Introduction to Computer Graphics

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Lecture # 16

Surfaces

We have been talking about curves so far. Today we are going to start on surfaces. As far as the motivation or requirement of studying surfaces is I suppose quite obvious because for most of the objects you would like to have the representation of the surface by which you can observe the shape of the object. Surfaces are basically nothing but some sort of a boundary representation of a 3D object. We are going to look at the various methods on generating these surfaces. These surfaces are primarily the boundary representation of these objects and as in the case of curves we desired the curves to be having the properties of being smooth.

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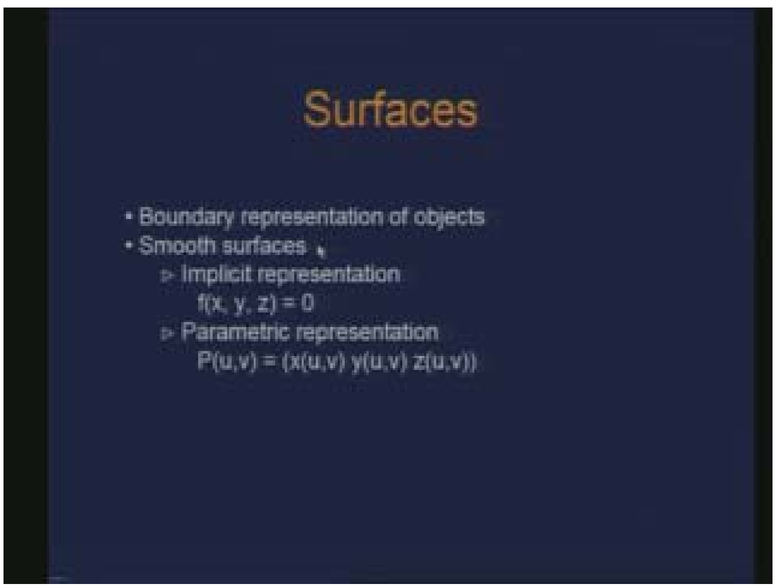

For instance just a polyline representation of a curve shape is not adequate in all situations. Therefore we looked at the various representations and methods of generating smooth curves. Similarly, here also what we would like to study is the generation of these smooth surfaces. As in the case of curves there are various representations. The two major representations are implicit representation where you have the representation of the surface as some function f(x, y, z) is equal to 0 so there is an inherent solution of the system of equation to be able to get the surface.

For instance, a plane is ax plus by plus cz plus d is equal to 0 is in its implicit form. And you also have general quadrics in implicit forms; ellipsoids, paraboloids they are the special cases of that quadric. And then we also have parametric representation. So, in parametric representation each coordinate (x, y, z) is represented in terms of the parameters. So, in case of curves we had a single parameter to be defined. So here we have the two parameters which basically define the domain of the parameter.

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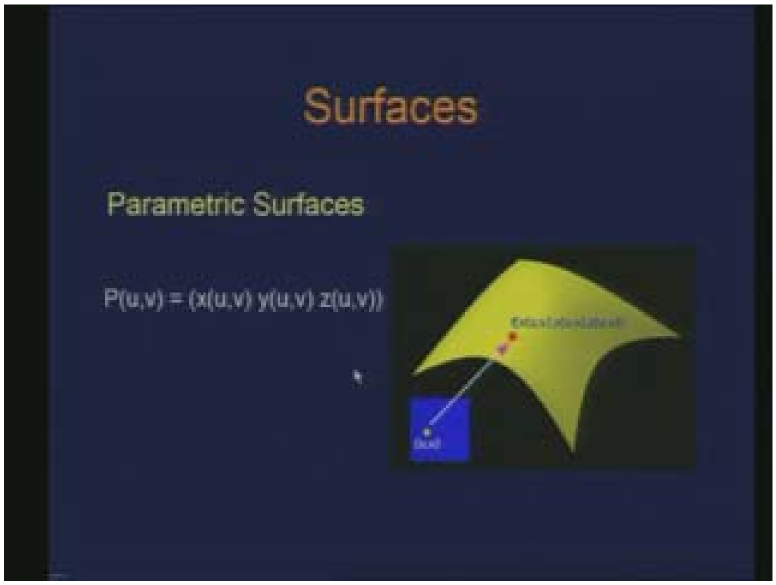

For instance, if I want to define a parametric surface in a rectilinear domain u ranging from 0 to 1 and v ranging from 0 to 1 a surface of this kind can be constructed. So again there are certain advantages of using implicit representation and there are certain disadvantages just as we had looked at in the cases of curve. For instance, if I want to find out whether a particular point lies on a surface or not through implicit surface it is very straightforward whereas in the case of parametric surface it is not that easy. So those advantages and disadvantages carry forward even in the case of surfaces.

We are basically going to look at the parametric surfaces. And just the way we had seen the parametric curves we would like to actually extend some of the notions we have studied in curves. Before we actually see similar constructions for instance as Bezier curves or B Splines let us look at some of the parametric surfaces which are simple to construct. For instance, let us look at the construction of surfaces using revolution, rotation. It is one of the simplest ways to generate surfaces. We have a two dimensional primitive or entity defined which you rotate about an axis. For instance, if you have just a point lying here in x y plane and you rotate that point about x axis you get a circle.

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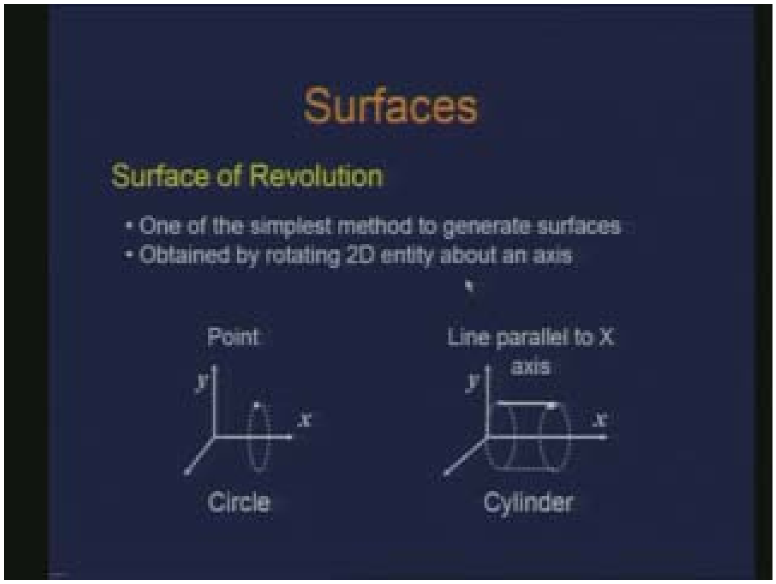

Similarly, if I just take a line defined in x y plane I rotate that line about x axis I get a cylinder. Hence these are very simple ways to have these surfaces constructed. And through the construction we also observe that if I take a cross section of the surface which is constructed using this method that is going to be a circle. If I take a cross section perpendicular to the axis of rotation that cross section is going to be a circle. So this line here is some sort of a generating curve for the surface of revolution which we are constructing.

Now, looking at this construction in parametric form what we are saying is that, if I have the parametric equation of the entity which is to be rotated that could be defined something like the way we had seen parametric curves P(t) defined in terms of x(t) y(t) z(t) and then you have a rotation of that entity. So that gives you the second parameter for the definition of the surface.

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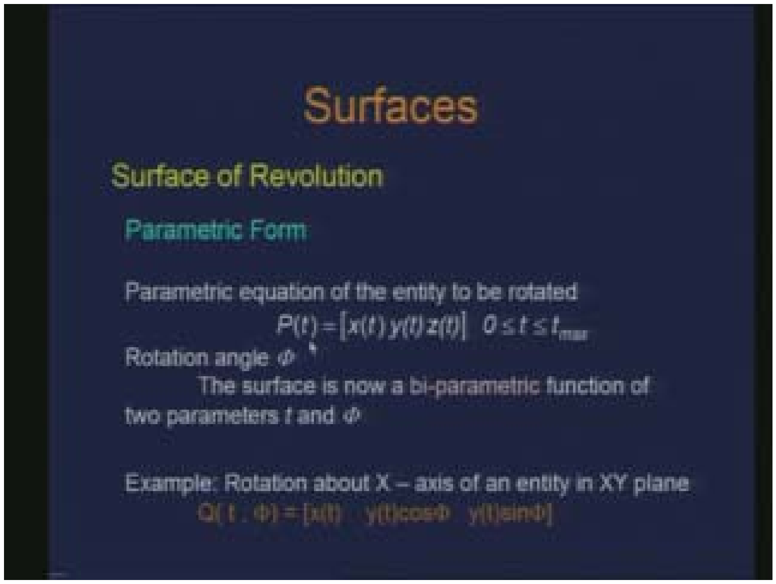

So in a way we form a bi parametric function which gives the surface of t and phi. An example is that if I rotate something about x axis which lies in x y plane then the phi parametric function of the surface which is constructed can be given as x(t) y(t) cos phi y(t) sin phi. It is a bi parametric function in t and phi so I can get any point using the parameter t and phi on that surface. So I can actually construct a sphere by rotating a semi circle centered at the origin and lying in XY plane.

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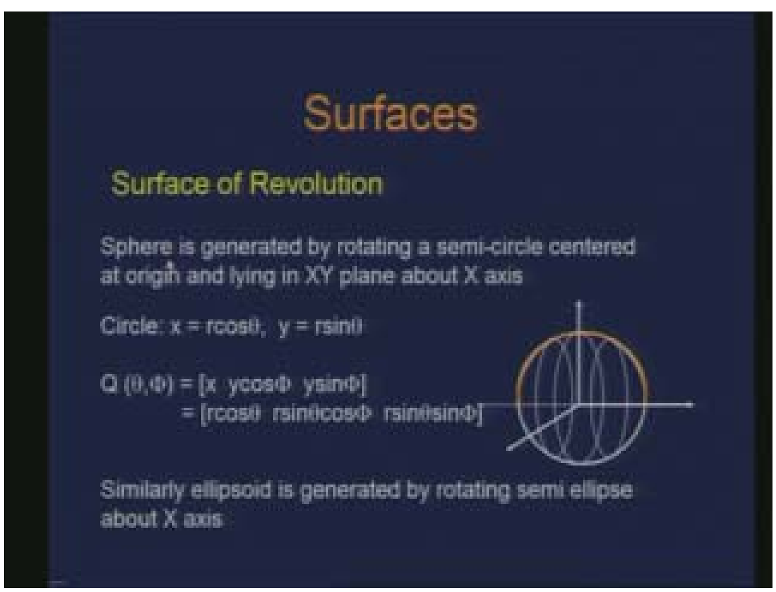

Therefore if I look at the definition of the circle in parametric form the parameter which is being used here is theta given as x is equal to rcos theta y rsin theta and when I apply the bi parametric function which is rotation by an angle phi about x axis I would get this so it is a bi parametric equation with the parameters theta and phi. Therefore with proper range of theta and phi I can construct this way. Similarly I can also construct shapes like ellipsoid by rotating a semi ellipse about x axis.

I can also construct a torus. There if I rotate a circle which is lying in x y plane and whose center is not at the origin so there is an offset from the origin of the center of the circle. So if I rotate the circle about x axis I again get the surface which is torus. In fact we can represent the circle again in a parametric form with an offset h and k so h and k is the location of the center in x y plane and by again using bi parametric function in theta and phi we can construct the surface.

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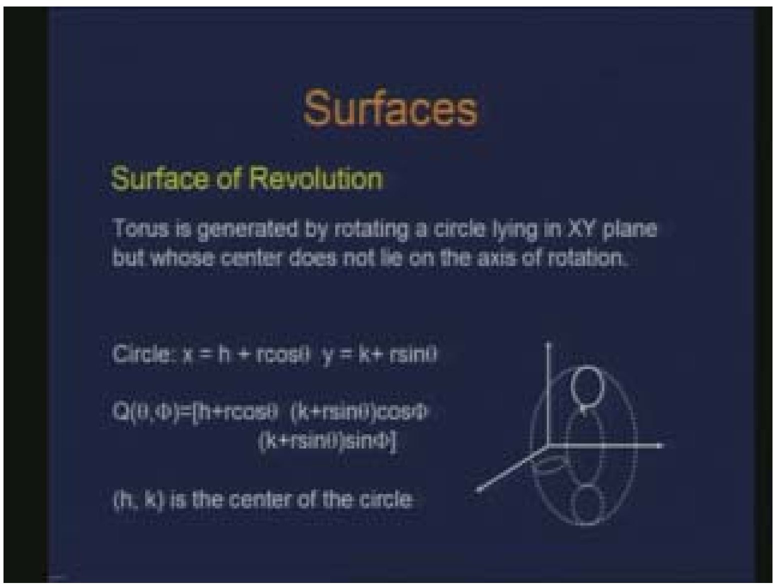

Now, if you want to represent this construction in a matrix form then all you are doing is, if I have a point located in an x y plane as point x y then the rotation about x axis causes this point to have the coordinates like x ycosphi ysinphi. Therefore through this transformation I can observe the matrix which causes this transformation and the matrix is this.

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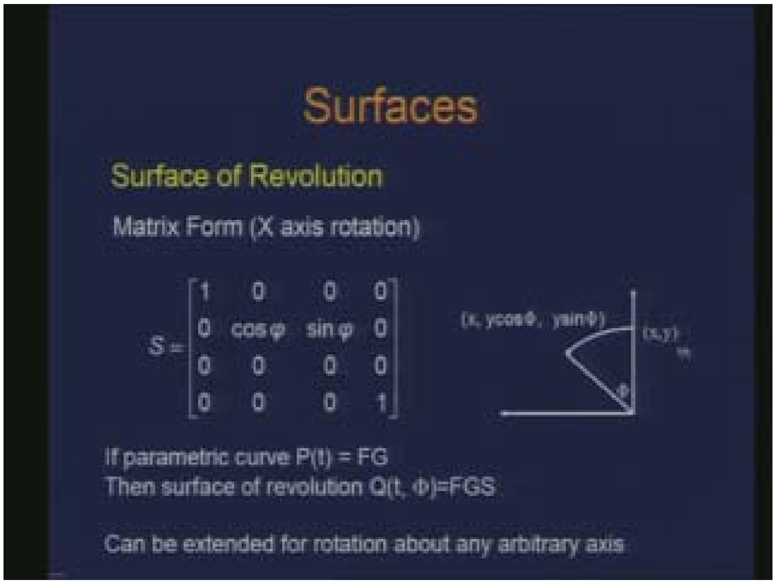

So you observe that a general rotation about x axis if you recall the form of that is very similar except that the fact we are using the point lying in x y plane does not require us to have the term 0. So this is the matrix which is causing this transformation. Now I can represent the surface using matrix. Now if you recall a parametric curve P(t) can also be represented in matrix form where I have these F which are nothing but some sort of blending functions where I can also construct those in the form of a monomial polynomials 1t t^2 t^3 and the associated coefficients and some information of geometry which is G. So these curves are represented in matrix form.

Now when I am generating the surface of revolution using this parametric curve all I need to do is use this matrix and multiply them. This gives me the surface Qt hi in matrix form. So far in most of the examples what we observed we saw the rotation about x axis. But that does not have to be the case. You can also construct a surface of revolution above using rotation about any arbitrary axis. So this matrix h which is being used here will be some other matrix which would cause the rotation about an arbitrary axis. And once again the parametric curve P(t) could also be any one of those parametric curves which you have studied. For instance, if you are trying to construct some sort of a wine glass you may actually construct the silhouette like a spline curve and do a rotation about an axis. Now let us look at other ways of constructing surfaces which are also parametric surfaces which are the sweep surfaces.

What we are trying to see here is that there is some parametric curve and we apply a transformation of that curve and that is what surface of revolution is. Similarly, here in sweep surfaces we have some transformation which is causing the sweep of some parametric entity. Here we sweep the curve which means traversing of an entity along a path. So again I can represent the generated surface as a bi parametric function using the parameter t and s where I have the curve or an entity defined through P(t) and this is what is causing the sweep T(s) that is the sweep transformation.

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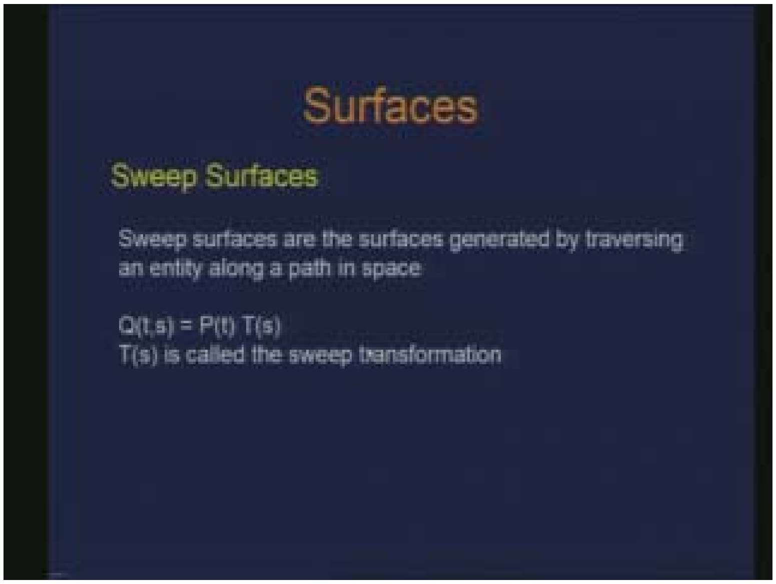

So one of the simpler transformation which you can see and construct surfaces is that if I use a translation sweep of a circle then it can generate a cylinder. So what I have is, this is my parametric curve P(t) and I am causing a transformation like a translation of this along some path or along some axis so this gives me cylinder. If I use translation sweep accompanied with scaling then I can generate frustums or cones or frustums. And if I add a rotational transformation I can generate things like helix, twisted elements etc.

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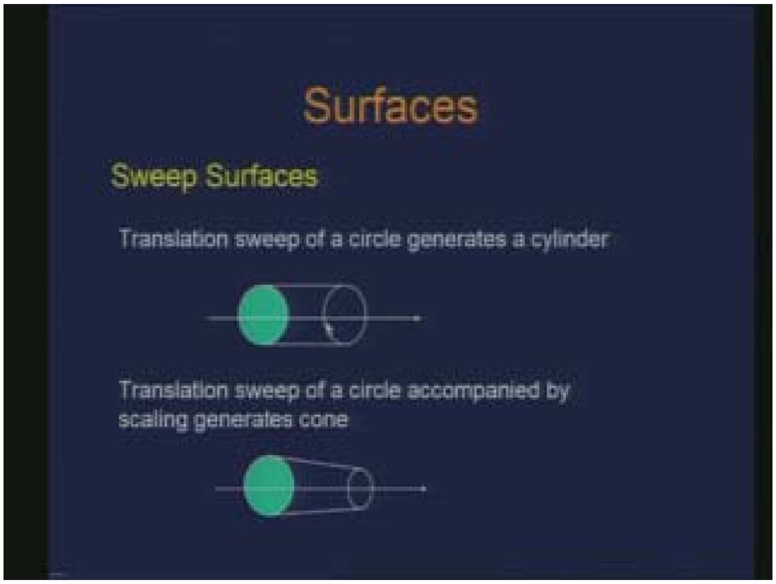

So all we are doing is taking some sort of a contour which is defined through P(t) and performing a traversal of that through a trajectory and that trajectory may be generated using transformations like translation or rotation and we may also cause transformation of the contour itself which is scaling or any other transformation. Therefore generalized cylinders can also be generated using a similar process. Surface of revolution can be considered as a special case also. That is also sort of a sweep process. So these are the surfaces which you are very familiar with and it is a very simple method of generating that.

Here is another example: I could also use P(t) as some spline or any other parametric curve. For instance if I have some cubic spline defined in x y plane and if I use a translation sweep then again I can use the matrix form of the transformation. So, again I can have the sweep surfaces in matrix form. So this is the translation factor I have and I can use some sort of a resolution definition of the translation which I am causing.

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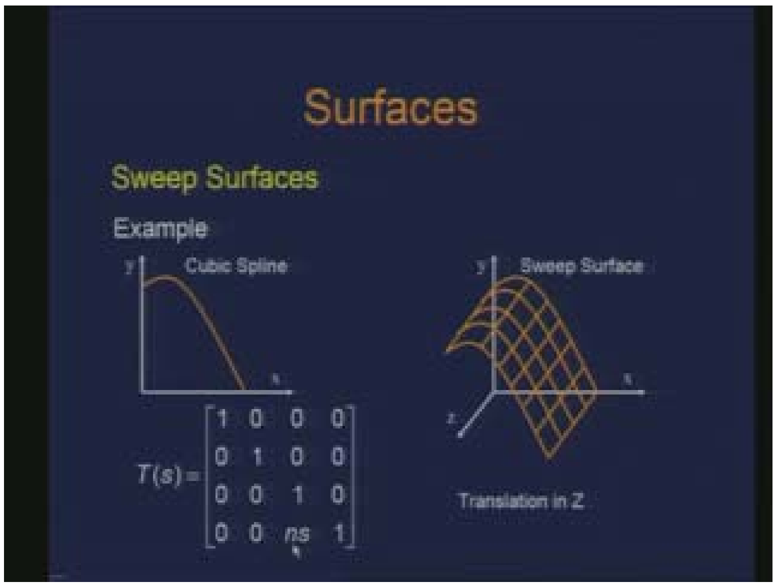

Each of these translated curves could be the unit resolution I am causing the translation….. 21:40 so n could be some number and s is the amount of translation you are doing in at one instance. So you can generate surfaces like that. Now when you have these sweep surfaces generated though we have not really looked at the methods of shading and rendering but there is an issue for what normal you would like to use for the purpose of rendering.

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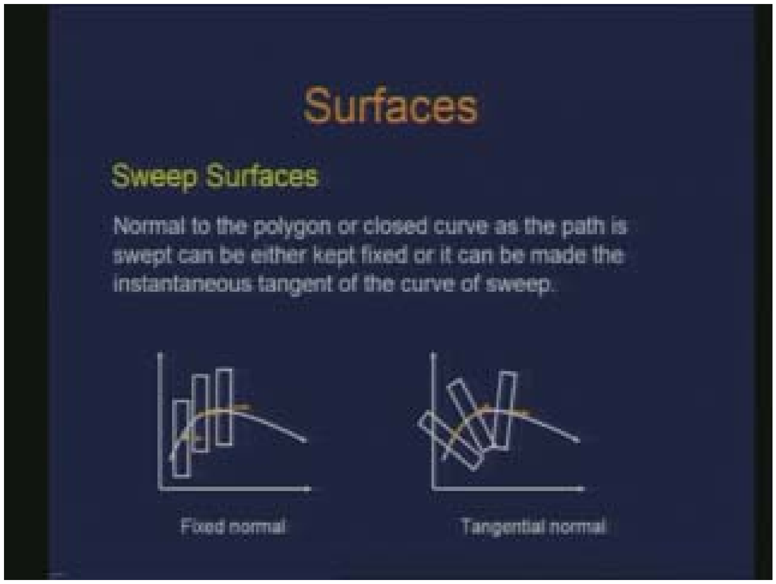

I have a contour or a polygon which is the contour for generating the sweep surface along a certain path. So I am basically doing a traversal of this polygon along this curve and in some way I will be rendering these polygons. Now the question comes that the normal of this contour or the polygon either I can use some fixed normal which is the normal of the contour at the beginning and then display this surface for the various points I have or I can also use some transformation of the normal which gives me the normal as a tangent to the curve along which I am traversing. This might be a better way of obtaining the surface to be displayed. This is more of a display issue of the surface which is constructed.

You are trying to align the normal of the contour with respect to the tangent of the trajectory you are using. This makes more chance. Therefore these are the two kinds of surfaces which are very simple to generate just using contour information and some transformation to those contours. Here are some methods which are in some way an extension of parametric curves. One of the simplest ways of defining surfaces is bi linear interpolation. Just the way we have linear interpolation to be the simplest way to get the curve between two end points.

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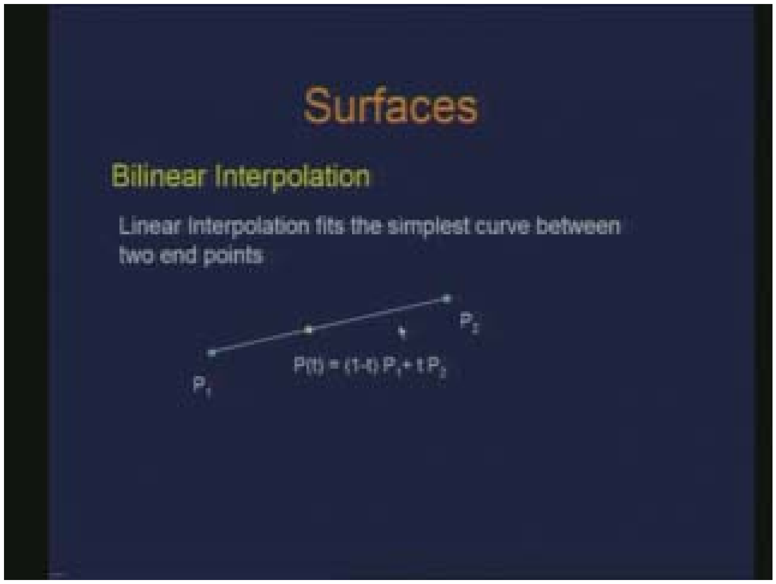

For instance I have a point P_1 I have a point P_2 then the simplest connection or a curve I can obtain between P_1 and P_2 is this line which is obtained through linear interpolation between these two points. In a similar way I have a simplest surface construction which is obtained using bi linear interpolation of the four corner points.

Therefore the problem here which is being posed is that the given four points b_{00}; b_{01}; b_{11} and b_{10} what is the simplest surface which interpolates these four points. That is what we are trying to construct. Just the way we had done the linear interpolation between two end points we can do a similar thing to construct a surface which would interpolate these four points. So, if I consider some parametric domain as (u v) a rectilinear parametric domain then I can use the same concept as linear interpolation and generate a bi linear surface which is to say that I have some two stage interpolation process so I can decompose the problem of two pass linear interpolation.

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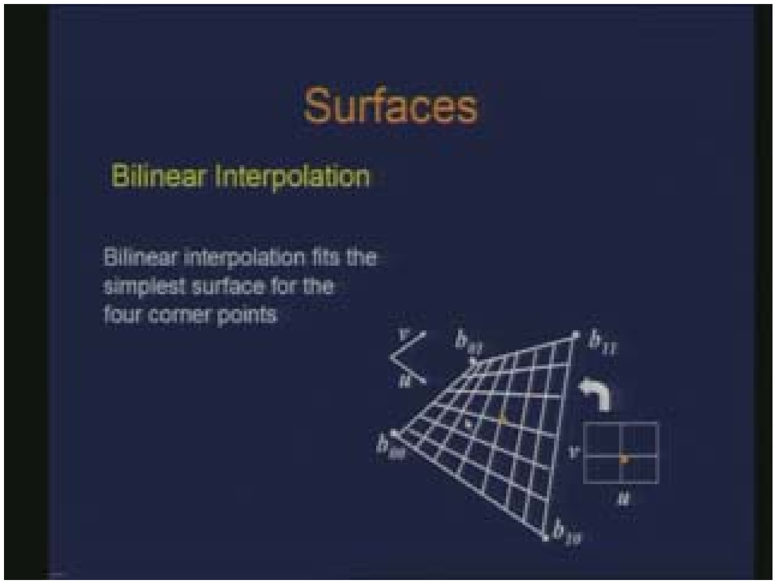

Therefore I have these four points and now what I do is, as far as the boundary of the surface is concerned that is a simple linear interpolation between the points.

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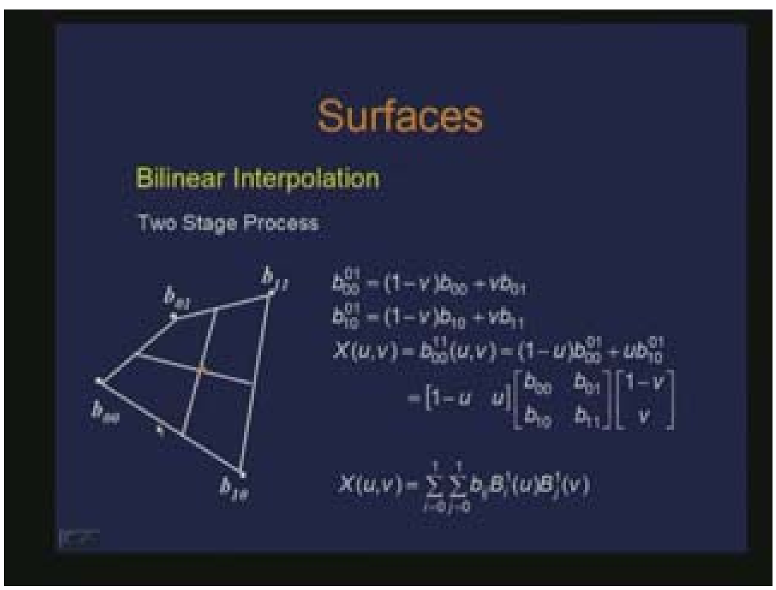

Now, for any internal point of the surface what I can do is I can use a two pass linear interpolation. The first pass enables me to get the point here which is the linear interpolation between b_{00} and b_{01}. Similarly, I would get a point here which is a linear interpolation between b_{10} and b_{11}. So I get some point here which I represent as b01, 10 which is nothing but the linear interpolation of b_{10} and b_{11}. Now what I do is, again I apply a linear interpolation in u the other parameter and I obtain this point which is the point on the surface.

So this surface if I represent as X(u, v) this is nothing but the linear interpolation of these resulting points. Again I can write this in some vectors and matrices and have [1 minus u u] and these are nothing but the geometric information which is nothing but the four corner points I have and the other polynomials. In this case they happen to be just a linear function 1 minus v and v. Again I can write again the surface X(u, v) as some sort of functions Bi's and Bj's along with some geometric information using Bij. So the surface which I obtain is a simple surface which interpolates all these four points. Now if you observe the iso parametric curve or lines, what do I mean by iso parametric curve or line? The curve which has the same value of u or v is the iso parametric curve or line.

So, if I go from here to here, if I have v is equal to constant then the line which I get here is or the curve which I get here is a line. Similarly, if I use u is equal to some constant value the curve which I have is a line. But the curves for u is equal to v is not a line because this is going to be then a quadratic function. Remember that these points are in space, these are not points in the plane. This is the way I can construct the bi linear surface which interpolates the four corner points. This is just a straightforward extension of linear interpolation.

The problem posed in the bi linear interpolation was, given four corner points I wanted to obtain the surface interpolating those four corner points. The problem here is that I am given two space curves C_1 and C_2 and I want to construct a surface which has the boundary as these two curves. In other words if I represent my surface as X(u, v) then the boundary corresponding to v is equal to 0 which is nothing but X(u, 0) is the curve given to me as C_1 and similarly the boundary corresponding to v is equal to 1 is given as C_2. Therefore apparently there are several ways you can do it.

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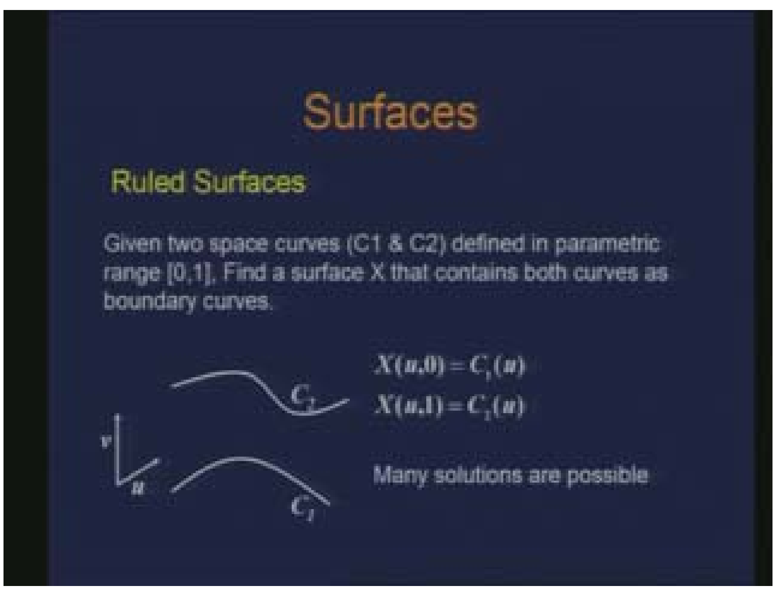

The problem is, these are the two space curves given to you and you want to construct a surface which has these two curves as the boundary curves. What is been suggested here is that if I take a point here and perhaps take a point here for the same length of the parameter and join them by line then I go to the next point on this curve for the same parameter t and I go there the other boundary and join them by line. Therefore if I have some kind of a ruler I just connect by these two points through a ruler the ruler being the line here. That is what is being done here and that is why we call the generated surface as the ruled surface. Therefore, all I am doing here is I take this point, connect it to the other point of the boundary. So it is just a traversal along the boundary curves or the contours and connect the points for the same parameter value which in turn means that I do a linear interpolation of the contours C_1 and C_2 in v if I have these contours defined as $C_1(u)$ and $C_2(u)$.

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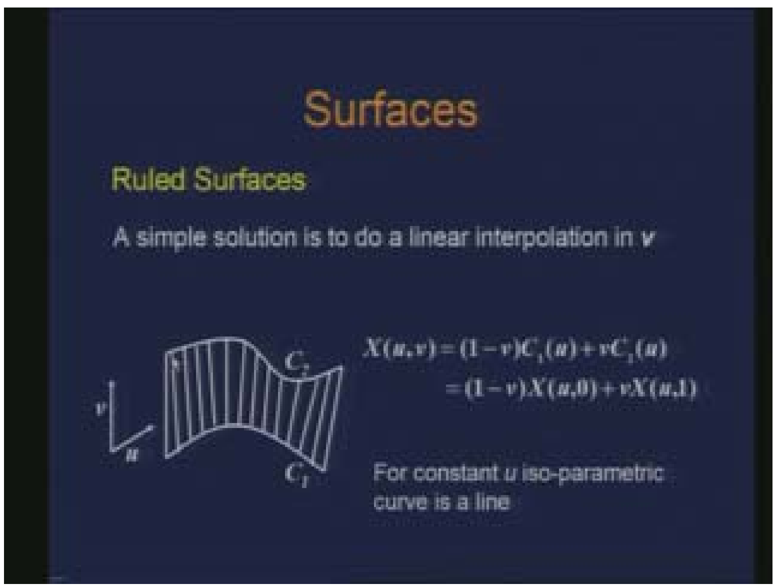

Again if I look at the iso parametric curve for constant u it is just a line. That is the simplest way of constructing a surface which interpolates the curves C_1 and C_2. Now I further change the problem. In the previous example I had only two boundary curves corresponding to the surface which I wanted to generate. Now I have now four boundary curves C_1 C_2 and D_1 D_2. In other words I have the surface or the boundary curve corresponding to v is equal to 0 as $C_1(u)$, for v is equal to 1 as $C_2(u)$, for u is equal to 0 as $D_1(v)$ and u is equal to 1 as $D_2(v)$. So these are the four boundary curves I am given. And the question we have is, we want to construct a surface which interpolates these boundary curves. That is, the boundary of the surface which I construct should have these as the boundary curves.

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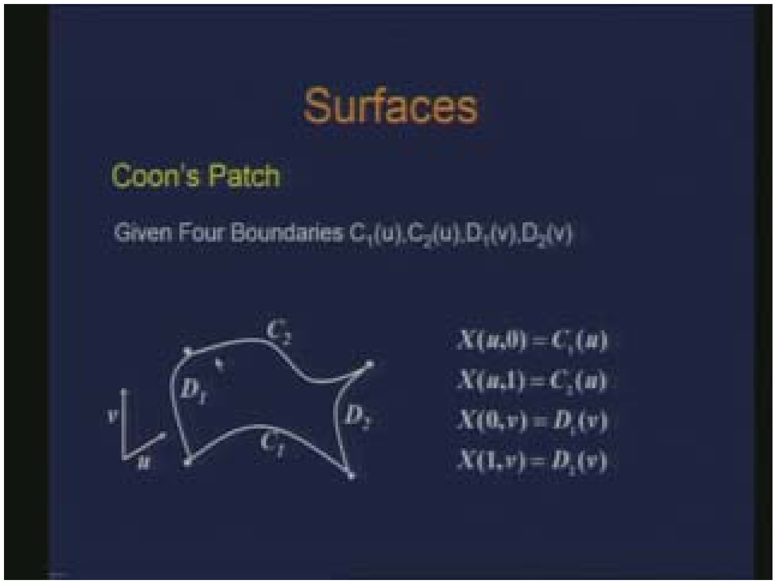

Let us try to do something using ruled surfaces. So, if I construct a ruled surface between $C_1(u)$ and $C_2(u)$ then I construct another ruled surface between D_1 and D_2 now what do I do? The fact that I have generated this and the fact I have generated the other ruled surface what happens if I just add them? When I add these two I get something like this. Through this ruled surface $r_C(I)$ was able to satisfy the constraint for the boundary curve as C_1 and C_2.

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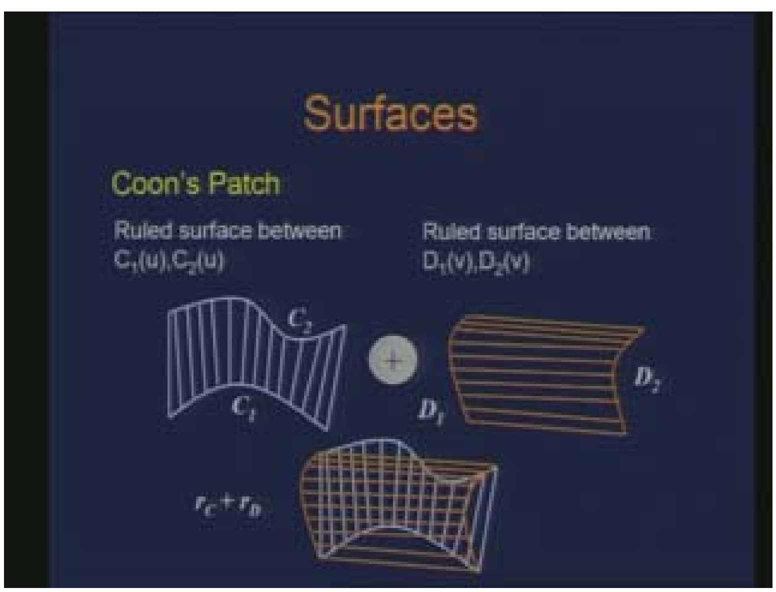

Similarly, I was able to satisfy here while using r_d the ruled surface between D_1 and D_2 for these curves. But as a consequence of heading them I am getting something extra here. I am getting these lines in addition to this curve and this is what is happening and similarly these lines in addition to this curve. Now the question is, can I do something to get rid of the extra which I have. The addition clearly gives me something extra. Now is there a way I can get rid of that extra thing. The extra things are these lines, when do you get lines?

We have seen the bi linear surfaces. What are bi linear surfaces? They actually connect these points through lines, these points through lines. So iso parametric curves for a given value of u and a given value of v are lines. That is what you have as the extra thing.

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So if I subtract this from the sum which I have done for the two ruled surfaces I will get what I want. Therefore I have the ruled surface which is obtained through C_1 and C_2, I call it r_c, this is another ruled surface which is obtained for the boundary curves D_1 and D_2 which is rd and this is what I have as the bi linear surface for the four corner points which I call as r_{CD}.

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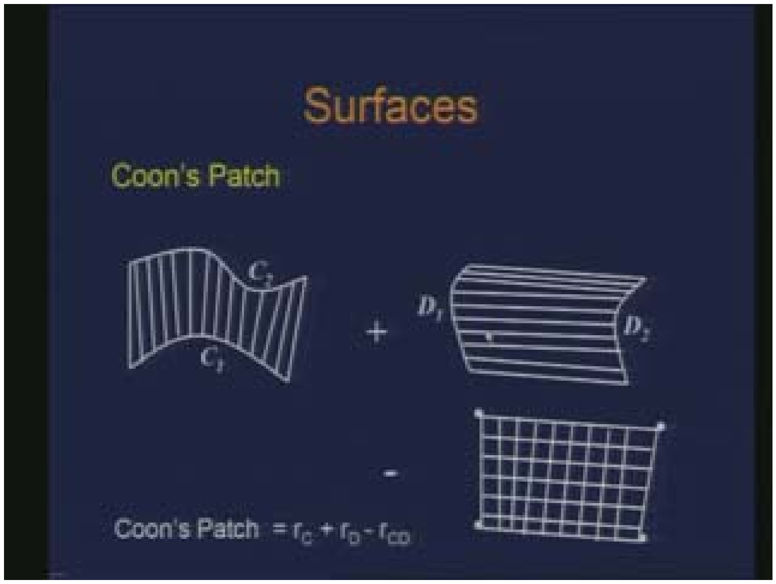

So the Coon’s patch is due to Coon and that is why it is called Coon’s patch. It is the resulting surface which interpolates the four boundary curves and which is obtained as the sum operation of the ruled surfaces r_C and r_D minus the bi linear surface. So mathematically if I want to write this, $r_C(u, v)$ is nothing but which is obtained as the linear interpolation of the boundary curves C_1 and C_2 which are nothing but X(u, 0) and X(u, 1). Similarly, $r_D(u, v)$ which is the other ruled surface obtained from the linear interpolation of the other boundary curves and r_{CD} is nothing but the bi linear surface.

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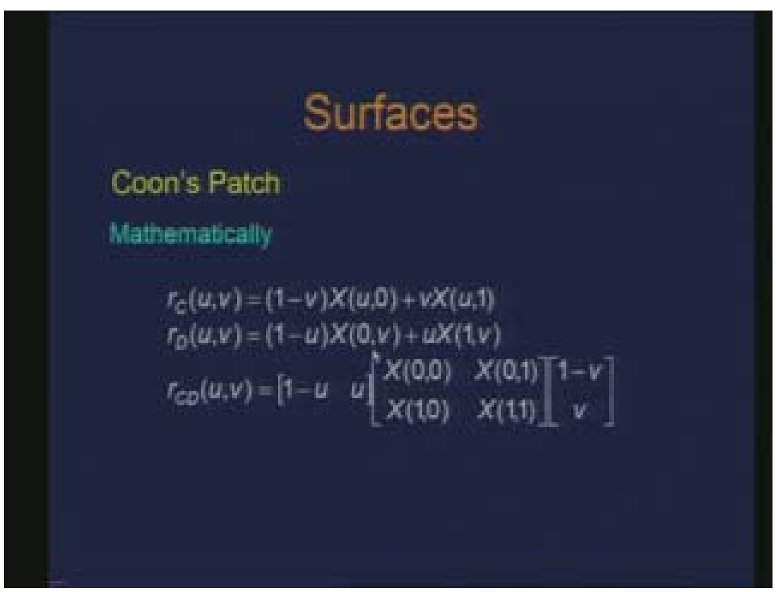

Then the resulting surface which is X(u, v) is nothing but r_C plus r_D minus r_{CD} which I have just substituted for each of these terms and got this.

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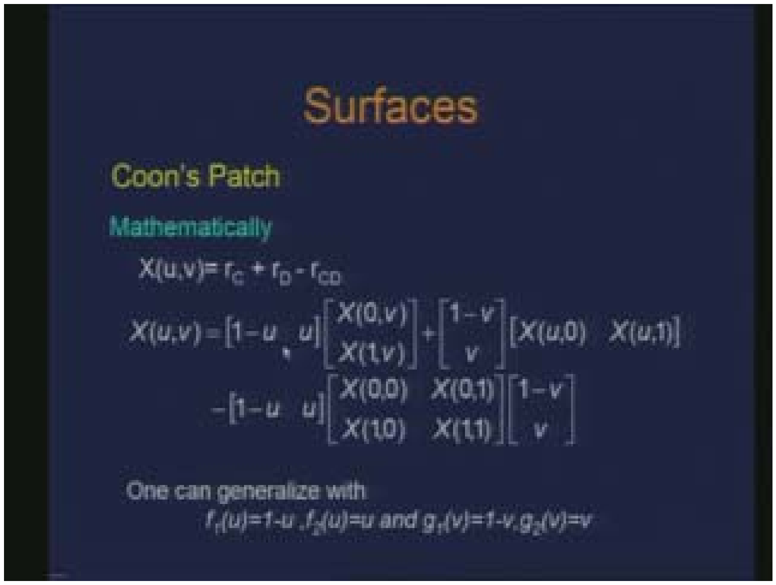

Now in fact if you observe these are the sort of blending functions which are being used here 1 minus u and u, 1 minus v and v. So one can even think of having some generalized form of this Coon's patch where I can define 1 minus u as some function as f_1(u) and u as some f_2(u) similarly 1 minus v as g_1(v) and v as g_2(v) and I can use then f_1, f_2, g_1, g_2 as the blending functions. So what we have seen is that how we obtain the interpolating surface for various scenarios. The bi linear surfaces obtained as an interpolating surface for given four corner points.

Ruled surfaces are simplest form of surfaces for obtaining surface interpolating two boundary curves and Coon's patches are the surfaces for obtaining the surfaces for four boundary curves and we can see that they can be represented in a very similar way as what we observed in the case of parametric curves.