Introduction to Computer Graphics
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Lecture - 32
Fractals (Contd...)

So, we will continue on fractals. Last time we were talking about fractals. In fact we looked at geometric fractals.

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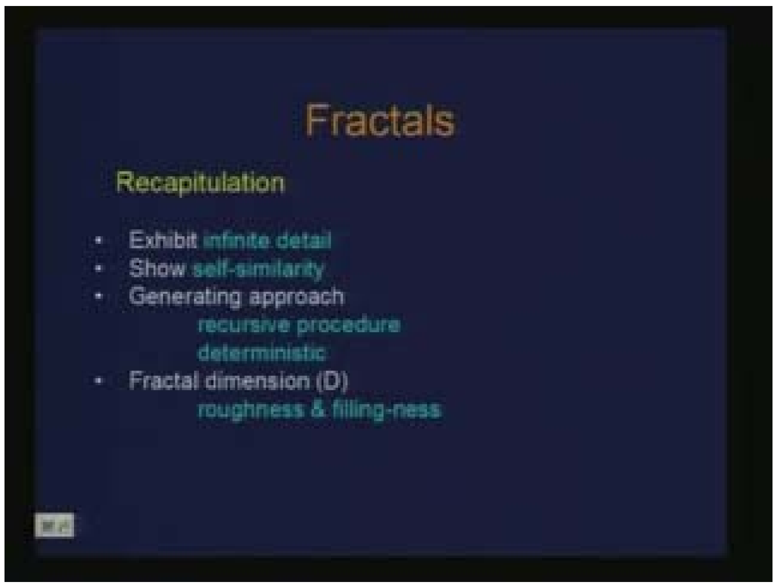

So just to recapitulate some of the properties and details we discussed about fractals. One of the things is that they exhibit infinite details which basically means that if I try to zoom in to the fractal structure I keep finding the details. So the details present inside the structure are at infinite level. So as you keep zooming in you keep finding details there. Therefore for the purpose of creating these fractals we put a limit to it because of the resources we have namely the computational space and things like that. Hence we cannot do this at infinite level.

And the other important property they exhibit is the self similarity. So the self similarity property is again referring to the situation where one is trying to zoom in to the structure. You observe some scaled copy of the same structure when you zoom in. So there is a self similarity from what it was originally as a structure and what you keep finding as in the details. In fact these two characteristics when one observes also find many of the natural phenomena.

For instance, you take a view of a mountain and you try to approach the mountain, you zoom in. So first of all you find lots of details coming out and the second thing is the kind of structures you observe has some similarity to the over all structure. This is known as self similarity. Many of these natural phenomena also show these characteristics. That is one of the reasons fractals are chosen for modeling natural phenomena. If we look at the process of their generation it turns out that it is a recursive procedure where you start from initial state and you consider a transformation function which could be a transformation in terms of a geometrical pattern or some other transformation and you keep applying that transformation function. So, there is a recursive application, a successive application of the same thing so it is a recursive procedure which adds details into the structure.

The other observation we had which was particular in the case of geometric fractals is that the process of generation is deterministic. You take a copy of the transformation and then reapply the same transformation. And it is deterministic which in fact renders in these geometrical fractals the regularity in what the pattern gives you. Though these fractals demonstrate irregularity in their structure but the way these get applied show regularity in that pattern. So there is regularity in the irregularity. That is because of the fact they are deterministic in the process of generation. And we also observe that they have a dimension like what we observe in Euclidian geometry but this dimension is referred to as fractal dimension because of the fact that this could be fractional as a number. So you can have a fractal dimension like 1.26, 1.89 and so on.

We also observe that the effect of this fractal dimension in some sense captures the amount of roughness of the structure which we obtain and also has a self filling characteristic. For instance, we observe that when a fractal dimension goes from 1 to 2 so you actually have a fractal dimension of 2 for a curve then the property of self filling ness is observed. So, when the fractal dimension is 2 the whole area where it gets applied it sort of self fills which intuitively matches to the dimension what we observe as the Euclidian dimension of 2 for a planar structure. These are some of the characteristics which we observed in the case of geometrical fractals.

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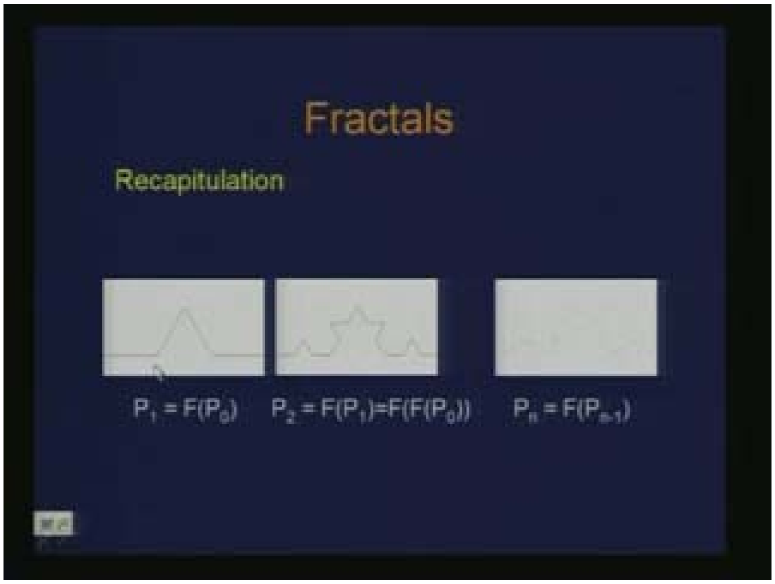

Again looking back to the construction of Von Koch Curve you have a pattern of this kind so we start from a state which was a straight line and this is the transformation which is applied which we are referring to as F and then you again apply this transformation to each of these segments and so on. This is the recursive procedure. Ultimately you get a rough pattern structure. This is the Von Koch Curve.

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Now the problem with them as we looked at is because of the nature of their generation which is deterministic they exhibit the self similarity property in exact form so they are exactly self similar structures. When we are referring to them as procedures for constructing natural phenomena where we observe that there is no deterministic pattern but in fact there is randomness into whatever we observe. Then there is some modification to this process where randomness is introduced. So the fractals which have some randomness to it are referred to as random fractals. So the whole idea is that the process of generation would now have some randomness in it which in turn renders a self similarity but in a statistical form.

When you zoom in a fractal structure which has randomness in it then what you observe is that the zoom in structure is seemingly similar to the original structure and not exactly similar. That is what we mean by statistical self similarity. There are number of ways in which these random fractals can be generated. In fact one of the features of the process of generation you observe is involving some sort of a subdivision mechanism.

Adding details in some sense is a method of subdivision. Then what you do at the level of subdivision is something different. But as a process you are doing some sort of a subdivision even when we are generating the geometric fractals. We will be looking at the various ways of process of subdivision which in turn gives you a generating approach. Let us look at the process of generation and one of the ways in which one can think of generating these random fractals is a mid point subdivision.

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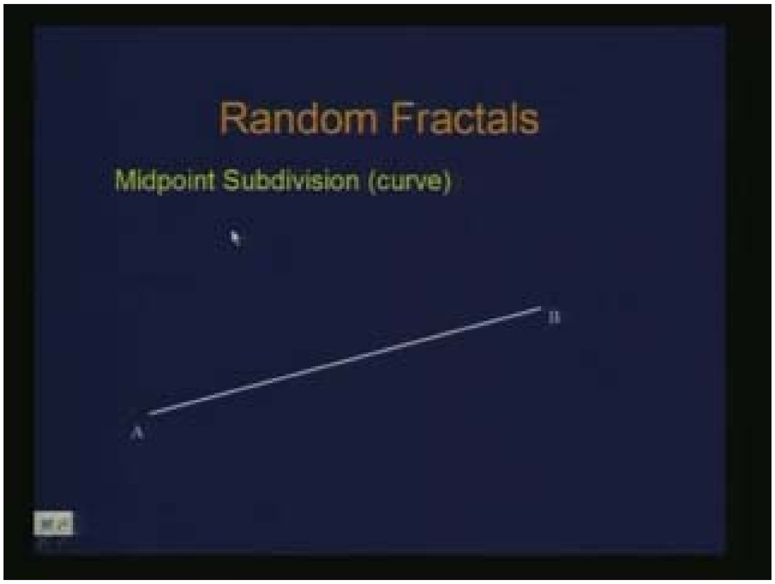

Here is an example where we are interested in creating fractal curves. So again one can start from a line A to B which is like an initiator to the fractal and you want to add details to this line similarly as we did in geometric fractals. When we are saying that it is a midpoint subdivision the idea here is that we would consider the midpoint of this line and do an action to that so that is why we call this as a midpoint subdivision. So we actually take this midpoint which is at M displace it to point C so this initial AB line now is transformed to two segments AC and BC.

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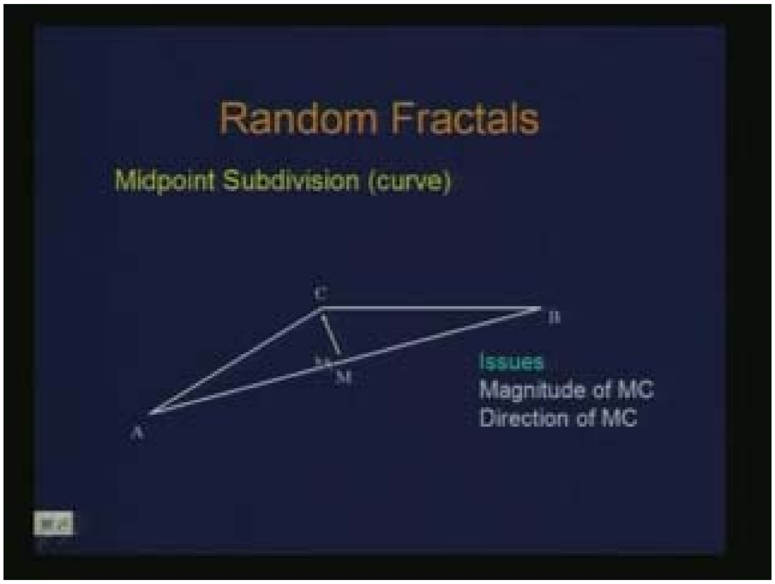

Now clearly there are issues concerning as to how do you displace, what is the magnitude of MC and the other issue is in what direction do you displace? These are the two basic issues. When we are displacing this M to C remember that this process has to continue. Next time what will happen is I will consider this as a segment, take the midpoint here and do a displacement of that. Similarly, this segment, take a midpoint of this and displace it. So at every level I am performing this process of subdivision and a displacement of that midpoint for that level. So at various levels this issue would arise, how much do I displace and in what direction do I displace. So one of the things which need to be looked at is that the displacement has to capture the level or the scale where I am. So at this level this is the displacement because of the fact that this is the scaling, this is the initial length I have so this has to in some sense scale with this.

Similarly, when I do a displacement here this has to scale with this and the second thing is the direction. Some suggest that you could do this displacement only in one direction which is in 2-D if considered this as X and this as Y then this is Y direction or it could be the direction of the normal of this segment. So it will be normal here and it will be normal here. The other parameter to the generation is the flipping of it. Remember we could actually go positive offset or we can also go negative offset. That is also an additional parameter, something like a flipping what we used in the construction of a dragon. These are the various parameters one can play with.

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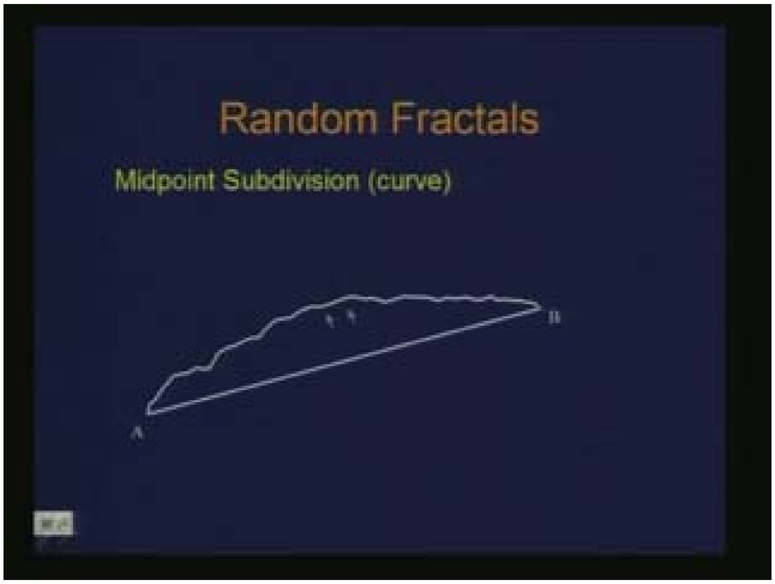

Therefore may be at the end of the whole thing we get something like this. So this seems to be of some interest if you are trying to use them for modeling some natural phenomena for instance coastal line unlike Von Koch curve which was extremely a regular structure.

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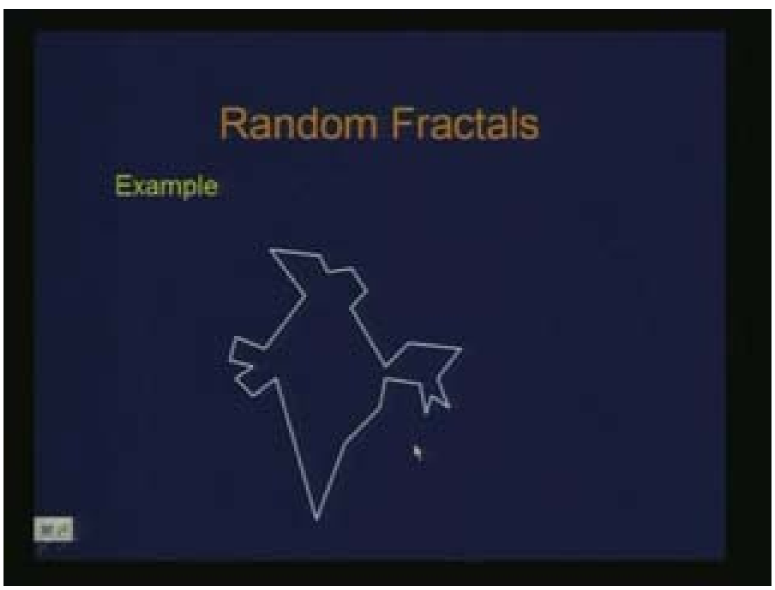

You consider having a coastal line of this where what you have done is you have supplied an initial structure which gives you a gross shape so it is like the initiator where you want to start the generation of fractal curves and then you apply this random fractal generation on the initiator so you may get this.

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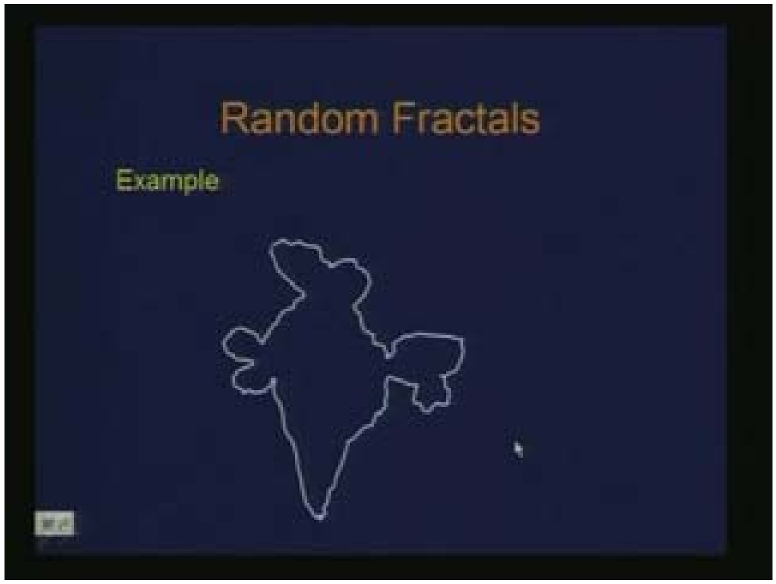

So clearly this should not be confused with the exact GIS data of Indian map. But if you are doing a simulation or doing some computer generated synthesis then this is a freely acceptable approximation. This was concerning the curves. A similar idea can be now used for surfaces. In fact there are number of ways we can look at the process of subdivision again.

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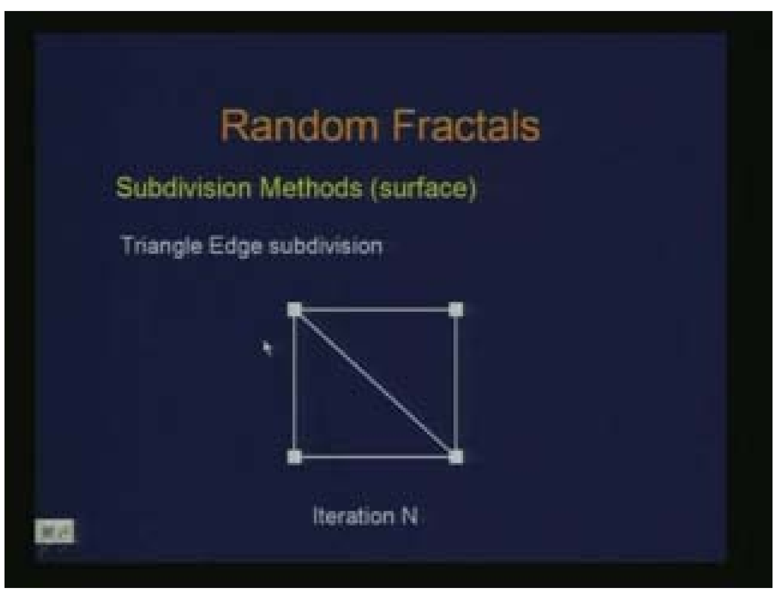

For example, you have an initiator of the kind where you divide or you construct the surface as collection of triangles. So at some iteration N you have a configuration like this. There is this triangle and there is another triangle here and you have the positional data of these. Now there is a type of triangle edge subdivision where you do something very similar to what you did in the case of curve where you obtain the midpoint of each of the edges in the triangle so this is the midpoint, this is the midpoint, this is the midpoint and so on. Now that becomes your next level of surface then appropriately you construct the triangles using those points.

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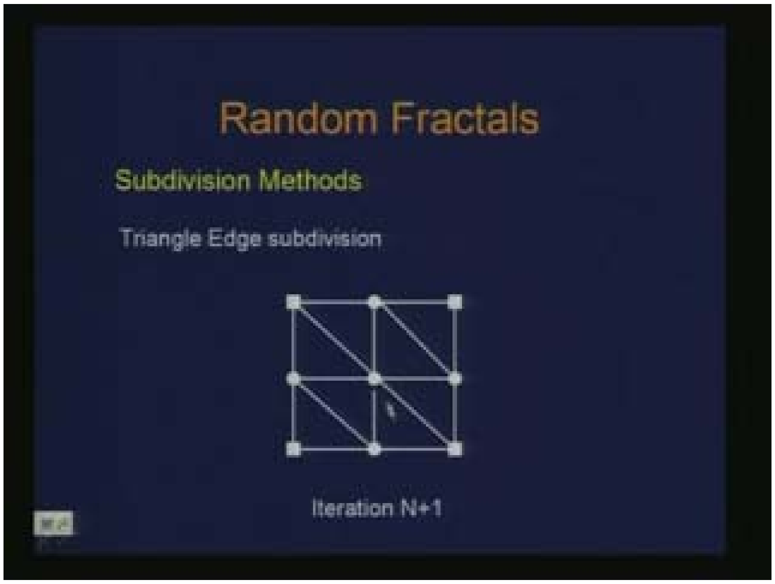

So the next iteration N plus 1 has got these additional points which are nothing but the midpoints of the individual edges of the triangle which are there and then you have these as new set of triangles and this can be repetitively done. From this iteration you can go to the next iteration where you subdivide each of the edges here and then again do a reconstruction of set of times. This is what is happening in a topological fashion as to how these triangles and points are being added.

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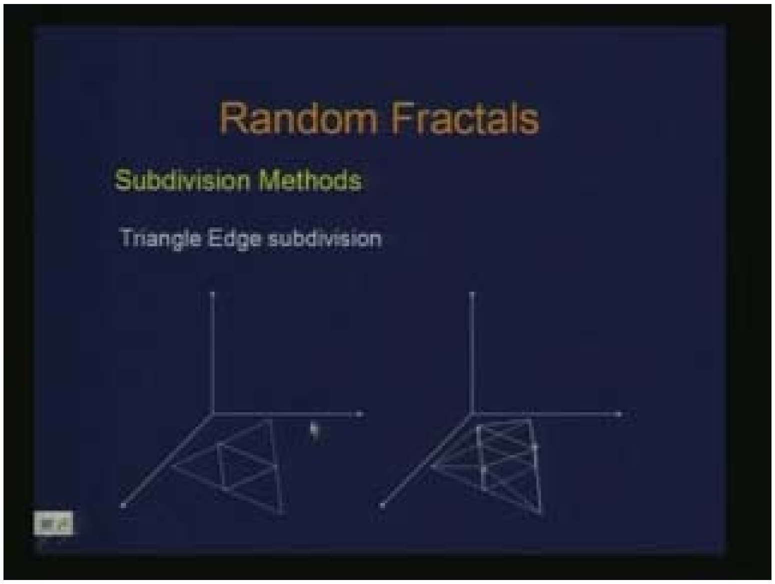

Again at a surface construction level for example if this is a triangle to start with, this is the initial triangle, these are the midpoints you had and because of these points this whole triangle becomes as four triangles so this is the topology of triangles which have been constructed. Now you have to also do the height assignment that is where the perturbation to the surface would come.

So what you are going to do is you are going to displace this point to a certain height, you are going to displace this point to a certain height, you are going to displace this point to a certain height something very similar to what was done in curves. You have this height, this height and this height. So the new surface is this triangle, this triangle, this triangle and this triangle. Again there are issues. What are the issues? One of the things is that the direction in which you displace these points. Therefore let us look at these issues. Again one of them is the amount of displacement. So the amount of displacement is relevant, we go to the next level where we take this triangle and we take this triangle.

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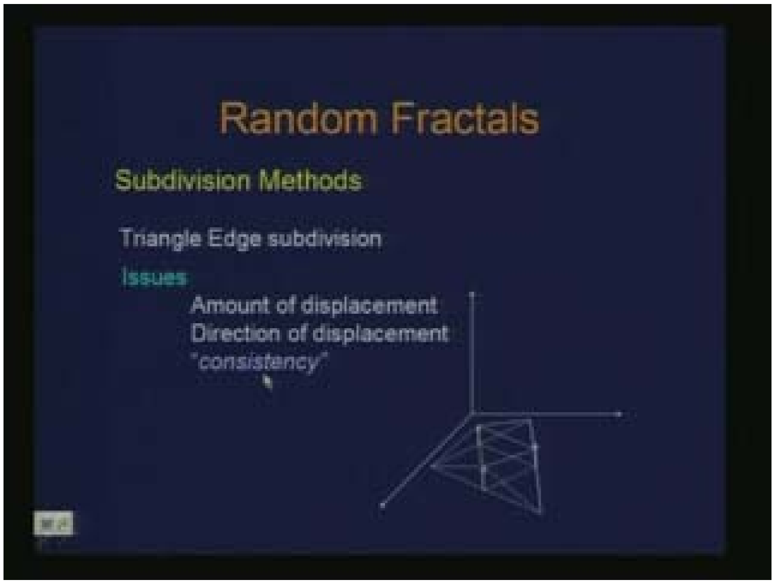

So there will be a midpoint here somewhere which would be shared between this and this so the amount by which I displace for this set of triangles has to be the same as for this otherwise there is going to be an inconsistency. And when I am using the randomness which is primarily obtained by some random number generator one has to be consistent in getting these random numbers or the random heights for the two adjoining triangles otherwise I would end up getting inconsistency. If you are using some seed based generation of random then one has to may be use the indices of the edge to determine the seed.

It has to associate the edge so that you are consistent in the height you assign. So once again if I have the mid point which is being shared and I displace this along the normal of this triangle and the other one along the normal of this triangle then I would have a rupture, I will need a hole so there also you require consistency. Therefore one has to address the issue of consistency whenever you are doing these both in terms of settings the amount of displacement and choosing the direction in which you displace. Here are some of the results. If you consider this as your base triangle this is at level two, so 0 level is the base triangle, level 1 is when you have four triangles and level 2 is after that so this is the level 2 and this is level 4 and so on so at the end you get something like this.

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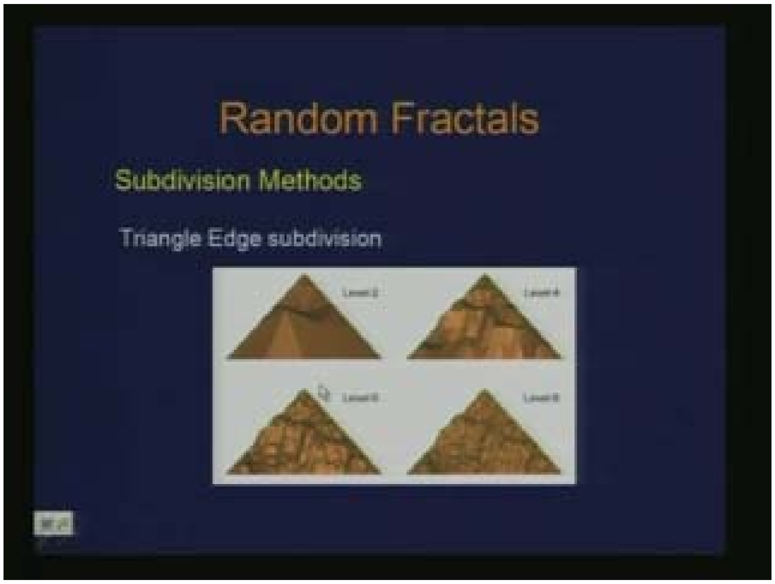

It does not still look like a mountain but it seems okay. Now what is the rendering or shading model one should be using for rendering fractals? Would you be doing Phong shading or would you be doing Gouraud shading or would you be doing flat shading? Actually you should be using flat shading. If you were using Phong shading or Gouraud shading what kind of result you would have had? If you were to use Gouraud shading and Phong shading what would that have done is in fact smoothened the rendering thereby removing the details so the whole purpose is lost. At one point you are trying to add details through subdivision and when it comes to rendering them you just do the reverse process but it does not work. So you would be actually using flat shading.

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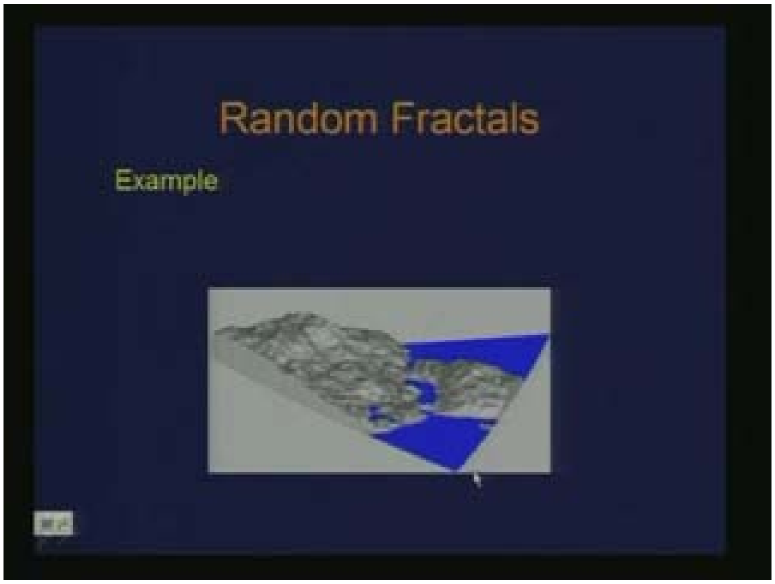

The previous one was one of those kinds you would have generated in your assignment. Here this was the base triangle and then these are all the perturbations added as this height during the subdivision.

What has been done is that this 0 height or the height at the base is given a blue color like something giving you an impression of water and again in fact when you change this height you can vary the color. So there could be different gradation of colors, it could be green to brown or whatever. So this looks reasonable. Let us discuss about some other ways of doing this subdivision. One was this triangle edge. There is another way which you can use for subdivision which is the diamond square method. What is diamond square method? This is the configuration you have at an iteration N so these are the points given to you.

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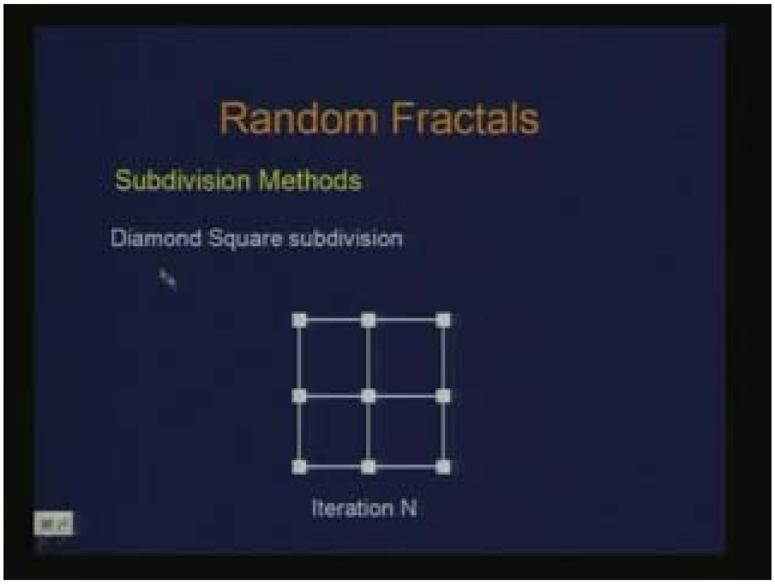

Now in the case of diamond square subdivision this is what happens topologically. You obtain this point which is the average of these four points. Similarly you obtain these points which would be the average of these four points. This point is the average of these four points and this point as the average of these four points.

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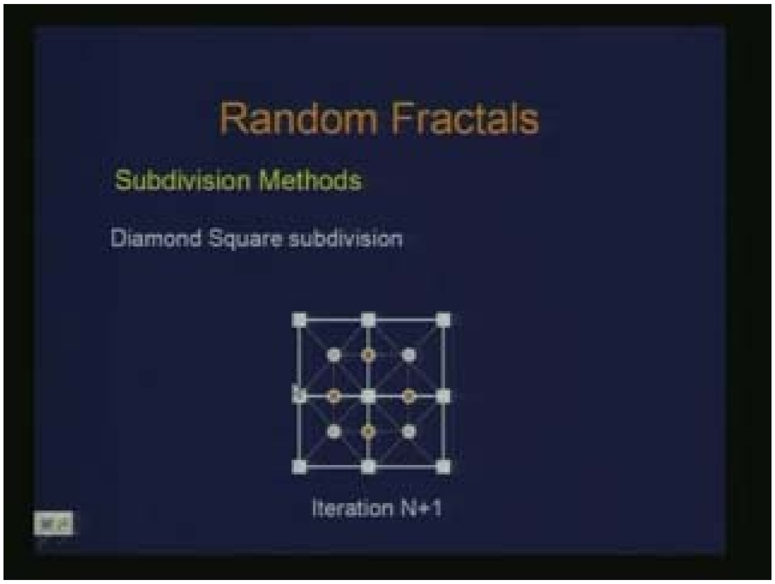

So there is a middle point for this square which is the average of the corner points. And then you have this point which is obtained from these two points. You have this point which is obtained from these two points, you have this point which is obtained from these points and so on. So what you could sort of construct is, this shape is on the top of this shape so these are sort of diamonds on the top of squares so that is why they are called as diamond square subdivision.

This is another way of doing the subdivision. One of the thing which is common between this and triangle edge subdivision is that you end up getting points onto the edges here. There is a point here, there is a point here, there is a point here and so on. Now to retain the consistency what might happen is that you may have some sort of an artificial edge getting appeared in the final structure to maintain the consistency because of the points which are being created now along this edge you would see some lining on the final structure. That is in some sense an artifact which may be there in the case of diamond square subdivision and also in the case of triangular subdivision. That is also referred to as creeze effect. It is an artificial creeze which is being formed at the edge. So in order to avoid this there is another scheme of subdivision which is called as square square subdivision.

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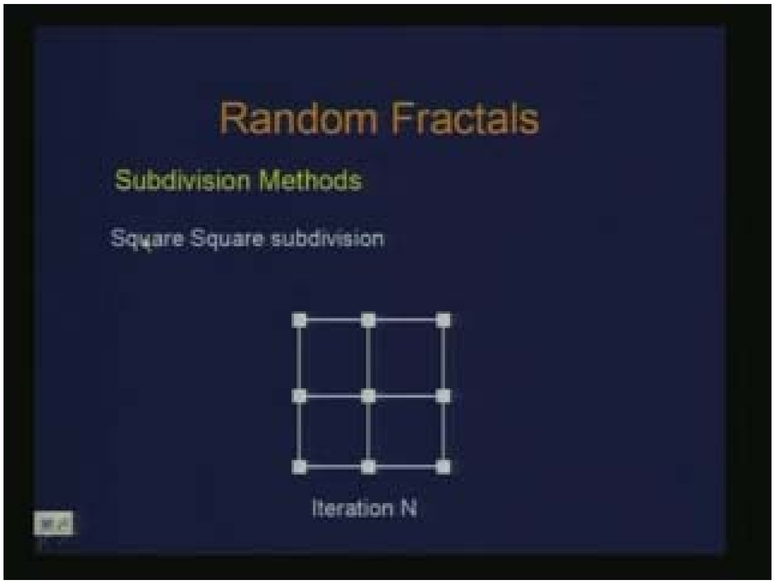

Again what is done here is you have these points at an iteration N so what we have to look at is what happens in the next iteration. This is what happens. Here we create these four points within these squares, another four points within this square and so on.

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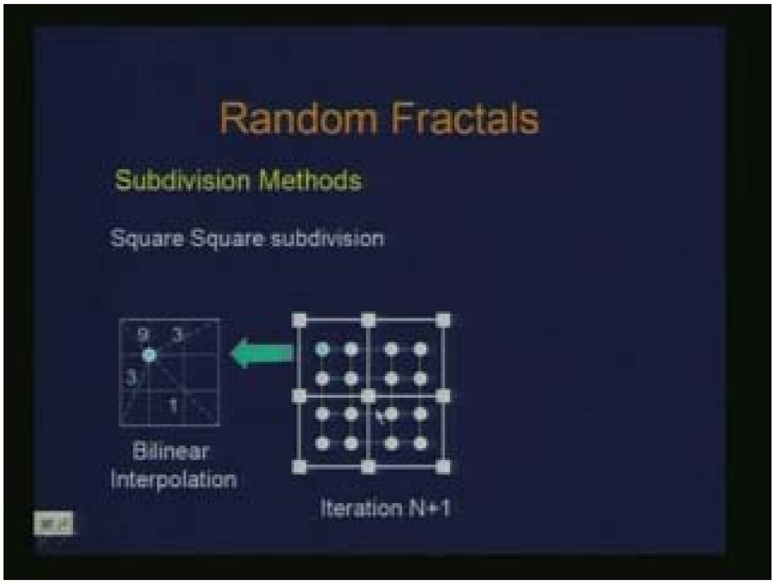

Basically this square is in some sense getting replaced by this square and so on. And these two squares are again joined so there is a square here, square here, square here so from squares you get squares structurally.

Now how do you get this point that is the question we should answer. Now this in fact is obtained using bilinear interpolation. These are the four corner points we have and we are trying to locate this point. Now let us say I just enumerate that as 0 0, 1 0, 1 1, 0 1. So this particular point is located at 1 by 3 and this particular point is located at 2 by 3 and so on.

So, if I perform a bilinear interpolation, one parse in this direction and another parse in this direction so this point can be located in some sense from the weights which are shown here like the weight of 9 3 3 1 which are nothing but the weight coefficients which one needs to apply to the respective points. This point is basically obtained as nine times this point, three times this point, three times this point and one time this point which you can further normalize divide the whole thing by 16. And in some sense it is all telling you the proximity of this point with respect to the four corner points which is what happens in bilinear interpolation anyways.

Similarly, the corresponding weights would get determined for this point which you are trying to obtain here and this point and this point. So each of these four points now can be computed using bilinear interpolation from the corner points of the square. Once you have obtained this point and all the points of the square then you have obtained a new configuration for the iteration N plus 1. Now again this can be repeated. What is happening is, in some sense from this boundary I have reduced the boundary to these. In further iteration I would come here somewhere. But if you want to preserve these boundaries then you have to do something extra within this range. So you can obtain extra points which would locate points here, locate points here and so on so you have extra squares for the boundary cases.

In fact if I were to use this process of repetitive bilinear interpolation to a curve or to a surface in this fashion like a subdivision what is the limiting surface we have? Here is an example of a curve, if I were to use some sort of a midpoint subdivision, this is the situation to start with. Now I am obtaining a point here and obtaining a point here which is the midpoint of the two.

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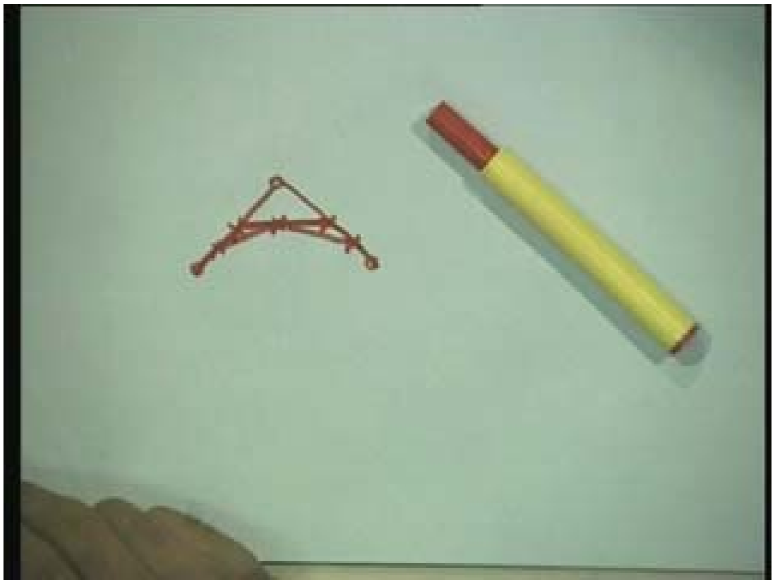

I also preserve the boundaries. So in some sense I have a new set of points like this. Again I can do the same thing so what happens is this. So what is the limiting curve I have? This is the subdivision of a Bezier curve, it is a quadratic curve, so it is a Bezier quadratic curve. You keep subdividing it.

Here what we are doing is we are actually perturbing them. Therefore this is what is happening topologically. But then you are perturbing the heights and that is what will create a structure like a terrain or a landscape. But this actually does not give you the artifact of creezing because every time you compute these new points these points are not aligned to one of the edges so this removes the artifact of creezing. This was due to given miller who introduced it in graph in 86 or so.

Here I am computing the height of this from this part and similarly the height for this with the same weight structure so there is no issue. Basically it is 1 by 16 times height of this, 3 by 16 times height of this, 9 by 16 times height of this and 3 by 16 times height of this. So there is no issue of consistency here. We have seen examples of coast lines, we have seen examples of landscape but this actually is a much better example now which even has some fogginess to the atmosphere.

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You can also do clouds in a similar manner. In the case of cloud what you need to change is the opacity. It is the opacity which would determine the thickness of the cloud. We can even do candies. Some slides contain examples of candies. It is the cover of the candy we have as an example and which can be generated using a similar process. Also stones and many other things like plants, trees which we have already seen. These were random fractals and we can see the relevance of these random fractals towards the modeling of natural phenomena. There is another kind of fractals which are also referred as algebraic fractals. The Mandelbrot's set or Mandelbrot's function is again some sort of a generating or a transformation function.

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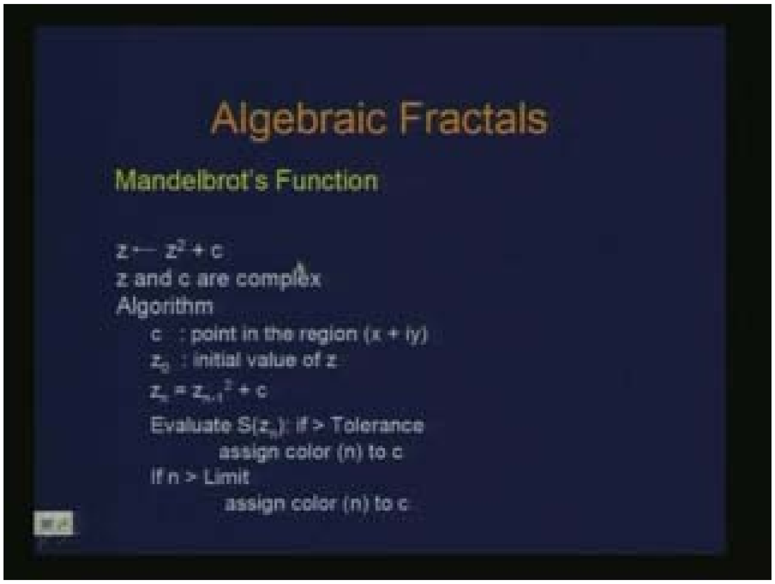

Here z is equal to z power 2 plus c where z and c both are complex, they both are complex numbers. So the idea here is when we are saying algebraic fractals we are basically doing a visualization of this function as to how this function iterates. So, for that there is a very simple algorithm. Consider c to be the point on the region you want to display. The fact that these are complex numbers they can be written as something like x plus iy which is the position in two dimensions. So if I consider c to be the point in question for the region to be displayed and z_0 given as an initial value to z so this is the function I am going to use so there has to some initial value of z which I call as z_0 then what is happening is the evaluation of this function which is z_n is equal to z_n minus 1 square plus c.

Then what you do is you examine the size of this zn, the size could be just the squares of the real part and the imaginary part so this is the size. So what you are like basically looking at is some sort of a divergence characteristic of this function z is equal to z^2 plus c. So clearly there is this self squaring which is happening to the function. One would imagine that this function would diverge soon. But it turns out that there are points that never grow after a certain value.

This is what actually gets defined as Mandelbrot's set. Here we are just looking at the mechanism of displaying this process. When we do the evaluation of this size if the size turns out to be a tolerance defined by you then you assign a color which is actually a function of this N which is the iteration count to this point c in the region. And if it turns out that it never grows after a certain size that means you have crossed a certain limit of number of iterations you assigned for then again you assign color to that N which is the limit of your iteration. So this would give you the set of those points which stops growing after a certain number of iterations.

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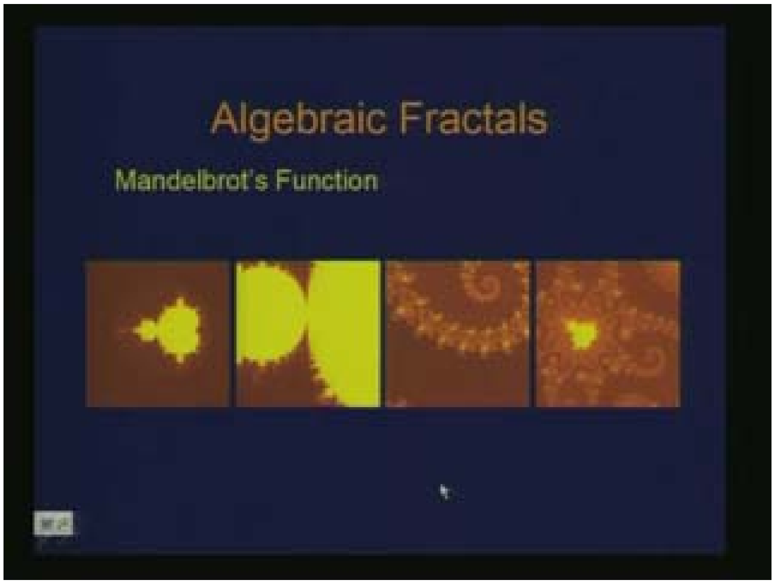

The kind of structures you would observe is something like this: This is the start where you have the region starting from minus 0.5 to minus 1.25 to 2 to 1.25 so there is a radius of 2.5 region and you obtain something like this. And these yellow points are the points which do not grow after a certain number of iterations. This is Mandelbrot's set.

This is the similarity of these fractals the algebraic fractals [….]49:52 the other fractals we have observed is that if you zoom into the structure if you take a small part here and try to zoom in here this is what you obtain. So, by zoom in what you mean is that you take a small region here increase the resolution of your computation for the region c you define and obtain this. This is what you get after zooming. And what you observe is that there are similar structures like this which are of the similar shape which you had obtained at the first level from where you have zoomed.

Again you can keep doing this process zooming in so you again you find details so there is infinite details to the structure that is one observation which is what is shared by other fractals. And also there is a recurrence or reappearance of a similar structure which you had started with. So there is this property of self similarity. For the purpose of looking at as a process of generating these patterns and structures they are in the same category.

We are perhaps ignoring the mathematics of this function. In fact there are people who have attempted looking at the generalization of this where z is actually coming as z to the alpha plus c rather taking a z power 2 plus c consider some power of z and this alpha could be an integer, it could be a real, it could be positive or it could be negative and interesting patterns can be generated using this.

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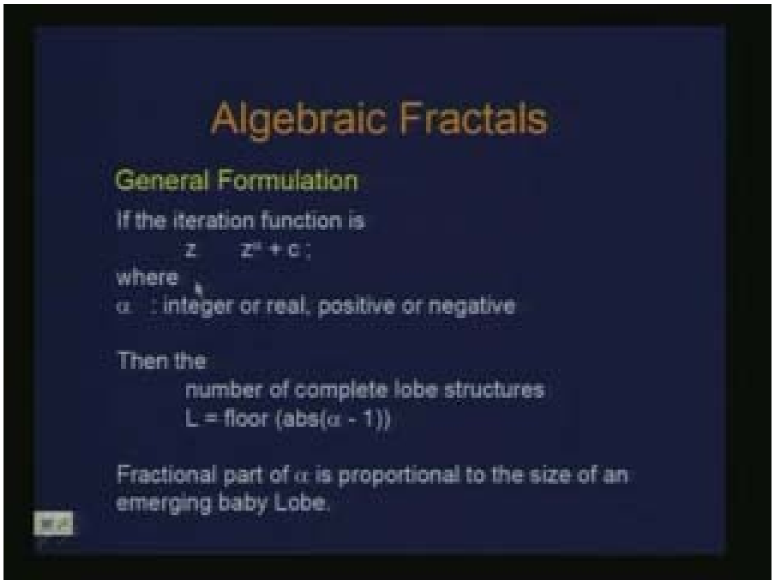

One characteristic which is observed is that you define L as a lobe structure. Now when you see here this is one lobe structure. So if I say this is one lobe structure then the number of these lobes structure is basically given as the floor of absolute value of alpha minus 1. So when I had alpha to be 2 then the number of structures I had is 1 then if I consider alpha to be 3 then the number of lobe structure is going to be 2 and so on. In these slides which are circulated around, you can see there are patterns where a different alpha is considered other than 2 and you have these various lobe structures. Another interesting observation is that even the fractional part signifies some sort of a beginning of the growth of a small lobe structure which is like a baby lobe structure. This is one lobe here (Refer Slide Time: 54:35) and this is another lobe and so on.

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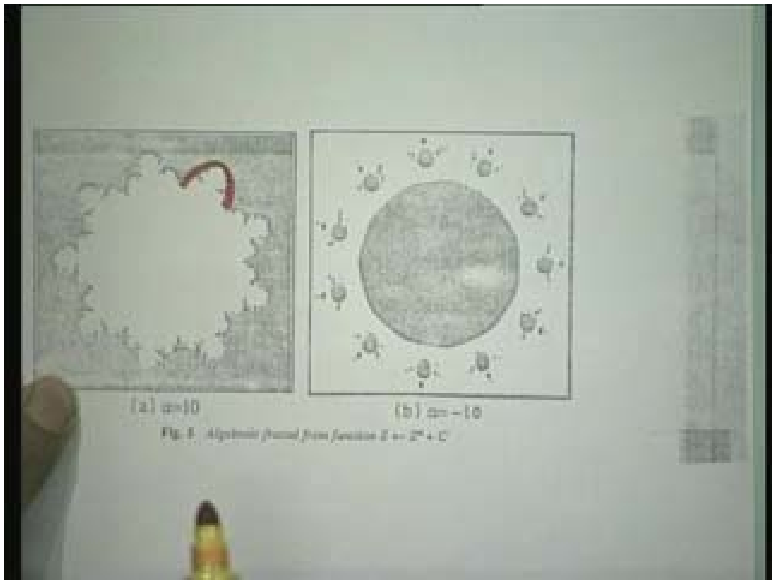

So, for alpha is equal to 10 you would have nine such structures so it is 1 2 3 4 5 6 7 8 9. And even in the case of negative number this alpha is minus 10 and this is what I am referring to as a lobe structure. There will be eleven of such like 1 2 3 4 5 6 7 8 9 10 11.

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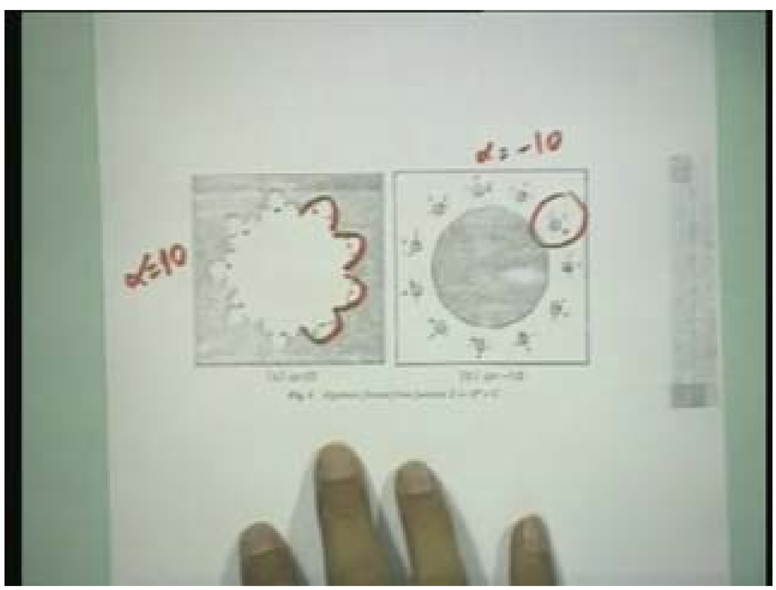

Anyway we are not actually looking at the implication of this number in terms of the mathematics behind it but we are just trying to display that. In fact there are very interesting properties with respect to what happens mathematically to this. So one can take other complex functions and see this .and clearly this would not happen to all structures. There are only certain structures which exhibit the property of self similarity and details inside so there is a class of functions which would give you this.