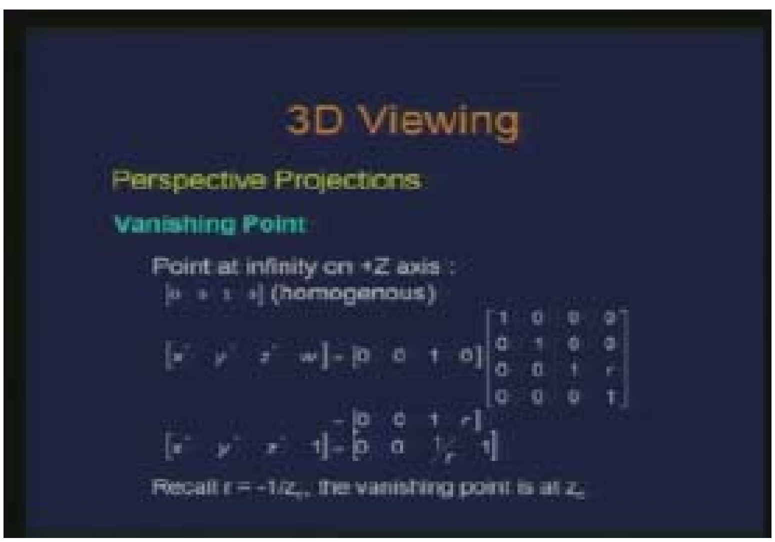

In this way I can obtain the vanishing point.

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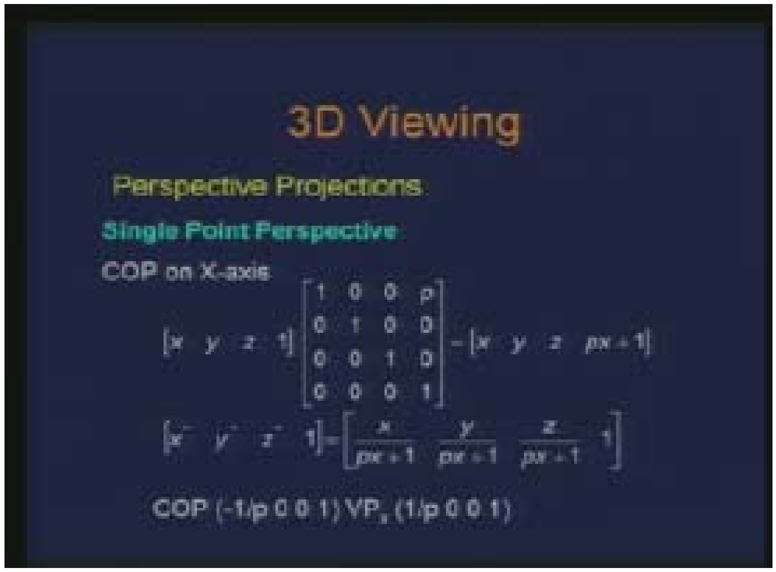

Similarly, this was the example where I had considered the single perspective so I call the single perspective transformation when one of the three entries here and the last column of the matrix is non zero. I had earlier considered the third row where the center of projection was along Z axis similarly I can consider the other axis. Here if I consider center of projection on X axis then I am taking about a non zero value here p and I have a very similar treatment which can give me the transformed point x y z 1 as this and correspondingly the center of projection is located at minus 1 by p 0 0 1 and the vanishing point along X axis is located at 1 by p 0 0 1. So I call the type of perspective transformation as a single point perspective transformation when one of the three entries here is non zero. So this is again when I would consider the second row now I get center of projection as this and this is VP_y.

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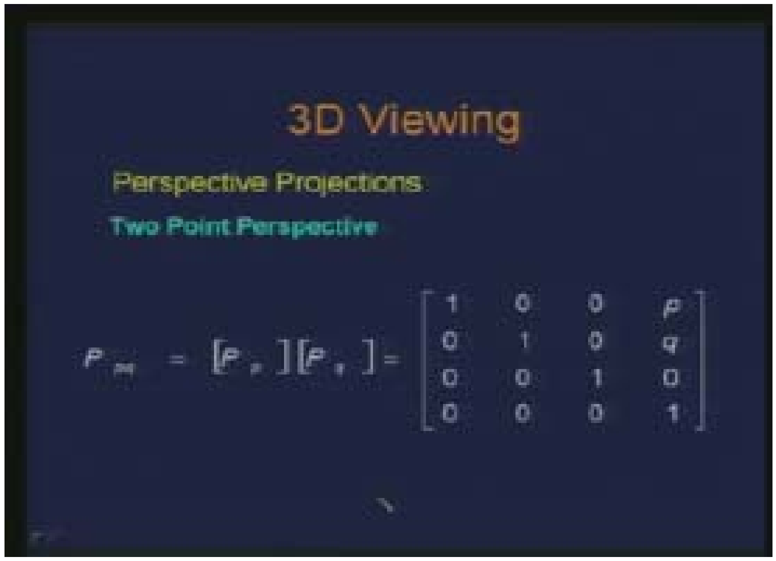

Now the idea can be extended of single point perspective to have a two point perspective just using the same framework of values in the matrix. So instead of having one of the values in the last column to be non zero let us say I have two values P and Q which can also be obtained by concatenating the individual matrices of one point perspective like P_p and P_q. If I have just multiplied them I would have obtain this again. This is to get a two point perspective. So this would give me the two centers of projections and correspondingly two principle vanishing points.

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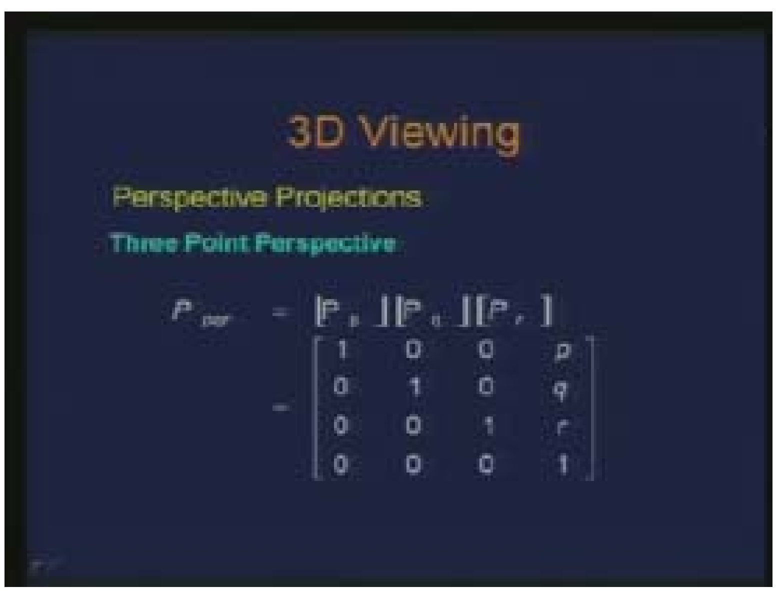

Now I can further extend this idea where all the three entries here are non zero p q r. And once again this can be obtained as concatenation of the three one point perspective transformation. And this gives me three centers of projections and the corresponding vanishing points.

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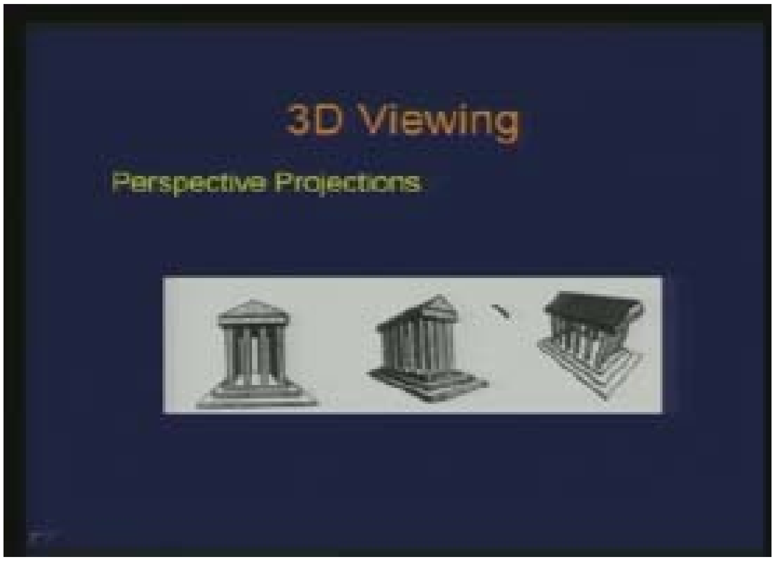

Here this is the example where I have as one point or single point perspective. So these lines of the columns are still parallel but these lines if you consider this to be as Z axis then they seem to meet somewhere. Therefore this is a one point or a single point perspective, this is a two point perspective. So here I still have the lines which are along the y axis that is still parallel whereas the other ones seem to converge this one and these ones. So this is a two point perspective and this is a three point perspective and none of the parallel lines now seem to remain parallel but they seem to converge at some point. Therefore again the idea of using one point two point three point perspective is to add wherever required the 3 dimensional viewing in the object.

What do you mean by two point perspective relating to what happens in the reality because even when you take a camera there is only one camera from the image perspective. This is just with respect to the matrix we have. So I basically looked at the entries in the last column and defined the various types of perspective there and correspondingly what we get is the principle vanishing points. But we can generate these even using single maybe a camera. If you actually apply certain transformations before you take a single perspective transformation you can create two or three point perspective.

So all it is saying is that if you apply a certain transformation to the object and then you take a single point perspective you may get two point perspective or you may get even three point perspective. That is the process of generating two point or three point perspective.

There I had considered the location of the position of the object in such a way that they were aligned to the principle axis and the vanishing points which I am referring to here are basically the principle vanishing points because so I am looking at the lines which are parallel to either of the coordinate axis. But you can actually get vanishing points for the lines which are not parallel to one of these coordinate axis and they still converge to a point. Hence there might be just vanishing points or sometimes they are also referred as tracing points.

If you take a point in (x, y) plane, treating it from the representation of a matrix was to define these single point, two point and three point perspective. When you are generating them that is when you have to generate such a kind of a projection you may still be using one camera and that is where your viewer or the camera is located. But you can create the effect of multiple vanishing points. So let us look at how to generate these perspective views.

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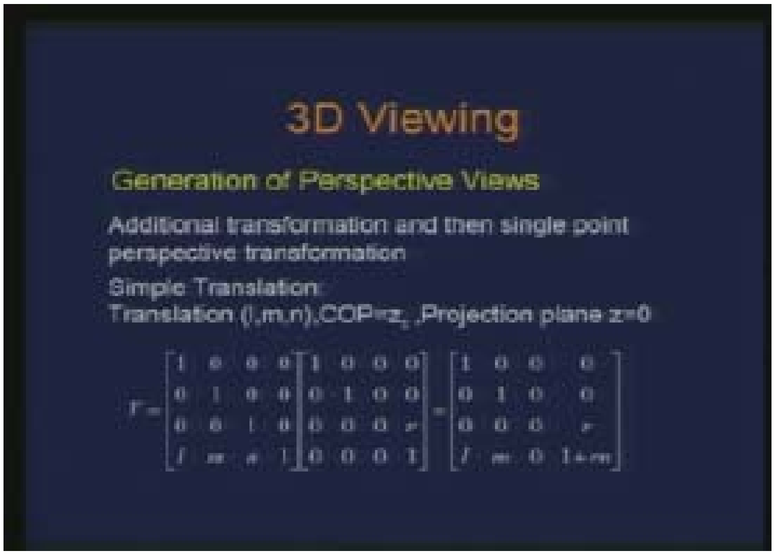

Even when we are talking about the aspect of enhancing the 3 dimensionality in viewing of the objects a single perspective may not be adequate and that is the idea. For instance, if I put an object or a cube in such a way that the cube is basically having all the edges parallel to the coordinate axis and I am viewing from the Z axis then what will happen is I will just see the front face considering there is a hidden line or hidden surface elimination process going on. Therefore all I will see is the first front face and nothing else and that would not give me any 3 dimensional viewing so I need to enhance that in some ways. So the way we can do is we can actually have additional transformation before we apply the single perspective transformation.

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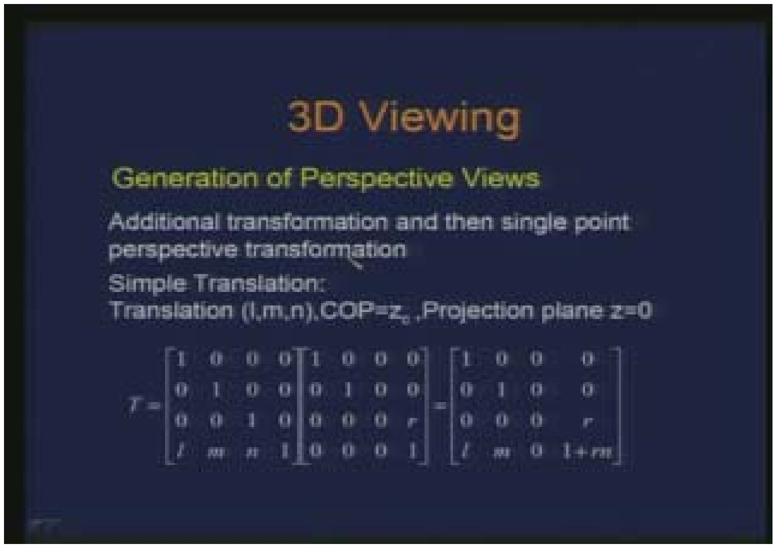

Hence let us consider a simple transformation like translation. If I apply a translation by the offset l m n to the object and again I consider the center of projection located at Z is equal to c and the projection plane Z is equal to 0 then I just concatenate these two matrices the translation matrix and the projection matrix and this is what I get.

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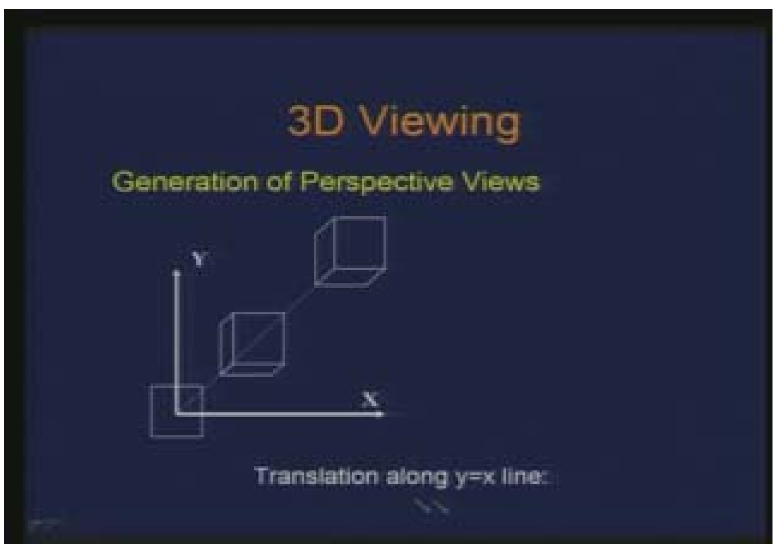

So the example of this, if I consider the translation along y is equal to x line so an object initially located here when translated along y is equal to x line may actually look like this or this which adds to the 3D viewing. So a simple translation is adding to the process of 3D image.

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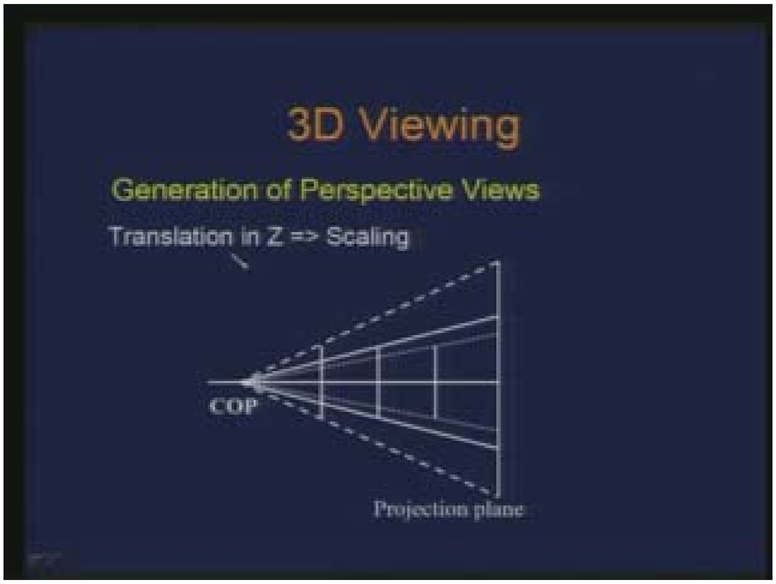

Similarly, if I have a translation in Z that is another aspect of translation would actually give me the notion of scaling and that we absorb as zooming in zooming out. This is one of the things we often absorb. The object here looks big and the object here will look small and this is when everything is in front of the projection plane if it goes behind the projection plane then the object becomes smaller than its original shape. So, just a simple addition of transformation of like translation can change the process of viewing. Further I add rotation.

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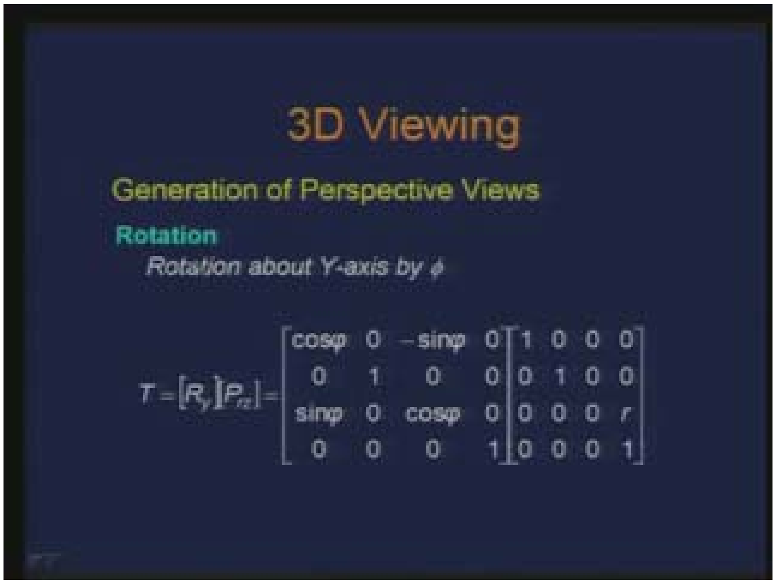

I do a rotation before I take the perspective transformation and now I am always doing perspective transformation which is a single point that is one point. Now I do a rotation about y axis by an angle phi. So the total transformation is basically the rotational transformation and the projection transformation which is this multiplied by this.

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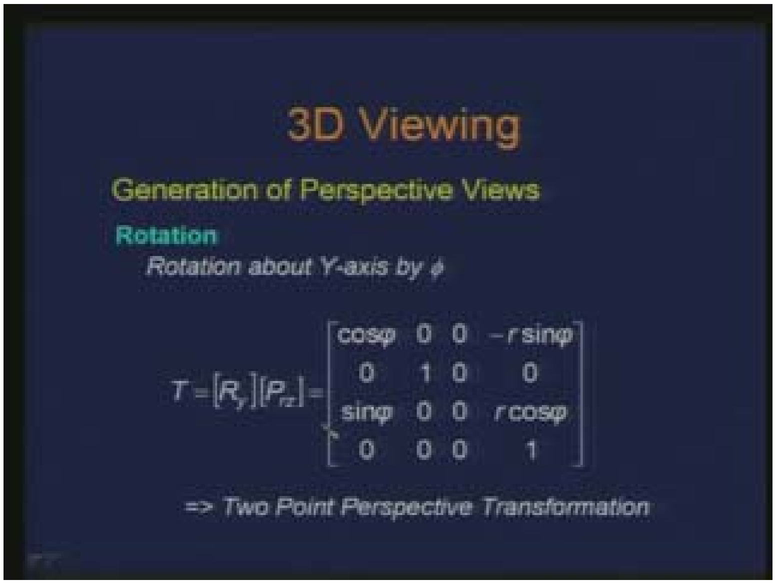

And if I see the resulting matrix this is what I get. In the last column we considered single point perspective. The only thing is I applied a transformation of rotation before I gave the perspective transformation. So now this gives me a two point perspective transformation. Therefore in this way I can have a two point perspective transformation using single camera.

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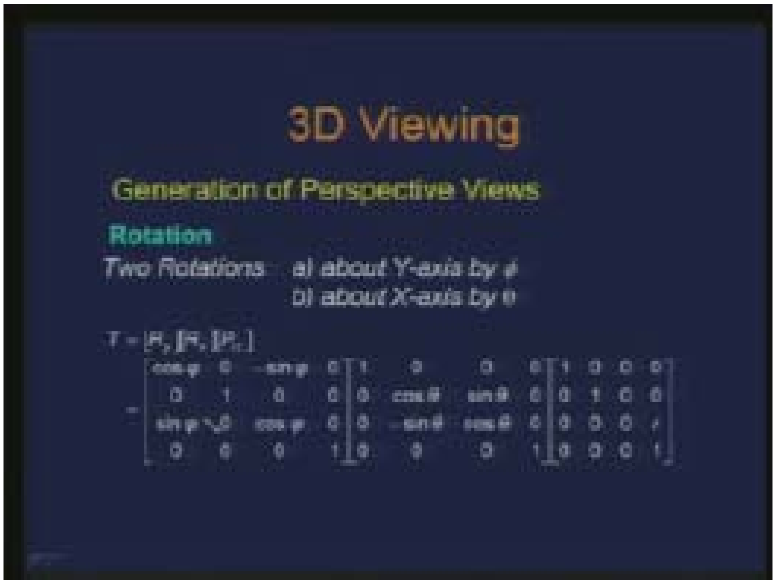

Similarly if I use two rotations about Y axis by an angel phi and about X axis about an angle theta so the total transformation is the rotation about Y, rotation about X and the projection transformation. So if I multiply these three matrices this is what I get.

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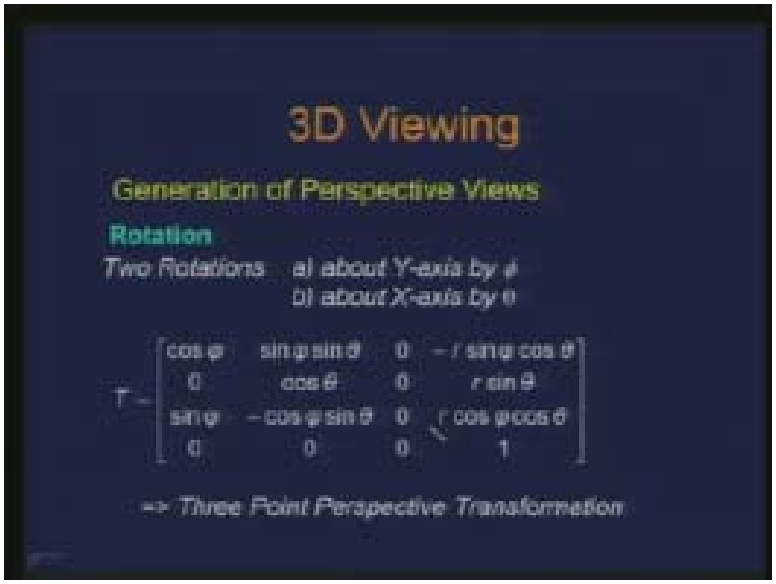

Now I have all those three entries non zero which basically means that I get a three point perspective transformation. So now using two rotations but still using a same single point perspective transformation I can give the effect of three point perspective transformation. This is how I can generate the various types of perspective transformation.